

Emergency-induced effects on high-speed railway networks: A complex network theory's perspective

Junfeng Ma* Shan Ma* Tao Peng* Weihua Gui*

* School of Automation, Central South University, Changsha 410083
China

Abstract: In this paper, we analyze the characteristics of high-speed railway networks in the presence of emergencies from a complex network theory's perspective. First, we represent the railway network by a graph where the nodes denote stations and the edges denote train flows. For a railway network system, the punctuality of trains and the number of trains running through stations are the two main factors for evaluating emergency-induced effects. We thus assign each edge of the graph a weight which is determined by these two key quantities. Then we propose a method to estimate the delay of trains and some metrics are introduced to analyze the properties of the railway network under disruptive events. These metrics may be also used to quantify the influences of the emergencies. Finally, examples are provided to illustrate the developed theory.

Keywords: Railway network, Complex network, Train delays

1. INTRODUCTION

The study of complex network theory starts from observing a collective phenomenon in the real world and then tries to build a suitable model to analyze its behavior. Networks such as communication and transportation networks often suffer from failures or attacks. For an interdependent network, the failure of one node or edge may lead to the failure of other interrelated nodes or edges. However, most local failures rarely result in the loss of global network structural properties due to the robustness of networks. One of the exciting topics that have attracted much attention in recent years is to analyze the robustness of networks under various types of destructions Dobson et al. (2007); Cai et al. (2015); Albert et al. (2000). In practice, network components such as nodes and edges might undergo failures, and the networks consequently lose their functionality. Thus, the investigation of the robustness of complex networks under failures is of significant importance for practical applications.

Since the small-world phenomenon was discovered, the research in complex networks has become a hot topic in many scientific branches Watts and Strogatz (1998); Latora and Marchiori (2001); Barabási and Albert (1999). In particular, transportation networks, such as airport networks Bagler (2008); Wang et al. (2011); Zhang et al. (2010), urban public transport networks Derrible and Kennedy (2010); Cipriani et al. (2012); Derrible (2012) and railway networks Vromans et al. (2006); Giua and Seatzu (2008); Zhang et al. (2016), have attracted increasing attention. Among transportation networks, high-speed

railway networks have experienced rapid development in recent years due to their enormous impact on the economy. High-speed railway networks often suffer from random or systematic component failures. Hence it is important to study the effects in detail to assist the improvement of the management efficiency. Failures in high-speed railway networks usually correspond to disruptive events with a certain duration time. It is interesting to note that railway networks exhibit some specific characteristics that do not appear in other types of networks. For example, when a disruptive event occurs in a railway network, it often results in the delay or cancellation of trains, and the disrupted stations and tracks can resume their services after some time. However, in other networks, it may result in the total removal of some edges or nodes. Therefore, it should be of significant interest to investigate high-speed railway networks in the presence of emergencies.

In this paper, we restrict our attention to high-speed railway networks (HSRN). Various aspects of HSRN have been investigated in recent years. For example, Vromans et al. (2006) performed a study on reliability and heterogeneity of railway networks, and found that homogeneous timetables can dampen the propagation of train delays. Giua and Seatzu (2008) provided a modular representation of railway networks, and used Petri nets to deal with the supervisory control problems of networks. Zhang et al. (2016) performed an empirical study on structural vulnerability and intervention of HSRN, and took a nearest-link approach to improve the connectivity and reliability of HSRN. As mentioned before, high-speed railway networks often suffer from disturbances, including natural hazards (strong wind, etc.) and component failures (electricity power failure, etc.). Such disturbances often result in the

* This work was supported in part by the National Natural Science Foundation of China (Grant Nos. 61790572, 61490702). Corresponding author: Shan Ma, Email: shanma.adfa@gmail.com

delay or cancellation of trains, thus could lead to huge financial losses.

One main contribution of this paper is to develop a framework for the characteristic analysis of Chinese high-speed railway network (CHRN) in the presence of emergencies using a complex network theory-based approach. Firstly, we represent the railway network by a graph where the nodes denote stations, and the edges denote train flows. Then we assign each edge of the graph a weight which is determined by the number of trains running through stations and the punctuality of the trains. A detailed analysis will be performed on the graph constructed from the high-speed train operation records within a typical weekday. By using some information such as train delays and the number of delayed trains, we define a new type of weight for the edges of the railway network. By analyzing the impact of delays in the high-speed rail network, a new weighted index is proposed for the evaluation of the vulnerability of networks. In order to illustrate our theoretical framework, we take the part of CHRN managed by Shanghai railway bureau as an example to assess the vulnerability of the network in the presence of failures. The result developed in this paper could be potentially useful for the evaluation of robustness of railway networks and may provide some insight into the improvement of emergency support capacity.

2. HIGH-SPEED RAILWAY NETWORK MODEL

We consider a high-speed railway network modelled as a directed graph $G = \{V, E\}$, where $V = \{1, 2, \dots, N\}$ is the set of nodes representing the railway stations and $E \subseteq V \times V$ is the set of edges. In this graph, (i, j) is an element of E if and only if there exists a train that arrives at station i before j and has stations i and j as its two consecutive stops. Associated with the graph is an adjacency matrix $A = [a_{ij}]$, $i, j = 1, \dots, N$, whose element $a_{ij} = 1$ if $(i, j) \in E$, and $a_{ij} = 0$ otherwise. The out-degree of a node i is defined as

$$k_{i,out} = \sum_{j=1}^N a_{ij}, \quad (1)$$

where N is the total number of nodes in set V . The in-degree of a node $k_{i,in}$ can be similarly defined. In- and out-degrees are two main topological metrics evaluating the connectivity and centrality of nodes in extensive networks. However, these two metrics have some drawbacks when applied to railway networks. Firstly, they do not take the number of trains traveling across a station into account, which is often considered to be an important indicator of a station's connectivity and centrality. Secondly, train delays in the occurrence of emergencies may also play an important role in defining a station's connectivity and centrality. Thus, a proper metric should also take train delays into account. Instead of using these two metrics $k_{i,in}$ and $k_{i,out}$, we here propose an alternative metric to describe the emergency-induced effects on station i :

$$h_i(G, G^*) = f(n_i, \tau_i, n_i^{delay}), \quad (2)$$

where G is the graph corresponding to the scheduled train timetable in the railway network, and G^* is the graph corresponding to the actual train timetable after emergencies; n_i is the number of trains scheduled to stop

at station i . The scheduled departure time for train m at station i is denoted by d_i^m . In the case of emergencies, train delays may occur. Suppose that the actual departure time for train m at station i is a_i^m . Then the departure delay of train m at station i is $\tau_i^m = a_i^m - d_i^m \geq 0$. Note that in this paper we assume the emergency is not too severe such that no train will be cancelled. Collecting all of the n_i train delays, we obtain $\tau_i = \{\tau_i^1, \tau_i^2, \dots, \tau_i^{n_i}\}$. n_i^{delay} denotes the number of nonzero elements in the set τ_i . We see from Eq. (2) that the metric $h_i(G, G^*)$ is a function of the number of scheduled trains n_i , the train delays τ_i and the number of delayed trains n_i^{delay} at station i . We mention that the function $f(\cdot)$ can be freely chosen based on different preferences. In this paper, we choose a linear function $f(n_i, \tau_i, n_i^{delay}) = \alpha \sum_{m=1}^{n_i} \tau_i^m + \beta n_i^{delay} + n_i$, where α and β are non-negative constants. For simplicity, we write $h_i(G, G^*)$ as h_i in the sequel. Compared to the in- and out-degrees $k_{i,in}$ and $k_{i,out}$, our definition h_i takes the number of trains and the punctuality of each train into account, and might be more appropriate for the evaluation of the importance of stations in the high-speed railway network under emergencies.

After analyzing the importance of nodes, we next turn our attention to edges. It is well known that the edges in the railway network are not equally important. We assign each edge between two nodes a weight to characterize its importance. Similar to Refs. Wang and Chen (2008); Baharan et al. (2011) where degree and betweenness centrality are used to estimate weights in networks, we introduce a weighting method based on the evaluation metric h_i . The weight of an edge (i, j) is defined as

$$w_{ij} = \begin{cases} (h_i h_j)^\theta & \text{if } a_{ij} \neq 0 \\ 0 & \text{otherwise} \end{cases}, \quad (3)$$

where $\theta > 0$ is a constant. The definition of weight is based on the intuition that an edge between two important end nodes is important in the graph. We can see that w_{ij} depends largely on the emergency-induced effects. If there is no deviation from the timetable for each train at stations i and j , we have $f(n_i, \tau_i, n_i^{delay}) = n_i$, hence the weight of the high-speed railway network reduces to $w_{ij} = a_{ij}(n_i n_j)^\theta$, which depends only on the number of trains scheduled to stop at stations i and j in this situation.

In order to give an intuitive explanation of the high-speed railway network, an example of a railway network with 8 stations and 16 railway lines is given in Fig. 1. The number in each circle represents the label of the corresponding station. In CHRN, most of the railway lines between two stations are double-track, and they do not affect each other in general. Without loss of generality, we only take the single-track into account. The scheduled train timetable is recorded in Tab. 1. According to the information extracted from Fig. 1 and Tab. 1, we can obtain the topology of the railway network graph, as shown in Fig. 2. The blockade of one railway line might influence the weight of several edges in our railway network. For example, the route of train B is $1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 8$, as shown in Tab 1. When the railway line $(5, 7)$ suffers from destructions, train B might be delayed at station 5, 7 and 8, which influences the weight of edges $(5, 7)$ and $(7, 8)$ as shown in Fig. 2.

Table 1. The scheduled train timetable corresponding to the train network in Fig. 1

train index	route	arrival time / departure time							
		station 1	station 2	station 3	station 4	station 5	station 6	station 7	station 8
train A	1 → 2 → 5 → 7	-/7 : 30	7 : 45/7 : 48	-	-	8 : 13/8 : 16	-	8 : 43/-	-
train B	1 → 2 → 5 → 7 → 8	-/7 : 40	7 : 55/7 : 58	-	-	8 : 23/8 : 26	-	8 : 53/8 : 56	9 : 21/-
train C	1 → 2 → 5 → 7 → 8	-/7 : 45	8 : 00/8 : 05	-	-	-	-	9 : 01/9 : 04	9 : 29/-
train D	3 → 5 → 7 → 8	-	-	-/7 : 45	-	8 : 04/8 : 07	-	8 : 30/8 : 34	9 : 02/-
train E	3 → 5 → 7 → 8	-	-	-/8 : 05	-	8 : 34/8 : 37	9 : 22/9 : 25	9 : 09/9 : 12	9 : 37/-
train F	3 → 5 → 7 → 8	-	-	-/8 : 19	-	8 : 48/8 : 51	9 : 37/9 : 41	-	9 : 51/-
train G	4 → 6 → 8	-	-	-	-/8 : 01	-	8 : 21/8 : 24	-	8 : 45/-
train H	4 → 6 → 7 → 8	-	-	-	-/8 : 08	-	8 : 44/8 : 47	9 : 17/9 : 21	9 : 45/-

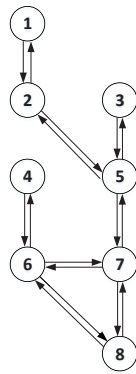


Fig. 1. An example of a high-speed railway network

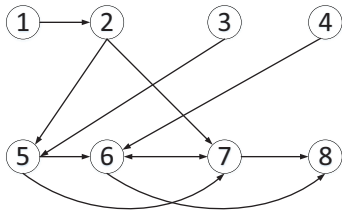


Fig. 2. Topology of the railway network shown in Fig. 1

3. EMERGENCY-INDUCED EFFECT ANALYSIS FOR HIGH-SPEED RAILWAY NETWORKS

This section will first introduce a method to estimate the train delays, and then propose a performance metric to evaluate the emergency-induced effects.

3.1 Train delay estimation

From Eqs. (2) and (3), in order to calculate the weight of an edge (i, j) , we need to obtain information such as the train delays τ_i, τ_j and the number of delayed trains n_i^{delay}, n_j^{delay} at the stations i and j . Though the location and duration of emergencies are not precisely known in practice, it is possible to make a good prediction of them based on historical train movements data. Using this idea, we first introduce a method to estimate the train delays and the number of delayed trains and then present a systemic analysis for the emergency-induced effects. The disturbance we consider in this paper is the blockage of

railway lines. Its definition has been given in Cacchiani et al. (2014), and the cancelation of trains is not taken into account. It is assumed that the start time and end time of the blockage are known, denoted by D_{ij}^{start} and D_{ij}^{end} , respectively. Here the subscript ij means that the blockage happens at the railway line between the stations i and j . The trains scheduled to run on the disrupted railway lines have to stop at a station until the disruption is over. This is based on the fact that for the high-speed railway system in China, trains cannot change the operational route without permission from the railway bureau. In addition, the number of trains that stop at a station can not exceed the capacity of the station. We assume the station capacity of the station i is p_i , which means at most p_i trains could stop at the station i at the same time.

Recall that the departure delay for a train m at station i is $\tau_i^m = a_i^m - d_i^m$, where d_i^m denotes the scheduled departure time, and it can be obtained from the scheduled timetable. Hence our objective in this section is to estimate the actual departure time a_i^m . Suppose that there are k delayed trains, and we record these delayed trains as a set $T = \{1, 2, 3, \dots, k\}$. Suppose the disruption event occurs in the segment $(i^*, i^* + 1)$. When there are sufficient station tracks for the delayed train m to dwell in station i^* , that is $m \leq p_{i^*}$, we have

$$a_{i^*}^m = \max \left\{ D_{i^*, i^*+1}^{end} + \sum_{s=1}^{m-1} l_{i^*}^s, d_{i^*}^m \right\}, \quad (4)$$

where $m \in T$, $l_{i^*}^s$ denotes the minimum time interval between trains s and $s + 1$ at station i^* . When $p_{i^*} < m \leq p_{i^*-1} + p_{i^*}$, it means that the train m has to stop at station $i^* - 1$ until the disruption is over. The actual departure time of the train m at station $i^* - 1$ in this time can be denoted by

$$a_{i^*-1}^m = \max \left\{ D_{i^*, i^*+1}^{end} + \sum_{s=1}^{m-p_{i^*}-1} l_{i^*-1}^s, d_{i^*-1}^m \right\}. \quad (5)$$

Similarly, we can calculate the primary train delays for other values of m . We next consider the delay of the trains on their subsequent paths. For a train m , we record its stations scheduled to stop as a set $S_m = \{i_s, i_s + 1, i_s + 2, \dots, i_e\}$ in order, where i_s denotes the station that the primary delay occurs, and i_e denotes the terminal station of train m . In order to estimate the delay of the trains on their subsequent paths, it is essential

to obtain the time supplement and buffer time from the scheduled timetable Goverde and Hansen (2013); Harrod et al. (2019). As shown in Fig. 3, the time supplement refers to the additional time beyond the minimum running time and dwelling time of trains. It keeps the punctuality of trains when railway networks suffer from disruptive events. Here u_i^m denotes the time supplement of train m between station $i - 1$ and i . The time supplement consists of dwell-time and running-time supplement. We define them as follows:

$$\begin{aligned} u_i^m &= u_{1i}^m + u_{2i}^m, \\ u_{1i}^m &= \max \{d_i^m - g_i^m - e_i^m, 0\}, \\ u_{2i}^m &= \max \{g_{i+1}^m - d_i^m - r_{i,i+1}^m, 0\}, \end{aligned} \quad (6)$$

where u_{1i}^m and u_{2i}^m denote the dwell-time supplement and running-time supplement, respectively; g_i^m denotes the scheduled arrival time of train m at station i ; e_i^m denotes the minimum dwell time of train m at station i ; $r_{i,i+1}^m$ denotes the minimum running time of train m between stations i and $i + 1$. Different from the time supplement, buffer time is the additional time beyond the minimum headway time between two trains, as shown in Fig. 3. When the leading train is delayed, the buffer time reduces the interference with the following trains. Let v_i^m denote the buffer time of station i between trains $m - 1$ and m . For two consecutive trains $m - 1$ and m , the headway time between them has to be larger than the minimum headway time for security. The definition of buffer time is given as follows:

$$v_i^m = \max \{d_i^m - d_i^{m-1} - l_i^{m-1}, 0\}. \quad (7)$$

Similar to Ref. Harrod et al. (2019), the delay of train m at station i will propagate to the following stations through the expression of individual delay given in Eqs. (4) and (5).

$$\tau_i^m = \max \{\tau_{i-1}^m - u_i^m, \tau_i^{m-1} - v_i^m, 0\}, \quad (8)$$

where $i - 1, i \in S_m$. From Eq. (8), we can see that a train m may recover from its own delay using the time supplement u_i^m , and the headway time between train m and the preceding train $m + 1$ can not exceed the minimum headway time. When either of these limits is exceeded, the train m at station $i \in S_m$ will experience a delay. In addition, we can find from Eq. (8) the larger one of the time supplement u_i^m and buffer time v_i^m determines the train delays.

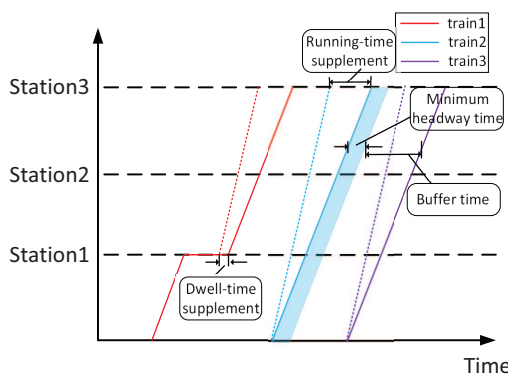


Fig. 3. Illustration of time supplement and buffer time

3.2 Emergency-induced effect evaluation

The analysis of emergency-induced effects can lead to the improvement of risk and crisis management for railway systems. As an essential metric, cumulative delay is commonly used to analyze the impact of an emergency in rescheduling problems Törnquist and Persson (2007); Zhan et al. (2015). Its expression is given as follows:

$$\Gamma = \sum_{k=1}^M \sum_{i=1}^N \tau_i^k, \quad (9)$$

where Γ denotes the cumulative delay, and M denotes the number of trains in the railway network. However, the metric Γ does not take the number of delayed trains and the scope of the emergency-induced effects into account. Hence it is not suitable to be used for the evaluation of the impact of the emergency. In order to give an appropriate description of emergency-induced effects, we need to specify another performance metric. The pre- and post-performance of the railway network under a disruptive event are both considered. From Eqs. 2 and 3, we construct a weight matrix of the railway network, denoted by $W = [w_{ij}]$. The pre-disruption performance of a railway network is measured by its weight matrix W , which can be obtained by the scheduled train timetable according to the definition. When a disruptive event happens, by estimating the train delays using the method mentioned above, we can obtain the weight matrix of the post-disruption railway network, denoted by W' . Then we define the emergency-induced effect metric as the changes of the weight matrix. Its expression is given as follows:

$$H(G, G^*) = \|W - W'\|_F, \quad (10)$$

where $H(G, G^*)$ denotes the emergency-induced effect metric, and $\|\cdot\|_F$ denotes the Frobenius norm. The definition of the metric $H(G, G^*)$ is based on the observation that if the deviation of trains from the scheduled timetable is small, then the effect of the disruptive event should be unserious. In addition, for the disruptive events that occur in the same location, when the events last longer, there will be more delayed trains and cumulative delays. Hence $H(G, G^*)$ is a non-decreasing function of the duration of the event. When there is no deviation from the scheduled timetable for every train in the railway networks, we have $H(G, G) = 0$.

4. EXAMPLES OF HIGH-SPEED RAILWAY NETWORKS

This section takes two high-speed railway networks as examples to illustrate the proposed methodology. The minimum headway time is assumed to be 3 min. The time supplement and buffer time are assumed to be both constants $u = v = 3$ min. In addition, the metric h_i in our examples is chosen as:

$$h_i = 0.1 \sum_{m=1}^{n_i} \tau_i^m + n_i^{delay} + n_i,$$

and the weight w_{ij} is chosen as:

$$w_{ij} = \begin{cases} h_i h_j & \text{if } a_{ij} \neq 0 \\ 0 & \text{otherwise} \end{cases}.$$

4.1 Example 1

Table 2. Train delays when the disruptive event occurs in segment (2, 5) from 7 : 30

duration of the disruptive event (min)	20	30	60	90
cumulative delay of train A (min)	2	20	80	140
cumulative delay of train B (min)	0	6	93	170
cumulative delay of train C (min)	0	0	58	118
cumulative delay of train D (min)	0	0	0	0
cumulative delay of train E (min)	0	0	0	0
cumulative delay of train F (min)	0	0	0	0
cumulative delay of all trains (min)	2	26	231	428

Table 3. Train delays when the disruptive event occurs in segment (5, 7) from 7 : 30

duration of the disruptive event (min)	20	30	60	90
cumulative delay of train A (min)	0	0	17	47
cumulative delay of train B (min)	0	0	16	76
cumulative delay of train C (min)	0	0	46	106
cumulative delay of train D (min)	0	0	42	102
cumulative delay of train E (min)	0	0	2	84
cumulative delay of train F (min)	0	0	0	111
cumulative delay of all trains (min)	0	0	123	526

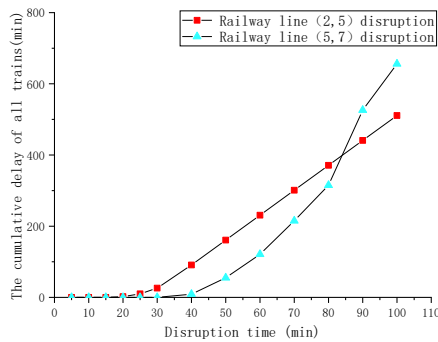


Fig. 4. The cumulative delay of trains under different disruption time

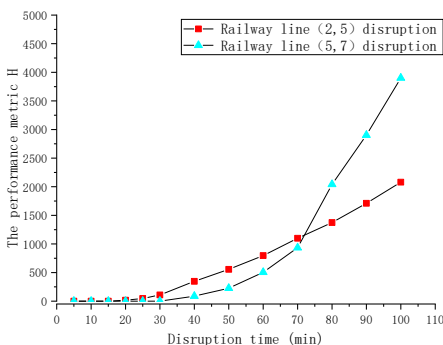


Fig. 5. The performance metric $H(G, G^*)$ under different disruption time

The first example is shown in Fig. 1, and the scheduled timetable of trains is given in Tab. 1. We consider two cases: 1) the segment (2, 5) is disrupted; 2) the segment (5, 7) is disrupted. The start time $D_{ij}^{start} = 7 : 30$, and the station capacity $p_i = 4$ for $i = 1, 2, \dots, 8$. Then we can estimate the train delays under different end time D_{ij}^{end} , as shown in Tabs. 2 and 3. It can be found that the cumulative

delay of trains is not only dependent on the duration and location of the emergency but also dependent on the train route and the scheduled timetable. When the disruption time is small, the train delays may be absorbed by the time supplement and buffer time. We can see from Fig. 4 that the cumulative delay grows with incremental increases in disruption time, slowly at first but quickly in the final phase. Fig. 5 shows the variation of the performance metric $H(G, G^*)$ when the disruption time grows. Comparing the two broken segments in Figs. 4 and 5, we find that the disruption event that occurs in different locations results in different consequences. In addition, when the disruption time is small, the effects of the blockage in segment (2, 5) are more serious than that in segment (5, 7). As the disruption time increases, the blockage in segment (5, 7) has a more serious impact. This is because there are more delayed trains when the segment (5, 7) is broken.

4.2 Example 2



Fig. 6. Topological map of a high speed railway network managed by Shanghai railway bureau. The red lines denote the selected locations of disruptive events.

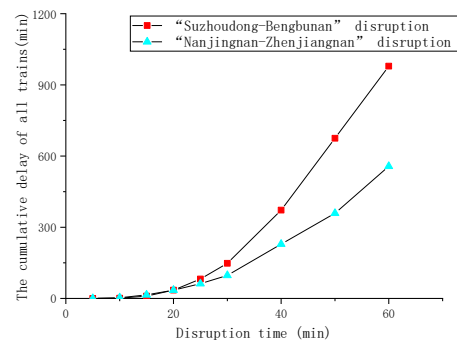


Fig. 7. Total delays of railway network under different disruption time

The second example is a real high-speed railway network, which consists of 115 stations and 236 railway lines managed by Shanghai railway bureau, as shown in Fig. 6. The corresponding scheduled train timetable could be extracted from “12306” website 12306 China Railway (2019) and the time supplement and buffer time could be calculated by Eqs. (6) and (7). For this example, we select two railway lines “Suzhoudong-Bengbunan” and

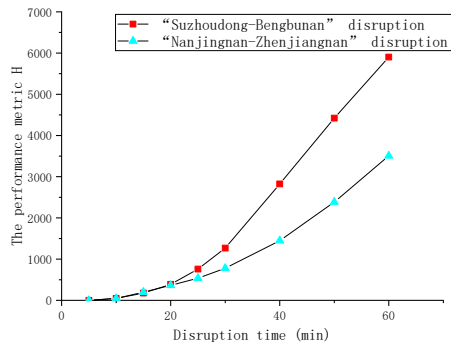


Fig. 8. The performance metric $H(G, G^*)$ under different disruption time

“Nanjingnan-Zhengjiangnan” as the locations of the disruptions, and analyze the delayed trains during 6 : 00 – 12 : 00. The start time of the disruptive event in this example is $D_{ij}^{start} = 9 : 00$. Figs. 7 and 8 show the cumulative delay and performance metric $H(G, G^*)$ for the part of CHRN managed by Shanghai railway bureau, respectively. It can be seen that the cumulative delay and performance metric $H(G, G^*)$ grow with the disruption time. Comparing the two curves in Figs. 7 and 8, we can find that the disruptive event occurs in the railway line “Suzhoudong-Bengbunan” has more serious consequences than “Nanjingnan-Zhenjiangnan” in the period of disruption time 9 : 00 – 10 : 00.

5. CONCLUSION

In this paper, we have proposed a quantified evaluation for the emergency-induced effects on high-speed railway networks. A metric has been proposed to analyze the importance of stations in railway networks. This metric takes the punctuality of trains and the number of delayed trains into account. Then we have defined the weight of edges in the railway networks and introduced a method to estimate the delay of trains. To qualify the emergency-induced effects, we have proposed a performance metric for analysis. Finally, we have provided two examples to illustrate the proposed methodology and analyzed the effects of disruptions with different locations and durations. The presented approach may provide useful guidance to the management of train networks under disruptive events.

REFERENCES

Albert, R., Jeong, H., and Barabási, A. (2000). Error and attack tolerance of complex networks. *Nature*, 340(1), 378–382.

Bagler, G. (2008). Analysis of the airport network of india as a complex weighted network. *Physica A: Statistical Mechanics and its Applications*, 387(12), 2972–2980.

Baharan, M., Mahmoudreza, B., Mahdi, J., and Mohammadali, S. (2011). Cascaded failures in weighted networks. *Physical Review E Statistical Nonlinear & Soft Matter Physics*, 84(2), 046114.

Barabási, A. and Albert, R. (1999). Emergence of scaling in random networks. *Science*, 286(5439), 509–512.

Cacchiani, V., Huisman, D., Kidd, M., Kroon, L., Toth, P., Veelenturf, L., and Wagenaar, J. (2014). An overview of recovery models and algorithms for real-time railway

rescheduling. *Transportation Research Part B: Methodological*, 63, 15–37.

Cai, Y., Cao, Y., Li, Y., Huang, T., and Zhou, B. (2015). Cascading failure analysis considering interaction between power grids and communication networks. *IEEE Transactions on Smart Grid*, 7(1), 530–538.

Cipriani, E., Gori, S., and Petrelli, M. (2012). Transit network design: A procedure and an application to a large urban area. *Transportation Research Part C: Emerging Technologies*, 20(1), 3–14.

Derrible, S. (2012). Network centrality of metro systems. *PloS one*, 7(7), e40575.

Derrible, S. and Kennedy, C. (2010). The complexity and robustness of metro networks. *Physica A: Statistical Mechanics and its Applications*, 389(17), 3678–3691.

Dobson, I., Carreras, B., Lynch, V., and Newman, D. (2007). Complex systems analysis of series of blackouts: Cascading failure, critical points, and self-organization. *An Interdisciplinary Journal of Nonlinear Science*, 17(2), 026103.

Giua, A. and Seatzu, C. (2008). Modeling and supervisory control of railway networks using petri nets. *IEEE Transactions on automation science and engineering*, 5(3), 431–445.

Goverde, R. and Hansen, I. (2013). Performance indicators for railway timetables. In *Proceedings of IEEE International Conference on Intelligent Rail Transportation*, 301–306. IEEE.

Harrod, S., Cerreto, F., and Nielsen, O. (2019). A closed form railway line delay propagation model. *Transportation Research Part C: Emerging Technologies*, 102, 189–209.

Latora, V. and Marchiori, M. (2001). Efficient behavior of small-world networks. *Physical Review Letters*, 87(19), 198701.

Törnquist, J. and Persson, J. (2007). N-tracked railway traffic re-scheduling during disturbances. *Transportation Research Part B: Methodological*, 41(3), 342–362.

Vromans, M., Dekker, R., and Kroon, L. (2006). Reliability and heterogeneity of railway services. *European Journal of Operational Research*, 172(2), 647–665.

Wang, J., Mo, H., Wang, F., and Jin, F. (2011). Exploring the network structure and nodal centrality of china’s air transport network: A complex network approach. *Journal of Transport Geography*, 19(4), 712–721.

Wang, W. and Chen, G. (2008). Universal robustness characteristic of weighted networks against cascading failure. *Physical Review E Statistical Nonlinear & Soft Matter Physics*, 77(2), 026101.

Watts, D. and Strogatz, S. (1998). Collective dynamics of ‘small-world’ networks. *Nature*, 393(6684), 440–442.

Zhan, S., Kroon, L., Veelenturf, L., and Wagenaar, J. (2015). Real-time high-speed train rescheduling in case of a complete blockage. *Transportation Research Part B: Methodological*, 78, 182–201.

Zhang, J., Hu, F., Wang, S., Dai, Y., and Wang, Y. (2016). Structural vulnerability and intervention of high speed railway networks. *Physica A: Statistical Mechanics and its Applications*, 462, 743–751.

Zhang, J., Cao, X., Du, W., and Cai, K. (2010). Evolution of Chinese airport network. *Physica A: Statistical Mechanics and its Applications*, 389(18), 3922–3931.

12306 China Railway. <https://www.12306.cn>.