Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Contract-based Hierarchical Model Predictive Control and Planning for Autonomous Vehicle

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Abstract: Planning and control of autonomous vehicles are becoming increasingly important for many applications. However, autonomous vehicles are often subject to disturbances and uncertainties, which become critical especially in cluttered and dynamic environments. To provide guaranteed constraints satisfaction, e.g. for collision avoidance, we propose a hierarchical model predictive control and planning approach. The moving horizon planning layer and the low-level model predictive controller agree on a "contract" (precision conditions). The high-level moving horizon planner is based on a mixed-integer programming formulation using a simplified model on a slow time scale, and constraint tightening. The autonomous vehicle itself is controlled by a lower-level tube-based model predictive controller. The decomposition of the control problem reduces the computational cost, enables real-time implementation while it allows to provide guarantees. To ensure compatibility between the levels and guarantee safety, we do explicitly consider the problem of recursive feasibility of the hierarchical controller ensuring constraint satisfaction and obstacle avoidance, despite the action of (unknown) disturbances. Simulation results illustrate the efficiency and applicability of the proposed hierarchical strategy.

Keywords: Moving Horizon Planning, Model Predictive Control, Contracts, Robustness.

1. INTRODUCTION

Autonomous vehicles are increasingly used in many applications, e.g. geological surveillance, agriculture, household cleaning and search and rescue missions e.g. (Bormann et al., 2018). Often, autonomous vehicles, e.g. unmanned aerial vehicles, are exposed to many disturbances (wind gusts), uncertainties (sensor noise, modeling error), and operating in dynamically changing environments.

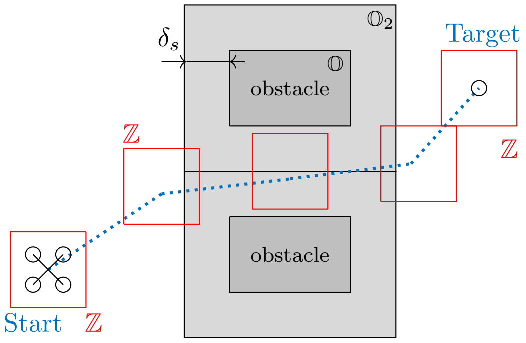
Neglecting the uncertainties and changing environment is critical and can result in performance degradation or complete failure. To overcome these challenges, robust planning approaches and control systems are needed to obtain safe and plausible references and control inputs to satisfy constraints and improve performance, e.g. (Mayne et al., 2005; Schouwenaars, 2006; Singh et al., 2017, 2018). While a combined planning and control approach is desirable, it can often not be implemented for complexity and computational feasibility reasons.

To address these challenges and reduce the complexity, we propose a hierarchical model predictive control (MPC) framework for planning and control in an uncertain and clutter environment, see Fig. 1. The proposed hierarchical structure effectively allows to decompose the complex task into separate subproblems that can be solved in real-time using the often limited on-board computational resources (Ibrahim et al., 2019; Koeln and Alleyne, 2018; Cowlagi and Tsiotras, 2012). The planner generates the reference for the low-level controller, while the dynamic obstacles can be taken into account. The low-level controller stabilizes the autonomous system and tracks the reference. Typically, such hierarchical approaches lack guarantees of consistency, e.g. constraint satisfaction between the planning and control layers which are normally time scale separated. The separation may for example produce an infeasible reference leading to unsafe behavior of the vehicle in the presence of uncertainty in the dynamic environment.

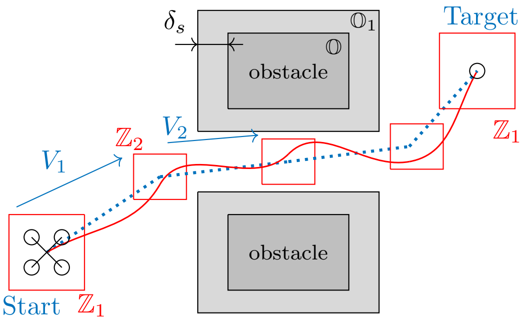
In this work, we propose a framework, see Fig. 2, that guarantees robustness and recursive feasibility, i.e. it maintains a guarantee of "consistency" between the different layers. The high-level planner operates on a moving horizon fashion to compute online a collision-free reference taking the constraint tightening into account. During the mission, encountered obstacles (static and dynamics) are enlarged by a safety bound to provide safe references to the low-level controller, see Fig. 1. The safety bounds are provided and ensured by the low-level controller. The planning problem using a simplified dynamic model is formulated as a mixed-integer programming (MILP). To track the generated reference, we use a robust tube based MPC (Mayne et al., 2005; Chisci et al., 2001). This MPC ensures robust constraint satisfaction despite uncertainties by an adequate level of accuracy bounded in a tube.

The organization of paper is structured as follows: Section 2 presents the problem formulation. The hierarchical moving horizon and control approach is outlined in Section 3.

⋆ MI and MK contributed equally to this work.



(a) No feasible solution with conservative constraint tightening/obstacle enlargement, even if there exists a feasible one.



(b) Reference planner can find a feasible solution with less conservative constraint tightening, i.e. obstacle enlargement. As uncertainty bounds can depend on the vehicle states (e.g. velocity): here $\mathbb{O}_1 \subset \mathbb{O}_2$ is smaller due to slower movement.

Fig. 1. Planner generates a feasible reference (dotted blue line) using a larger sampling time but the low-level controller is unable to follow this reference exactly (red line). While the planner cannot find feasible solution with conservative constraint tightening/obstacle enlargement e.g. Fig. 1a, even if there exists a feasible solution by adjusting the vehicle velocity Fig. 1b.

Section 4 presents simulation results. Section 5 summarizes the findings and the outlooks.

2. PROBLEM FORMULATION

We consider the control and motion planning for an autonomous vehicle. The overall objective is to navigate an autonomous vehicle from Start point to Target avoiding obstacles, see Fig. 1. For simplicity, we focus on a linear dynamics. The system is furthermore subject to bounded additive disturbances:

$$x(k+1) = Ax(k) + Bu(k) + \omega(k), \tag{1a}$$
$$y(k) = Cx(k), \tag{1b}$$
$$x(k) \in \mathbb{X},\ u(k) \in \mathbb{U}. \tag{1c}$$

Here $x(k)$ is the vehicle's state, $u(k)$ the control input and $y(k) \in \mathbb{R}^p$ is the output corresponding to the vehicle's positions and $\omega(k)$ is an unknown, but bounded disturbance. The sets restricting the state and input, i.e. $\mathbb{X} \subset \mathbb{R}^n$ and $\mathbb{U} \subset \mathbb{R}^m$, are convex and compact.

In addition to the state and control constraints (1c) the autonomous vehicle needs to avoid obstacles: the output $y(k)$/the vehicle's position should not touch obstacles modeled by a set of convex, compact polytopes (denoted by $\mathbb{O}$):

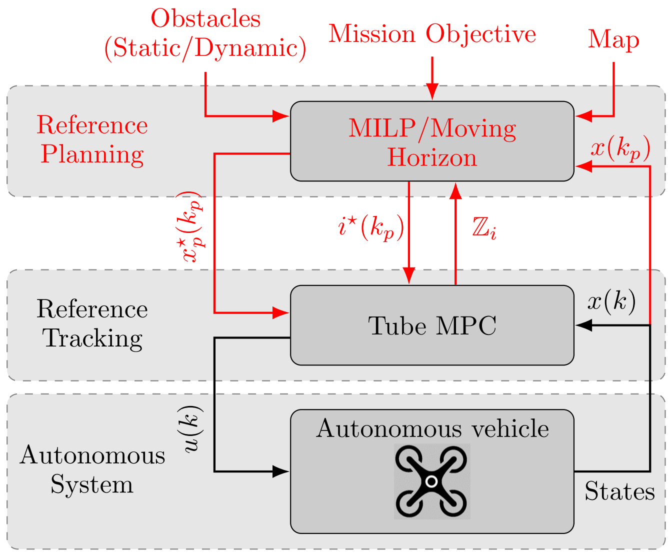


Fig. 2. Illustration of the hierarchical moving horizon control strategy and the signals transmitted between reference planning and the tracking controller. In general the low-levels (black) utilizes a faster sampling frequency than the high level planner (red).

$$y_k \notin \mathbb{O},\ \mathbb{O} = \{\{E_i y(k) < f_i\},\ i = 1, \ldots, H\}, \tag{2}$$

where H is the obstacles number. Note that the set of constraints are nonconvex and can be equivalently reformulated as

$$\forall i \in \{1, \ldots, H\},\ \exists j \in \{1, ..., q_i\} \text{ s.t. } E_{i,j} y(k) \geq f_{i,j}, \tag{3}$$

which can be efficiently handled utilizing a MILP framework using the so called big M approach, see Appendix A.

We assume that the bounds on the disturbance $\omega(k)$ depend on the state $x(k)$, e.g. velocity. For example, if the autonomous vehicle moves with a slow speed, the worst case disturbance might be smaller. This state-dependent disturbances might be due to increasing uncertainty or due to the low-level controller. We assume that there are N_ω different operating regions:

Assumption 1. (State-dependent disturbance bounds) If $x(k) \in \mathbb{X}_i \subseteq \mathbb{X}$, then $\omega(k) \in \mathbb{W}_i$, where $\mathbb{X}_i$ and $\mathbb{W}_i$ are convex, compact polytopes.

Note that the sets $\mathbb{X}_i$ can overlap, i.e. one can have that $\mathbb{X}_i \cap \mathbb{X}_j \neq \emptyset$ for $i \neq j$.

Remark 1. This assumption could be generalised for both state constraints $x(k) \in \mathbb{X}_i \subseteq \mathbb{X}$ and input constraints $u(k) \in \mathbb{U}_i \subseteq \mathbb{U}$.

2.1 Proposed control scheme

For motion planning and control of the autonomous vehicle we propose a hierarchical control scheme, see Fig. 2. The upper level planner generates a reference to navigate the vehicle around the obstacles to a desired point. This path will be determined such that all constraints (1c) will be satisfied and all obstacles will be avoided (3) for all possible disturbance realizations.

To do so, the upper-level planner uses a simplified model of the form

$$x_p(k+1) = A_p x_p(k) + B_p u_p(k), \tag{4}$$

to determines a path for the autonomous vehicle which is controlled by the lower level controller, see Fig. 2. This path will be determined such that all constraints (1c) will be satisfied and all obstacles will be avoided (3) for all possible disturbance realizations. The lower level reacts to disturbances and aims to implement the planned reference with a specific guaranteed accuracy, while considering the constraints. The lower level controller uses a possibly more detailed model (1) to track the reference. As the obstacles are handled by the planner, the lower level controller does not (directly) need to consider them, which allows an efficient and fast implementation of the lower level controller overcoming non-convex constraints.

In this work we assume that the planning model is obtained from the real model (1) using a larger sampling time:

$$A_p = A^M, \qquad B_p = \sum_{i=0}^{M-1} A^i B. \tag{5}$$

With respect to the real dynamics (1a) and the planing dynamics (4) we make the following assumption.

Assumption 2. (Controllability) The pairs (A_p, B_p) and (A, B) are controllable.

3. ROBUST HIERARCHICAL MOVING HORIZON PLANNING CONTROL WITH GUARANTEE

To guarantee the consistency and compatibility between the high-level planner and the vehicle dynamics controlled by the low-level controller we propose a robust hierarchical strategy (c.f. Fig. 2). In comparison to other approaches, our strategy defines the interaction between the planning and the tracking control as contracts, inspired by (Bäthge et al., 2018; Lucia et al., 2016, 2015; Blasi et al., 2018).

A contract specifies the capabilities (uncertainty bound), which the planning can request from the tracking control: the lower level controller can guarantee that the error between planned reference and the real movements will stay within a specific bounds. Basically, the lower level controller guarantees

$$x((k+1)M) - x_p(k+1) \in \mathbb{Z}_i, \tag{6}$$

if $x \in \mathbb{X}_i$, i.e. the state is inside the corresponding operation region i, see Assumption 1. The contracts are known by both control levels and they depend on the design of the low-level controller and the (partly) selectable uncertainty bound. Thus the planner can consider and exploit the capability of the low-level controller in the planning optimization problem. Therefore, the planner computes and transmits to the low-level controller not only the reference but also selects the required maximum discrepancy due to the choice of $\mathbb{Z}_i$. Then the reference planner can improve the performances by switching between different operation regions, see e.g. Fig. 1.

In both layers, we exploit the potentialities of moving horizon MPC (Grüne and Pannek, 2017; Findeisen and Allgöwer, 2002), which is an efficient control strategy to handle constraints, such as speed, acceleration limits and/or obstacle avoidance; for a wide varieties of dynamic systems, in presence of model uncertainties and noise. MPC has received significant attention for a wide range of applications besides stabilization e.g. reference planning (Howard et al., 2014; Ibrahim et al., 2019), and path following control (Matschek et al., 2019).

3.1 Moving Horizon based Reference Planning using MILP

To provide guarantee despite the hierarchical separation between the planner and the control (c.f. Fig. 2) we develop a robust reference planning and tracking framework that guarantees feasibility and constraint satisfaction, including collision avoidance, despite the uncertainty. Thus we can handle the challenges associated with uncertainty in the environmental and system dynamics and the geometry of the environment.

We extend and improve the moving horizon planning (Ibrahim et al., 2019, 2020), which is inspired by a series of MILP formulations (Trodden and Richards, 2008; Pinto and Afonso, 2017; Richards and How, 2006). In the reference planning, the nonconvex constraints due to obstacle avoidance are handled using a MILP formulation introducing several binary decision variables, as:

$$\min_{x,u,i} J_p(\{x_p\}, \{u_p\}), \tag{7a}$$

s.t. ($\forall j \in \{0, \ldots, N-1\}$)

$$\begin{aligned}
x_p(k_p+j+1|k_p) =& A_p x_p(k_p+j|k_p) && (7b)\\
&+ B_p u_p(k_p+j|k_p), \\
x(k_p M) - x_p(k_p|k_p) \in& \mathbb{Z}_i, && (7c)\\
x_p(k_p+j|k_p) \in& \mathbb{X}_i \ominus \mathbb{Z}_i, && (7d)\\
u_p(k_p+j|k_p) \in& \mathbb{U} \ominus K\mathbb{Z}_i, && (7e)\\
C x_p(k_p+j|k_p) \notin& \mathbb{O}_i \oplus C\mathbb{Z}_i, && (7f)\\
(x_p(k_p+j|k_p), u_p(k_p+j|k_p)) \in& \mathbb{I}_i, && (7g)\\
x_p(k_p+N|k_p) \in& \mathbb{X}_i^f. && (7h)
\end{aligned}$$

Here N is the planning horizon and i the selected operation region, and $(k_p+j|k_p)$ denotes the prediction of a value at time k_p+j made at time k_p. Note that we use the time variable k_p for the typically slower time scale operation. We utilize the planning objective function:

$$J_\mathrm{p} = \|x_\mathrm{Target} - x(k_p+N)\|_\infty + \sum_{j=k_p}^{k_p+N-1} \|u(j)\|_\infty.$$

Here, the stage cost minimize the control input $\|u(j)\|_\infty$, while the terminal cost penalizes the distance to the target point x_Target at the end of the planning horizon.
The sets $\mathbb{Z}_i$ are convex, compact polytopes and depend on the lower level closed loop tracking accuracy achieved for the operation region i.

The terminal constraint (7h) and the inter-sample constraints (7g) are nonconvex, depend also on the operation region i. With respect to the inter-sample constraints (7g) we make the following assumption to guarantee that the lower level controller can satisfy the constraints at all time, even so it can operate on a slower time scale.

Assumption 3. (Inter-sample constraints) The set $\mathbb{I}_i$ defining the inter-sample constraints are determined such that $(x_p, u_p) \in \mathbb{I}_i$ implies for $l = 1, \ldots, M-1$:

$$A^l x_p + \sum_{m=0}^{l-1} A^m B u_p \in \mathbb{X}_i \ominus \mathbb{Z}_i, \tag{8a}$$

$$C(A^l x_p + \sum_{m=0}^{l-1} A^m B u_p) \notin \mathbb{O}_i \oplus C\mathbb{Z}_i. \tag{8b}$$

Clearly, a trivial choice is to choose $\mathbb{I}_i$ directly as (8), which might slightly increases the computational effort. However this is not always necessary, e.g. one can use alternative approaches to enlarge the obstacles, see Appendix A.

The terminal set $\mathbb{X}_i^f$ is assumed to be a positive invariant set satisfying all constraints:

Assumption 4. (Terminal sets) There exists a terminal control law $\kappa^f(x_p)$ and terminal sets $\mathbb{X}_i^f$ such that if $x_p \in \mathbb{X}_i^f$, then $\forall l = 1, \ldots, M-1$

$$A_p x_p + B_p \kappa^f(x_p) \in \mathbb{X}_i^f, \tag{9a}$$

$$x_p \in \mathbb{X} \ominus \mathbb{Z}_i, \tag{9b}$$

$$\kappa^f(x_p) \in \mathbb{U} \ominus K\mathbb{Z}_i, \tag{9c}$$

$$(x_p, \kappa^f(x_p)) \in \mathbb{I}_i, \tag{9d}$$

$$Cx_p \notin \mathbb{O}_i \oplus C\mathbb{Z}_i. \tag{9e}$$

Note that the terminal set is nonconvex due to the obstacle avoidance (9e) and the inter-sample constraints (9d).
A rather straight forward choice is to focus on admissible, nominal steady states $x_p = A_p x_p + B_p \kappa^f(x_p)$ for the terminal sets, i.e. points where the autonomous vehicle can stop. This can for unmanned aerial vehicles possibly be satisfied.

The upper level planner sends the selected operation region $i^\star$ and the following inter-sampled reference to the lower level controller

$$x_{ref}(kM + j) = A^j x_p^\star(k_p|k_p) + \sum_{m=0}^{j} A^{m-1} B u_p^\star(k_p|k_p). \tag{10}$$

Proposition 1. (Recursive Feasibility) Let Assumptions 1-4 hold. If the moving horizon planning problem (7) is feasible at time k_p and the lower level controller guarantees for (10) $x(k) - x_{ref}(k) \in \mathbb{Z}_{i^\star}$, then (7) is feasible at $k_p + 1$.

Proof. Let us denote the optimal solution of (7) as $x_p^\star(k_p|k_p), \ldots, u_p^\star(k_p|k_p), \ldots, i^\star$.

To verify the above result consider for the optimization problem at $k_p + 1$ the following initial guess based on the previous solution and the terminal control law κ^f

$$\begin{aligned}
i =& i^\star, \\
x_p(k_p + j|k_p + 1) =& x_p^\star(k_p + j|k_p),\ j = 1, \ldots, N, \\
x_p(k_p + N + 1|k_p + 1) =& A_p x_p^\star(k_p + N|k_p) \\
&+ B_p \kappa^f(x_p^\star(k_p + N|k_p)), \\
u_p(k_p + m|k_p + 1) =& u_p^\star(k_p + m|k_p),\ m = 1, \ldots, N-1, \\
u_p(k_p + N|k_p + 1) =& \kappa^f(x_p^\star(k_p + N|k_p)).
\end{aligned}$$

One can verify straightforwardly that this initial guess is feasible (but suboptimal), i.e. that all constraints of (7) are feasible at $k_p + 1$ using the properties of the terminal set $\mathbb{X}^f$ and the guarantee on the low-level control accuracy.■

Remark 2. (Planing without feedback) *In principle one can modify the approach such that the initial constraint* (7c) *is only enforced at the begin* ($k = 0$) *and use the equality constraint* $x_p^\star(k_p{+}1|k_p) = x_p(k_p{+}1|k_p{+}1)$ *instead of* (7c) *for* $k_p > 0$. *This removes the feedback from the plant to the planning, which enables a computationally more efficient planing, but in general leads to a decreased control performance.*

Problem (7) is nonconvex and can be reformulated using the big-M method (Ibrahim et al., 2019, 2020) to obtain an efficiently solvable MILP, see Appendix A.

3.2 Robust Model Predictive Tracking Control

To track the generated reference we propose to use a robust tube based MPC (Mayne et al., 2005; Chisci et al., 2001) based on the (fast) real system dynamics (1). The proposed tube based MPC utilizes a nominal prediction dynamics (state z, input v) starting from the current real state

$$z(k + j + 1|k) = Az(k + j|k) + Bv(k + j|k), \tag{11a}$$

$$z(k|k) = x(k), \tag{11b}$$

for predicting the effect of future disturbances $w(k + j)$ which is taken into account using the fictitious, auxiliary control law

$$u(k + j|k) = v(k + j|k) + K(x(k + j) - z(k + j|k)), \tag{12}$$

where K is a control gain, such that $A + BK$ is Schur stable. Note that one does not need to use only one control gain, see (Kögel and Findeisen, 2020a,b). The difference

$$e(k + j|k) = x(k + j) - z(k + j|k)$$

between the predictions made using (11) and the real system can be bounded in form of sets with the above auxiliary control law.

In detail, if the system is in the i-th operation region, then $e(k + j|k) \in \mathbb{E}_i(j)$ where

$$\mathbb{E}_i(j + 1) = (A + BK)\mathbb{E}_i(j) \oplus \mathbb{W}_i, \quad \mathbb{E}_i(0) = \{0\}. \tag{13}$$

As $j \to \infty$ the sets size $\mathbb{E}_i$ increases, but is bounded by $\mathbb{E}_i(j) \subseteq \mathbb{Z}_i$, where $\mathbb{Z}_i$ is the (minimum) robust positive invariant set:

$$\mathbb{Z}_i \supseteq (A + BK)\mathbb{Z}_i \oplus \mathbb{W}_i. \tag{14}$$

The low-level MPC predicts until the next planning instant, so it uses a cyclic horizon $L_k = M - \mathrm{rem}(k, M)$, see (Kögel and Findeisen, 2013). In detail, if k is a multiple of M, then $L_k = M$. Otherwise, L_k is chosen such that $L_k < M$ and $k{+}L_k$ is a multiple of M. This means that the horizon shrinks between planing instants and is increased at the next planing instant again to length M.

The low-level MPC predicts a (nominal) state trajectory

$$\mathbf{z}(k) = \{z(k|k), \ldots, z(k + L_k|k)\},$$

and nominal input trajectory

$$\mathbf{v}(k) = \{v(k|k), \ldots, v(k + L_k - 1|k)\},$$

which are consistent with the nominal dynamics (11) and subject to the following constraints

$$z(k + j|k) \in \mathbb{X}_i \ominus \mathbb{E}_i(j), \tag{15a}$$

$$v(k + j|k) \in \mathbb{U} \ominus K\mathbb{E}_i(j), \tag{15b}$$

$$Cz(k + j|k) \in \{Cx_{ref}(k + j) \oplus C(\mathbb{Z}_i \ominus \mathbb{E}_i(j))\}, \tag{15c}$$

$$z(k + L_k|k) \in \{x_{ref}(k + L_k) \oplus (\mathbb{Z}_i \ominus \mathbb{E}_i(L_k))\}. \tag{15d}$$

Note that the (convex) state and input constraints (1c) are directly included in these constraints. In contrast the nonconvex obstacle avoidance constraints are considered by requiring that the lower level controller enforces the guaranteed accuracy on the output and at the final step on the entire state, which results in convex constraints.

The low-level MPC uses the objective function

$$J_t(\mathbf{z}(k), \mathbf{v}(k)) = \sum_{j=k}^{k+L_k-1} \|x_{ref}(j) - z(j|k)\|_Q^2 + \|v(j|k)\|_R^2 + \|x_{ref}(k+L_k) - z(k+L_k|k)\|_P^2, \quad (16)$$

where $Q \in \mathbb{R}^{n\times n}$, $P \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ represent the positive definite weighting matrix for the states and the inputs respectively, used to penalizes the deviation error from the reference x_{ref}.

In summary, to determine the input $u(k) = v^\star(k|k)$ the lower level MPC solves the following optimization problem

$$\min_{\mathbf{v}(k)} J_t(\mathbf{z}(k), \mathbf{v}(k)) \text{ s.t. } (11), (15). \quad (17)$$

Note that optimization problem is based on the current state as well as the reference and the operation region determined by the upper level planner. The optimization problem is a convex quadratic program and can be solved efficiently, even on computationally limited hardware.

For the closed loop we have the following properties.

Proposition 3. (Constraint satisfaction) Let Assumptions 1, 3 hold. If the lower level MPC problem (17) is feasible, then the constraints (1c) are satisfied: $x(k) \in \mathbb{X}_i \subseteq \mathbb{X}$ and $u(k) \in \mathbb{U}$ and the obstacles are avoided, i.e. (3) holds.

Proof. From the constraints (11), (12) and (15) we have that $z^\star(k|k) = x(k)$, as $v^\star(k|k) = u(k)$, $Cz^\star(k|k) = y(k)$. Together with $\mathbb{E}_i(0) = \{0\}$ and Assumption 1 this implies that vehicle constraint (1c) is satisfied. Moreover, $y(k) \in x_{ref}(k) \oplus \mathbb{Z}_i$, which implies together with (7f) and (8) and that the avoidance constraint $y_k \notin \text{int}(\mathbb{O}_i)$, i.e. (3) holds. ■

Proposition 4. (Recursive feasibility of the control scheme) Let Assumptions 1-4 hold. If the planner (7) is feasible at $k = 0$, then for the closed loop system consisting of the upper level moving horizon planer (7), the lower level controller (17), and the plant dynamics (1) the optimization problems (7), (17) are feasible for any $k > 0$.

Proof. The proof has three parts: first we show that feasibility of the planning problem (7) implies feasibility of the low-level controller (17), second that feasibility of (17) implies feasibility of (17) (if $k+1$ is not a multiple of M) or the planning problem (7) (otherwise).

If the upper-level planning problem (7) is feasible at k, then using the lower level input trajectory

$$v(k+j|k) = u_p^\star(k|k) + K((A+BK)^j x(k) - x_{ref}(j)),$$

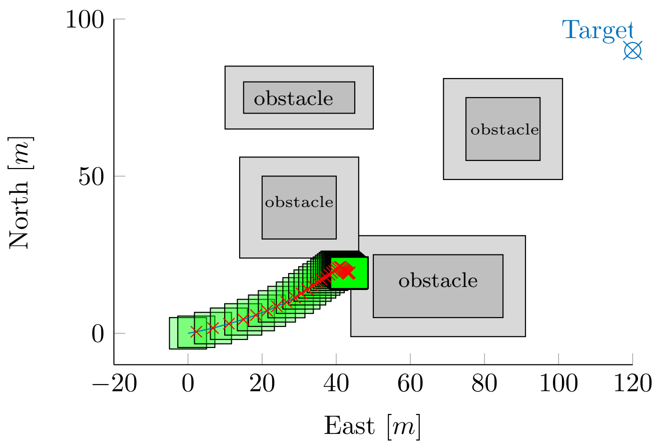
where $j = 0, \ldots, M-1$ results in a state trajectory satisfying all constraints due to the constraint tightening utilized in the upper and lower level optimization problems.

If $k+1$ is not a multiple of M, i.e. no planing takes place and the horizon $L(k)$ shrinks, then due to the design of the set $\mathbb{E}_i(j)$ at $k+1$ a feasible nominal state trajectory $\mathbf{z}(k+1)$ and a nominal input trajectory $\mathbf{v}(k+1)$, satisfying (11) and (15), exists:

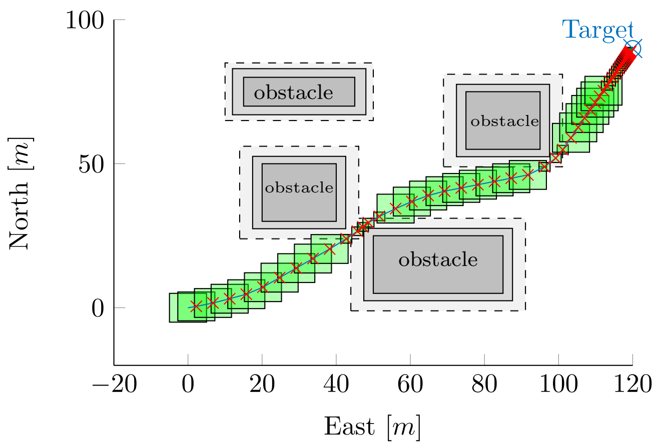
$$z(k+j|k+1) = z^\star(k+j|k) + (A+BK)^{j-1} w(k),$$
$$v(k+j|k+1) = v^\star(k+j|k) + K(A+BK)^{j-1} w(k).$$

If $k+1$ is a multiple of M, then the feasibility of (17) at k implies that $x(k) - x_{ref}(k) \in \mathbb{Z}_{i^\star}$, which together with Proposition 1 implies that the planning problem (7) is feasible. ■

Remark 5. We note assumed that the available constraints $\{\mathbb{X}_i, \mathbb{Z}_i\}$ are fixed over time at the initial time. In principle, they can also be changed/adapted over time, e.g. due to changing weather conditions for UAV's. This will be subject to future conditions.



(a) No feasible solution with conservative constraint tightening.



(b) The reference planner can find a feasible solution.

Fig. 3. Contract choice achieves less conservative results with enlarged feasible region (Fig. 3b) compared to the case of constant worst-case uncertainty (Fig. 3a).

4. SIMULATION RESULTS

To illustrate the efficiency of the proposed robust hierarchical MPC, we consider a linear vehicle model with a different sampling time for the planning $T_p = 1s$ and tracking control $T_t = 0.1s$, i.e. $M = 10$. The prediction horizon for planing is $N = 15$.

Fig. 3 provides a comparison between the proposed approach and the case considers a constant worst-case uncertainty. As depicted in Fig. 3a, there is no feasible solution with conservative constraint tightening (obstacle enlargement), even if there exists a feasible one, that is happen due to the short planning horizon. Nevertheless a large horizon would require a large computational time which it is often unfeasible for onboard implementation. On another hand, the reference planner can find a feasible solution with less conservative constraint tightening switching between possible choices, see Fig. 3b.

The contract choice enables the planner to find a feasible solution by adjusting the vehicle velocity to operate at the state region/contract with smaller uncertainty bound. The reference planner decides which state region is activated via the decision variables d_1 and d_2, see Fig. 4. Consequently, the controller adjusts the vehicle velocity V_t according to the decision. As we can see, the vehicle is moving at fast speed up to the time $11s$ when it is close to the obstacle. At this time, the planner activates the state region with less uncertainty bound via making the decision variable $d_1 = 1$ and deactivate the state region with large uncertainty bound via making the decision variable $d_2 = 0$. Thus, the planner find a feasible solution and then accelerate again after pass over the obstacles. This situation is repeated again at time $26s$.

Both robust MPC formulation were formulated with YALMIP (Löfberg, 2004) and solved via Gurobi (Optimization, 2014), while the tube MPC is implemented using the MPT toolbox (Kvasnica et al., 2004).

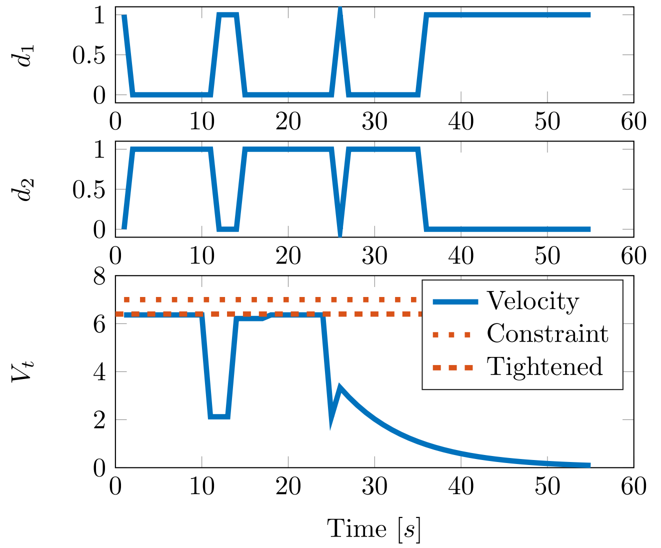


Fig. 4. The reference planner selects different contracts in the planning to achieve the desired performance.

5. CONCLUSION AND OUTLOOKS

We have proposed a combined moving horizon planner and robust model predictive controller. In the hierarchical strategy, the upper and lower levels exploit "contract" (guaranteed precision levels). To ensure compatibility between these levels and to guarantee safety, we do facilitate recursive feasibility of the hierarchical controller by suitable constraint. The reference planning is formulated as a MILP considering constraints tightening. The constraints tightening −achievable performance− is calculated at the lower-level controller, based on the capabilities of the autonomous vehicle.

Utilizing a different controller with different precision provides signification advantages, e.g. allow for less conservative results with enlarged feasible region compared to assuming a constant worst-case uncertainty. This decomposition of the control problem reduces computational cost, enables real-time implementation for robust control in autonomous vehicle. The efficiency of the proposed hierarchical approach is demonstrated via some simulation results in a clutter environment.

Possible extensions are the consideration of faster sampling to achieve better performance, for example exploiting direct hardware implementation of the lower level, e.g. by (Lucia et al., 2017). Further extension can be to consider nonlinear dynamics based on ellipsoidal tube MPC (Villanueva et al., 2017; Hu et al., 2018). In this case, the lower controller online sends upwards the tube parameterization, therefore the planner can predict a possible uncertainty evaluation over the planning horizon.

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Appendix A. OPERATING REGION SCHEDULER

According to the Assumption 1, the uncertainty set $\omega \in \mathbb{W}_i$ defines the state-dependent disturbance bounds for every operating region $\mathbb{X}_i$ For each, the low-level tube MPC guarantees the bounds $\mathbb{Z}_i$ on the tracking error.

Consequently, the planner can exploit as additional degree of freedom the operating regions for the lower-level controller, which are defined as state constraints sets:

$$\mathbb{X}_i \ominus \mathbb{Z}_i \equiv \{x | F_i x \leq G_i\}, \quad \forall i \in \{1, \cdots, N_\omega\},$$

here, N_ω is the number of the operating region, i.e. different velocity range.

The scheduling of the operating regions is formulated as linear constraints representing M-sided polygons exploiting the so called big M method (Schouwenaars, 2006):

$$F_i x \leq G_i + M_{\text{big}}(1 - d_i(k)), \quad \forall i \in \{1, \cdots, N_\omega\}.$$

Here M_{big} is a sufficiently large positive number to relax the constraints when the i-th region is not activated in the prediction horizon. $d_i(k)$ is a binary decision variable used to decide which region is active.

When this happens, the binary decision variable $d_i(k)$ can be set equal to one, which labels the constraint i-th being activated at time k. To insure that one region is activated at ever planning sample, we impose the extra constraint:

$$\sum_{i=1}^{N_\omega} d_i(k) = 1.$$

As results, the planning constraint $\mathbb{X}_i$ is tightened, i.e. the obstacle boundary $\mathbb{O}_i$ are enlarged via the tracking error set $\mathbb{Z}_i$.

Obstacle Avoidance Constraints:

The non-convex constraint (3) can be approximated by convex polygons introducing extra binary variables $b_s^i(k)$:

$$\forall s \in \{1, \ldots, S\}, \forall k \in \{1, \ldots, N\}, i \in \{1, ..., N_\omega\}$$
$$\big(p_x(k) - p_x^o(k)\big) \cos \frac{2\pi s}{S} + \big(p_y(k) - p_y^o(k)\big) \sin \frac{2\pi s}{S} \geq \delta_{\text{safe}}^i - M_{\text{big}} b_s^i(k),$$

where an extra constrain

$$\sum_{s=1}^{S} b_s^i(k) \leq S - d_i(k)$$

is imposed to ensure that at least one constraint is active for region i, i.e. $d_i(k) = 1$.

δ_{safe}^i is a minimum separation distance between the autonomous vehicle $\big(p_x(k), p_y(k)\big)$ and the obstacle position $\big(p_x^o(k), p_y^o(k)\big)$ at time step k for the region i.

Consequently, the planner has additional degree of freedom to adjust the obstacle enlargement δ_{safe}^i by controlling the vehicle velocity, see Fig. 3.