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An Extension of Barbalat's Lemma with its Application to Synchronization of a Class of Switched Networked Nonlinear Systems ⋆

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Abstract: This paper investigates the leader-following synchronization problem of uncertain Euler-Lagrange multi-agent systems subject to communication delays, disturbances and uniformly connected switching networks. The current settings cause great challenges to the solvability of the problem. To tackle these technical challenges, we make an extension to Barbalat's lemma. Based on the certainty equivalence principle, we propose a novel adaptive distributed control law and apply the generalized Barbalat's lemma to the synchronization problem. The effectiveness of the main result is demonstrated by an application to synchronization control of practical multiple mechanical systems.

Keywords: Euler-Lagrange system, distributed control, leader-following synchronization, disturbances, switching network.

1. INTRODUCTION

Inspired by more and more complex practical engineering tasks, cooperative control of multi-agent systems becomes a hot research topic in the field of systems and control. The synchronization control problem is one of the most fundamental cooperative control problems and all kinds of dynamics have been considered in the synchronization problem (cf.,Olfati-Saber and Murray (2004); Jadababaie et al. (2003)). Among them, the synchronization problem of uncertain Euler-Lagrange multi-agent systems has attracted many researchers' interest (cf., Ren (2009); Cai and Huang (2014); Chen and Kai (2018); Chung and Slotine (2009); Chen et al. (2015); Sun et al. (2007); Lu and Liu (2018b); Yang et al. (2017); Chen and Lewis (2011); Cai and Huang (2016); Lu and Liu (2019)).

The static networks are first considered in the synchronization problem of Euler-Lagrange systems, see Ren (2009); Chung and Slotine (2009); Chen et al. (2015); Sun et al. (2007); Chen and Lewis (2011) However, communication between agents is often disrupted by environment disturbances and/or internal instability of communication devices. Thus, the switching networks have attracted considerable attention in the synchronization problem of uncertain Euler-Lagrange multi-agent systems. In particular, the leader-following synchronization problem of multiple uncertain Euler-Lagrange systems is studied in Cai and Huang (2014) under uniformly connected switching networks by a distributed observer approach. Later, the similar problem is further studied in Cai and Huang (2016) by proposing an adaptive distributed observer. Recently, the synchronization problem of uncertain Euler-Lagrange multi-agent systems is investigated in Lu and Liu (2018a), where both the time-varying communication delays and switching networks are considered.

The leader-following synchronization problem of uncertain Euler-Lagrange multi-agent systems can be viewed a generalized version of the adaptive tracking control problem of a single uncertain Euler-Lagrange system. It is known that Barbalat's lemma has been shown to be an effective tool in dealing with the adaptive tracking problem of a single uncertain Euler-Lagrange system (cf.,Slotine and Li (1991, 1988)). However, the switching network invalidates the application of Barbalat's lemma to the leader-following synchronization problem of uncertain Euler-Lagrange system. To tackle this technical challenge, the generalized Barbalat's lemma developed in Su and Huang (2012) is adopted in these existing results Cai and Huang (2014), Cai and Huang (2016) and Lu and Liu (2018a) on leader-following synchronization of multiple uncertain Euler-Lagrange systems subject to switching networks. The generalized Barbalat's lemma in Su and Huang (2012) is effective in that it relaxes some conditions on the function in barbalat's lemma from continuous to piecewise continuous.

In this paper, we further study the leader-following synchronization problem of uncertain Euler-Lagrange multi-agent systems. In particular, the communication delays, disturbances and switching networks are all taken into consideration. It is noted that the effect of the disturbance in

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the dynamics of Euler-Lagrange systems has seldom been considered in these existing works (Cai and Huang, 2014), (Cai and Huang, 2016) and (Lu and Liu, 2018a). In fact, due to the existence of the disturbance, the generalized Barbalat's lemma in Su and Huang (2012) cannot be used to solve the current problem. To be precise, an adaptive distributed control law is first constructed based on the certainty equivalence principle. Then, as in existing works (cf., Cai and Huang (2014, 2016); Lu and Liu (2018a)), a Lyapunov-like function is introduced and it needs to show that the time derivative of this function tends to zero asymptotically. However, the disturbance compensation brings an additional function in the derivative of the Lyapunov-like function. It cannot be obtained that the time derivative of this Lyapunov-like function is non-positive in each time interval as in Cai and Huang (2014), Cai and Huang (2016) and Lu and Liu (2018a). As a consequence, the condition in the generalized Barbalat's lemma in Su and Huang (2012) is not satisfied. It precludes the application of the generalized Barbalat's lemma to the current problem. To overcome this challenge, we develop a new technical lemma, which further extends the generalized Barbalat's lemma in Su and Huang (2012). As its application, it is shown that the leader-following synchronization problem of uncertain Euler-Lagrange multi-agent systems subject to communication delays, disturbances and uniformly connected switching networks can be solved by the proposed distributed control law.

The remainder of this note is organized as follows. Section 2 presents an extension of Barbalat's lemma. Section 3 formulates the synchronization problem of switched networked uncertain Euler-Lagrange systems and provides its solution. One application to synchronization control of multiple mechanical systems is given in Section 4 to illustrate our design. Finally, some conclusions are made to end this note in Section 5.

Notation. For $x_i \in \mathbb{R}^{n_i \times m}$, $i = 1, \ldots, n$, $\mathrm{col}(x_1, \ldots, x_n) = [x_1^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}]^{\mathrm{T}}$. For $A_i \in \mathbb{R}^{p\times m}$, $i = 1, \ldots, n$, block diag$\{A_1, \ldots, A_n\}$ denotes a block matrix with its i-th diagonal entry being A_i and all other entries being zero. Denote a piecewise constant switching signal by a map $\sigma : [0, +\infty) \to \mathcal{P} = \{1, 2, ..., \rho\}$, where the switching instants t_i, $i = 0, 1, 2, \ldots$, satisfy $t_{i+1} - t_i \geq \tau_D$ for some positive real number τ_D, $\mathcal{P}$ is called the switching index set and τ_D is called the dwell time. For some positive scalar h, denote by $\mathcal{C}([-h, 0], \mathbb{R}^n)$ the Banach space of continuous functions mapping the interval $[-h, 0]$ into $\mathbb{R}^n$ endowed with the supremum norm. $||A||$ denotes the induced norm of the matrix A by the Euclidean norm. Throughout this paper, the derivative of a function always refers to its upper right-hand derivative.

2. AN EXTENSION OF BARBALAT'S LEMMA

It is known that Barbalat's lemma is an effective tool in stability analysis of non-autonomous systems. For non-autonomous system with piecewise continuous dynamics, a generalized version of Barbalat's Lemma has been developed in Su and Huang (2012). In this section, we further relax the condition in the Su and Huang (2012) and establish the following lemma.

Lemma 2.1. Given the sequence $\{t_k\}$ with $t_0 = 0$, $t_{k+1} - t_k \geq \tau_D > 0, k = 0, 1, 2, \ldots$, suppose that $V(t) : [0, +\infty) \to \mathbb{R}$ satisfies

1) $V(t)$ is lower bounded;

2) $V(t)$ is twice differentiable on each time interval $[t_k, t_{k+1})$, and

$$\dot{V}(t) = -U(t) + N^{\mathrm{T}}(t)D(t) \tag{1}$$

where $U(t) = N^{\mathrm{T}}(t)N(t)$, and $D(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$;

3) $\dot{U}(t)$ is bounded over $[0, +\infty)$ in the sense that there exists a positive constant K_0 such that

$$\sup_{t_k \leq t < t_{k+1}, k=0,1,2,\ldots} |\dot{U}(t)| \leq K_0. \tag{2}$$

Then, $\lim_{t\to\infty} \dot{V}(t) = 0$.

The proof of this Lemma is omitted due to space limit.

Remark 2.1. It is worth mentioning that Barbalat's Lemma (Slotine and Li, 1991) is extended in Su and Huang (2012) where $\dot{V}(t)$ can be piecewise continuous and $\ddot{V}(t)$ is required to be bounded over $[0, +\infty)$. Lemma 2.1 can be viewed a slight extension of the result in Su and Huang (2012). In particular, the condition $\dot{V}$ is non-positive in Su and Huang (2012) is relaxed to condition $\dot{V}(t) = -U(t) + N^{\mathrm{T}}(t)D(t)$ for a non-negative function $U(t)$ and $D(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$. In addition, the condition $\ddot{V}(t)$ is bounded over $[0, +\infty)$ in Su and Huang (2012) is relaxed to $\dot{U}(t)$ is bounded over $[0, +\infty)$.

3. SYNCHRONIZATION PROBLEM OF SWITCHED NETWORKED EULER-LAGRANGE SYSTEMS

3.1 Problem Formulation

Consider a group of uncertain Euler-Lagrange systems with their dynamics being described as follows:

$$\begin{aligned} &M_i(q_i(t))\ddot{q}_i(t) + C_i(q_i(t), \dot{q}_i(t))\dot{q}_i(t) + G_i(q_i(t)) \\ &= u_i(t) + d_i(t) \\ &q_i(\theta) = q_i^0(\theta), \dot{q}_i(\theta) = \dot{q}_i^0(\theta),\ \theta \in [-\tau, 0],\ i = 1, \ldots, N \end{aligned} \tag{3}$$

where $q_i(t) \in \mathbb{R}^n$ is the generalized position vector, $\dot{q}_i(t) \in \mathbb{R}^n$ is the generalized velocity vector, $M_i(q_i(t)) \in \mathbb{R}^{n\times n}$ is the inertia matrix and is symmetric positive definite, $C_i(q_i(t), \dot{q}_i(t))\dot{q}_i(t) \in \mathbb{R}^n$ is the Coriolis and centripetal force vector, $G_i(q_i(t)) \in \mathbb{R}^n$ is the gravity vector, $u_i(t) \in \mathbb{R}^n$ is the generalized force vector, $d_i(t) \in \mathbb{R}^n$ is the disturbance, τ denotes the upper bound of the time-delay involved, $q_i^0(\theta) \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$ and $\dot{q}_i^0(\theta) \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$.

The desired generalized position vector $q_0(t) \in \mathbb{R}^n$ and the disturbance $d_i(t) \in \mathbb{R}^n$ are assumed to be generated by a linear autonomous system as follows:

$$\begin{aligned} &\dot{v}(t) = Sv(t) \\ &q_0(t) = Dv(t), d_i(t) = E_i v(t) \\ &v(\theta) = v^0(\theta),\ \theta \in [-\tau, 0] \end{aligned} \tag{4}$$

where $v(t) \in \mathbb{R}^q$, $v^0(\theta) \in \mathcal{C}([-\tau, 0], \mathbb{R}^q)$, $S \in \mathbb{R}^{q\times q}$, $D \in \mathbb{R}^{n\times q}$ and $E_i \in \mathbb{R}^{n\times q}$ are some constant matrices.

It is known that the dynamics of the Euler-Lagrange system has the following properties (Slotine and Li, 1991).

Property 1. The matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.

Property 2. For all $x, y \in \mathbb{R}^n$, $M_i(q_i)x + C_i(q_i, \dot{q}_i)y + G_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\theta_i$, where $Y_i(q_i, \dot{q}_i, x, y) \in \mathbb{R}^{n\times p}$ is a known regression matrix, and $\theta_i \in \mathbb{R}^p$ is a constant vector consisting of the uncertain parameters of (3).

As in Cai and Huang (2014), the system composed of (3) and (4) can be viewed as a multi-agent system with (4) as the leader and the N subsystems (3) as followers. Given a piecewise constant switching signal $\sigma(t)$, the communication network for this multi-agent system is described by a switching digraph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$. In particular, the node set $\mathcal{V} = \{0, 1, \ldots, N\}$ with node 0 denoting system (4) and node i, $i = 1, \ldots, N$, denoting i-th subsystem of (3). The edge $(j, i) \in \mathcal{E}_{\sigma(t)}$ if and only if the information of agent j is accessible to the control law u_i at the instant of time t. Let $\mathcal{A}_{\sigma(t)}$ be the weighted adjacency matrix of the digraph $\mathcal{G}_{\sigma(t)}$ (Godsil and Royle, 2001). In particular, $\mathcal{A}_{\sigma(t)} = [a_{ij}(t)] \in \mathbb{R}^{(N+1)\times(N+1)}$, $i, j = 0, 1, \ldots, N$, where $a_{ii}(t) = 0$, $i = 0, 1, \ldots, N$ and $a_{ij}(t) = 1$, $i = 1, \ldots, N, j = 0, 1, \ldots, N$, if and only if $(j, i) \in \mathcal{E}_{\sigma(t)}$. The neighbor set of agent i at the instant of time t is defined as $\mathcal{N}_i(t) = \{j|(j, i) \in \mathcal{E}_{\sigma(t)}\}$.

We introduce the distributed control law as follows:

$$\begin{aligned} u_i(t) =&\phi_i(q_i(t), \dot{q}_i(t), \zeta_i(t), \zeta_j(t - \tau_{ij}(t)), j \in \mathcal{N}_i(t)) \\ \dot{\zeta}_i(t) =&\psi_i(q_i(t), \dot{q}_i(t), \zeta_i(t), \zeta_j(t - \tau_{ij}(t)), j \in \mathcal{N}_i(t)) \\ \zeta_i(\theta) =&\zeta_i^0(\theta), \theta \in [-\tau, 0], i = 1, \ldots, N \end{aligned} \tag{5}$$

where $\zeta_i(t) \in \mathbb{R}^{n_{\zeta_i}}$ for some positive integer n_{ζ_i}, $\tau_{ij}(t)$ are the communication delays and are piecewise continuous satisfying $0 \le \tau_{ij}(t) \le \tau$ for some positive real number τ, $\zeta_i^0(\theta) \in \mathcal{C}([-\tau, 0], \mathbb{R}^{n_{\zeta_i}})$, ϕ_i and ψ_i are some functions to be determined.

Now, the leader-following synchronization problem for the uncertain Euler-Lagrange multi-agent system can be described as follows.

Problem 3.1. Given multiple uncertain Euler-Lagrange systems (3), the leader system (4), and a switching digraph $\mathcal{G}_{\sigma(t)}$, design a distributed control law of the form (5) such that, for all piecewise continuous and bounded communication delays $0 \le \tau_{ij}(t) \le \tau$, all initial conditions $v^0(\theta)$, and $q_i^0(\theta)$, $\dot{q}_i^0(\theta)$, $\zeta_i^0(\theta)$, $\theta \in [-\tau, 0]$, $i = 1, \ldots, N$, $q_i(t)$ and $\dot{q}_i(t)$ exist for all $t \ge 0$ and satisfy

$$\lim_{t\to\infty}(q_i(t) - q_0(t)) = 0, \text{ and } \lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0, i = 1, \ldots, N. \tag{6}$$

To solve the problem, two assumptions are first given.

Assumption 3.1. The matrix S is marginally stable.

Assumption 3.2. There exists a subsequence $\{l_k\}$ of $\{l : l = 0, 1, \ldots\}$ with $t_{l_{k+1}} - t_{l_k} < \nu$ for some positive ν such that every node $i = 1, \ldots, N$ is reachable from the node 0 in the union graph $\bigcup_{j=l_k}^{l_{k+1}-1} \mathcal{G}_{\sigma(t_j)}$.

Remark 3.1. It is noted that the leader system (4) can generate a large class of reference signals such as multi-tone sinusoidal signals (Cai and Huang, 2016). Assumption 3.2 is also called uniformly connected condition (cf., Jadababaie et al. (2003), Su and Huang (2012)). It may be the mildest condition on network connectivity.

3.2 Solvability of the Synchronization Problem

Let us first introduce a dynamic compensator as follows:

$$\begin{aligned} \dot{\xi}_i(t) =&S\xi_i(t) + \mu \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(e^{S\tau_{ij}(t)}\xi_j(t - \tau_{ij}(t)) \\ &- \xi_i(t)) \\ \xi_i(\theta) =&\xi_i^0(\theta), \theta \in [-\tau, 0], i = 1, \ldots, N \end{aligned} \tag{7}$$

where $\xi_i(t) \in \mathbb{R}^q$, $\xi_0(t) = v(t)$, $\xi_i^0(\theta) \in \mathcal{C}([-\tau, 0], \mathbb{R}^q)$, and μ is an arbitrary positive real number.

Let

$$\dot{q}_{ri}(t) = DS\xi_i(t) - \alpha(q_i(t) - D\xi_i(t)) \tag{8}$$

where α is an arbitrary positive real number.

Then, by (8),

$$\ddot{q}_{ri}(t) = DS\dot{\xi}_i(t) - \alpha(\dot{q}_i(t) - D\dot{\xi}_i(t)). \tag{9}$$

By Property 2, there exists a known matrix $Y_i(t) = Y_i(q_i(t), \dot{q}_i(t), \ddot{q}_{ri}(t), \dot{q}_{ri}(t)) \in \mathbb{R}^{n\times p}$ and an unknown constant vector $\theta_i \in \mathbb{R}^p$ such that for $i = 1, \ldots, N$,

$$Y_i(t)\theta_i = M_i(q_i(t))\ddot{q}_{ri}(t) + C_i(q_i(t), \dot{q}_i(t))\dot{q}_{ri}(t) + G_i(q_i(t)). \tag{10}$$

Define

$$s_i(t) = \dot{q}_i(t) - \dot{q}_{ri}(t). \tag{11}$$

Then, the adaptive distributed control law of the form (5) with $\zeta_i(t) = \mathrm{col}(\hat{\theta}_i(t), \xi_i(t))$ is defined as follows:

$$\begin{aligned} u_i(t) =& - K_i s_i(t) - E_i\xi_i(t) + Y_i(t)\hat{\theta}_i(t) \\ \dot{\hat{\theta}}_i(t) =& - \Gamma_i Y_i^{\mathrm{T}}(t)s_i(t) \\ \dot{\xi}_i(t) =&S\xi_i(t) + \mu \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(e^{S\tau_{ij}(t)}\xi_j(t - \tau_{ij}(t)) \\ &- \xi_i(t)),\ i = 1, \ldots, N \end{aligned} \tag{12}$$

where $\hat{\theta}_i(t) \in \mathbb{R}^p$, $\Gamma_i \in \mathbb{R}^{p\times p}$ and $K_i \in \mathbb{R}^{n\times n}$ are some positive definite matrices.

Now, under the distributed control law (12), the synchronization problem of the Euler-Lagrange multi-agent system can be solved. The main result is summarized by the following theorem.

Theorem 3.1. Consider the multi-agent system composed of (3) and (4), and a switching digraph $\mathcal{G}_{\sigma(t)}$. Then, under Assumptions 3.1 and 3.2, the leader-following synchronization problem can be solved by the adaptive distributed control law (12).

Proof: Let $\hat{\xi}_i(t) = \xi_i(t) - \xi_0(t)$ and $\tilde{\xi}_i(t) = e^{-St}\hat{\xi}_i(t)$, $i = 0, 1, \ldots, N$. Then,

$$\begin{aligned} \dot{\tilde{\xi}}_i(t) =&\mu \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(\tilde{\xi}_j(t - \tau_{ij}(t)) - \tilde{\xi}_i(t)) \\ \tilde{\xi}_i(\theta) =&\tilde{\xi}_i^0(\theta), \theta \in [-\tau, 0],\ i = 1, \ldots, N \end{aligned} \tag{13}$$

where $\tilde{\xi}_0(t) \equiv 0$, and $\tilde{\xi}_i^0(\theta) \in \mathcal{C}([-\tau, 0], \mathbb{R}^q)$.

Note the communication delays $\tau_{ij}(t)$ are piecewise continuous and satisfy $0 \le \tau_{ij}(t) \le \tau$ for some positive real number τ. Thus, under Assumption 3.2, by Lemma .1, we have that the origin of the switched time-delay system (13) is exponentially stable. That is,

$$\lim_{t\to\infty} \tilde{\xi}_i(t) = 0 \tag{14}$$

exponentially.

Furthermore, under Assumption 3.1, $||e^{St}|| \leq Q_0$ for some constant $Q_0 > 0$, which together with (14) implies that, for $i = 1, \ldots, N$,

$$\lim_{t\to\infty} \hat{\xi}_i(t) = 0 \tag{15}$$

exponentially.

Furthermore, it can also be verified that, for $i = 1, \ldots, N$,

$$\lim_{t\to\infty} \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(e^{S\tau_{ij}(t)}\xi_j(t-\tau_{ij}(t)) - \xi_i(t)) = 0 \tag{16}$$

exponentially. It implies that

$$\lim_{t\to\infty} \dot{\hat{\xi}}_i(t) = 0 \tag{17}$$

exponentially.

By (15) and (17),

$$\lim_{t\to\infty}(D\xi_i(t)-q_0(t)) = 0, \text{and} \lim_{t\to\infty}(D\dot{\xi}_i(t)-\dot{q}_0(t)) = 0 \tag{18}$$

exponentially.

By (11),

$$\dot{q}_i(t) = s_i(t) + \dot{q}_{ri}(t),\ \ddot{q}_i(t) = \dot{s}_i(t) + \ddot{q}_{ri}(t). \tag{19}$$

For $i = 1, \ldots, N$, substituting (19) into (3) yields

$$\begin{aligned} &M_i(q_i(t))(\dot{s}_i(t) + \ddot{q}_{ri}(t)) + C_i(q_i(t), \dot{q}_i(t))(s_i(t) + \dot{q}_{ri}(t)) \\ &+ G_i(q_i(t)) = -K_i s_i(t) - E_i\hat{\xi}_i(t) + Y_i(t)\hat{\theta}_i(t). \end{aligned} \tag{20}$$

Under the distributed control law (12), by (10) and (20), it can be obtained that

$$\begin{aligned} M_i(q_i(t))\dot{s}_i(t) + C_i(q_i(t), \dot{q}_i(t))s_i(t) = &-K_i s_i(t) - E_i\hat{\xi}_i(t) \\ &+ Y_i(t)\tilde{\theta}_i(t) \end{aligned} \tag{21a}$$

$$\dot{\hat{\theta}}_i(t) = -\Gamma_i Y_i^{\mathrm{T}}(t)s_i(t) \tag{21b}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $i = 1, \ldots, N$.

Let $q = \mathrm{col}(q_1, \ldots, q_N)$, $\dot{q} = \mathrm{col}(\dot{q}_1, \ldots, \dot{q}_N)$, $\hat{\xi} = \mathrm{col}(\hat{\xi}_1, \ldots, \hat{\xi}_N)$, $s = \mathrm{col}(s_1, \ldots, s_N)$, $\tilde{\theta} = \mathrm{col}(\tilde{\theta}_1, \ldots, \tilde{\theta}_N)$, $\hat{\theta} = \mathrm{col}(\hat{\theta}_1, \ldots, \hat{\theta}_N)$, $K = \text{block diag}\{K_1, \ldots, K_N\}$, $E = \text{block diag}\{E_1, \ldots, E_N\}$, $\Gamma = \text{block diag}\{\Gamma_1, \ldots, \Gamma_N\}$, $Y = \text{block diag}\{Y_1, \ldots, Y_N\}$, $M(q) = \text{block diag}\{M_1(q_1), \ldots, M_N(q_N)\}$, and $C(q, \dot{q}) = \text{block diag}\{C_1(q_1, \dot{q}_1), \ldots, C_N(q_N, \dot{q}_N)\}$, Then, (21a) and (21b) can be put as follows:

$$\begin{aligned} M(q(t))\dot{s}(t) + C(q(t), \dot{q}(t))s(t) = &-Ks(t) - E\hat{\xi}(t) \\ &+ Y(t)\tilde{\theta}(t) \end{aligned} \tag{22}$$
$$\dot{\hat{\theta}}(t) = -\Gamma Y^{\mathrm{T}}(t)s(t).$$

Let

$$V(t) = \frac{1}{2}(s^{\mathrm{T}}(t)M(q(t))s(t) + \tilde{\theta}^{\mathrm{T}}(t)\Gamma^{-1}\tilde{\theta}(t)) \tag{23}$$

which is a continuous function. Then, by Property 1 and (22), the time derivative of $V(t)$ satisfies

$$\begin{aligned} \dot{V}(t) =& s^{\mathrm{T}}(t)M(q(t))\dot{s}(t) + \frac{1}{2}s^{\mathrm{T}}(t)\dot{M}(q(t))s(t) \\ &+ \tilde{\theta}^{\mathrm{T}}(t)\Gamma^{-1}\dot{\tilde{\theta}}(t) \\ =& s^{\mathrm{T}}(t)\big(-C(q(t), \dot{q}(t))s(t) - Ks(t) - E\hat{\xi}(t) \\ &+ Y(t)\tilde{\theta}(t)\big) + \frac{1}{2}s^{\mathrm{T}}(t)\dot{M}(q(t))s(t) + \tilde{\theta}^{\mathrm{T}}(t)\Gamma^{-1}\dot{\tilde{\theta}}(t) \\ =& -s^{\mathrm{T}}(t)Ks(t) - s^{\mathrm{T}}(t)E\hat{\xi}(t). \end{aligned} \tag{24}$$

Denote $U(t) = s^{\mathrm{T}}(t)Ks(t)$. Note that $V(t) \geq 0$, $U(t) \geq 0$ and $\lim_{t\to\infty} \hat{\xi}(t) = 0$ exponentially. Conditions 1) and 2) in Lemma 2.1 are satisfied. It can be verified that $V(t)$ is bounded for all $t \geq 0$, which implies that $s(t)$ and $\tilde{\theta}$ are bounded for all $t \geq 0$.

By the definition of switching signal $\sigma(t)$, $t_{k+1} - t_k \geq \tau_D$, $k = 0, 1, \ldots$, for some $\tau_D > 0$. Then, by (7), (9) and (11), $s(t)$ is differentiable on each time interval $[t_k, t_{k+1})$ for all $k \geq 0$. Thus, $\dot{U}(t)$ is differentiable on each time interval $[t_k, t_{k+1})$ for all $k \geq 0$.

Under Assumption 3.1, $v(t)$ is bounded for all $t \geq 0$. Thus, by (15), $\xi_i(t)$, $i = 1, \ldots, N$, is bounded for all $t \geq 0$. It follows from (7) that $\dot{\xi}_i(t)$, $i = 1, \ldots, N$, is bounded for all $t \geq 0$.

By (8) and (11),

$$\dot{q}_i(t) = -\alpha q_i(t) + (DS\xi_i(t) + \alpha D\xi_i(t) + s_i(t)). \tag{25}$$

Since α is a positive real number, by (25), $q_i(t)$ and $\dot{q}_i(t)$ are bounded. Then, by (8) and (9), both $\dot{q}_{ri}(t)$ and $\ddot{q}_{ri}(t)$ and thus, by (10), $Y_i(q_i(t), \dot{q}_i(t), \ddot{q}_{ri}(t), \dot{q}_{ri}(t))$ is bounded on each time interval $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$.

Note that $s_i(t)$ is differentiable on each time interval $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$. By (21a), we can conclude that $\dot{s}_i(t)$ is bounded on each time interval $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$.

Note $\dot{U}(t) = 2s^{\mathrm{T}}(t)K\dot{s}(t)$. Since both $s(t)$ and $\dot{s}(t)$ are bounded on each time interval $[t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$, $\dot{U}(t)$ is bounded over $[0, +\infty)$ in the sense that there exists a positive constant K_0 such that

$$\sup_{t_k \leq t < t_{k+1}, k=0,1,2,\ldots} |\dot{U}(t)| \leq K_0. \tag{26}$$

That is, condition 3) in Lemma 2.1 holds. Therefore, by Lemma 2.1, $\lim_{t\to\infty} \dot{V}(t) = 0$. In view of (24), it follows that $\lim_{t\to\infty} s(t) = 0$.

By (7), (8) and (11),

$$\begin{aligned} &\dot{q}_i(t) - D\dot{\xi}_i(t) \\ =& -\alpha(q_i(t) - D\xi_i(t)) \\ &- \mu D \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(e^{S\tau_{ij}(t)}\xi_j(t-\tau_{ij}(t)) - \xi_i(t)) + s_i(t) \end{aligned}$$

which is input-to-state stable viewing $(q_i(t) - D\xi_i(t))$ as the state and $(-\mu D \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(e^{S\tau_{ij}(t)}\xi_j(t-\tau_{ij}(t)) - \xi_i(t)) + s_i(t))$ as the input. Since $\alpha > 0$, $\lim_{t\to\infty} s(t) = 0$ and $\lim_{t\to\infty} \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(e^{S\tau_{ij}(t)}\xi_j(t-\tau_{ij}(t)) - \xi_i(t)) = 0$ by (16), it can be obtained that $\lim_{t\to\infty}(q_i(t) - D\xi_i(t)) = 0$ and $\lim_{t\to\infty}(\dot{q}_i(t) - D\dot{\xi}_i(t)) = 0$. Therefore, the proof is completed by invoking (18). □

Remark 3.2. The classical Barbalat's lemma is not applicable to the convergence analysis of $\dot{V}(t)$. It is obviously that due to the existence of switching networks, by (7), (9), and (11), the function $s_i(t)$ is not continuously differentiable for $t \in [0, \infty)$. Thus, by (24), $\dot{V}(t)$ is not continuously differentiable for $t \in [0, \infty)$, which implies that $\dot{V}(t)$ cannot be uniformly continuous in t. As a consequence, one of the conditions required in the classical Barbalat's lemma does not hold.

Remark 3.3. As stated in Remark 3.2, the function $s_i(t)$ is not continuously differentiable for $t \in [0, \infty)$. Thus, $s_i(t)$ cannot be uniformly continuous in t. As a result, the classical Barbalat's lemma cannot be applied to the convergence analysis of $s_i(t)$.

Remark 3.4. It is worth mentioning that the leader-following synchronization problem of multiple uncertain Euler-Lagrange systems under uniformly connected condition has been considered in Cai and Huang (2014, 2016) and Lu and Liu (2018a). In fact, the switching network results in a switched closed-loop system. As a result, Barbalat's lemma cannot be used to solve the problem in these existing works. Instead, the generalized Barbalat's lemma proposed in Su and Huang (2012) is employed to solve the problem. However, the effect of the disturbance in the dynamics of Euler-Lagrange systems is not considered in Cai and Huang (2014, 2016) and Lu and Liu (2018a). To deal with the disturbance, the adaptive control law is designed based on the certainty equivalence principle. As in Cai and Huang (2014, 2016) and Lu and Liu (2018a), the Lyapunov-like function $V(t)$ is constructed. Due to the effect of the disturbance, an additional item appears in the time derivative of $V(t)$, that is, $(-s^{\mathrm{T}}(t)E\hat{\xi}(t))$ in (24). As a consequence, we cannot obtain that $\dot{V}(t)$ is non-positive and the condition required in generalized Barbalat's lemma in Su and Huang (2012) does not hold. This motivates our result in Section 2.

4. EXAMPLE

Consider a group of three-link cylindrical arms whose dynamics are described by Euler-Lagrange systems of form (3) (Lewis et al., 1993), where

$$q_i(t)=\begin{bmatrix} q_{i1}(t) \\ q_{i2}(t) \\ q_{i3}(t) \end{bmatrix}, M_i=\begin{bmatrix} m_{i1}+m_{i2}q_{i3}^2(t) & 0 & 0 \\ 0 & m_{i2}+m_{i3} & 0 \\ 0 & 0 & m_{i3} \end{bmatrix}$$
$$C_i = \begin{bmatrix} m_{i3}q_{i3}(t)\dot{q}_{i3}(t) & 0 & m_{i3}q_{i3}(t)\dot{q}_{i1}(t) \\ 0 & 0 & 0 \\ -m_{i3}q_{i3}(t)\dot{q}_{i1}(t) & 0 & 0 \end{bmatrix}, d_i = E_i v$$
$$G_i = \begin{bmatrix} 0 \\ (m_{i2}+m_{i3})gq_{i2}(t) \\ 0 \end{bmatrix}, \theta_i = \begin{bmatrix} m_{i1} \\ m_{i2} \\ m_{i3} \end{bmatrix}, \ i = 1, 2, 3, 4. \tag{27}$$

The leader system is in the form of (4) with

$$S = \begin{bmatrix} 0_{3\times3} & I_3 \\ -I_3 & 0_{3\times3} \end{bmatrix}, D = [I_3 \ 0_{3\times3}]$$
$$E_i = \begin{bmatrix} i & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & i/2 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \end{bmatrix}, \ i = 1, 2, 3, 4. \tag{28}$$

It is easy to verify that Assumption 3.1 holds.

The communication network of the five agents $\mathcal{G}_{\sigma(t)}$ is dictated by the switching signal $\sigma(t)$ as follows:

$$\sigma(t) = \begin{cases} 1, \text{ if } sT \le t < (s+\frac{1}{4})T \\ 2, \text{ if } (s+\frac{1}{4})T \le t < (s+\frac{1}{2})T \\ 3, \text{ if } (s+\frac{1}{2})T \le t < (s+\frac{3}{4})T \\ 4, \text{ if } (s+\frac{3}{4})T \le t < (s+1)T \end{cases} \tag{29}$$

where $s = 0, 1, 2, \ldots$. The four digraphs $\mathcal{G}_i$, $i = 1, 2, 3, 4$ are illustrated in Fig. 1(Lu and Liu, 2018a). It can be verified that Assumption 3.2 is satisfied. The communication delays are $\tau_{ij}(t) = \frac{1}{2}(sin(\frac{1}{i}t))^2 \ \ sec$, $i = 1, 2, 3, 4$ (Lu and Liu, 2018a).

The adaptive distributed control law of the form (12) is designed, where $\mu = 1$, $K_i = 40I_3$, and $\Gamma_i = I_3$, $i = 1, \ldots, 4$. The simulation is conducted with $\theta_1 = \mathrm{col}(1.1, 2.1, 0.1)$, $\theta_2 = \mathrm{col}(1.2, 2.2, 0.2)$, $\theta_3 = \mathrm{col}(1.3, 2.3, 0.2)$, $\theta_4 = \mathrm{col}(1.4, 2.4, 0.3)$ (Lu and Liu, 2018a), $g = 9.8 \ m/s^2$, $T = 1 \ sec$, and all initial conditions being randomly chosen from the interval $[-1 \ 1]$. The position tracking error and the velocity tracking error of all followers are shown in Fig. 2 and Fig. 3, respectively. It can be found that leader-following synchronization of the four Euler-Lagrange systems is achieved.

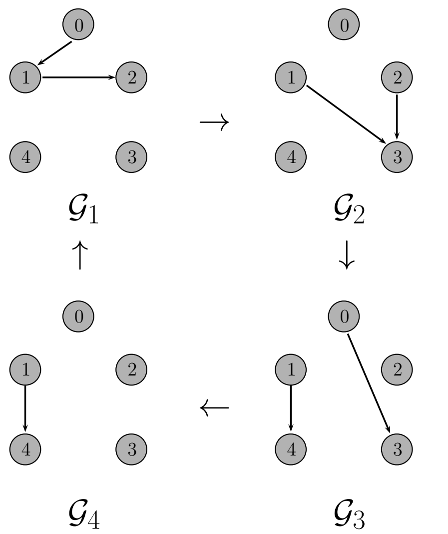

Fig. 1. Communication network $\mathcal{G}_{\sigma(t)}$ with $\mathcal{P} = \{1, 2, 3, 4\}$.

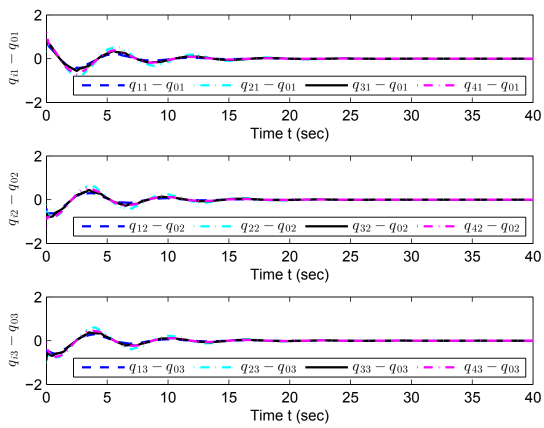

Fig. 2. The position tracking error of all followers.

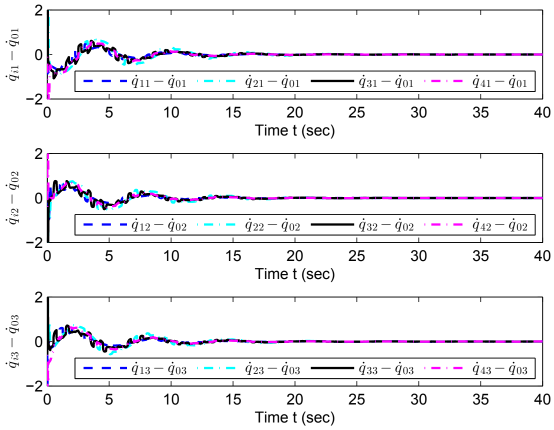

Fig. 3. The velocity tracking error of all followers.

5. CONCLUSION

In this paper, we have investigated the leader-following synchronization problem of uncertain Euler-Lagrange multi-agent systems subject to nonuniform time-varying communication delays, disturbances and uniformly connected switching networks. In particular, we have sightly extended the generalized Barbalat's lemma in Su and Huang (2012) to a more general version. Then, we have applied the newly generalized Barbalat's lemma to the synchronization problem of uncertain Euler-Lagrange multi-agent systems. By proposing a novel adaptive distributed control law, it has been demonstrated that leader-following synchronization of uncertain Euler-Lagrange multi-agent systems can be achieved.

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A TECHNICAL LEMMA

Lemma .1. (Corollary 3.1 in Lu and Liu (2017)) Consider the switched time-delay system

$$\begin{aligned} \dot{\chi}_i(t) &= \mu \sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(\chi_j(t - \tau_{ij}(t)) - \chi_i(t)) \\ \chi_i(\theta) &= \chi_i^0(\theta), \theta \in [-\tau, 0], \ i = 1, \ldots, N \end{aligned} \tag{.1}$$

where $\chi_0(t) \equiv 0$, $\chi_i(t) \in \mathbb{R}^m$, $i = 1, \ldots, N$, $\chi_i^0(\theta) \in \mathcal{C}([-\tau, 0], \mathbb{R}^m)$, and μ is an arbitrary positive real number. $\tau_{ij}(t)$ are piecewise continuous and satisfy $0 \leq \tau_{ij}(t) \leq \tau$ for an arbitrary positive real number τ. Then, under Assumption 3.2, the origin of (.1) is exponentially stable.