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Output Tracking Control Based on Output Feedback with Adaptive PFC for Discrete-Time Systems

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Abstract: This paper provides an output tracking control system design strategy based on an output feedback control with an adaptive parallel feedforward compensator (PFC) for discrete-time systems. In the proposed method, a PFC is introduced for non-almost strictly positive real (ASPR) systems in order to guarantee the stability of the designed adaptive control system. The PFC parameters are adaptively adjusted to remain the ASPR-ness of the resulting augmented system with the PFC. Moreover, in order to attain output tracking, a two-degree-of-freedom output feedback control system with an adaptive neural network (NN) feedforward control is designed. The stability of the resulting adaptive control system is analyzed theoretically and the effectiveness of the proposed method is confirmed through numerical simulations.

Keywords: Output tracking, Adaptive control, Output feedback control, ASPR, Parallel feedforward compensator

1. INTRODUCTION

Adaptive controls can keep the high control performance by automatically adjusting the controller parameters according to unknown and variable parameters of the controlled system (Landau (1979); Goodwin and Sin (1984); Narendra and Annaswamy (1989)). In particular, adaptive output feedback controls based on almost strictly positive realness (ASPR-ness) of the system including simple adaptive control (SAC) have a simple structure compared with the conventional adaptive controls (Mizumoto and Iwai (1996); Kaufman et al. (1997)). These adaptive control methods can design the controller without the knowledge of the order of the controlled system and reduce the number of adjusting parameters. Moreover, these methods are applied to process and diesel combustion systems (Mizumoto et al. (2015); Fujii et al. (2019)).

The system is called ASPR if there exists a static output feedback such that the resulting closed-loop system is strictly positive real (SPR)(Kaufman et al. (1997)). The conditions for the discrete-time systems to be ASPR have been provided as follows: (1) the system is minimum-phase, (2) the system has a relative degree of 0, (3) the high-frequency gain of the system is positive. Unfortunately, since the most practical systems do not satisfy these conditions, these conditions for the system to be ASPR impose a severe restriction to practical applications of the ASPR based adaptive controls. One simple method to solve the issue of the conditions imposed on the controlled system is the introduction of a parallel feedforward compensator (PFC) (Bar-Kana (1987); Iwai and Mizumoto (1994); Fradkov (1996)). In this method, the PFC is introduced in parallel with the non-ASPR controlled system so as to render the resulting augmented system ASPR. Most of the existing PFC design methods provide the static PFC. Therefore, in the case where the property of the controlled system changes during operation, it is difficult to maintain the ASPR-ness of the augmented system with the static PFC. Moreover, in the conventional PFC design methods, some kind or another information of the controlled system such as a nominal model are required to design the PFC.

With this in mind, adaptive-type PFC design schemes have been proposed for continuous-time and discrete-time systems (Takagi and Mizumoto (2015); Fujii and Mizumoto (2018)). In these methods, the PFC parameters are adaptively adjusted by using only input/output data of the controlled system. Thus, these PFC design methods do not require the detail information of the controlled system and can maintain the ASPR-ness of the augmented system even when the property of the controlled system changes. However, in the ASPR based output feedback control system using the PFC, the tracking performance degenerates since the control system is designed for the augmented system with the PFC instead of the original controlled system. The result shown in Takagi and Mizumoto (2015); Fujii and Mizumoto (2018) are only for the stabilization problem of the resulting control system and the influence of the PFC output causes the degradation of the control performance. Therefore, for continuous-time systems, the output tracking control system with the adaptive PFC has been investigated and the tracking performance is improved (Mizumoto et al. (2018)). However, for discrete-time systems, the tracking performance of the control system with the adaptive PFC has not been studied.

In this paper, an output tracking control system design scheme based on the output feedback with adaptively adjusted PFC is proposed for discrete-time systems. In the adaptive PFC design method in Fujii and Mizumoto (2018), the number of the adjusted parameters is redundant, and the structure of the adaptive PFC is complicated. In this paper, the number of the PFC parameters to be estimated is reduced and the adaptive PFC which has a simple structure is proposed. Moreover, in order to attain the output tracking of the original controlled system, a feedforward control system is added, and a two-degree-of-freedom output feedback control system is designed. In the feedforward control system, an adaptive neural network (NN) control input is designed in this paper (Ge et al. (2002); Mizumoto et al. (2010)). The stability of the resulting adaptive control system is analyzed and the effectiveness of the proposed method is verified through numerical simulations.

2. PROBLEM STATEMENT

Let us consider a n-th order SISO discrete-time linear system with the state-space representation:

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + \boldsymbol{b}u(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}(k) \end{aligned} \tag{1}$$

where $\boldsymbol{x}(k) \in R^n$ is the state vector, $y(k), u(k) \in R$ are the output and the input of the system. The transfer function of (1) is denoted by $G(z)$.

A PFC: $H(z)$ is introduced in parallel with the system $G(z)$ as seen in Fig. 1 and the PFC denoted by $H(z, \boldsymbol{\rho}, d)$ is parameterized by $\boldsymbol{\rho}$ and d. Suppose that the PFC parameterized by $\boldsymbol{\rho}$ and d satisfies the following assumption.

Assumption 1. $H(z, \boldsymbol{\rho}, d) = d$ with $\boldsymbol{\rho} = \boldsymbol{0}$

The augmented system denoted by $G_a(z)$ can be given by

$$G_a(z, \boldsymbol{\rho}, d) = G(z) + H(z, \boldsymbol{\rho}, d). \tag{2}$$

If the augmented system with the PFC is ASPR, one can design stable output feedback control systems based on the ASPR-ness of the considered system. However, in the case where the controlled system is unknown and/or is changing during operation, designing the static PFC which maintains the ASPR-ness of the resulting augmented system is difficult. The first objective in this paper is to design an adaptive and simpler PFC so as to match the designed augmented system to the given desired ASPR model $G_a^*(z)$.

To design an adaptive NN feedforward control system, we impose the following assumptions.

Assumption 2. A reference signal $y_r(k)$ which the output $y(k)$ of the controlled system is required to track is generated by the following neural stable exosystem:

$$\begin{aligned} \boldsymbol{\omega}(k+1) &= \boldsymbol{p}(\boldsymbol{\omega}(k)) \\ y_r(k) &= q(\boldsymbol{\omega}(k)) \end{aligned} \tag{3}$$

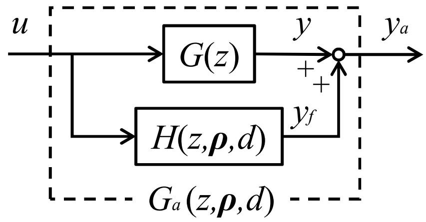

Fig. 1. Augmented system with PFC

where $\boldsymbol{\omega}(k) \in R^{n_\omega}$ is the state vector and $\boldsymbol{p}(\boldsymbol{0}) = \boldsymbol{0},\ q(0) = 0$.

Assumption 3. There exist an ideal state vector $\boldsymbol{x}^*(k)$ and an ideal input $v^*(k)$ which attain perfect tracking such that

$$\begin{aligned} \boldsymbol{x}^*(k+1) &= A\boldsymbol{x}^*(k) + \boldsymbol{b}v^*(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}^*(k) \equiv y_r(k) \end{aligned} \tag{4}$$

and they are given by the functions of $\boldsymbol{\omega}(k)$ such as $\boldsymbol{x}^*(k) = \boldsymbol{\pi}(\boldsymbol{\omega}(k))$ and $v^*(k) = c(\boldsymbol{\omega}(k))$.

The second objective in this paper is to design a two-degree-of-freedom output feedback control system with the adaptive NN feedforward control.

3. CONTROL SYSTEM DESIGN

3.1 Ideal PFC

Define the ideal output of the desired ASPR model: $G_a^*(z)$ with any input $u(k)$ by

$$y_a^*(k) = G_a^*(z)[u(k)] \tag{5}$$

where the notation of $y(k) = W(z)[u(k)]$ denotes the output of the system $W(z)$ with the input $u(k)$.

Then the ideal output of the PFC can be obtained by

$$y_f^*(k) = y_a^*(k) - y(k) \tag{6}$$

using the available output of the controlled system.

Suppose that the n_h-th order ideal PFC model is expressed by the following transfer function:

$$H^*(z) = \frac{d^* z^{n_h} + b_{n_h}^* z^{n_h-1} + \cdots + b_2^* z + b_1^*}{z^{n_h} + a_{n_h}^* z^{n_h-1} + \cdots + a_2^* z + a_1^*}. \tag{7}$$

Moreover, a parametric representation of the ideal PFC can be represented by

$$\begin{aligned} y_f^*(k) &= H^*(z)[u(k)] \\ &= \boldsymbol{\rho}^{*T}\boldsymbol{z}(k) + d^*u(k) \end{aligned} \tag{8}$$

where

$$\boldsymbol{\rho}^* = \left[\, -a_{n_h}^* \ \cdots \ -a_1^* \ b_{n_h}^* \ \cdots \ b_1^* \,\right]^T \tag{9}$$

$$\begin{aligned} \boldsymbol{z}(k) = \big[\, & y_f^*(k-1) \ \cdots \ y_f^*(k-n_h) \\ & u(k-1) \ \cdots \ u(k-n_h) \,\big]^T. \end{aligned} \tag{10}$$

In the method in Fujii and Mizumoto (2018), $\boldsymbol{\rho}^*$ and d^* are adaptively estimated. However, the parameter d^* of the direct input feedthrough term is the same as that of the desired ASPR model. Therefore, since the desired ASPR model is given by the controller designer, d^* is known. In the following, the PFC parameter $\boldsymbol{\rho}^*$ will be adaptively estimated without adjusting d^* to design the simpler adaptive PFC.

3.2 Approximation of Ideal Input by RBF NN

Under Assumption 2 and 3, the ideal feedforward control input can be approximated based on the Radial Basis Function (RBF) neural network (NN) by

$$v_{nn}(k) = W^T S(\boldsymbol{\omega}(k)) \tag{11}$$

where $W = [w_1, \ \cdots, \ w_l]^T \in R^l$ is the weight vector, l is the NN node number, and $S(\boldsymbol{\omega}) = [s_1(\boldsymbol{\omega}), \ \cdots, s_l(\boldsymbol{\omega})]^T$ is the basis function vector. The commonly used RBFs are the Gaussian functions as follows:

$$s_i(\boldsymbol{\omega}) = \exp\left[-\frac{(\boldsymbol{\omega}-\boldsymbol{\mu}_i)^T(\boldsymbol{\omega}-\boldsymbol{\mu}_i)}{\eta_i^2}\right] \tag{12}$$
$$i = 1, \ 2, \ \cdots, \ l$$

where $\boldsymbol{\mu}_i = [\mu_{i1}, \ \cdots, \ \mu_{in_\omega}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function.

There exists an ideal weight vector W^* such that

$$W^* \triangleq \arg\min_{W\in R^l}\{\sup_{\boldsymbol{\omega}\in\Omega_\omega} |v^* - W^T S(\boldsymbol{\omega})|\} \tag{13}$$

for a sufficiently large NN node number l and a compact set $\Omega_\omega \subset R^{n_\omega}$ (Ge et al. (2002)).

Then, the ideal input $v^*(k)$ can be approximated by

$$v^*(k) = W^{*T} S(\boldsymbol{\omega}) + \varepsilon(\boldsymbol{\omega}), \ |\varepsilon(\boldsymbol{\omega})| \le \varepsilon^* \tag{14}$$

where $\varepsilon(\boldsymbol{\omega})$ is the NN approximation error.

Here, we impose the following assumption.

Assumption 4. For a given NN node number l, there exists an ideal weight vector W^* that satisfies (13) for all $\boldsymbol{\omega} \in \Omega_\omega$.

In the following, we will adaptively adjust the ideal weight vector W^*.

3.3 Ideal Control System

Define a PFC output signal with a parameter $\boldsymbol{\rho}$ by

$$\begin{aligned} y_f(k,\boldsymbol{\rho},d^*) &= H(z,\boldsymbol{\rho},d^*)[u(k)] \\ &= \boldsymbol{\rho}^T\boldsymbol{z}(k) + d^*u(k). \end{aligned} \tag{15}$$

It follows from (8) that $y_f(k,\boldsymbol{\rho}^*,d^*) = y_f^*(k)$ with $\boldsymbol{\rho} = \boldsymbol{\rho}^*$. Moreover, define a PFC output with a parameter $\boldsymbol{\rho}$ and a feedforward input $v(k)$ by

$$\begin{aligned} y_{fv}(k,\boldsymbol{\rho},d^*) &= H(z,\boldsymbol{\rho},d^*)[v(k)] \\ &= \boldsymbol{\rho}^T\boldsymbol{z}_v(k) + d^*v(k) \end{aligned} \tag{16}$$

$$\begin{aligned} \boldsymbol{z}_v(k) = [\, & y_{fv}(k-1) \ \cdots \ y_{fv}(k-n_h) \\ & v(k-1) \ \cdots \ v(k-n_h)\,]^T. \end{aligned} \tag{17}$$

It follows that $y_{fv}(k,\boldsymbol{\rho}^*,d^*) = y_{fv}^*(k)$.

Then, the ideal PFC output with a feedback input $u_e(k) = u(k) - v(k)$ can be expressed by

$$\begin{aligned} \bar{y}_f^*(k) &= H(z,\boldsymbol{\rho}^*,d^*)[u_e(k)] \\ &= y_f(k,\boldsymbol{\rho}^*,d^*) - y_{fv}(k,\boldsymbol{\rho}^*,d^*) \\ &= y_f^*(k) - y_{fv}^*(k). \end{aligned} \tag{18}$$

In the case where the ideal PFC parameter $\boldsymbol{\rho}^*$ is known, the ideal two-degree-of-freedom controller which attain output tracking can be designed as follows:

$$\begin{aligned} u^*(k) &= u_e^*(k) + v^*(k) \\ u_e^*(k) &= -\theta^*\bar{e}_a^*(k) \end{aligned} \tag{19}$$

where $\bar{e}_a^*(k) = \bar{y}_a^*(k) - y_r(k)$, $\bar{y}_a^*(k) = y(k) + \bar{y}_f^*(k)$, and θ^* is the ideal feedback gain such that the resulting closed-loop system is SPR.

However, since the controlled system is unknown, the ideal PFC parameter $\boldsymbol{\rho}^*$ and the ideal feedforward control input $v^*(k)$ are unknown. In the following, we propose an adaptive controller in which $\boldsymbol{\rho}^*$ and $v^*(k)$ are adaptively adjusted.

3.4 Adaptive Control System

Define the following signals based on the PFC outputs defined in (8) and (16) by using a ASPR model $G_a^*(z)$:

$$\begin{aligned} y_f(k) &= G_a^*(z)[\boldsymbol{\rho}(k)^T\bar{\boldsymbol{z}}(k)] + d^*u(k) \\ \bar{\boldsymbol{z}}(k) &= G_a^*(z)^{-1}[\boldsymbol{z}(k)] \end{aligned} \tag{20}$$

and

$$\begin{aligned} y_{fv}(k) &= G_a^*(z)[\boldsymbol{\rho}(k)^T\bar{\boldsymbol{z}}_v(k)] + d^*v(k) \\ \bar{\boldsymbol{z}}_v(k) &= G_a^*(z)^{-1}[\boldsymbol{z}_v(k)] \end{aligned} \tag{21}$$

where $\boldsymbol{\rho}(k)$ is an adaptively adjusted parameter vector of $\boldsymbol{\rho}^*$.

Then, design the following output signal as the adaptive PFC output:

$$\bar{y}_f(k) = y_f(k) - y_{fv}(k). \tag{22}$$

Using these signals, the adaptive control system with the adaptive PFC and the adaptive NN feedforward control is designed as follows (See Fig. 2):

$$u(k) = u_e(k) + v(k) \tag{23}$$

$$u_e(k) = -\theta\bar{e}_a(k) - \rho_z\|\bar{\boldsymbol{z}}_v(k)\|^2\bar{e}_a(k) - \rho_e\bar{e}_a(k) \tag{24}$$

$$\bar{e}_a(k) = \bar{y}_a(k) - y_r(k), \ \bar{y}_a(k) = y(k) + \bar{y}_f(k)$$

$$v(k) = \begin{cases} v_{\min} & (\hat{W}(k)^T S(\boldsymbol{\omega}(k)) < v_{\min}) \\ \hat{W}(k)^T S(\boldsymbol{\omega}(k)) & \\ & (v_{\min} \le \hat{W}(k)^T S(\boldsymbol{\omega}(k)) \le v_{\max}) \\ v_{\max} & (v_{\max} < \hat{W}(k)^T S(\boldsymbol{\omega}(k))) \end{cases} \tag{25}$$

where θ is a large feedback gain such that the resulting closed-loop system is SPR and $\hat{W}(k)$ is the adaptively adjusted NN weight vector of W^* in (14). The second and third terms in (24) are additional feedback terms of maintaining the stability of the obtained adaptive control

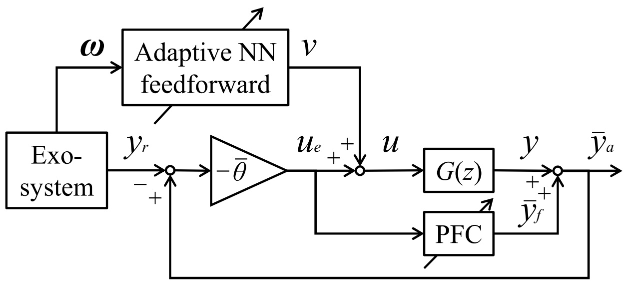

Fig. 2. Block diagram of adaptive control system

system. $v_{\max}$ and $v_{\min}$ are maximum and minimun values of the feedforward control input saturation constraint. $\boldsymbol{\rho}(k)$ and $\hat{W}(k)$ are adaptively adjusted by the following parameter adjusting laws:

$$\boldsymbol{\rho}(k) = \bar{\sigma}_\rho \boldsymbol{\rho}(k-1) - \bar{\sigma}_\rho \Gamma_\rho \left(\bar{\boldsymbol{z}}(k) - \bar{\boldsymbol{z}}_v(k)\right) \bar{e}_a(k) \quad (26)$$

$$\bar{\sigma}_\rho = \frac{1}{1+\sigma_\rho}, \ \sigma_\rho > 0, \ \Gamma_\rho = \Gamma_\rho^T > 0$$

$$\hat{W}(k) = \bar{\sigma}_W \hat{W}(k-1) - \bar{\sigma}_W \Gamma_W S(\boldsymbol{\omega}(k)) \bar{e}_a(k) \quad (27)$$

$$\bar{\sigma}_W = \frac{1}{1+\sigma_W}, \ \sigma_W > 0, \ \Gamma_W = \Gamma_W^T > 0.$$

As for the feedback control input, the practicable equivalent input can be obtained without the causality problem as follows:

$$\begin{aligned} u_e(k) = &-\frac{\bar{\theta}(k)}{1+\bar{\theta}(k)d^*}\big(y(k) + G_a^*(z)[\boldsymbol{\rho}(k)^T \bar{\boldsymbol{z}}(k)] \\ &-G_a^*(z)[\boldsymbol{\rho}(k)^T \bar{\boldsymbol{z}}_v(k)] - y_r(k)\big) \end{aligned} \quad (28)$$

where $\bar{\theta}(k) = \theta + \rho_z \|\bar{\boldsymbol{z}}_v(k)\|^2 + \rho_e$.

It should be noted that, the augmented error signal $\bar{e}_a(k)$ can be equivalenty obtained by using the available signals without the causality problem.

3.5 Analysis of Obtained Control System

Concerning the boundedness of all signals in the proposed adaptive control system, we have the following theorem.

Theorem Under Assumptions 1 to 4, all signals in the resulting control system with control inputs given in (23) to (25) with the parameter adjusting laws given in (26) and (27) are bounded.

Proof We first derive an error system of the obtained control system. The adaptive PFC output $\bar{y}_f(k)$ can be represented by

$$\begin{aligned} \bar{y}_f(k) &= \bar{y}_f(k) - \bar{y}_f^*(k) + \bar{y}_f^*(k) \\ &= (y_f(k) - y_{fv}(k)) - \big(y_f^*(k) - y_{fv}^*(k)\big) + \bar{y}_f^*(k) \\ &= G_a^*(z)[\Delta\boldsymbol{\rho}(k)^T \bar{\boldsymbol{z}}(k)] - G_a^*(z)[\boldsymbol{\rho}(k)^T \bar{\boldsymbol{z}}_v(k)] \\ &\quad -d^* v(k) + y_f^*(k) \end{aligned} \quad (29)$$

where $\Delta\boldsymbol{\rho}(k) = \boldsymbol{\rho}(k) - \boldsymbol{\rho}^*$. Therefore, the augmented output $\bar{y}_a(k)$ can be obtained by

$$\begin{aligned} \bar{y}_a(k) &= y(k) + \bar{y}_f(k) \\ &= y_a^*(k) - y_f^*(k) + \bar{y}_f(k) \\ &= G_a^*(z)[u_e(k)] + G_a^*(z)[v(k) - v^*(k)] + G(z)[v^*(k)] \\ &\quad +H^*(z)[v^*(k)] + G_a^*(z)[\Delta\boldsymbol{\rho}(k)^T \bar{\boldsymbol{z}}(k)] \\ &\quad -G_a^*(z)[\boldsymbol{\rho}(k)^T \bar{\boldsymbol{z}}_v(k)] - d^* v(k) \end{aligned} \quad (30)$$

where $G_a^*(z) = G(z) + H^*(z)$. Define the new PFC output as follows:

$$y_{fv}^{**}(k) = H^*(z)[v^*(k)] = \boldsymbol{\rho}^{*T} \boldsymbol{z}_v^{**}(k) + d^* v^*(k). \quad (31)$$

Then, the augmented output $\bar{y}_a(k)$ can be expressed by

$$\begin{aligned} \bar{y}_a(k) &= G_a^*(z)[u_e(k)] + G_a^*(z)[v(k) - v^*(k)] \\ &\quad +G(z)[v^*(k)] + G_a^*(z)[\Delta\boldsymbol{\rho}(k)^T \bar{\boldsymbol{z}}(k)] \\ &\quad +\boldsymbol{\rho}^{*T}\left(\boldsymbol{z}_v^{**}(k) - \boldsymbol{z}_v(k)\right) + d^* v^*(k) \\ &\quad -G_a^*(z)[\Delta\boldsymbol{\rho}(k)^T \bar{\boldsymbol{z}}_v(k)] - d^* v(k). \end{aligned} \quad (32)$$

Taking into account the fact that $G(z)[v^*(k)] = y_r(k)$, the error system is obtained as follows:

$$\begin{aligned} \bar{e}_a(k) &= \bar{y}_a(k) - y_r(k) \\ &= G_a^*(z)[u_e(k) + \Delta W(k)^T S(\boldsymbol{\omega}(k)) + \varepsilon(\boldsymbol{\omega}(k)) \\ &\quad +\Delta\boldsymbol{\rho}(k)^T\left(\bar{\boldsymbol{z}}(k) - \bar{\boldsymbol{z}}_v(k)\right) + \boldsymbol{\rho}^{*T}\left(\bar{\boldsymbol{z}}_v^{**}(k) - \bar{\boldsymbol{z}}_v(k)\right) \\ &\quad +d^* \Delta\bar{v}(k)] \end{aligned} \quad (33)$$

where $\Delta W(k) = \hat{W}(k) - W^*$, $\bar{\boldsymbol{z}}_v^{**}(k) = G_a^*(z)^{-1}[\boldsymbol{z}_v^{**}(k)]$, $\Delta\bar{v}(k) = G_a^{*-1}(z)[\Delta v(k)]$, $\Delta v(k) = v^*(k) - v(k)$.

Since the closed-loop system with the control input given in (24) for $G_a^*(z)$ is SPR, the error system can be represented by

$$\begin{aligned} \bar{e}_a(k) &= G_s(z)[-\rho_z \|\bar{\boldsymbol{z}}_v(k)\|^2 \bar{e}_a(k) - \rho_e \bar{e}_a(k) \\ &\quad +\Delta W(k)^T S(\boldsymbol{\omega}(k)) + \varepsilon(\boldsymbol{\omega}(k)) \\ &\quad +\Delta\boldsymbol{\rho}(k)^T\left(\bar{\boldsymbol{z}}(k) - \bar{\boldsymbol{z}}_v(k)\right) \\ &\quad +\boldsymbol{\rho}^{*T}\left(\bar{\boldsymbol{z}}_v^{**}(k) - \bar{\boldsymbol{z}}_v(k)\right) + d^* \Delta\bar{v}(k)] \end{aligned} \quad (34)$$

where $G_s(z) = \frac{\theta G_a^*(z)}{1+\theta G_a^*(z)}$ is a SPR system.

Define a realization of $G_s(z)$ by $(A_s, \boldsymbol{b}_s, \boldsymbol{c}_s, d_s)$, the state space representation of the error system is expressed by

$$\boldsymbol{x}_s(k+1) = A_s \boldsymbol{x}_s(k) + \boldsymbol{b}_s u_s(k) \quad (35)$$

$$\bar{e}_a(k) = \boldsymbol{c}_s^T \boldsymbol{x}_s(k) + d_s u_s(k) \quad (36)$$

where $u_s(k) = -\left(\rho_z \|\bar{\boldsymbol{z}}_v(k)\|^2 + \rho_e\right) \bar{e}_a(k) + \Delta W(k)^T S(\boldsymbol{\omega}(k)) + \varepsilon(\boldsymbol{\omega}(k)) + \Delta\boldsymbol{\rho}(k)^T\left(\bar{\boldsymbol{z}}(k) - \bar{\boldsymbol{z}}_v(k)\right) + \boldsymbol{\rho}^{*T}\left(\bar{\boldsymbol{z}}_v^{**}(k) - \bar{\boldsymbol{z}}_v(k)\right) + d^* \Delta\bar{v}(k)$.

Since $G_s(z)$ is SPR, there exist symmetric positive definite matrices $P = P^T > 0$, $Q = Q^T > 0$, an appropriate vector $\boldsymbol{l}$, and a scalar w such that the following Kalman-Yakubovich-Popov (KYP) lemma is satisfied.

$$\begin{aligned} A_s^T P A_s - P &= -Q - \boldsymbol{l}\boldsymbol{l}^T \\ A_s^T P \boldsymbol{b}_s &= \boldsymbol{c}_s - \boldsymbol{l} w \\ \boldsymbol{b}_s^T P \boldsymbol{b}_s &= 2d_s - w^2 \end{aligned} \quad (37)$$

Now, consider the following positive definite function $V(k)$:

$$\begin{aligned} V(k) &= V_1(k) + V_2(k) + V_3(k) \\ V_1(k) &= \boldsymbol{x}_s(k)^T P \boldsymbol{x}_s(k) \\ V_2(k) &= \bar{\sigma}_\rho \Delta\boldsymbol{\rho}(k-1)^T \Gamma_\rho^{-1} \Delta\boldsymbol{\rho}(k-1) \\ V_3(k) &= \bar{\sigma}_W \Delta W(k-1)^T \Gamma_W^{-1} \Delta W(k-1). \end{aligned} \tag{38}$$

Define the difference of $V(k)$ by $\Delta V(k) = V(k) - V(k-1)$. The difference of $V_1(k)$ can be evaluated using the KYP lemma by

$$\begin{aligned} \Delta V_1(k) \leq & -\boldsymbol{x}_s(k-1)^T Q \boldsymbol{x}_s(k-1) \\ & -2\rho_z \|\bar{\boldsymbol{z}}_v(k-1)\|^2 \bar{e}_a(k-1)^2 - 2\rho_e \bar{e}_a(k-1)^2 \\ & +2\bar{e}_a(k-1)\Delta\boldsymbol{\rho}(k)^T(\bar{\boldsymbol{z}}(k-1) - \bar{\boldsymbol{z}}_v(k-1)) \\ & +2\bar{e}_a(k-1)\Delta W(k-1)^T S(\boldsymbol{\omega}(k-1)) \\ & +2\bar{e}_a(k-1)\boldsymbol{\rho}^{*T}(\bar{\boldsymbol{z}}_v^{**}(k-1) - \bar{\boldsymbol{z}}_v(k-1)) \\ & +2\bar{e}_a(k-1)\left(d^*\Delta\bar{v}(k-1) + \varepsilon(\boldsymbol{\omega}((k-1)))\right). \end{aligned} \tag{39}$$

The differences of $V_2(k)$ and $V_3(k)$ can be obtained as follows with any positive constans δ_1 and δ_2.

$$\begin{aligned} \Delta V_2(k) \leq & -\left(\bar{\sigma}_\rho^{-1} - \bar{\sigma}_\rho - \delta_1\right)\Delta\boldsymbol{\rho}(k-1)^T \Gamma_\rho^{-1} \Delta\boldsymbol{\rho}(k-1) \\ & -2\bar{e}_a(k-1)\Delta\boldsymbol{\rho}(k-1)^T(\bar{\boldsymbol{z}}(k-1) - \bar{\boldsymbol{z}}_v(k-1)) \\ & +\frac{\sigma_\rho^2}{\delta_1}\boldsymbol{\rho}^{*T}\Gamma_\rho^{-1}\boldsymbol{\rho}^* \end{aligned} \tag{40}$$

$$\begin{aligned} \Delta V_3(k) \leq & -\left(\bar{\sigma}_W^{-1} - \bar{\sigma}_W - \delta_2\right)\Delta W(k-1)^T \Gamma_W^{-1} \Delta W(k-1) \\ & -2\bar{e}_a(k-1)\Delta W(k-1)^T S(\boldsymbol{\omega}(k-1)) \\ & +\frac{\sigma_W^2}{\delta_2} W^{*T}\Gamma_W^{-1} W^*. \end{aligned} \tag{41}$$

Finally, the difference of $V(k)$ can be evaluated by

$$\begin{aligned} \Delta V(k) \leq & -\boldsymbol{x}_s(k-1)^T Q \boldsymbol{x}_s(k-1) \\ & -\left(\bar{\sigma}_\rho^{-1} - \bar{\sigma}_\rho - \delta_1\right)\Delta\boldsymbol{\rho}(k-1)^T \Gamma_\rho^{-1} \Delta\boldsymbol{\rho}(k-1) \\ & -\left(\bar{\sigma}_W^{-1} - \bar{\sigma}_W - \delta_2\right)\Delta W(k-1)^T \Gamma_W^{-1} \Delta W(k-1) \\ & -\left(2\rho_z - \delta_3\right)\|\bar{\boldsymbol{z}}_v(k-1)\|^2 |\bar{e}_a(k-1)|^2 \\ & -\left(2\rho_e - \delta_4 - \delta_5\right)|\bar{e}_a(k-1)|^2 \\ & +\frac{\sigma_\rho^2}{\delta_1}\boldsymbol{\rho}^{*T}\Gamma_\rho^{-1}\boldsymbol{\rho}^* + \frac{\sigma_W^2}{\delta_2} W^{*T}\Gamma_W^{-1} W^* + \frac{1}{\delta_3}\|\boldsymbol{\rho}^*\|^2 \\ & +\frac{1}{\delta_4}\|\boldsymbol{\rho}^*\|^2 \bar{\boldsymbol{z}}_{v,max}^{**2} + \frac{1}{\delta_5}\left(d^*|\Delta\bar{v}|_{max} + |\varepsilon^*|\right)^2 \end{aligned} \tag{42}$$

where $\|\bar{\boldsymbol{z}}_v^{**}(k-1)\| \leq \bar{\boldsymbol{z}}_{v,max}^{**}$ and δ_3 to δ_5 are any positive constants. Since the desired ASPR model $G_a^*(z)$ has stable zeros, $G_a^{*-1}(z)$ is stable. Therefore, $\Delta\bar{v}(k)$ is bounded under the input saturation constraint and it follows $|\Delta\bar{v}(k-1)| \leq |\Delta\bar{v}|_{\max}$. Under Assumption 3 and 4, the maximum NN approximation error $|\varepsilon^*|$ is a constant value.

Consequently, considering positive constants δ_1 to δ_5 such that $\bar{\sigma}_\rho^{-1} - \bar{\sigma}_\rho - \delta_1 > 0$, $\bar{\sigma}_W^{-1} - \bar{\sigma}_W - \delta_2 > 0$, $2\rho_z - \delta_3 > 0$, $2\rho_e - \delta_4 - \delta_5 > 0$, it follows that there exist appropriate design parameters σ_ρ, σ_W, ρ_z, ρ_e such that all the signals in the control system are bounded.

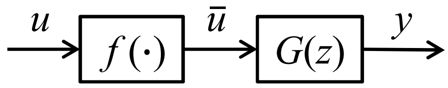

Fig. 3. Hammerstein nonlinear system

4. VALIDATION THROUGH NUMERICAL SIMULATIONS

The effectiveness of the proposed scheme is verified through the following numerical simulations. A controlled object is given by the Hammerstein nonlinear system as depicted in Fig. 3. The linear dynamics system $G(z)$ is given as

$$G(z) = \frac{3.179 \times 10^{-5} z + 3.159 \times 10^{-5}}{z^2 - 1.981z + 0.981},$$

and the static nonlinearity $f(\cdot)$ is given as follows:

$$\bar{u}(k) = u(k) + 0.5u^2(k) + 0.25u^3(k).$$

For this system, the design parameters for the output feedback control system with the adaptive PFC are given by

$$G_a^*(z) = \frac{0.5z}{z - 0.5},\ \theta + \rho_e = 1.0 \times 10^5,\ \rho_z = 1.0 \times 10^4$$
$$\Gamma_\rho = \mathrm{diag}[0.02, 0.02, 0.02, 0.02],\ \sigma_\rho = 1.0^{-4}.$$

The order of the designed PFC is 2. The reference signal $y_r(k)$ which the output of the controlled system is required to follow is given by the output of the following exo-system:

$$\begin{aligned} \omega(k+1) &= \omega(k),\ \omega(0) = 1 \\ y_r(k) &= G_{ref}(z)[\alpha\omega] \\ G_{ref}(z) &= \frac{0.00995}{z - 0.99} \\ \alpha &= \begin{cases} 5 & (100 \leq k < 3100) \\ 10 & (3100 \leq k < 6100) \\ 5 & (6100 \leq k < 7500) \\ 15 & (9100 \leq k < 12000). \end{cases} \end{aligned}$$

The design parameters for the adaptive NN feedforward control system are applied as follows:

$$l = 1,\ \mu_1 = 0,\ \eta_1 = 10,\ \Gamma_W = 4.0 \times 10^2,\ \sigma_W = 1.0 \times 10^{-5}$$
$$v_{\min} = -10,\ v_{\max} = 10.$$

The initial values of the estimated parameters in the proposed control system are set by $\boldsymbol{\rho}(0) = \boldsymbol{0},\ \hat{W}(0) = 0.$

Fig. 4 shows the results with only the output feedback control method with the adaptive PFC. The augmented output tracked the reference signal and the designed adaptive control system with the adaptive PFC could maintain the stability even when the static property of the controlled system changed. Designing the proposed adaptive PFC can reduce the information of the controlled system for designing the control system compared with the conventional PFC design methods which provide the static PFC. Moreover, the proposed method can design the stable control system easily with a simple structure. However, in the control result of the original controlled system, the degeneration of the tracking performance was caused by the PFC.

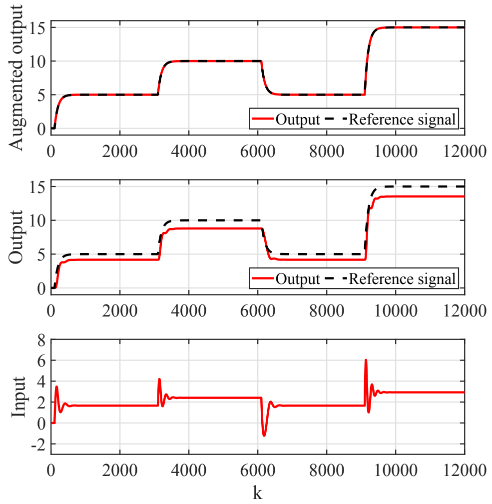

Fig. 4. Simulation results with only feedback control

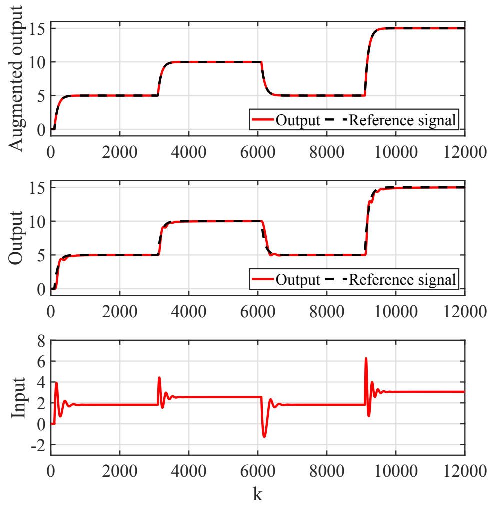

Fig. 5. Simulation results with proposed method

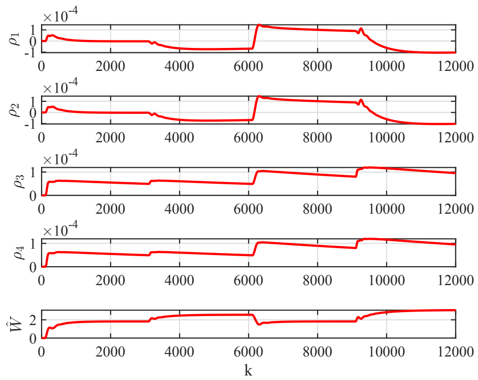

Fig. 6. Ajusted parameters ($\boldsymbol{\rho}(k)$ and $\hat{W}(k)$)

Fig. 5 and Fig. 6 show the results with the proposed two-degree-of-freedom control method. The obtained adaptive control system was stable. Moreover, the output of the original controlled system tracked the reference signal accurately. Therefore, the tracking performance can be improved by designing the proposed adaptive control system with the adaptive NN feedforward control.

5. CONCLUSIONS

An output tracking control system design scheme based on the ASPR based output feedback control with the adaptive PFC was proposed for discrete-time systems. In the proposed method, in order to guarantee the stability of the control system based on the ASPR-ness, the adaptive PFC which has s simpler structure was designed. Moreover, in order to attain output tracking, the two-degree-of-freedom control system with the adaptive feedforward control system based on the RBF NN was designed. The stability of the resulting adaptive control system was analyzed and the effectiveness of the proposed scheme was verified through numerical simulations.

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