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Enhancements of the numerical simulation algorithm for natural gas networks based on node potential analysis

Jens Rüdiger

Technical University of Applied Sciences Wildau, Hochschulring 1
15745 Wildau, Germany (Tel: +49 3375 508 754, e-mail: ruediger@ th-wildau.de).

Abstract: A reliable energy supply for the economy of every country is a matter of national importance. Powerful simulation tools for natural gas networks are essential for operators of gas networks. In this paper, enhancement algorithms of previous developed node potential analysis algorithm are presented. These enhancement algorithms are used for a reasonable setting of initial values in the numerical gas net simulation algorithm. The setting of the initial values has a significant influence on the convergence behavior of the numerical simulation. The presented enhancement algorithms are explained and simulation results are evaluated.

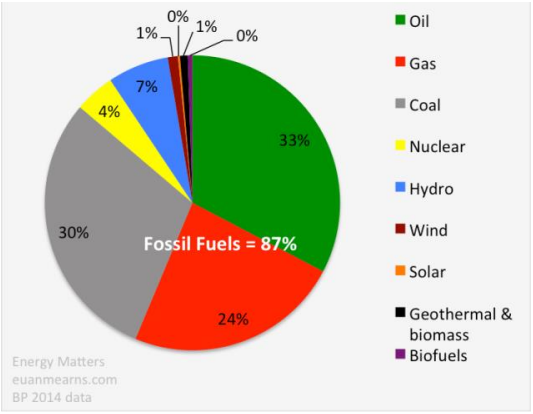
Keywords: energy supply, gas net, pipelines, natural gas, simulation, numerical algorithm

1. INTRODUCTION

A reliable energy supply for the economy of every country is a matter of national importance. Requirements for energy supply systems are efficiency of operation, reliability, cost-efficiency for customers and environmental compatibility. The aspect of sustainability has become a crucial significance in the worldwide discussion of climatic change.

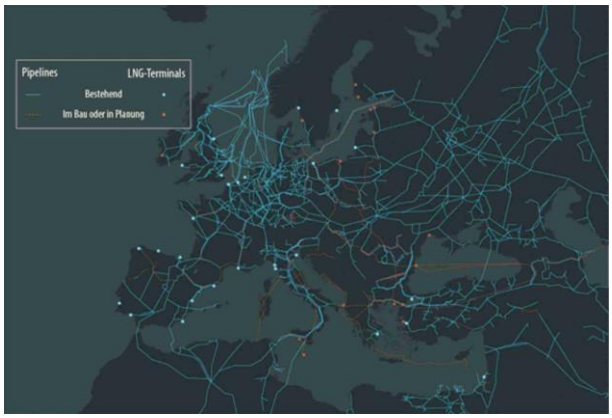
Large natural gas resources, spread over all continents, are still readily available, which makes natural gas to a major energy contributor with about 24% of the overall energy-consumption worldwide (2018) [BP]. In addition, transport of natural gas is easier to perform than other energy carriers such as crude oil. A further important advantage of natural gas is the low carbon dioxide emission of combustion in comparison with other fossil fuels. The energy production by gas generates only 45% of CO2 emission of coal. It is expected that the importance of natural gas will be growing in the next years.

Fig. 1: Global energy consumption 2013 [BP plc]



Natural gas can be delivered as liquefied natural gas (LNG) or by pipeline systems. Germany is supplied by natural gas from Siberia in Russia or the North Sea. Currently Germany has no LNG terminal. Therefore the gas transport is carried out by pipeline systems with up to transcontinental dimensions and under participation of many countries. This complicates the gas grid management tremendously. It makes extended requirements of stability and abilities to short term gas storage necessary. In order to cope with this present development, efficient network simulation tools will be needed.

Fig. 2: Gas pipelines in Europe 2017 (red: in construction) [zukunft erdgas]



The currently most used algorithms in simulation tools are algorithms based on Hardy-Cross or Mc Ilroy [Cerbe], [Eberhard], [Horlacher]. However they do not meet the above mentioned extended new requirements on modern network simulation, because they were not originally developed for these applications.

For this reason, the author had developed a new algorithm for gas net simulation based on node potential analysis. This work was published and presented in previous papers [Rüdiger, 2006]. Subsequently the algorithm was extended to simulate gas with real gas behaviour [Rüdiger, 2011]. The extension had been made by embedding GERG-88-Equations into the algorithm. It could be shown that this simulation

algorithm can generate results that are very close to the reality. These topics are only briefly explained in this paper to understand the extensions of the algorithm.

In this paper the focus is on newly developed extensions of the node potential algorithm by algorithm to generate the initial values. These different methods are explained and investigated by simulation with a reference network [Cerbe]. Further simulations benchmark these methods by increasing complexity of nets.

2. THE BASICS OF FLUID DYNAMICS

This chapter gives a glance at the basics of fluid dynamic to understand the algorithm explained in the next chapter. The continuity equation (1) describes the conservation of mass in a flow:

$$\left(\frac{\partial(\rho \cdot A)}{\partial t}\right)_x + \left(\frac{\partial(\rho \cdot A \cdot v)}{\partial x}\right)_t = 0 \quad (1)$$

In a stationary flow eq. (1) can be simplified to:

$$\dot{m} = \rho \cdot A \cdot v = const. \quad (2)$$

Equation (2) is the continuity equation. It describes the conservation of mass flow. By a constant density of the fluid $\rho = const.$ follows the flow-through equation (3):

$$\dot{V} = A \cdot v = const. \quad (3)$$

From the energy conservation law can be derived the Bernoulli Equation (4). It describes the pressure conditions in a flow.

$$p_1 + \varrho \cdot g \cdot h_1 \cdot v_1^2 = p_2 + \varrho \cdot g \cdot h_2 \cdot v_2^2 = const. \quad (4)$$

This equation only applies in a flow without friction. However, in a real flow, the friction has to be considered as it causes pressure loss.

The pressure loss Δp is the main objective of the gas net simulation. In a pipe it is determined by the Darcy Equation (5):

$$\Delta p = \lambda \cdot \frac{l}{d} \cdot \frac{\rho}{2} \cdot v^2 = \lambda \cdot \frac{8 \cdot \rho}{\pi^2} \cdot \frac{l}{d^5} \cdot \dot{V}^2 \quad (5)$$

Parameter λ in (5) is the resistance coefficient. The accurate determination of it is a major issue in fluid dynamic. The parameter λ depends on the kind of flow as well as the geometrical dimensions of the pipe. Generally, for the determination of λ has to be distinguished between laminar and turbulent flow. The model equations (6)-(11) in Table 1, describe the different states of flow.

All equations in Table 1 are nonlinear equations. Equations no. 9 and 11 cannot be solved further in an explicit way. In the simulation algorithm, this equations are solved by the use of iterative methods. The dependency of λ on the kind of flow is illustrated in the $Re - \lambda$ chart in Figure 3.

A characteristic to specify the present state of fluid flow and consequently the appropriate model equation out of (6) – (11) leads to the Reynold's number Re:

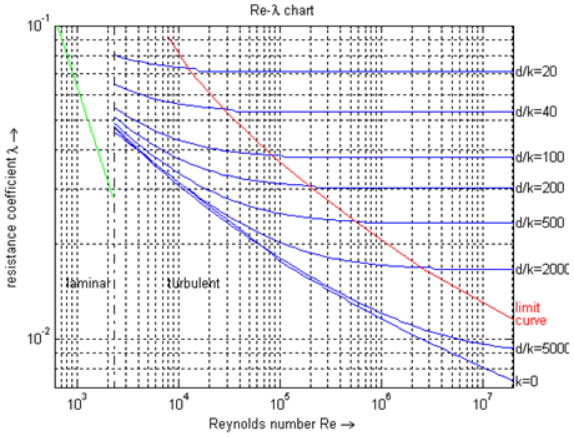
$$Re = \frac{v \cdot d}{\nu} = \frac{4 \cdot \dot{V}}{\pi \cdot d \cdot \nu} \quad (12)$$

The below described transition from laminar to turbulent flow begins at $Re \geq 2320$.

Table 1: Model equations for the determination of λ

	Equation	name	*Re*	
6	$\lambda = \frac{64}{Re}$	Hagen-Poisseuill	<2320	
7	$\lambda = 0.3164 \cdot Re^{-0.25}$	Blasius	2320 …10^5	k=0
8	$\lambda = 0.0032 + 0.221 \cdot Re^{-0.237}$	Nikuradse	10^5 … 2.5· 10^6	k=0
9	$\frac{1}{\sqrt{\lambda}} = 2 \cdot \lg\left(Re\sqrt{\lambda}\right) - 0.8$	Prandtl u. Karman	>2.5· 10^6	k=0
10	$\frac{1}{\sqrt{\lambda}} = 2 \cdot \lg\frac{d}{k} + 1.14$	Nikuradse	$> \frac{200d}{\sqrt{\lambda}\,k}$	rough
11	$\frac{1}{\sqrt{\lambda}} = -2 \cdot \lg\left(\frac{2.51}{Re\sqrt{\lambda}} + 0.269\frac{k}{d}\right)$	Prandtl u. Colebrook	$< \frac{200d}{\sqrt{\lambda}\,k}$	smooth - rough

Fig. 3. Re-λ chart



3. THE POTENTIAL ANALYSIS SIMULATION ALGORITHM

The basic idea for the development of an algorithm for gas network simulation is to use the principles of node potential algorithm in electrical engineering. This is due the non-linearities in fluid engineering only rudimentary possible.

In the next chapter the node potential algorithm in electrical engineering and the development of algorithm is briefly explained [Rüdiger 2006]. The analogy and the differences to fluid dynamics and the implementation of the algorithm are illustrated in the subsequent chapters.

3.1 The node potential analysis in electrical engineering

The requirement to every network calculation algorithm is to set up a system of independent equations for the demanded variables. In the node analysis a reference node with a fixed electrical potential is defined. Based on this the node potential voltages to every other node of the network (DC) are determined:

$$U_{diff} = U_{node} - U_0 \tag{13}$$

by the reference potential $U_0 = 0$ hence:

$$U_{diff} = U_{node} \tag{14}$$

By these node potentials (n-1) node equations can be set up. The currents in the node equation are the currents of the input sources and the currents caused by the potential difference between the several nodes connected by conductances. These equations can be expressed as a vector equation (15):

$$\overline{G} \cdot \overline{U} = \overline{I} \qquad \text{with:} \tag{15}$$

$\overline{G}$: matrix of conductance

$\overline{U}$: vector of node potentials (demanded variables)

$\overline{I}$: vector of input currents (input sources)

The elements of $\overline{G}$ can easily be determined from the conductances g of the network according to the scheme:

$$\overline{G}_{ii} = \sum_{1}^{j} g_{ij} \qquad \forall i, j \wedge i \neq j \tag{16}$$

$$\overline{G}_{ij} = -g_{ij} \tag{17}$$

By resolving the node equation (15) the vector of the node potentials can be achieved. Finally, the demanded currents and voltages in the branches of the network can be calculated by the obtained node potentials.

This algorithm can be extended for all kinds of input sources, i.e. ideal or real voltage- and current sources. This is with focus on gas net simulation a very important issue.

3.2 Analogies and differences between electrical engineering and flow dynamics

The development of the algorithm is based on the analogy to electrical engineering. Thereby following analogies for the pressure loss and the flow rate are used:

electrical engineering	↔	flow dynamics
U	↔	Δp
I	↔	$\dot{V}$

(18)

The pressure loss is determined by the Darcy Equation (5) and corresponds to the Ohm's law:

$$U = R \cdot I \quad \leftrightarrow \quad \Delta p = \lambda \cdot \frac{8 \cdot \rho}{\pi^2} \cdot \frac{l}{d^5} \cdot \dot{V}^2 \tag{19}$$

In the equations above a major difference has been revealed. In Ohm's law the voltage drop has a linear dependence from the current. In opposition to that, the pressure loss in a pipe shows a nonlinear, quadratic dependence on the flow:

$$\Delta p = f(\dot{V}^2) \tag{20}$$

This first kind of nonlinearity is the main challenge for the development of the algorithm, explained in the next subsection. The second nonlinearity appears in the resistance:

$$R \quad \leftrightarrow \quad R^* = \lambda \cdot \frac{8 \cdot \rho}{\pi^2 \cdot d^5} \tag{21}$$

The resistance coefficient λ in (21) is also a nonlinear dependency, as explained in section 2. It has to be determined by numerical methods.

$$\lambda = f(\dot{V}, k, l, d) \tag{22}$$

3.3 Derivation of the node potential algorithm in flow dynamics

The assumption for the application of the node analysis of electrical engineering is linear behaviour. The challenge for the derivation of the algorithm in flow dynamics is to find a way to resolve the $\dot{V}^2$. The basic approach is to split $\dot{V}^2$ in (5) and allocate one $\dot{V}$ to the resistance R.

$$\Delta p = \lambda \cdot \frac{8 \cdot \rho}{\pi^2} \cdot \frac{l}{d^5} \cdot \dot{V}^2$$

$$\Downarrow \qquad \Delta p = R \cdot \dot{V}$$

$$\Delta p = \underbrace{\lambda \cdot \frac{8 \cdot \rho}{\pi^2} \cdot \frac{l}{d^5} \cdot \dot{V}}_{R} \; \dot{V}$$

$$R = \lambda \cdot \frac{8 \cdot \rho}{\pi^2} \cdot \frac{l}{d^5} \cdot \dot{V} \tag{23}$$

Thus the equation of flow dynamics has been changed in an equivalent form of Ohm's law in electrical engineering. The consequence however is the dependency of the resistance R from the flow. In the iteration this is the flow $\dot{V}_{t-1}$ from the preceding iteration step. So it can be formulated the iteration rule for the algorithm:

$$\Delta p_{(t)} = \lambda_{(t-1)} \cdot \frac{8 \cdot \rho \cdot l}{\pi^2 \cdot d^5} \cdot \dot{V}_{(t-1)} \cdot \dot{V}_{(t)} \tag{24}$$

By this iteration rule the node potential algorithm for gas networks can be developed. First the conductance g for the separate branches of the network is needed. It is the reciprocal of the resistance R.

$$g = \frac{1}{R} \tag{25}$$

As above mentioned, g are necessary to generate the matrix $\overline{G}$. By matrix $\overline{G}$ and the vector of input flows $\dot{V}_{Input}$ the potential pressures for every node i can be calculated.

$$\overline{p}_{i0,(t)} = \overline{G}^{-1}_{(t-1)} \cdot \overline{\dot{V}}_{Input} \tag{26}$$

The wanted pressure loss $\Delta p_{ij,(t)}$ between the nodes i and j can be determined by the potential pressures.

$$\Delta p_{ij,(t)} = p_{i0,(t)} - p_{j0,(t)} \tag{27}$$

Out of it the volume flow in the branch can be calculated.

$$\dot{V}_{ij,(t)} = \sqrt{\left| \frac{\pi^2 \cdot d^5}{\lambda_{ij,(t-1)} \cdot 8 \cdot l \cdot \rho} \cdot \Delta p_{ij,(t)} \right|} \tag{28}$$

These iterations have to be done until the iteration limit is fulfilled. The reasonable setting of the initial values is the objective of the next chapter. The extension of the algorithm to real gas behaviour is in this paper completely omitted. It is referred to the elaborate explanation of it in [Rüdiger 2011]

4. DEVELOPMENT OF ALGORITHMS FOR INITIAL VALUES GENERATION

The setting of the initial values is essential for solving numerical algorithm. A reasonable setting can minimize the

number of iteration steps and thereby minimize the processing time. This aspect is for simulation of gas nets with a great number of nodes crucial.
The first method is the **maximum method**. By this method the gas flow of all input and output nodes are scanned for the maximum flow. This maximum flow is assumed as initial flow for all connections in the network. The assumption is that a flow in a pipe connection cannot be higher than the maximum input.

$$\forall\, k\colon\ \dot{V}_{k\,(0)} = max\{\dot{V}_{in}\} \tag{29}$$

However this method assumes the maximum flow in all pipe connections which is impossible. This method does not consider the topology of the network. For this reason the method has been refined to include the topology of the net in the determination of initial values.
The next simulations have been done by arbitrarily chosen initial values $\dot{V}_{k\,(0)}$ which were only limited by the maximum input flow (**random method**).

$$rand\left\{0 < \dot{V}_{k\,(0)} < \max\{\dot{V}_{in}\}\right\} \tag{30}$$

The results were not satisfactory. Consequently it has been developed further methods to determine the initial values systematically.
The first approach to consider the topology has been done by dividing the input flow by the number of pipe connections m (**number of pipes method**).

$$\dot{V}_{k\,(0)} = \frac{max\{\dot{V}_{in}\}}{m} \tag{31}$$

Simulation results (chapter 5) have shown that this method brings only a small or no improvement. It depends on the topology of the net. This method has no advantage if the network contains stub connections. Stub connections are connections which are not included in meshes. For example the connections 1-2 or 12-14 in the reference net [Fig. 4].

The next developed method (**number of branches method**) has tried to consider the topology of the net. To set the initial values the maximum input flow has been divided by the half of the number of branches z. The half number is applied because it is assumed that in the net every branch has an opposite branch where the two pipes come together.

$$\dot{V}_{k\,(0)} = \frac{max\{\dot{V}_{in}\}}{z/2} \tag{32}$$

In the reference net the number of branches is $z = 5$. But there is also one brunch which leads into a stub connection. For this reason the assumption is not absolutely exact.

Lastly a method which assumes the flow in all connections as a laminar flow has been developed. This **laminar flow method** is an approach from considerations of fluid mechanics and not from the topology of nets. Laminar flow is easier to calculate as turbulent flow. In recent publications have been shown that the operation point of real gas nets is in most cases close to the laminar flow [Ruediger 2011].
By means of the formulas for determining the friction coefficient λ for laminar flow (equation (6)(, the definition of Reynoldsnumber Re (equation (12)) and the law for steady flow the friction coefficient λ can be determined:

$$\lambda = \frac{16\,\pi \cdot d \cdot \nu}{\dot{V}} \tag{33}$$

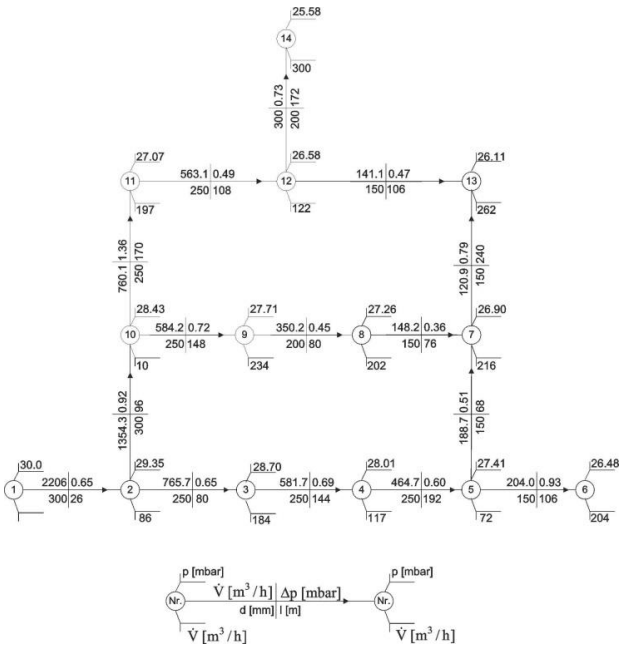
By this λ the conductance g can be calculated. The flow $\dot{V}$ could be eliminated in this equation in the same way like in the explanation of node potential algorithm above.

$$g = \frac{1}{\lambda} \cdot \frac{\pi^2 \cdot d^5}{8\,l \cdot \rho \cdot \dot{V}} = \frac{\pi \cdot d^4}{128\,l \cdot \nu \cdot \rho} \tag{34}$$

5. SIMULATION RESULTS

The simulations shall investigate the influence of different initial values at the performance of the algorithm. The simulations have been carried out in the same order like the different methods were explained in chapter 4.
The gas net used for simulation is an example taken from [Cerbe]. The input pressure in the simulations is 30 mbar. This is the pressure commonly used in supply nets.
The simulation shows respectively in the first figures the mean of relative change of flow and the number of used iteration steps. In the second figures the change of flow in the first 10 pipe connections is illustrated by different colors for every iteration step.
The consideration of single pipe connection makes the final evaluation of the different methods easier. The reasons for the different convergence behaviour can be more clearly seen by the flow in the single pipes.

Fig 4.: simulation network [Cerbe]



- maximum method:

Fig. 5: Simulation with maximum method: flow change and number of iterations

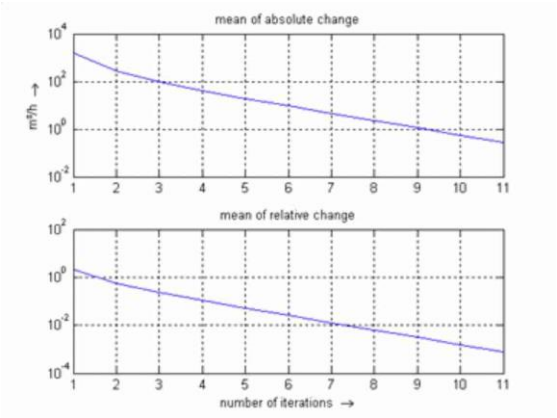
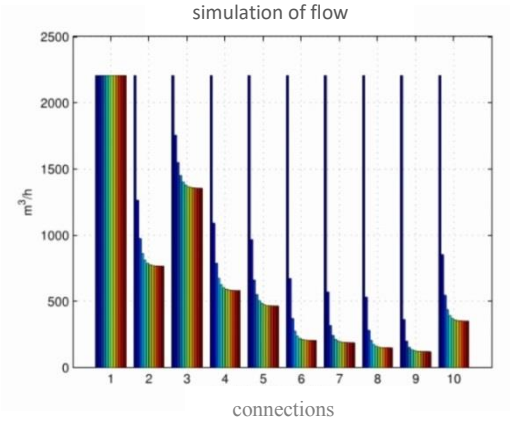


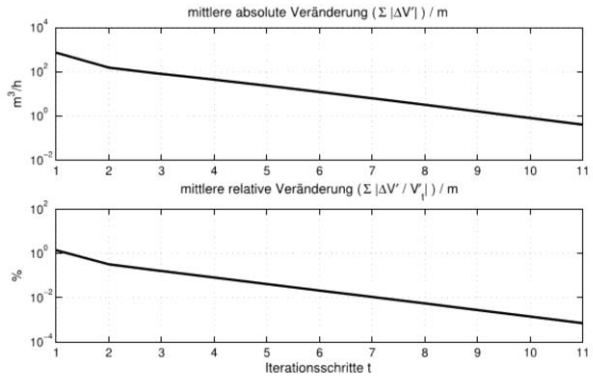
Fig. 6: Simulation with maximum method: flow in pipe connections 1-10



The convergence behaviour by the maximum method is very poor. It takes even for this very simple reference net 11 iteration step (Fig. 5). In figure 6 with the single connections can be seen that apart from the first connection in all other connections the initial value is far away from the final value. This method makes only sense in nets with less meshes.

- random method

Fig. 7: Simulation with random method: flow change and number of iterations

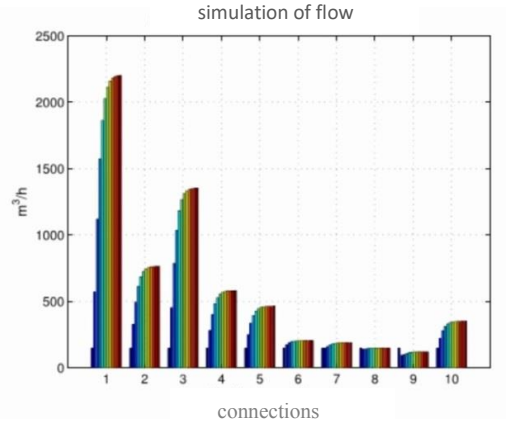


The random method is the easiest to perform method but the result is with 11 iteration steps also very poorly.

- number of pipes method

This method takes also 11 iteration steps and shows no progress in comparison to the two other methods. It can be seen that the initials value for the first connections is far too low. The gas flow in these connections is still not split enough.

Fig. 8: Simulation with number of pipes method: flow in pipe connections 1-10



- number of branches method:

Fig. 9: Simulation with number of branches method: flow change and number of iterations

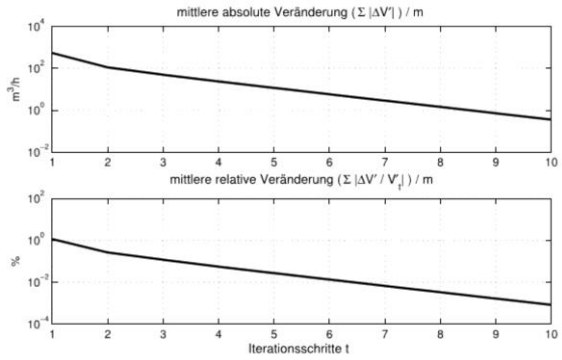
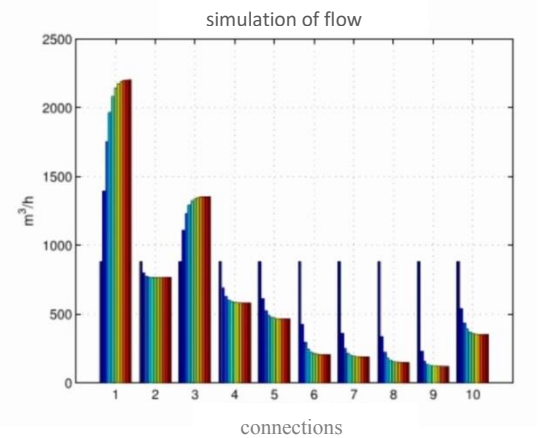


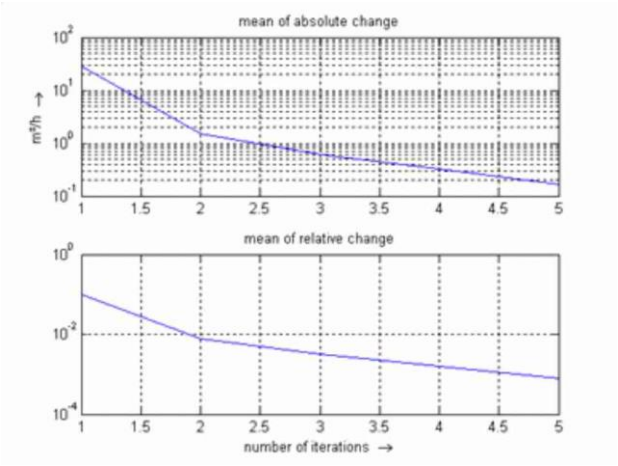
Fig. 10: Simulation with number of branches method: flow in pipe connections 1-10



The number of branches method shows with 10 iteration steps a very small improvement. This method can be applied for simulation of nets with many meshes.

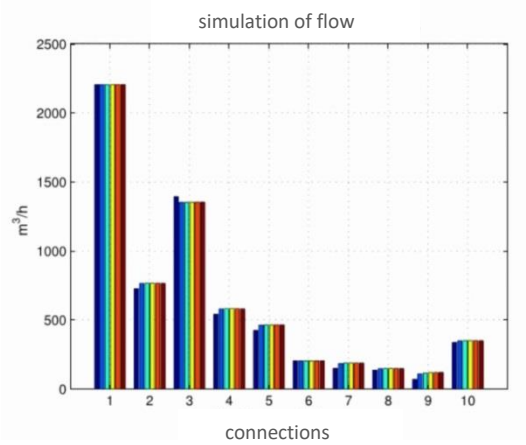
- laminar flow method:

Fig. 11: Simulation with laminar flow method



The laminar flow method shows with only 5 iteration steps the best performance of all methods. It shows the best performance in the simulation of real networks.

Fig. 12: Simulation with laminar flow method: flow in pipe connections 1-10



The performance of the algorithm can not only investigated by the setting of initial values. Furthermore these investigations have to be seen in connection with the complexity of the network. In the last part the results of the simulation in dependency of number of nodes and initial values shall be illustrated.
The first figure shows the number of nodes versus the number of iteration steps with different set initial values.
It is remarkable that the number of iterations by a strongly increasing number of nodes is relatively constant.

In contrast the simulation of processing time increases by the number of nodes. In both simulations the setting of initial values by the laminar flow method shows much better convergence behaviour.

Fig. 13: number of iterations with laminar (- - -) and max. (—) initial values versus node numbers

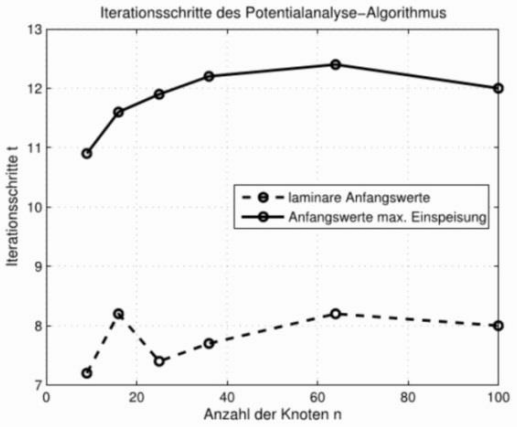
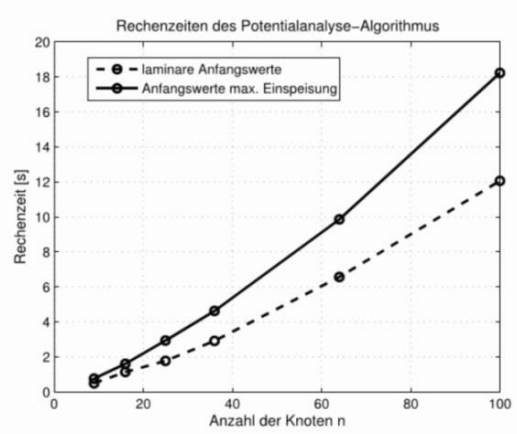


Fig. 14: processing time with laminar (- - -) and max. (—) initial values versus node numbers



6. CONCLUSIONS

In this paper enhancements of the developed node potential algorithm with initial value settings methods have been explained. The developed setting methods have been investigated and evaluated. Furthermore the influence of initial values and the complexity of networks with higher number of node have been investigated.
Resulting of these simulations the algorithm with laminar flow settings has proofed to be the method with best convergence characteristic. Future work could be focused on modelling underground gas storage.

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