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Synthesis of an Attitude Control System for Unmanned Underwater Vehicle Using H-infinity Approach

Gavrilina E. A.*, Chestnov V. N.**

*Bauman Moscow State Technical University, Moscow, Russia
(e-mail: ekaterina.a.gavrilina@gmail.com).
** V. A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences,
Moscow, Russia, (e-mail: vnchest@yandex.ru)

Abstract: Recently problems requiring control unmanned underwater vehicles (UUV) at large angles of inclination (pitch and roll), become more frequent. Traditional attitude control systems use Euler angles. However, the performance of traditional systems decreases with the increasing of the tilt angles, which delays their use for new tasks. To solve this problem, stability analysis of the UUV's attitude control system according to the generalized Nyquist stability criterion is carried out. The analyses showed that the stability of the system depends on the UUV inclination along the roll. However, at large angles of inclination, the roll channel is subject to perturbations from the yaw and pitch channels. The roll control system synthesis is solved as the H_∞ - optimization problem with the requirements of low sensitivity to perturbations from other channels. The simulation results on the full non-linear UUV Aqua-MO model confirmed the efficiency of the approach in question and demonstrated the best quality in comparison with PD controller. The obtained stability condition and synthesis approach allow to expand the working angles and improve the quality of the existing UUV control systems. These results are useful for the development of new systems as well.

Keywords: Autonomous underwater vehicles, remotely operated vehicles, robust controller synthesis, attitude control, Euler angles, linear multivariable systems, H-infinity control, decoupling problems.

1. INTRODUCTION

Traditionally unmanned underwater vehicles (UUV) operate at small angles of inclination (pitch and roll). However, the UUV application area is expanding, which means that new tasks and requirements appear. For example, during mine countermeasures, (Reed et al. (2010)), remotely operated vehicle (ROV) works with objects located on the bottom, walls, access to which may be difficult due to the geography features, the presence of underwater structures. To complete the tasks, it is necessary to work from a close distance, which is possible with a large ROV inclination by the pitch.

Another example is the docking of a hybrid ROV with the ship hull or another working surface. Hybrid ROV are effective in solving problems of failure detection, surface preparation in underwater conditions, see Serebrenniy et al. (2019), Hayato and Toshikazu (2017). Their propulsion system allows to control the movement in the water due to thrusters, and the movement on the working surface due to the additional propulsion system (tracks, wheels). One of the stages of the operation of such ROV is docking with the working surface, for which the ROV is rotated through a large angle of pitch or roll. Figure 1 illustrates the docking process on the example of the ROV "Iznos", developed at Bauman Moscow State Technical University (BMSTU) see Gladkova et al. (2020) and Gavrilina et al. (2019).

It is clear that directional maneuverability in the entire range of angles is necessary for UUVs that conduct surveys of enclosed spaces (caves, tunnels, ports). For example, autonomous underwater vehicle (AUV) UX-1 (see Suarez Fernandez et al. (2019)) was designed for exploration and mapping tunnels of underground flooded mines. Control with a large pitch angle is necessary for movement along vertical tunnels. AUV SUNFISH (see Richmond et al. (2018)) was designed for exploration and mapping of complex 3D spaces. Controllability over the entire range of tilt angles is necessary for maneuvering in confined spaces, and is also used to increase the efficiency of the underwater sonar SLAM systems, since the AUV can freely aim sensors in different orientations. Other examples of UUVs, controlled at large pitch and roll angles may be found in Sakagami et al. (2011), Ferreira et al. (2012), Tolstonogov et al (2019).

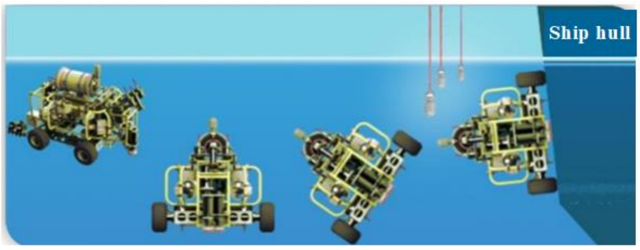


Fig. 1 Docking process of ROV Iznos

Solving these tasks with traditional UUV (with small angles of inclination) though will necessitate the following:

- to equip the UUVs with rotary devices for cameras or to duplicate sensors;

- to use more complex manipulators or duplicate them;
- to reduce the UUV's size.

Duplication of devices, the complication of the manipulators' design will lead to an increase in the UUV cost, an increase in its dimensions. The reduction in size for increasing in maneuverability complicates the design, reduces the size of the AUV batteries. At the same time, controllability in the entire range of attitude angles provides additional advantages: it allows to operate in confined spaces, improves maneuverability, allows to orient sensors, tools freely, expands the working area of the manipulator and allows to perform contact operations with objects located at an angle. Thus, the most effective solution is the development of UUVs, designed to be controlled over the entire range of attitude angles.

However, the development of such UUVs requires the use of control systems that are operable in the entire range of orientation angles. At the moment, the issue of constructing an attitude control system of UUVs is not sufficiently developed. In the works of Fjellstad and Fossen (1994), Antonelli (2007), quaternion-based control systems are constructed. At the same time, the modes of UUV movements at large angles of inclination (for example, yaw rotations at pitch angles of more than 45°) are not studied in the works; the dynamics of the propulsors is not taken into account during the analysis. The development of a control system for large angles and its field tests were reviewed by Suarez Fernandez et al. (2019) and Sakagami et al. (2011). However, passive pitch stabilization was used to control the UUV's incline. In this paper, we consider motion control only at the expense of UUV's thrusters.

Nowadays, the issue of the traditional attitude control systems (using Euler angles) operability is not sufficiently studied at large angles of inclination despite the solution to this problem is practically significant, as it will allow to use the accumulated experience and expand the working angles of the existing UUV control systems of the legal acts.

The main reason for refusing to use Euler angles is the presence of a singular point (pitch ± 90 °), in which the degeneracy of kinematic equations takes place and the problem of attitude description ambiguity appears (the yaw is indistinguishable from roll) see Goldstain (1959). However, algorithms that can eliminate disadvantages of Euler angles have been developed: Ozgoren (2019), Singla et al. (2005). In addition the problem of synthesizing an attitude control system is considered for AUV tilting along the pitch by ±90° in the work of Ferreira et al. (2012).

The operation of a traditional control system at large angles of inclination was investigated in Gavrilina et al. (2019). It is shown in the work that the degree of connectivity between the yaw, pitch, and roll channels increases with an increase in the tilt angles. The dynamic errors appear at cooperation. In this case, the roll channel is most susceptible to influence from other channels (dynamic error reaches 50°). To solve the problem, a decomposition algorithm is proposed that improves the control system's quality. It was confirmed by experiments on real UUVs (dynamic error is reduced to 5°). However, the system stability was not investigated in the work, and during the synthesis of the roll control channel, it was not taken into account that it was subjected to perturbations from other channels.

In other words, Euler angles have limitations and quaternions are used in the development of attitude control systems. Since the great number of existing UUV control systems are based on Euler angles, methods eliminating their limitations have been developed. Therefore, the development of attitude control systems based on Euler angles for UUVs operating at large angles of inclination is practically significant and yet this issue has not been studied in sufficient detail. There are no stability analyses of the system at large inclinations and an approach to synthesis that takes into account perturbations between channels.

This paper studies the stability of the original and decomposed systems. The stability study is conducted using the generalized Nyquist criterion. The obtained stability condition depends on the UUV inclination along the roll, while the stability of the roll control channel does not depend on changes in other orientation angles. The synthesis is considered as an H_∞- optimization task, designed so that the resulting control system has low sensitivity to perturbations from the yaw and pitch channels and has sufficient stability margins. For practical reasons the recommendations for simplifying the resulted controller are also given in the paper. Verification of the results is carried out by mathematical modeling methods on a full non-linear model of the "Aqua MO" ROV. The results of the algorithm operation are compared with the PD controller.

The work is structured as follows. In Section 2, the equations of the mathematical model of the UUV are given. In Section 3, a study of the stability of traditional attitude control system is conducted. In Section 4 the problem of synthesis of roll angle control system using the H_∞-approach is solved. The results of an experimental study of the obtained approach to constructing the attitude control system on the full nonlinear mathematical model of the UUV are presented in Section 5.

2. MATHEMATICAL MODEL

The mathematical model should qualitatively indicate the UUV features as a controlled object and remain applicable for analytical studies. In this paper we use for modeling the full nonlinear UUV model based on Fossen (2002). The non-linear model is linearized for the worst conditions in terms of stability and is used for synthesis and analysis. Mathematical model includes a kinematic model, a dynamic model and a propulsion model.

2.1. Coordinate System and Kinematics of UUV Motion

Two coordinate systems have been considered: body-fixed frame $Oxyz$ and intermediate frame $Ox_gy_gz_g$. Vertex O is aligned with the UUV (pole) centre of mass. Intermediate frame axis is directed parallel to the North-East-Down (NED) coordinate system. Axis Ox_g is directed to the north, Oy_g - to the east and Oz_g – downwards normal to the Earth's surface. Body-fixed frame Ox axis is directed from aft to fore, Oy axis is directed to the starboard, Oz axis is directed from the top to bottom. $Oxyz$ orientation regardless $Ox_g y_g z_g$ is described by sequential rotations at yaw ψ, pitch θ and roll ϕ angles respectively.

Kinematic equations are described by Euler equations:

$$\dot{\eta} = P(\theta, \phi)\nu, \theta \neq \pm 90^0 \quad (1)$$

where $P(\theta, \phi) = \begin{bmatrix} 1 & tan(\theta)\sin(\phi) & tan(\theta)\cos(\phi) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{bmatrix}$,

$\dot{\eta} = [\,\dot{\phi}\;\dot{\theta}\;\dot{\psi}\,]^T$ – vector of roll, pitch and yaw angular velocity respectively, $\nu = [\omega_x\;\omega_y\;\omega_z]^T$ – vector of UUV angular velocity about the axes Ox, Oy, Oz.

2.2. Dynamics of UUV Motion

The UUV dynamics model is considered for low speeds. A complete non-linear model of a UUV dynamics is presented in Fossen (2002) and has the following form:

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau, \quad (2)$$

where M - a UUV and added inertia matrix in the form:

$$M = diag\{I_x - K_{\dot{\omega}_x}, I_y - M_{\dot{\omega}_y}, I_z - N_{\dot{\omega}_z}\},$$

- I_x, I_y, I_z – UUV moments of inertia and $K_{\dot{\omega}_x}, M_{\dot{\omega}_y}, N_{\dot{\omega}_z}$ – added inertia moments about Ox, Oy, Oz axes respectively;

- $C(\nu)$ – centripetal and Coriolis terms matrix (for UUV and added inertia):

$$C(\nu) = \begin{bmatrix} 0 & (I_z - N_{\dot{\omega}_z})\omega_z & -\left(I_y - M_{\dot{\omega}_y}\right)\omega_y \\ -(I_z - N_{\dot{\omega}_z})\omega_z & 0 & (I_x - K_{\dot{\omega}_x})\omega_x \\ \left(I_y - M_{\dot{\omega}_y}\right)\omega_y & -(I_x - K_{\dot{\omega}_x})\omega_x & 0 \end{bmatrix}.$$

- $D(\nu)$- a matrix of hydrodynamic damping:

$$D(\nu) = -diag\left\{ \mathrm{K}_{\omega_x}, \mathrm{M}_{\omega_y}, \mathrm{N}_{\omega_z} \right\} -$$
$$-diag\left\{ \mathrm{K}_{\omega x|\omega_x|}|\omega_x|, \mathrm{M}_{\omega_y|\omega_y|}|\omega_y|, \mathrm{N}_{\omega_z|\omega_z|}|\omega_z| \right\},$$

where $\mathrm{K}_{\omega_x}, \mathrm{M}_{\omega_y}, \mathrm{N}_{\omega_z}, \mathrm{K}_{\omega_x|\omega_x|}, \mathrm{M}_{\omega_y|\omega_y|}, \mathrm{N}_{\omega_z|\omega_z|}$ – a UUV hydrodynamic damping coefficients;

- $g(\eta)$ – vector of hydrostatic moments acting on UUV. A mandatory requirement for the considered UUVs is minimization of metacentric height. For this reason, to simplify the UUV mathematical model, it is assumed that the UUV center of mass coincides with the center of buoyancy and moments from hydrostatic forces do not affect the UUV;

- $\tau = [\tau_x\;\tau_y\;\tau_z]^T$ – moment of force vector created by a propulsion system.

The linearization of equation (2) is held for the worst case in terms of stability (for $\omega_x{}^* = \omega_y{}^* = \omega_z{}^* = 0$). The obtained equations are transformed according to Laplace and the following transfer functions of a UUV rotational motion are obtained:

$$W_{UUVi} = \frac{\omega_i(p)}{\tau_i(p)} = \frac{K_{UUVi}}{T_{UUVi}p+1} \quad (3)$$

where $T_{\mathrm{UUVi}} = \frac{\mathrm{I_i} - A_{\dot{\omega}_i}}{-\mathrm{A}_{\omega_i} - 2\mathrm{A}_{|\omega_i|\omega_i}\omega_i{}^*}$, $K_{\mathrm{UUV_i}} = \frac{1}{-\mathrm{A}_{\omega_i} - 2\mathrm{A}_{|\omega_i|\omega_i}\omega_i{}^*}$,

$i = x, y, z$; $\mathrm{A} = \mathrm{K}, \mathrm{M}, \mathrm{N}$ respectively; $\omega_i{}^*$- linearization parameter; p – Laplace parameter.

2.3. A Propulsion Model

A propulsion model is considered for the case when a UUV is in a mooring mode. In accordance with the study Egorov (2002) dynamics of the propulsion could be described by a first-order aperiodic link:

$$W_{Pi} = \frac{\tau_i(p)}{U_i(p)} = \frac{K_{Pi}}{T_{Pi}p+1}, \mathrm{i} = \mathrm{x, y, z} \quad (4)$$

where T_{Pi}, K_{Pi} – time constant and propulsion gain, that control movement on an i-th channel, respectively, U_i – voltage applied to propulsion. In accordance with equations (3), (4) for further calculations UUV mathematical model in body-fixed frame could be written in a following form:

$$W_i = \frac{\omega_i(p)}{U_i(p)} = W_{Pi}(p)\mathrm{W}_{UUV\mathrm{i}}(p), \mathrm{i} = \mathrm{x, y, z}. \quad (5)$$

3. STABILITY ANALYSIS

3.1. Traditional Approach to Attitude Control of the UUV

Figure 2 shows the block diagram of the conventional UUV attitude control system.

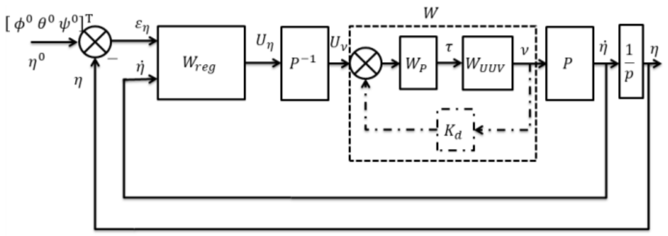


Fig. 2. The block diagram of the traditional attitude control system of the UUV

The mismatch between the desired and current values in the roll, pitch and yaw, given by the vector $\varepsilon_\eta = [\,\varepsilon_\phi\;\varepsilon_\theta\;\varepsilon_\psi]^T$, and the current angular velocities $\dot{\eta} = [\dot{\phi}\;\dot{\theta}\;\dot{\psi}]^T$ go to the controllers of the separate control channels of roll, pitch and yaw of the UUV represented by the W_{reg} matrix. The generated control signals of the yaw, pitch and roll control channels $U_\eta = [\,U_\phi\;U_\theta\;U_\psi]^T$ enter the matrix P^{-1} and are converted into control signals $\mathrm{U}_\nu = [U_x\;U_y\;U_z]^T$ relative to the coordinate system associated with the UUV. Then they are fed to the local control loop for the angular velocity or fed to the propulsion system of the UUV.

The control action, in accordance with control signals are generated for the propulsion system, is the following:

$$\begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} = P^{-1} \begin{bmatrix} W_{reg\phi 1}\varepsilon_\phi - W_{reg\phi 2}\dot{\phi} \\ W_{reg\theta 1}\varepsilon_\theta - W_{reg\theta 2}\dot{\theta} \\ W_{reg\psi 1}\varepsilon_\psi - W_{reg\psi 2}\dot{\psi} \end{bmatrix} - \begin{bmatrix} K_{dx}\omega_x \\ K_{dy}\omega_y \\ K_{dz}\omega_z \end{bmatrix}, \quad (6)$$

where $P^{-1} = \begin{bmatrix} 1 & 0 & -sin(\theta) \\ 0 & cos(\phi) & sin(\phi)\,cos(\theta) \\ 0 & -\sin(\phi) & cos(\phi)\,cos(\theta) \end{bmatrix}$.

There are two ways for constructing a UUV attitude control system: in the first case, damping velocity based control is carried out according to the angular velocity vector $\dot{\eta}$, i.e. $W_{reg\eta 2} \neq 0, K_{dv} = 0$. In the second case, damping velocity based control is carried out due to the angular velocity vector ν, i.e. $K_{dv} \neq 0, W_{reg\eta 2} = 0$. In Gavrilina et al. (2019), an open-loop transfer matrix was obtained for the second case:

$$\eta = \frac{1}{\mathrm{p}} W(p)\varepsilon_\eta \tag{7}$$

where

$$W(p) = \begin{bmatrix} W_{reg\phi}W_x & \frac{1}{2}\mathrm{s}(2\phi)\tan\theta W_{reg\theta}\left(W_y - W_z\right) & \mathrm{s}(\theta)\,W_{reg\psi}\left(W_y\,\mathrm{s}^2(\phi) + W_z\,\mathrm{c}^2(\phi) - W_x\right) \\ 0 & W_{reg\theta}\left(W_z\,\mathrm{s}^2(\phi) + W_y\,\mathrm{c}^2(\phi)\right) & \frac{1}{2}\mathrm{s}(2\phi)\,\mathrm{c}(\theta)\,W_{reg.\psi}\left(W_y - W_z\right) \\ 0 & \frac{\mathrm{s}(2\phi)}{2\mathrm{c}(\theta)}W_{reg\theta}\left(W_y - W_z\right) & W_{reg\psi}(W_y\,\mathrm{s}^2(\phi) + W_z\,\mathrm{c}^2(\phi)) \end{bmatrix}$$

s – sin, c – cos, W_x, W_y, W_z- transfer functions (5) of that part of the system that is between the matrices P and P^{-1}.

However, the transfer matrix (7) can be used to describe the first way for constructing the control system for zero desired values ($\psi^0 = \theta^0 = \phi^0 = 0$):

$$\dot{\eta} = W(p)\varepsilon_\eta \tag{8}$$

The type of the transfer matrix allows concluding that the UUV attitude control system is multivariable, and with the increase of the roll and pitch angles the following features appear: if the roll angle increases, the parameters of the diagonal elements of the transfer matrix of the system change; if the pitch angle increases, the influences between the channels increases; the roll control loop is the most susceptible to influences of other channels.

3.2. Decomposition Algorithm

The structure of the control system (7), (8) is such that the application of any special techniques to the synthesis of separate channel controllers will not solve the problem of mutual influences between channels. Often, for such systems, a decomposition algorithm that brings the transfer matrix of the system to a diagonal form is obtained, see Zubov et al. (2017, 2019).

The control law with the decomposition algorithm obtained in Gavrilina et al. (2019) and has the form similar to (5), but the matrix P^{-1} is replaced by the matrix P'^{-1} of the following form:

$$P'^{-1} = \begin{bmatrix} 1 & 0 & -sin(\theta)\frac{W'_z(p)}{W'_x(p)} \\ 0 & cos(\phi) & sin(\phi)\,cos(\theta)\frac{W'_z(p)}{W'_y(p)} \\ 0 & -\sin(\phi)\frac{W'_y(p)}{W'_z(p)} & cos(\phi)\,cos(\theta) \end{bmatrix} \tag{9}$$

where $W'_x(p), W'_y(p), W'_z(p)$ – parameters of the decomposing algorithm, which are defined as estimates of the transfer functions $W_x(p), W_y(p), W_z(p)$ of the UUV. When using the decomposition algorithm, the transfer matrix W (p) takes the form:

$$W(p) = \begin{bmatrix} \mathrm{W}_{\mathrm{reg}\phi}\mathrm{W}_{\mathrm{x}}(p) & W_{reg.\theta}(p)W_{12}(p) & W_{\mathrm{reg}\psi}(p)W_{13}(p) \\ 0 & W_{reg.\theta}(p)W_{22}(p) & W_{\mathrm{reg}\psi}(p)W_{23}(p) \\ 0 & W_{reg.\theta}(p)W_{32}(p) & W_{\mathrm{reg}\psi}(p)W_{33}(p) \end{bmatrix}, \tag{10}$$

where $W_{12}(p) = \frac{1}{2}\sin(2\phi)\tan(\theta)\left(W_y(p) - \frac{W_z(p)W_{y'}(p)}{W_{z'}(p)}\right)$,

$W_{13}(p) = \sin(\theta)\left(\left(W_z(p)\cos^2(\phi) + \frac{W_y(p)W_{z'}(p)}{W_{y'}(p)}\sin^2(\phi)\right) - \frac{W_x(p)W_{z'}(p)}{W_{x'}(p)}\right)$, $W_{22}(p) = W_y\cos^2(\phi) + \frac{W_z(p)W_{y'}(p)}{W_{z'}(p)}\sin^2(\phi), W_{23}(p) = \sin(2\phi)\cos\theta\left(W_y(p) - \frac{W_z(p)W_{y'}(p)}{W_{z'}(p)}\right), W_{32}(p) = \frac{\sin(2\phi)}{2\cos(\theta)}\left(W_y(p) - \frac{W_z(p)W_{y'}(p)}{W_{z'}(p)}\right), W_{33}(p) = (W_z(p)\cos^2(\phi) + \frac{W_y(p)W'_z(p)}{W'_y(p)}\sin^2(\phi))$.

If the parameters of the decomposition algorithm are defined exactly: $W'_i(\mathrm{p}) = W_i(p), \mathrm{i} = \mathrm{x,y,z}$, then the transfer matrix (10) takes a diagonal form and the control channels become independent of each other. In case the parameters are not accurately determined, then the conclusions made for the transfer matrix of the original system (7), (8) are valid for the transfer matrix of the decomposed system. The transfer matrix of the original system can be obtained from the transfer matrix (10), if the parameters of the decomposition algorithm are set to unit values:

$$W'_x(p) = W'_y(p) = W'_z(p) = 1 \tag{11}$$

3.3. Stability Analysis

Figure 3 shows the block diagram for the analysis of the stability of the attitude control system of the UUV for the zero desired values. This block diagram is suitable for the analysis of stability of both the control system of the first and second types.

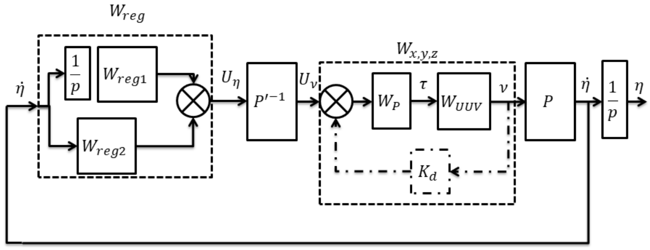


Fig.3. The block diagram for the stability analysis

The necessary and sufficient conditions for the stability of the UUV attitude control system according to the generalized Nyquist stability criterion is used, in accordance with the theorem given in Desoer and Wang (1980). It is necessary to obtain relations for the eigenvalues $\lambda_i(p)$ of the transfer matrix $W(p)$ for the analysis.

Since, when condition (11) is fulfilled, from the transfer matrix of the decomposed system it is possible to obtain the transfer matrix of the original system, analysis of the matrix (10) will make it possible to obtain stability conditions for both the decomposed and the not decomposed system. The eigenvalues $\lambda_i(p)$ of the system (7) are found from the following equation:

$$\det\left[\lambda E_3 - \frac{1}{\mathrm{p}} W(p)\right] = 0, \qquad (12)$$

where E_3 – 3x3 identity matrix. The characteristic equation of the system will take the form:

$$\left[\lambda^2 - \frac{\lambda}{p}\left(sin^2(\phi)\left(W_y - \frac{W_{y'}}{W_{z'}}W_z\right)\left(\frac{W_{z'}}{W_{y'}}W_{\mathrm{reg}\psi} - W_{\mathrm{reg}\theta}\right) + W_z W_{\mathrm{reg}\psi} + W_y W_{\mathrm{reg}\theta}\right) + \frac{1}{p^2} W_{\mathrm{reg}\theta} W_{\mathrm{reg}\psi} W_y W_z\right]\left[\lambda - \frac{1}{p} W_{\mathrm{reg}\phi} W_x\right] = 0 \qquad (13)$$

Matrix eigenvalues:

$$\begin{cases} \lambda_{1,2} = \frac{1}{2p}\left(W \pm \sqrt{W^2 - 4W_{\mathrm{reg}\psi}W_{\mathrm{reg}\theta}W_y W_z}\right), \\ \qquad \lambda_3 = \frac{1}{p} W_{\mathrm{reg}\phi} W_x, \end{cases} \qquad (14)$$

where $W = sin^2(\phi)\left(W_y - \frac{W_{y\prime}}{W_{z\prime}}W_z\right)\left(\frac{W_{z\prime}}{W_{y\prime}}W_{\mathrm{reg}\psi} - W_{\mathrm{reg}\theta}\right) + W_z W_{\mathrm{reg}\psi} + W_y W_{\mathrm{reg}\theta}$, in addition $\theta \neq \pm 90^0$.

From the relations for eigenvalues it can be concluded the following. The eigenvalue corresponding to the roll control channel, λ_3 does not depend on the roll and pitch angles of the UUV. Therefore, the synthesis of the roll control system can be carried out independently of the yaw and pitch channels.

A pair of eigenvalues $\lambda_{1,2}$, corresponding to the yaw and pitch control channels, in the general case does not depend on the UUV pitch angle, but depends on the roll angle. In the case of a not decomposed system, a pair of eigenvalues $\lambda_{1,2}$ does not depend on the roll angle if the transfer functions of the rotation of the UUV about the Oy, Oz axes or the transfer functions of the controllers at the yaw $W_{\mathrm{reg}\psi}$ and the pitch $W_{\mathrm{reg}\theta}$ coincide, i.e. $\mathrm{W_y} = W_z$ or $W_{\mathrm{reg}.\psi} = W_{\mathrm{reg}\theta}$ and the stability conditions of the separate yaw and pitch control channels are fulfilled.

If the elements of the decomposing algorithm $W_y'(p)$, $W_z{}'(p)$ are defined exactly, then the stability of the system depends only on the stability of the separate yaw channels, pitch and roll. The accuracy of the determination of $W_x'(p)$does not affect the stability of the system. In the case of inaccurate determination of the parameters of the decomposing algorithm, the roll remains the channel most susceptible to influences from the control actions along the yaw and pitch. At the same time with an increase in the roll angle, the stability conditions of the yaw and pitch channels change.

Thus, the synthesis of the roll channel should be carried out with the requirement to low sensitivity to disturbances from other the yaw and pitch channels and during synthesis can be considered independently of them.

4. THE ROLL CONTROL SYSTEM SYNTHESIS

One of the main requirements for the roll control channel is low sensitivity to perturbations from other channels. For the successful implementation of the obtained control actions in practice, it is necessary that the constructed control system have sufficient stability margins. One of the common ways to solve such problems is to use the H_∞ approach to the synthesis of controllers, see Zhou et al. (1996), Belov and Andrianova (2017), Chestnov (2019).

4.1. Problem Statement

The equations of the linearized roll channel are written in standard form, for the case when the pitch angle is zero:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 U_x \\ y &= Cx \end{aligned} \qquad (15)$$

where $x = \left[\phi\ \dot{\phi}\ \tau_x\right]^T$ is the vector of the variables of the state of the UUV, U_x - voltage applied to thrusters that control rotation about Ox axis; w is the external perturbation on the side of the yaw and pitch channels, brought to the input of the propulsion system of the UUV, y is the vector of the measured variables,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{\mathrm{K}_{\omega_x|\omega_x|}}{I_x - K_{\dot{\omega}_x}} & \frac{1}{I_x - K_{\dot{\omega}_x}} \\ 0 & 0 & -\frac{1}{T_{prx}} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{K_{prx}}{T_{prx}} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{K_{prx}}{T_{prx}} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

The equations of the desired controller are:

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y \\ u &= C_c x_c + D_c y \end{aligned} \qquad (16)$$

where x_c is the state vector of the controller, the order of which does not exceed the order of the control plant, the matrices A_c, B_c, C_c, D_c of the controller will be obtained when solving the synthesis problem for control system. Controller synthesis will be carried out based on the following system requirements:

1. The magnitude of the dynamic error is limited by the value Δ_ϕ;

2. The resulting control system must have a stability margins of the main and speed loops of at least 10 dB for gain margin (GM) and 50 ° for phase margin (PM);

3. The transient response in the system must meets the requirements for settling time of $t_{st} \leq t_{st}^0$.

4.2. Synthesis of the Controller

In Chestnov (2019), a synthesis method that meets the above requirements was proposed. Figure 4 shows the block diagram for the synthesis of the controller.

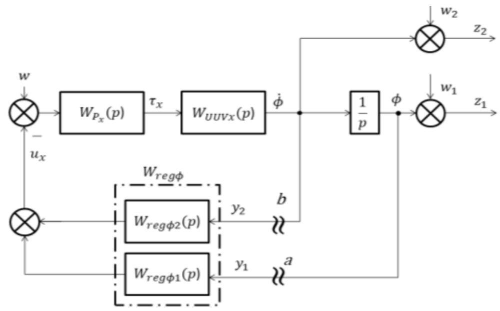


Fig. 4. Block-diagram for the closed-loop system

There are two controllable variables: z_1 – angle and z_2 - angular velocity; w_1, w_2 are fictitious external perturbations, w is the vector of perturbing influences from the side of the other channels, which make up the extended vector of external perturbations $\overline{w} = [\,w_1\, w_2\, w]^T$.

When solving the H_∞- optimization problem, the norm of the transfer matrix $T_{z\overline{w}}$ of the system is minimized from the vector of extended external perturbations $\overline{w}$ to the vector of controlled variables $z = [z_1\, z_2]^T$. For the transfer matrix $T_{z\overline{w}}$, and, therefore, for each of its elements, the following condition will be satisfied:

$$||T_{z\overline{w}}||_\infty \leq \gamma, \tag{17}$$

where γ is a given or minimized parameter.

The equations of the generalized plant for the considered problem in standard form:

$$\begin{aligned} \dot{x} &= A_g x + B_g \begin{bmatrix} \overline{w} \\ u_x \end{bmatrix} \\ z &= C_1 x + D_{11}\overline{w} + D_{12}u_x \\ y &= Cx + D_{21}\overline{w} + D_{22}u_x \end{aligned} \tag{18}$$

where the matrices of the generalized plant are:

$$A_g = A,\, B_g = [B_0\, B_0\, B_1\, B_2],\, B_0 = [0\; 0\; 0]^T,$$

$$C_g = \begin{bmatrix} C_1 \\ C \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$D_g = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}, D_{11} = \begin{bmatrix} 1\;0\;0 \\ 0\;1\;0 \end{bmatrix}, D_{12} \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, D_{22} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Thus, the proposed problem statement will make it possible to obtain a roll channel controller that provides a small dynamic error in the joint operation of the channels.

5. SIMULATION STUDY

Consider an example of applying the proposed approach to the synthesis of attitude control system on the AQUA-MO ROV. The parameters of the AQUA-MO ROV model are were obtained in Egorov (2002) and are shown in Table 1.

At the first stage, we carry out the synthesis of the roll channel control system according to the proposed technique. At the next stage, we calculate the parameters of the decomposition algorithm and synthesize the control system by the method widely used in practice, described in Egorov (2002). The results are comparable.

Table 1. AQUA-MO ROV PARAMETERS

i	T_{Pi},s	K_{Pi}, $\frac{N\cdot m}{V}$	$J_i - A_{\dot{\omega}_i}$, $kg\cdot m^2$	$A_{\omega i}$ $kg\cdot m^2$	$A_{\omega_i\lvert\omega_i\rvert}$ $\frac{kg\cdot m^2}{s}$	K_{UUVi}, $\frac{rad}{s\cdot N\cdot m}$	T_{UUVi}, s
x	0.15	33	55	-1200	-80	0,013	0,69
y	0,1	35	280	-1200	-120	0,008	2,3
z	0,2	50	323	-1200	-110	0,009	2,94

5.1. *Synthesis of the Roll Channel Using the H_∞ Approach*

When calculating the parameters of the controller, used the Robust Control Toolbox Matlab package with the linear matrix inequality (LMI) technique was used (*hinflmi* function). Control system requirements:

$$\Delta_\phi \leq 5^0, \mathrm{GM} \geq 10\; dB\,, \mathrm{PM} \geq 50^0, t_{st} \leq 2\; s \tag{19}$$

The resulting controller has the following values of matrices and transfer functions:

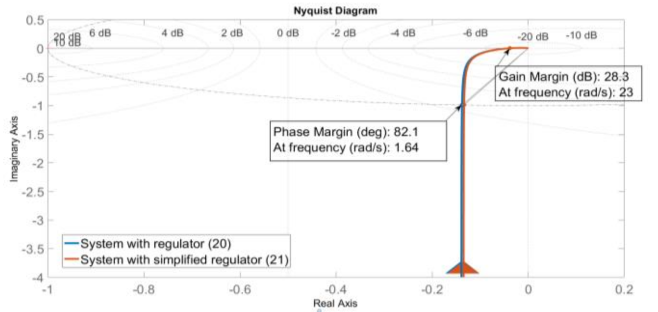
$$A_c = \begin{bmatrix} 14.2412 & -10.8619 & 105.2716 \\ -117.7602 & 1.7740 & -195.5293 \\ -450.0358 & 92.1511 & -665.3548 \end{bmatrix},$$

$$B_c = 1000 \cdot \begin{bmatrix} 0.0108 & -0.2722 \\ 0.8550 & -0.1679 \\ -0.7538 & -1.7050 \end{bmatrix},$$

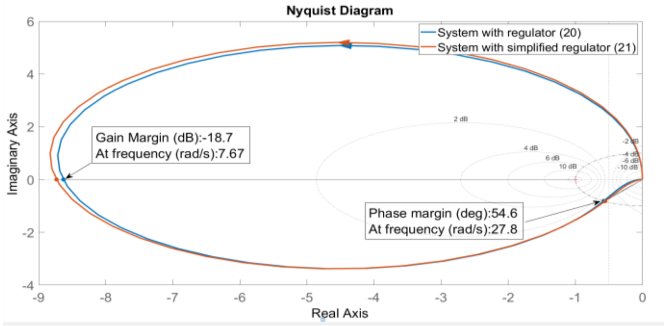
$$C_c = [-65.3828\;\; 18.8144\; - 138.9760], D_c = [0\; 0].$$

$$\begin{aligned} W_{reg\phi 1} &= \frac{1.2014\cdot 10^5\,(s^2+54.91s+1266)}{(s+556.2)\,(s+69.19)\,(s+23.99)}, \\ W_{reg\phi 2} &= \frac{2.5159\cdot 10^5(s+22.53)(s+15.53)}{(s+556.2)(s+69.19)(s+23.99)} \end{aligned} \tag{20}$$

The value of the optimization parameter is $\gamma = 1.001$. The settling time $t_{st} =$ 1.6 s, which meets the performance requirements of the control system (19). The system stability margins for roll angle loop PM = 82.1 °, GM = 28.3 dB, the transient response has an aperiodic character, the crossover frequency of 1.64 rad/s. The speed loop has PM = 54.6°, GM = 18.7 dB, and a loop crossover frequency of 27.8 rad/s. Figure 5 shows the Nyquist plot of the system open in position (point "a" in the Fig. 4) and speed (point "b" in the Fig. 4). The system meets the requirements for stability margins.



(a)



(b)

Fig. 5 Nyquist plot of the initial and simplified systems in open position (a) and in speed (b)

The resulting controllers are of the 3rd order. The numerator and denominator of the resulting controllers contain links with small time constants in comparison with the inversed crossover frequency, the influence of which can be neglected. In addition, the numerator and denominator $W_{reg\phi 2}$ contain links with close time constants that can be shortened. After simplification, the controllers will take the following form:

$$W_{reg\phi 1} = \frac{216\,(s^2 + 54.91s + 1266)}{(s+69.19)(s+23.99)}, W_{reg\phi 2} = \frac{425(s+15.53)}{(s+69.19)} \qquad (21)$$

Figure 5 shows the Nyquist plot of the simplified system open in position and speed. The plots almost match. The stability margins, settling time and other system parameters of the system didn't change. At the same time the regulator has a lower order and it is easier to implement it in practice.

5.2. Nonlinear System Simulation

Consider the structure of a control system with damping speed feedback based on $\dot{\psi}, \dot{\theta}, \dot{\phi}$ (first type (8)) and a decomposition algorithm. When modelling, a simplified decomposition algorithm is used, see Gavrilina et al. (2019). The calculated values of the decomposition algorithm parameters: $W_x' = 0.2884$, $W_y' = 0.04243$, $W_z' = 0.02797$. The modelling of the operation of the algorithms is carried out on a full non-linear model of the ROV. For comparison, the parameters of the PD - controllers were selected according to a technique common in practice and described in Egorov (2002).

At the initial stage, the quality of the separate channels is checked. Initial state of the ROV corresponded to zero values of roll, pitch and yaw. For checking quality of separate channels the desired orientation of the ROV was:

for yaw channel: $\psi^0 = 45^0, \theta^0 = 0^0, \phi^0 = 0$;

for pitch channel: $\psi^0 = 0^0, \theta^0 = 60^0, \phi^0 = 0^0$;

for roll channel: $\psi^0 = 0^0, \theta^0 = 0^0, \phi^0 = 10^0$.

Settling time for yaw, pitch and roll channels with PD-controllers was $t_\psi = 0{,}65\ s$, $t_\theta = 0.62\ s$, $t_\phi = 0.72\ s$, overshoot of the transient response was not more than 5%. Settling time for roll channel with H_∞ controller was $t_\phi = 1.6\ s$.

At the next stage, modelling of the joint work of the attitude control channels was carried out, at which the desired orientation was: $\psi^0 = 45^0$, $\theta^0 = 60^0$, $\phi^0 = 10^0$. For 3 seconds the ROV was set to rotate 90 ° in the yaw, so the desired orientation became: $\psi^0 = -45^0$, $\theta^0 = 60^0$, $\phi^0 = 10^0$. Figure 6 shows transient response in a control system with a controller constructed in accordance with the traditional approach and with the approach proposed in this work.

Figure 6 (a) shows the transient response of the control system with the traditional PD-controllers. The settling time of the system has increased in comparison with the operation of separate channels: $t_\psi = 1.25\ s, t_\theta = 0.75, t_\phi = 1.9\ s$. The dynamic error in the roll channel was 20 °, and the dynamic error in the pitch control channel was 3 °.

Figure 6 (b) shows transient responses of control system with PD – controllers in the yaw and pitch channels and controller (20) in the roll channel. The dynamic error in the roll channel is 2 °, i.e. decreased by 10 times. The dynamic error in the pitch channel also decreased to 2.5 °. Settling times improved: $t_\psi = 1\ s, t_\theta = 0.62, t_\phi = 1.8\ s$.

Figure 6 (c) shows the results of the operation of the attitude control system with a simplified controller in the roll channel (22). The type of transient processes is almost similar to processes in a control system without simplifications shown in Fig. 6 (b), with the exception that the dynamic error in the roll channel decreased to 1.8 °.

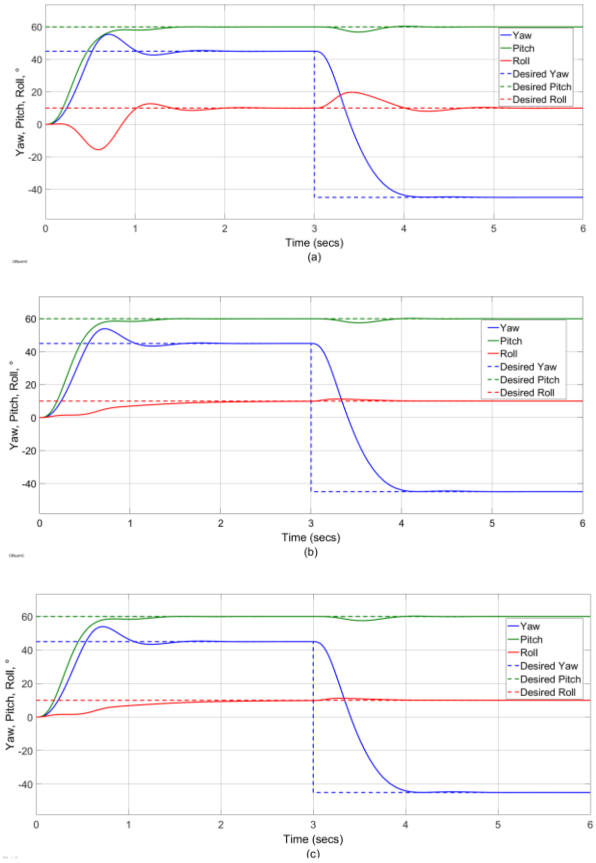


Fig.6. Transient responses in the system for the traditional approach (a), H_∞ −synthesis of the roll channel (b) H_∞ −synthesis of the roll channel with simplification of the regulator (c)

6. CONCLUSION

The stability condition for a traditional and decomposed control system for UUV orientation has been obtained. It is shown that the control system stability does not depend on the UUV pitch (for the case when $\theta \neq \pm 90^0$), but depends on the roll. Since the stability of the roll control channel does not depend on the angles of the yaw and the pitch of the UUV, it can be considered separately during synthesis. However, the synthesis should take into account that the roll control channel is affected by perturbations from other channels.

The roll control system synthesis problem has been solved as an H_∞ - optimization problem and is designed to provide low sensitivity to perturbations from other channels. Tests of the controller on a full ROV Aqua-MO non-linear model confirmed the effectiveness of the proposed solution (the dynamic error is reduced by 10 times in comparison to the PD controller). Moreover, the resulting system has sufficient stability margins (PM = 82.1 °, GM = 28.3 dB). An approach for simplifying the regulator is proposed for practical implementation.

The results will improve the quality of existing attitude control systems and increase the maneuverability of ROVs and AUVs. The obtained stability condition can be used to develop new regulators of the UUV yaw channel and pitch, and the proposed approach to the synthesis of the roll channel is applicable to other UUV control channels.

It is important to note that the resulting system has limitations: inoperability for a pitch angle of ± 90 °. It is of interest for further studies.

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