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Anti-Disturbance State Feedback Controller Based on Disturbance Reconstruction for Underactuated Overhead Crane ⋆

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Abstract: The paper deals with the control problems for the underactuated overhead crane system with lumped disturbance. First, two-stage state transformations are made and the system is eventually transformed into a linear-like cascade system under the transformations. In order to cope with the lumped unknown inputs, an identical unknown input reconstruction method is developed and the reconstruction is based on an interval observer. And then, for the equivalent linear-like cascade system, an anti-disturbance (or anti-unknown input) state feedback controller is designed by introducing the unknown input reconstruction into the controller. Simulation results are given to show the effectiveness of our methods.

Keywords: Underactuated overhead crane, Interval observer, Unknown input reconstruction, Anti-disturbance state feedback controller.

1. INTRODUCTION

Overhead crane system, as a typical under actuated system, has been extensively studied by researchers and received more and more attention from the control purpose, especially when it suffers from uncertainties and disturbances Chang and Lie (2012); Sun and Fang (2012); Zavari et al. (2014); Hilhorst et al. (2016); Wu et al. (2017); Chang (2007); Zhao and Gao (2012); Chen et al. (2016); Smoczek and Szpytko (2017). Firstly, several approaches have been proposed to enhance tracking performance of the system by translating the nonlinear dynamic system to a linear one Zavari et al. (2014); Hilhorst et al. (2016). For example, paper Zavari et al. (2014) thoroughly investigates LTI control designing method for LPV system and the application on an overhead crane system with a varying cable length. And the multi-H_∞ controller makes a trade-off between reference tracking and disturbance rejection present in the overhead crane system. An LMI framework based fixed-order multi-objective H_2/H_∞ controller for discrete-time LPV system has been presented in paper Hilhorst et al. (2016) , and the practical viability of the approach is demonstrated by experimental validations on a lab-scale overhead crane with varying cable lengths. Consider the fact that when translating a nonlinear system into a linear one, the system uncertainty and disturbance make the transforming procedure more complex and transformation may even produce extra unknown information to the system, nonlinear controllers have been employed and developed for the overhead crane systems Wu et al. (2017); Chang (2007); Zhao and Gao (2012). For instance, in paper Wu et al. (2017), trajectory tracking of a multi-input multi-output under actuated nonlinear system with dead-zone band and time delays is synthesized and analyzed using robust control technique based on adaptive fuzzy control, and a tower crane system is used as a case study in the simulations to validate the effectiveness and robustness of the proposed control scheme. An effective all-purpose adaptive fuzzy controller for the crane, which only based on using trolley position and swing angle information instead of on the complex dynamic model of the crane system, is developed in Chang (2007). By modeling the complex nonlinear dynamic system of the crane as a three-rule T-S fuzzy model with a saturated input, paper Zhao and Gao (2012) investigates the problem of a T-S fuzzy-model-based state-feedback controller design method of a nonlinear overhead crane system with input delay and actuator saturation.

Because of the strong robustness against the system uncertainties and external disturbances imposing on the system, sliding mode control (SMC) techniques have been applied widely to the crane systems Lu et al. (2018); Sankaranarayanan and Mahindrakar (2009); Park et al. (2014). In paper Sankaranarayanan and Mahindrakar (2009), a sliding mode control algorithm is developed to robustly stabilize a class of underactuated mechanical systems that are not linearly controllable and violate Brockett's necessary condition for smooth asymptotic stabilization of the equilibrium, with parametric uncertainties. Since the sway dynamics are disturbed by the trolley acceleration and hoisting velocity, antisway control law is designed based on the sliding mode control and a fuzzy uncertainty observer is constructed and incorporated into the sliding-mode antisway control law to cope with system uncertainties such as system parameter variations, unknown actuator

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nonlinearities, unmodeled dynamics, and external disturbances Park et al. (2014). To overcome the drawback of the conventional SMC where a finite time convergence cannot be reached, an advanced sliding mode control named as terminal sliding mode control (TSMC) has been developed very recently Van et al. (2017); Van (2018); Mien et al. (2019).

In the present paper, we dedicate to design an anti-disturbance state feedback controller for the underactuated overhead crane system by developing a disturbance reconstruction method and introducing the reconstruction into the controller. The remainders of the paper is organized as follows: In section 2, mode construction and two mode transformation ways are discussed. In section 3, an interval observer is designed for providing the upper and lower boundary estimations of one of the system state variable and an unknown input reconstruction method is developed based on the interval observer, and then an anti-unknown input state feedback controller is designed. Simulation results are presented in section 4. And finally, some conclusions are drawn in section 5.

2. BACKGROUND STATEMENTS

In this section, the overhead H_∞ crane mode is given and its two equivalent transformations are made. An overhead crane consists of a trolley moving along a horizontal axis and a load hung from a flexible wire, as illustrated in Fig. 1, where M is the trolley mass, m is the payload mass, l is the wire length, $x(t)$ is the trolley position, $\theta(t)$ is the swing angle, and u is the control input applied to the trolley. The overhead crane system can be represented by the following dynamics equation Lu et al. (2018):

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u + d \\ ml^2\ddot{\theta} + ml\ddot{x}\cos\theta + mgl\sin\theta = 0 \end{cases} \tag{1}$$

where $d(t)$ denotes the lumped disturbance exerted on the system. In order to proceed with the subsequent analysis, the following new state variables are first defined Lu et al. (2018):

$$\begin{cases} \mu_1 = x - x_d + l\ln\left(\dfrac{1+\sin\theta}{\cos\theta}\right) \\ \mu_2 = \dot{x} - \dot{x}_d + \dfrac{l\dot{\theta}}{\cos\theta} \\ \mu_3 = -g\tan\theta \\ \mu_4 = -\dfrac{g\dot{\theta}}{\cos^2\theta} \end{cases} \tag{2}$$

where x_d is the reference trajectory for the trolley, which serves to ensure small initial value for μ_1.

Lemma 1. Under the state transformation of (2), the original system (1) is rearranged as follows:

$$\begin{cases} \dot{\mu}_1 = \mu_2 \\ \dot{\mu}_2 = \mu_3 + \delta(\mu) - \ddot{x}_d \\ \dot{\mu}_3 = \mu_4 \\ \dot{\mu}_4 = f(\mu) + h(\mu)(u+d) \end{cases} \tag{3}$$

where

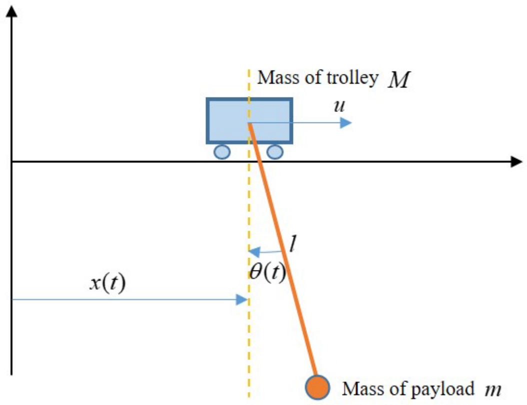

Fig. 1. Overhead crane system

$$\begin{cases} \delta(\mu) = -\dfrac{l\mu_3\mu_4^2}{(g^2+\mu_3^2)^{3/2}} \\ f(\mu) = \dfrac{-2gl(M+m\sin^2\theta)\dot{\theta}^2\sin\theta}{l(M+m\sin^2\theta)\cos^3\theta} \\ \qquad + \dfrac{mgl\dot{\theta}^2\sin\theta\cos^2\theta + (M+m)g^2\sin\theta\cos\theta}{l(M+m\sin^2\theta)\cos^3\theta} \\ h(\mu) = \dfrac{g}{(M+m\sin^2\theta)l\cos\theta} \end{cases} \tag{4}$$

Proof.

$$\begin{aligned} \dot{\mu}_1 &= \dot{x} - \dot{x}_d + l\frac{\cos\theta}{1+\sin\theta}\frac{\cos^2\theta + (1+\sin\theta)\sin\theta}{\cos^2\theta}\dot{\theta} \\ &= \dot{x} - \dot{x}_d + l\frac{1}{1+\sin\theta}\frac{\cos^2\theta + \sin^2\theta + \sin\theta}{\cos\theta}\dot{\theta} \\ &= \dot{x} - \dot{x}_d + l\frac{1}{1+\sin\theta}\frac{1+\sin\theta}{\cos\theta}\dot{\theta} \\ &= \dot{x} - \dot{x}_d + \frac{l}{\cos\theta}\dot{\theta} = \mu_2 \end{aligned}$$

That is the first equation in (3) holds. For the second equation in (3), on the one hand, we can compute out that

$$\begin{aligned} \dot{\mu}_2 &= \ddot{x} - \ddot{x}_d + l\frac{\dot{\theta}^2\sin\theta + \ddot{\theta}\cos\theta}{\cos^2\theta} \\ &= \frac{\ddot{x}\cos^2\theta + l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta}{\cos^2\theta} - \ddot{x}_d \\ &= \frac{\left(\ddot{x}\cos\theta + l\ddot{\theta}\right)\cos\theta + l\dot{\theta}^2\sin\theta}{\cos^2\theta} - \ddot{x}_d \end{aligned}$$

From the second equation of the original system (1), we have $\ddot{x}\cos\theta + l\ddot{\theta} = -g\sin\theta$. So

$$\begin{aligned} \dot{\mu}_2 &= \frac{\left(\ddot{x}\cos\theta + l\ddot{\theta}\right)\cos\theta + l\dot{\theta}^2\sin\theta}{\cos^2\theta} - \ddot{x}_d \\ &= \frac{-g\sin\theta\cos\theta + l\dot{\theta}^2\sin\theta}{\cos^2\theta} - \ddot{x}_d \end{aligned} \tag{5}$$

On the other hand, since

$$\begin{aligned} \delta(\mu) &= -\frac{l\mu_3\mu_4^2}{(g^2+\mu_3^2)^{3/2}} = \frac{g^3 l\dot{\theta}^2\tan\theta}{\left(g^2+g^2\tan^2\theta\right)^{3/2}\cos^4\theta} \\ &= \frac{l\dot{\theta}^2\tan\theta}{\left(1+\tan^2\theta\right)^{3/2}\cos^4\theta} = \frac{l\dot{\theta}^2\sin\theta}{\cos^2\theta} \end{aligned}$$

Thus we have

$$\begin{aligned}\mu_3+\delta(\mu)-\ddot{x}_d &= -g\tan\theta+\frac{l\dot{\theta}^2\sin\theta}{\cos^2\theta}-\ddot{x}_d \\ &= \frac{l\dot{\theta}^2\sin\theta-g\sin\theta\cos\theta}{\cos^2\theta}-\ddot{x}_d\end{aligned} \tag{6}$$

Now, comparing (5) and (6) gives the second equation of (3). The third equation in (3) is straightforward. For the fourth equation in (3), first we have

$$\dot{\mu}_4=-g\frac{2\dot{\theta}^2\sin\theta+\ddot{\theta}\cos\theta}{\cos^3\theta} \tag{7}$$

Besides, the original system (1) can be rewritten equivalently as

$$\begin{bmatrix} M+m & ml\cos\theta \\ ml\cos\theta & ml^2\end{bmatrix}\begin{bmatrix}\ddot{x}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}u+d+ml\dot{\theta}^2\sin\theta \\ -mgl\sin\theta\end{bmatrix}$$

Thus, we have

$$\begin{aligned}\begin{bmatrix}\ddot{x}\\ \ddot{\theta}\end{bmatrix} &= \begin{bmatrix} M+m & ml\cos\theta \\ ml\cos\theta & ml^2\end{bmatrix}^{-1}\begin{bmatrix}u+d+ml\dot{\theta}^2\sin\theta \\ -mgl\sin\theta\end{bmatrix} \\ &= \frac{1}{ml^2(M+m\sin^2\theta)} \\ &\quad\times\begin{bmatrix} ml^2 & -ml\cos\theta \\ -ml\cos\theta & M+m\end{bmatrix}\begin{bmatrix}u+d+ml\dot{\theta}^2\sin\theta \\ -mgl\sin\theta\end{bmatrix} \\ &= \frac{1}{ml^2(M+m\sin^2\theta)} \\ &\quad\times\begin{bmatrix} ml^2(u+d)+m^2l^3\dot{\theta}^2\sin\theta+m^2l^2g\sin\theta\cos\theta \\ -(ml\cos\theta)(u+d)-m^2l^2\dot{\theta}^2\cos\theta\sin\theta-(M+m)mgl\sin\theta\end{bmatrix}\end{aligned}$$

which implies that

$$\ddot{\theta}=\frac{-ml\dot{\theta}^2\sin\theta\cos\theta-(M+m)g\sin\theta-(\cos\theta)(u+d)}{l(M+m\sin^2\theta)} \tag{8}$$

Substituting (8) into (7) yields

$$\begin{aligned}\dot{\mu}_4 &= \frac{-2gl(M+m\sin^2\theta)\dot{\theta}^2\sin\theta+mgl\dot{\theta}^2\sin\theta\cos^2\theta}{l(M+m\sin^2\theta)\cos^3\theta} \\ &\quad+\frac{(M+m)g^2\sin\theta\cos\theta+(g\cos^2\theta)(u+d)}{l(M+m\sin^2\theta)\cos^3\theta} \\ &= \frac{-2gl(M+m\sin^2\theta)\dot{\theta}^2\sin\theta+mgl\dot{\theta}^2\sin\theta\cos^2\theta}{l(M+m\sin^2\theta)\cos^3\theta} \\ &\quad+\frac{(M+m)g^2\sin\theta\cos\theta}{l(M+m\sin^2\theta)\cos^3\theta}+\frac{g}{l(M+m\sin^2\theta)\cos\theta}(u+d) \\ &= f(\mu)+h(\mu)(u+d)\end{aligned}$$

Lemma 2. If we introduce a further transformation $x_1=\mu_1$, $x_2=\mu_2$, $x_3=\mu_3+\delta(\mu)-\ddot{x}_d$ and $x_4=\mu_4+\dot{\delta}(\mu)-\dddot{x}_d$, then we have

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=x_3\\ \dot{x}_3=x_4\\ \dot{x}_4=f(\mu)+h(\mu)u+\omega\end{cases} \tag{9}$$

where

$$\omega=h(\mu)d+\ddot{\delta}-\ddddot{x}_d \tag{10}$$

is the lumped unknown input.

The proof of Lemma 2 is straightforward and is omitted here.

Remark 1. The major purpose of the present paper is to design an effective controller such that the state variable $x(t)$ can track the desired trajectory $x_d(t)$ asymptotically, while controlling the oscillation of the payload swing angle, i.e., $\theta(t)$, to zero. In view of the new defined state variables as in (2), the controlling purpose is equivalently to reach $[\,\mu_1\ \mu_2\ \mu_3\ \mu_4\,]^T\to[\,0\ 0\ 0\ 0\,]^T$. Comparing the dynamic system (9) with (3), it can be seen that if $[\,x_1\ x_2\ x_3\ x_4\,]^T\to[\,0\ 0\ 0\ 0\,]^T$, then $[\,\mu_1\ \mu_2\ \mu_3\ \mu_4\,]^T\to[\,0\ 0\ 0\ 0\,]^T$ can be fulfilled. Therefore, the control purpose turns out to be designing an proper controller such that system (9) being asymptotically stable. For this reason, we call equivalent system (9) the designed system.

3. UNKNOWN INPUT RECONSTRUCTION AND CONTROLLER DESIGN

In this section, we are dedicated to develop a new unknown input reconstruction method based on equivalent system (9) via an interval observer which can produces the upper and lower boundary estimation of state x_4. Interval observer is a relatively new technology which can deal with the disturbance without topical assumption but only require the boundary information and can be easily apply for fault detection Zhang et al. (2019b). After this, also based on system (9), and an anti-disturbance state feedback controller is designed. For the designs, the following assumptions are necessary.

Assumption 1. All the system state variables $x(t)$, $\dot{x}(t)$, $\theta(t)$ and $\dot{\theta}(t)$ are measureable.

Assumption 2. The lumped external disturbance exerted on the system $d(t)$ is bounded in that $|d(t)|\le d_{01}$, and the boundary of d_{01} is a known constant.

Assumption 3. $|\theta(t)|\le\theta_{01}<\frac{\pi}{2}$, $\left|\dot{\theta}(t)\right|\le\theta_{02}$ and $\left|\ddot{\theta}(t)\right|\le\theta_{03}$, and θ_{01}, θ_{02} and θ_{03} are known constants.

Assumption 4. The reference input signal has forth order derivative and all of them are bounded, and moreover, $|\ddddot{x}_d|\le d_{02}$, where d_{02} is a known constant.

Lemma 3. Under Assumptions 2-4, we have $|\omega|\le d_0$, where

$$d_0=\frac{gd_{01}}{Ml\cos\theta_0}+\frac{6\theta_{02}^2+2\theta_{03}}{\cos^4\theta_0}l+d_{02} \tag{11}$$

Proof. Firstly,

$$|h(\mu)d|=\frac{g|d|}{(M+m\sin^2\theta)l|\cos\theta|}\le\frac{gd_{01}}{Ml\cos\theta_0}$$

Secondly, since

$$\begin{aligned}\delta(\mu) &= -\frac{l\mu_3\mu_4^2}{(g^2+\mu_3^2)^{3/2}}=\frac{g^3l\tan\theta}{\left(g^2+g^2\tan^2\theta\right)^{3/2}\cos^4\theta} \\ &= \frac{l\tan\theta}{\left(1+\tan^2\theta\right)^{3/2}\cos^4\theta}=\frac{l\sin\theta}{\cos^2\theta}\end{aligned}$$

Therefore it is not difficult for us to get

$$\dot{\delta}(\mu)=l\frac{\cos^3\theta+2\sin^2\theta\cos\theta}{\cos^4\theta}\dot{\theta}=l\frac{1+\sin^2\theta}{\cos^3\theta}\dot{\theta}$$

and

$$\ddot{\delta}(\mu) = l\frac{1+\sin^2\theta}{\cos^3\theta}\ddot{\theta} + l\frac{2\sin\theta\cos\theta\cos^3\theta + 3(1+\sin^2\theta)\sin\theta\cos^2\theta}{\cos^6\theta}\dot{\theta}^2 = l\left(\frac{\sin\theta(5+\sin^2\theta)}{\cos^4\theta}\dot{\theta}^2 + \frac{1+\sin^2\theta}{\cos^3\theta}\ddot{\theta}\right) = l\frac{\dot{\theta}^2\sin\theta(5+\sin^2\theta) + \ddot{\theta}(1+\sin^2\theta)\cos\theta}{\cos^4\theta}$$

And this implies

$$\left|\ddot{\delta}(\mu)\right| = l\left|\frac{\dot{\theta}^2\sin\theta(5+\sin^2\theta) + \ddot{\theta}(1+\sin^2\theta)\cos\theta}{\cos^4\theta}\right| \le l\frac{\left|\dot{\theta}\right|^2\left|\sin\theta(5+\sin^2\theta)\right| + \left|\ddot{\theta}\right|\left|(1+\sin^2\theta)\cos\theta\right|}{\cos^4\theta} \le \frac{6\theta_{02}^2 + 2\theta_{03}}{\cos^4\theta_{01}}l$$

Therefore, we have

$$|\omega| = \left|h(\mu)d + \ddot{\delta} - \dddot{x}_d\right| \le |h(\mu)d| + \left|\ddot{\delta}\right| + |\dddot{x}_d| \le d_0$$

, where d_0 is determined by (11). Now, for $\dot{x}_4 = f(\mu) + h(\mu)u + \omega$, consider system

$$\begin{cases} \dot{\hat{x}}_4^+ = x_4 - \hat{x}_4^+ + f(\mu) + h(\mu)u + d_0 \\ \dot{\hat{x}}_4^- = x_4 - \hat{x}_4^- + f(\mu) + h(\mu)u - d_0 \end{cases} \quad (12)$$

Lemma 4. System (12) is an interval observer for dynamic system $\dot{x}_4 = f(\mu) + h(\mu)u + \omega$ in that if the initial states satisfy $\hat{x}_4^-(0) \le x_4(0) \le \hat{x}_4^+(0)$, then we have $\hat{x}_4^-(t) \le x_4(t) \le \hat{x}_4^+(t)$ for all $t \ge 0$.

Proof. Denote $e_4^+(t) = \hat{x}_4^+(t) - x_4(t)$ and $e_4^-(t) = x_4(t) - \hat{x}_4^-(t)$ as the upper and lower boundary estimation error, respectively. It is not difficultly for us to get $\dot{e}_4^+ = -e_4^+ + d_0 - \omega$ and $\dot{e}_4^- = -\dot{e}_4^- + \omega + d_0$. Therefore, we have

$$\begin{cases} e_4^+(t) = \exp(-t)\cdot e_4^+(0) + \int_0^t \exp(-(t-\tau))\cdot(d_0 - \omega(\tau))\,d\tau \\ e_4^-(t) = \exp(-t)\cdot e_4^-(0) + \int_0^t \exp(-(t-\tau))\cdot(d_0 + \omega(\tau))\,d\tau \end{cases} \quad (13)$$

From $\hat{x}_4^-(0) \le x_4(0) \le \hat{x}_4^+(0)$, we know that $e_4^+(0) \ge 0$ and $e_4^-(0) \ge 0$. Moreover, from Lemma 3, we know that $d_0 - \omega(\tau) \ge 0$ and $d_0 + \omega(\tau) \ge 0$. Therefore, we can conclude that $\begin{bmatrix} e_4^+(t) \\ e_4^-(t) \end{bmatrix} \ge 0$ for all $t \ge 0$ based on (13), and this implies that $\hat{x}_4^-(t) \le x_4(t) \le \hat{x}_4^+(t)$ for all $t \ge 0$.

Next, based on the interval observer (12), we are going to propose an unknown input reconstruction method for the unknown input ω. $\hat{x}_4^-(t) \le x_4(t) \le \hat{x}_4^+(t)$ for all $t \ge 0$ implies that there exists a time varying scalar $\alpha(t)$ satisfying $0 \le \alpha(t) \le 1$, such that

$$x_4(t) = \alpha(t)\hat{x}_4^+(t) + (1-\alpha(t))\hat{x}_4^-(t)$$

or

$$x_4(t) = \alpha(t)\left(\hat{x}_4^+(t) - \hat{x}_4^-(t)\right) + \hat{x}_4^-(t) \quad (14)$$

In fact,

$$\alpha(t) = \frac{x_4(t) - \hat{x}_4^-(t)}{\hat{x}_4^+(t) - \hat{x}_4^-(t)} \quad (15)$$

satisfies (14). From (14), $\dot{x}_4 = \dot{\alpha}\left(\hat{x}_4^+ - \hat{x}_4^-\right) + \alpha\left(\dot{\hat{x}}_4^+ - \dot{\hat{x}}_4^-\right) + \dot{\hat{x}}_4^-$. Based on (12), we have

$$\dot{x}_4 = \dot{\alpha}\left(\hat{x}_4^+ - \hat{x}_4^-\right) + \alpha\left(\hat{x}_4^- - \hat{x}_4^+ + 2d_0\right) + x_4 - \hat{x}_4^- + f(\mu) + h(\mu)u - d_0 = x_4 + (\dot{\alpha} - \alpha)\left(\hat{x}_4^+ - \hat{x}_4^-\right) - \hat{x}_4^- + f(\mu) + h(\mu)u + (2\alpha - 1)d_0$$

Combining it with the fourth equation in (9) gives

$$\omega = x_4 + (\dot{\alpha} - \alpha)\left(\hat{x}_4^+ - \hat{x}_4^-\right) - \hat{x}_4^- + (2\alpha - 1)d_0 \quad (16)$$

where $\alpha(t)$ is calculated by (15). In order to get the $\alpha(t)$, we resort to the second-order supper-twisting algorithm given by reference Levant (1998):

$$\begin{cases} \dot{\hat{\alpha}} = \phi_1, \ \phi_1 = -\lambda_1|\hat{\alpha} - \alpha|^{1/2}\mathrm{sign}(\hat{\alpha} - \alpha) + \hat{\dot{\alpha}} \\ \dot{\hat{\dot{\alpha}}} = -\lambda_2\mathrm{sign}(\hat{\dot{\alpha}} - \phi_1) \end{cases} \quad (17)$$

Then $\hat{\alpha}$ and $\hat{\dot{\alpha}}$ will be the identical estimations of the α and $\dot{\alpha}$ in a finite time, respectively. Now based on (16), an identical estimation of $\omega(t)$ in a finite time is developed as

$$\hat{\omega} = x_4 + \left(\hat{\dot{\alpha}} - \alpha\right)\left(\hat{x}_4^+ - \hat{x}_4^-\right) - \hat{x}_4^- + (2\alpha - 1)d_0 \quad (18)$$

where $\hat{\dot{\alpha}}$ is the identical estimation of $\dot{\alpha}$ in a finite time produced by the second-order supper-twisting algorithm (17). After the identical estimation of the unknown input reconstruction of the ω has been got, the anti-unknown input state feedback controller design can be carried out.

Remark 2. The unknown input reconstruction determined by (18) can estimate its actual value given by (16) or (10) identically in a finite time. It should be further emphasized that known control input signal $u(t)$, which needs us to design such that the closed-loop system be stable, is decoupled from the unknown input reconstruction, and this feature is significant because it is this reconstruction feature that makes the anti-unknown input controller design feasible by introducing the reconstruction directly into the controller instead of eliminating the disturbance by introducing Sliding Mode controller such as Zhang et al. (2019a). And this is just what we plan to do next.

First, in view of auxiliary control input $u_a = f(\mu) + h(\mu)u + \omega$, system (9) is a linear system in form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + bu_a$, where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Obviously, $(\mathbf{A}, b)$ is controllable. Therefore, we can design the auxiliary control input by state feedback such that the closed-loop system is asymptotically stable. i.e. $u_a = k\mathbf{x}$, where $k \in \mathbf{R}^{1\times 4}$ is chosen such that all the eigenvalues of $\mathbf{A} + bk$ are with negative real parts. Next, based the unknown input reconstruction determined by (18), the actual controller can be designed as

$$u = h^{-1}(\mu)(k\mathbf{x} - f(\mu) - \hat{\omega}) \quad (19)$$

Theorem 1. The closed-loop system of (9) under controller (19) is asymptotically stable after a finite time.

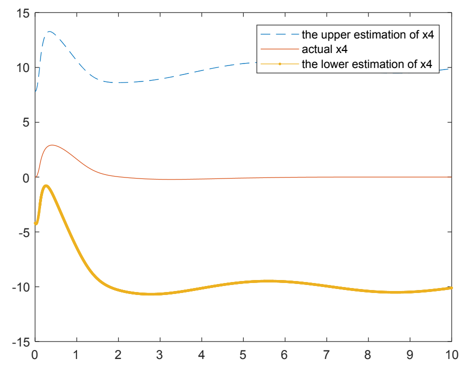

Fig. 2. The interval estimation of $x_4(t)$.

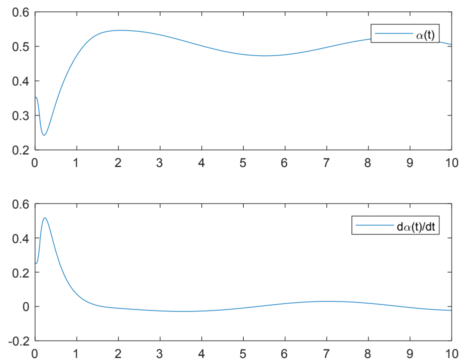

Fig. 3. Curves of $\alpha(t)$ and $\dot{\alpha}(t)$.

Proof. Substituting (19) into (9) yields $\dot{\mathbf{x}} = (\mathbf{A} + bk)\mathbf{x} + b\tilde{\omega}$, where $\tilde{\omega} = \omega - \hat{\omega}$. Since $\tilde{\omega} = 0$ after a finite time and all the eigenvalues of $\mathbf{A} + bk$ are with negative real parts, therefore the closed-loop system is asymptotically stable.

4. SIMULATION

In this section, we offer the simulation results of the underactuated overhead crane system to test the effectiveness of our design method. For the simulation, the parameters of the system are set as: $M = 100kg$, $m = 10kg$, $l = 4m$ and the target position for the trolley is set as $x_d(t) \equiv 5m$. The lumped disturbance is assumed to be $d(t) = 30 \sin t$. In order to get the identical estimation of the lumped unknown input ω consisting of the lumped disturbance, the second-order derivative of $\delta(\mu)$ determined by the first equation of (4) and the third order derivative information of the reference input $x_d(t)$, we need to design the interval observer for $\dot{x}_4 = f(\mu) + h(\mu)u + \omega$ and it is given by (12). The interval estimation of x_4 is shown in Fig. 2, where we can see that the interval estimation performance is satisfactory.

For the unknown input reconstruction purpose, the $\alpha(t)$ which is determined by (15) is needed to be calculated and the $\dot{\alpha}(t)$ is also needed to be produced by the second-order twisting algorithm (17) in advance. The curves of both the $\alpha(t)$ and $\dot{\alpha}(t)$ are plotted in Fig. 3. Now after have gotten the interval estimation of the state x_4 and the identical estimation of the $\dot{\alpha}(t)$, one can get the unknown input reconstruction for $\omega(t)$, and the reconstruction performance is reflected by Fig. 4. After we have completed the above procedures, we can carry out the anti-unknown input state feedback controller by (19). The state feedback gain is set as $k = [-12\ -22\ -18\ -7]$ and under the state feedback gain, the eigenvalues of the closed-loop system are placed on to $-1 \pm j$, -2 and -3. Under the controller given by (19), the trajectories of the system's states, $x(t)$, $\dot{x}(t)$, $\theta(t)$ and $\dot{\theta}(t)$, are shown in Fig. 5. From Fig. 5, we find that the horizontal trolley can tracking the reference signal of $x_d(t) \equiv 5$, while the velocity of the horizontal trolley, $\dot{x}(t)$, the payload swing angle, $\theta(t)$, and the velocity of the payload swing angle, $\dot{\theta}(t)$, are all driven to their equilibriums under the controller. Anyway, the simulation results show that our design method has satisfactory performances.

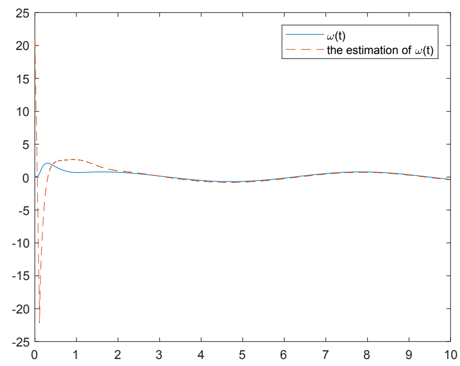

Fig. 4. The unknown input reconstruction of $\omega(t)$.

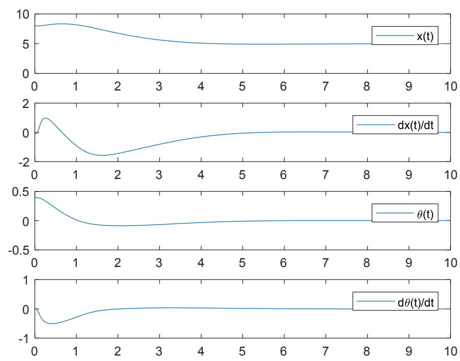

Fig. 5. Trajectories of $x(t)$ and $\dot{x}(t)$, $\theta(t)$ and $\dot{\theta}(t)$.

5. CONCLUSION

In the paper, we dedicate to design an anti-disturbance controller for the underactuated overhead crane system by developing an identical disturbance or unknown input reconstruction method based on an interval observer. We find that disturbance or unknown input reconstruction can identically estimate the actual value in a finite time, and moreover, the reconstruction can decouple the known control input. It is the decoupling of the unknown input reconstruction and the known control input that makes the anti-unknown state feedback controller design feasible. How to promote the design method to general underactuated systems will be our next considerations.

REFERENCES

Chang, C.Y. (2007). Adaptive fuzzy controller of the overhead cranes with nonlinear disturbance. *IEEE Transactions on Industrial Informatics*, 3(2), 164–172. doi:10.1109/tii.2007.898433.

Chang, C.Y. and Lie, H.W. (2012). Real-time visual tracking and measurement to control fast dynamics of overhead cranes. *IEEE Transactions on Industrial Electronics*, 59(3), 1640–1649. doi:10.1109/tie.2011.2163916.

Chen, H., Fang, Y., and Sun, N. (2016). A swing constraint guaranteed mpc algorithm for underactuated overhead cranes. *IEEE-Asme Transactions on Mechatronics*, 21(5), 2543–2555. doi:10.1109/tmech.2016.2558202.

Hilhorst, G., Pipeleers, G., Michiels, W., Oliveira, R.C.L.F., Peres, P.L.D., and Swevers, J. (2016). Fixed-order linear parameter-varying feedback control of a lab-scale overhead crane. *IEEE Transactions on Control Systems Technology*, 24(5), 1899–1907. doi: 10.1109/tcst.2016.2535260.

Levant, A. (1998). Robust exact differentiation via sliding mode technique. *Automatica*, 34(3), 379–384. doi: 10.1016/s0005-1098(97)00209-4.

Lu, B., Fang, Y., and Sun, N. (2018). Continuous sliding mode control strategy for a class of nonlinear underactuated systems. *IEEE Transactions on Automatic Control*, 63(10), 3471–3478. doi:10.1109/tac.2018.2794885.

Mien, V., Mavrovouniotis, M., and Ge, S.S. (2019). An adaptive backstepping nonsingular fast terminal sliding mode control for robust fault tolerant control of robot manipulators. *IEEE Transactions on Systems Man Cybernetics-Systems*, 49(7), 1448–1458. doi: 10.1109/tsmc.2017.2782246.

Park, M.S., Chwa, D., and Eom, M. (2014). Adaptive sliding-mode antisway control of uncertain overhead cranes with high-speed hoisting motion. *IEEE Transactions on Fuzzy Systems*, 22(5), 1262–1271. doi: 10.1109/tfuzz.2013.2290139.

Sankaranarayanan, V. and Mahindrakar, A.D. (2009). Control of a class of underactuated mechanical systems using sliding modes. *IEEE Transactions on Robotics*, 25(2), 459–467. doi:10.1109/tro.2008.2012338.

Smoczek, J. and Szpytko, J. (2017). Particle swarm optimization-based multivariable generalized predictive control for an overhead crane. *IEEE-Asme Transactions on Mechatronics*, 22(1), 258–268. doi: 10.1109/tmech.2016.2598606.

Sun, N. and Fang, Y. (2012). New energy analytical results for the regulation of underactuated overhead cranes: An end-effector motion-based approach. *IEEE Transactions on Industrial Electronics*, 59(12), 4723–4734. doi:10.1109/tie.2012.2183837.

Van, M. (2018). An enhanced robust fault tolerant control based on an adaptive fuzzy pid-nonsingular fast terminal sliding mode control for uncertain nonlinear systems. *IEEE-Asme Transactions on Mechatronics*, 23(3), 1362–1371. doi:10.1109/tmech.2018.2812244.

Van, M., Ge, S.S., and Ren, H. (2017). Finite time fault tolerant control for robot manipulators using time delay estimation and continuous nonsingular fast terminal sliding mode control. *IEEE Transactions on Cybernetics*, 47(7), 1681–1693. doi:10.1109/tcyb.2016.2555307.

Wu, T.S., Karkoub, M., Wang, H., Chen, H.S., and Chen, T.H. (2017). Robust tracking control of mimo underactuated nonlinear systems with dead-zone band and delayed uncertainty using an adaptive fuzzy control. *IEEE Transactions on Fuzzy Systems*, 25(4), 905–918. doi:10.1109/tfuzz.2016.2586970.

Zavari, K., Pipeleers, G., and Swevers, J. (2014). Gain-scheduled controller design: Illustration on an overhead crane. *IEEE Transactions on Industrial Electronics*, 61(7), 3713–3718. doi:10.1109/tie.2013.2270213.

Zhang, J., Zhu, F., Karimi, H.R., and Wang, F. (2019a). Observer-based sliding mode control for ts fuzzy descriptor systems with time delay. *IEEE Transactions on Fuzzy Systems*, 27(10), 2009–2023. doi: 10.1109/tfuzz.2019.2893220.

Zhang, X., Zhu, F., and Guo, S. (2019b). Actuator fault detection for uncertain systems based on the combination of the interval observer and asymptotical reduced-order observer. *International Journal of Control*, 1–9. doi:10.1080/00207179.2019.1620329.

Zhao, Y. and Gao, H. (2012). Fuzzy-model-based control of an overhead crane with input delay and actuator saturation. *IEEE Transactions on Fuzzy Systems*, 20(1), 181–186. doi:10.1109/tfuzz.2011.2164083.