Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Robust Adaptive Sliding Mode Controller for Wearable Robots

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Abstract: This paper concerns adaptive sliding-mode control for wearable robots with a human in the loop. The exoskeletons are wearable robots in interaction with different users. The proposed approach supposes that the dynamic model of the exoskeleton-human system is unknown except for some classical bounded properties. The controller guarantees the closed-loop convergence with an embedded-in estimation of unknown dynamics and uncertainties. The stability analysis of the system is demonstrated using the Lyapunov theory. Experimentation on an upper arm exoskeleton was conducted in order to exhibit the effectiveness of the proposed control method. The results show good tracking of the desired trajectories, which can be used in the assistive phase of the functional rehabilitation.

Keywords: Sliding mode, Adaptive control, Exoskeleton.

1. INTRODUCTION

Since a decade, exoskeletons or wearable robots continue to attract academia and industry interest, with different application domains. In the medical domain, exoskeletons are expected to deliver more effective rehabilitation therapies (Burger, 2000). The challenge is to provide appropriate assistance for patients with disabilities and the elderly not only in hospitals but also in their own homes. However, given the complexity of the exoskeleton as an electromechanical system, one of the research problem concerns the control system design, taking into account the safety of the wearer. Human-exoskeleton interactions lead to different challenging problems for the control design as the dynamic of the user's limb is practically unknown and varies from one user to another. The robustness of the exoskeleton controller plays a significant role to obtain satisfactory performances. The designed controller must be robust against dynamic changes, external disturbances and the state of the exoskeleton wearer. Several control strategies for exoskeletons have been proposed in the literature, as for instance: Adaptive control (Pehlivan et al., 2016), Fuzzy and backstepping control (Chen et al., 2017), Admittance control (Culmer et al., 2010), Impedance control and reinforcement learning (Li et al., 2017) and EMG-based control (Loconsole et al., 2014). A recent review on control strategies for upper limb exoskeletons can be found in (Proietti et al., 2016).

The Sliding Mode (SM) control is a robust technique for the control of systems with uncertainties and bounded disturbances (Yuri et al., 2014). Conventional SMC uses linear sliding surface which can only achieve asymptotic stability of the system during the sliding mode phase (Perruquetti and Barbot, 2002). More advanced techniques such as Terminal SM (TSM) control have been proposed in order to guarantee finite time convergence to zero of the tracking error (Zhihong et al., 1994). The Fast TSM (FTSM) surface has been introduced to further reduce the finite-settling-time (Madani et al., 2014). The TSM and FTSM techniques use fractional power in sliding surface that may lead to singularities. A Nonsingular TSM (NTSM) control has been proposed to overcome the singularity problem (Madani et al., 2016). An Integral TSM (ITSM) control has been proposed to completely eliminate singularities (Riani et al., 2018).

In this paper, a new adaptive ITSM controller is proposed for exoskeletons in order to perform passive movements. The main objective is to compute the torques of the wearable robot with fast time convergence towards the desired joint positions and velocities. The proposed sliding mode control is completely free of singularity, a well known phenomenon in the TSM control. The human and wearable-robot are seen as an unknown global interconnected system. An adaptive method is proposed to guarantee the convergence with embedded-in estimation of the unknown dynamics and uncertainties of the system. To validate the proposed scheme, experiments are carried out with a healthy subject using an upper limb exoskeleton to perform trajectories that correspond to passive arm movements.

The paper is organized as follows: In second II, the description of the considered dynamic model is presented. Section III describes the development of the proposed approach. Section IV constitutes the analysis of performing passive movements with the implementation of the proposed controller for an upper limb exoskeleton. The last section is the conclusion of this work.

2. DYNAMIC MODEL

Consider an n-link exoskeleton-human system with pivot joints governed by the following dynamic model described by the Lagrange-Euler equation:

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + H(q,\dot{q}) = u + \tau^{hum} + \tau^{ext}, \qquad (1)$$

where

$$H(q,\dot{q}) = G(q) + F(\dot{q}), \qquad (2)$$

with $q = q(t) \in \mathbb{R}^n$ denoting the joint position, $\dot{q} = \frac{d}{dt}q(t) \in \mathbb{R}^n$ the joint velocity, $\ddot{q} = \frac{d^2}{dt^2}q(t) \in \mathbb{R}^n$ the joint acceleration, $M(q) = M^T(q) \in \mathbb{R}^{n\times n}$ the positive definite inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ the centrifugal and Coriolis matrix, $G(q) \in \mathbb{R}^n$ the gravity torque, $F(\dot{q}) \in \mathbb{R}^n$ the friction torque, $u \in \mathbb{R}^n$ the actuated torque applied by the controlled actuators of the exoskeleton, $\tau^{hum} \in \mathbb{R}^n$ the torque provided by the human subject, and $\tau^{ext} \in \mathbb{R}^n$ the external torque, which can include the unmodeled effects.

In this paper, the notation $\|.\|$ denotes the Frobenius norm [1] . Since the system (1) is considered with only pivot joints, then the well known following properties are verified for any $q \in \mathbb{R}^n$ and $\dot{q} \in \mathbb{R}^n$ (Khalil and Dombre, 2002):

Property 1. *For any $x \in \mathbb{R}^n$, the matrix $M(q)$ is bounded by*

$$\mu_{1M}\,\|x\|^2 \le x^T M(q)\, x \le \mu_{2M}\,\|x\|^2\,, \qquad (3)$$

where $\mu_{1M}, \mu_{2M} > 0$ are some constants.

Property 2. *The matrix $[\dot{M}(q) - 2C(q,\dot{q})]$ is skew symmetric, then*

$$\tfrac{1}{2}\, x^T \dot{M}(q)\, x = x^T C(q,\dot{q})\, x, \qquad (4)$$

for any $x \in \mathbb{R}^n$.

Property 3. *The matrix $C(q,\dot{q})$ is bounded by $\|C(q,\dot{q})\| \le \mu_C\|\dot{q}\|$ where $\mu_C > 0$ is some constant.*

Property 4. *The vector $G(q)$ is bounded by $\|G(q)\| \le \mu_G$ where $\mu_G > 0$ is some constant.*

The friction vector $F(\dot{q})$ is defined as the gradient of the dissipation energy of the system with respect to $\dot{q}$. It is the result of numerous physical phenomena such as the geometry of the contacts, the nature of the lubricants, the properties of bulk, and surface materials on the bodies. The exact formulation of this torque is practically difficult to obtain. In this work, the following assumption for $F(\dot{q})$ is considered:

Assumption 1. *The vector $F(\dot{q})$ is bounded by $\|F(\dot{q})\| \le \mu_{1F} + \mu_{2F}\|\dot{q}\|$ where $\mu_F > 0$ is some constant.*

Obviously, the humans torques τ^{hum} are provided by natural muscles. They can not deliver infinite torques. On the other hand, the external torques τ^{ext} are completely unknown. Then, the following assumptions for τ^{hum} and τ^{ext} are adopted:

Assumption 2. *The vector τ^{hum} is bounded by $\|\tau^{hum}\| \le \mu_{\tau^{hum}}$ where $\mu_{\tau^{hum}} > 0$ is some constant.*

Assumption 3. *The vector τ^{ext} is bounded by $\|\tau^{ext}\| \le \mu_{\tau^{ext}}$ where $\mu_{\tau^{ext}} > 0$ is some constant.*

In the rest of this paper, the arguments (q), $(\dot{q})$ and $(q,\dot{q})$ will be omitted for the sake of clarity. Therefore, $M(q)$, $C(q,\dot{q})$ and $H(q,\dot{q})$ will be simply presented by M, C and H respectively.

[1] The Frobenius norm of x is defined by $\|x\| = \sqrt{\mathrm{tr}(x^T x)}$ where tr(.) is the trace function.

3. CONTROLLER DESIGN

The main objective of the proposed controller is to ensure the tracking of $q(t)$ and $\dot{q}(t)$ to their desired trajectories $q_d(t)$ and $\dot{q}_d(t)$ respectively in finite-time. The following additional assumptions are needed:

Assumption 4. *The position $q(t)$ and velocity $\dot{q}(t)$ are known for all time t.*

Assumption 5. *The desired trajectory $q_d(t)$ is an admissible trajectory which is twice differentiable with respect to time t and $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are bounded.*

The controller design will be done in the following two main steps that will be detailed in the sequel sections of the paper:

- Select an nonlinear switching manifold so that the closed-loop system in sliding mode guarantees the convergence to the equilibrium point in finite-time.
- Design an adaptive control law that guarantees the exact reachability of the selected sliding surface in finite-time.

The following lemmas will be used in the stability analysis of the proposed approach:

Lemma 1. *(Hale, 1969) Let $V(t)$ be a continuously differentiable scalar positive-definite function that satisfies the following differential inequality:*

$$\dot{V}(t) \le -\lambda\, V^{\gamma}(t),\ \forall t \ge t_0,\ V(t_0) \ge 0, \qquad (5)$$

where $\lambda > 0$ and $0 < \gamma < 1$ are some constants. Then, for any given t_0, the function $V(t)$ satisfies

$$\begin{array}{c} V^{1-\gamma}(t) \le V^{1-\gamma}(t_0) - \lambda(1-\gamma)(t-t_0), \\ \text{for } t_0 \le t < t_1, \end{array} \qquad (6)$$

and $V(t) = 0$ for $t \ge t_1$ where t_1 given by

$$t_1 = t_0 + \frac{V^{1-\gamma}(t_0)}{\lambda(1-\gamma)}. \qquad (7)$$

Lemma 2. *(Bhat and Bernstein, 2005) Let the following second-order system:*

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -k_2\, sgn(x_2)\, |x_2|^{\gamma} - k_1\, sgn(x_1)\, |x_1|^{\frac{\gamma}{2-\gamma}} \end{cases} \qquad (8)$$

where $x_1, x_2 \in \mathbb{R}$ and k_1,k_2,γ are some constants such as $0 < \gamma < 1$. Then, the origin $(x_1, x_2) = (0, 0)$ is a globally finite-time-stable equilibrium k_1,k_2 are chosen such that the polynomial $r^2 + k_2\, r + k_1$ is Hurwitz.

Lemma 3. *(Beckenbach and Bellman, 1961) Let the scalar constants $0 < \gamma < 1$ and $a_1, \ldots, a_n > 0$. Then, the following Jensen's inequality holds:*

$$\Big(\sum\nolimits_{i=1}^{n} a_i\Big)^{\gamma} \le \sum\nolimits_{i=1}^{n} a_i^{\gamma}. \qquad (9)$$

3.1 Switching manifold selection

Let $e = (q_d - q)$ and $\dot{e} = (\dot{q}_d - \dot{q})$ be the tracking position and velocity errors respectively. The selected nonlinear switching law is given by

$$s(t) = \dot{e}(t) + \alpha \int_0^t \dot{e}^{\frac{q}{p}}(\mu)\, d\mu + \beta \int_0^t e^{\frac{q}{2p-q}}(\mu)\, d\mu + \eta, \qquad (10)$$

where $p > q > 0$ with p and q are some positive odd integers, $\alpha = \mathrm{diag}(\alpha_1, ..., \alpha_n) \in \mathbb{R}^{n\times n}$ and $\beta = \mathrm{diag}(\beta_1, ..., \beta_n) \in \mathbb{R}^{n\times n}$ are some positive diagonal matrices, chosen such that the polynomials $r^2 + \beta_i\, r + \alpha_i$ for $i = 1, ..., n$ are Hurwitz, and η is some constant vector.

Remark 1. *The sliding manifold (10) contains terms* $\dot{e}^{\frac{q}{p}}$ *and* $e^{\frac{q}{2p-q}}$ *with fractional powers which satisfy* $0 < \frac{q}{p} < 1$ *and* $0 < \frac{q}{2p-q} < 1$*. The relative degree of the used sliding manifold s is 1 meaning that the control u has to appear explicitly in the first time derivative* $\dot{s}$*. Thanks to the integration parts of s in (10), the calculation of the time derivative of* $\dot{e}^{\frac{q}{p}}$ *and* $e^{\frac{q}{2p-q}}$ *are not required. Indeed, in the case of a time derivation of* $\dot{e}^{\frac{q}{p}}$ *and* $e^{\frac{q}{2p-q}}$*, the terms* $\frac{q}{p}\,\dot{e}^{\frac{q}{p}-1}\ddot{e}_i$ *and* $\frac{q}{2p-q}\,e^{\frac{q}{2p-q}-1}\dot{e}$ *appear and the singularity may occur due to the negative powers* $(\frac{q}{p}-1)$ *and* $(\frac{q}{2p-q}-1)$*. This problem will not arise by choosing the sliding manifold (10). Therefore, the proposed controller will be completely singularity-free.*

The derivative of the switching law (10) with respect to time, gives

$$\dot{s} = \ddot{e} + \alpha\,\dot{e}^{\frac{q}{p}} + \beta\,e^{\frac{q}{2p-q}}. \tag{11}$$

From the dynamic model (1) and the derivative (11), the following equation is obtained:

$$M\,\dot{s} + C\,s = M\,\ddot{q}_r + C\,\dot{q}_r + H - u - \tau^{hum} - \tau^{ext}, \tag{12}$$

where $\dot{q}_r$ and $\ddot{q}_r$ are the reference trajectories given by

$$\begin{cases} \dot{q}_r = s + \dot{q} \\ \ddot{q}_r = \ddot{q}_d + \alpha\,\dot{e}^{\frac{q}{p}} + \beta\,e^{\frac{q}{2p-q}} \end{cases} \tag{13}$$

Let the constants $\bar{M} \in \mathbb{R}^{n\times n}$, $\bar{C} \in \mathbb{R}^{n\times n}$ and $\bar{H} \in \mathbb{R}^n$ be the nominal values of $M(q)$, $C(q,\dot{q})$ and $H(q,\dot{q})$ respectively. Then, the following equation holds:

$$M\,\dot{s} + C\,s = \bar{M}\,\ddot{q}_r + \bar{C}\,\dot{q}_r + \bar{H} - u + \xi, \tag{14}$$

where ξ is given by

$$\xi = (M-\bar{M})\,\ddot{q}_r + (C-\bar{C})\,\dot{q}_r + (H-\bar{H}) - \tau^{hum} - \tau^{ext}. \tag{15}$$

The first equation of (13) gives $\dot{q} = \dot{q}_r - s$ then $\|\dot{q}\| \leq \|\dot{q}_r\| + \|s\|$. Hence, the vector ξ can be bounded by the following inequality according to the properties and the assumptions of the previous section:

$$\begin{aligned} \|\xi\| \leq\ & \mu_1 + \mu_2\,\|s\| + \mu_3\,\|s\|\|\dot{q}_r\| \\ & + \mu_4\,\|\dot{q}_r\| + \mu_5\,\|\dot{q}_r\|^2 + \mu_6\,\|\ddot{q}_r\|, \end{aligned} \tag{16}$$

where $\mu_1, ..., \mu_6 > 0$ are some suitable positive constants.

The previous inequality (16) results in the following relation:

$$\delta + \|\xi\| \leq \theta^T \phi(s, \dot{q}_r, \ddot{q}_r), \tag{17}$$

where the function $\phi(s, \dot{q}_r, \ddot{q}_r) \in \mathbb{R}^6_+$ is defined by

$$\phi(s, \dot{q}_r, \ddot{q}_r) = \begin{bmatrix} 1 \\ 1 + \|s\| \\ 1 + \|s\|\|\dot{q}_r\| \\ 1 + \|\dot{q}_r\| \\ 1 + \|\dot{q}_r\|^2 \\ 1 + \|\ddot{q}_r\| \end{bmatrix}, \tag{18}$$

the vector $\theta = [\theta_1, ..., \theta_6]^T \in \mathbb{R}^6_+$ is formed by

$$\begin{gathered} \theta_1 = \delta + \mu_1 - \sum_{i=2}^{6} \mu_i, \\ \theta_2 = \mu_2,\ \theta_3 = \mu_3,\ \theta_4 = \mu_4, \\ \theta_5 = \mu_5,\ \theta_6 = \mu_6, \end{gathered} \tag{19}$$

and δ is some arbitrary positive constant satisfying the inequality

$$\delta > -\mu_1 + \sum_{i=2}^{6} \mu_i. \tag{20}$$

In the rest of the paper, to simplify the writing of the equations, the arguments $(s, \dot{q}_r, \ddot{q}_r)$ will be omitted. So the function $\phi(s, \dot{q}_r, \ddot{q}_r)$ will be replaced by the simple notation ϕ. This function will be used in the proposed control law to ensures the stability of the closed-loop. The following lemma regarding this function ϕ will be used in the stability proof of the proposed controller.

Lemma 4. *Let x be any positive vector in* $\mathbb{R}^6_+$*. Then, the following inequality holds:*

$$x^T \phi \geq \|x\|, \tag{21}$$

Proof. *The function* ϕ *defined in (18) can be rewritten as an addition of two positive terms in the form*

$$\phi = \phi_1 + \phi_2, \tag{22}$$

where

$$\phi_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } \phi_2 = \begin{bmatrix} 0 \\ \|s\| \\ \|s\|\|\dot{q}_r\| \\ \|\dot{q}_r\| \\ \|\dot{q}_r\|^2 \\ \|\ddot{q}_r\| \end{bmatrix}. \tag{23}$$

Since x is defined as a positive vector, then it is obvious that $x^T\phi_1 = \|x\|_1$ *and* $x^T\phi_2 \geq 0$ *where* $\|x\|_1$ *denotes the one-norm*[2] *of x. Therefore, the computing of the product* $x^T\phi$ *gives*

$$x^T\phi = x^T\phi_1 + x^T\phi_2 = \|x\|_1 + x^T\phi_2 \geq \|x\|_1. \tag{24}$$

Recall that in this paper, the notation $\|.\|$ *denotes the Frobenius norm which here* $\|x\|$ *is the same as the standard two-norm because x is a vector. Using the well known property* $\|x\|_1 \geq \|x\|$ *in the equation (24), the inequality (21) comes. This completes the proof of the lemma 4.*

3.2 Sliding mode controller

In this section, a control law is proposed to achieve and maintain the sliding mode by assuming that the nominal values $\bar{M}$, $\bar{C}$, $\bar{H}$ and the vector θ are known. The following theorem is proposed:

Theorem 1. *Consider the system (1), the properties 1-4, the assumptions 1-5, and the switching law s given by (10), the trajectory* $(q, \dot{q})$ *tracks the desired one* $(q_d, \dot{q}_d)$ *in finite-time under the controller*

$$u = \bar{M}\,\ddot{q}_r + \bar{C}\,\dot{q}_r + \bar{H} + v, \tag{25}$$

where v is a switching control term defined as

$$v = \theta^T\phi\,\frac{s}{\|s\|}. \tag{26}$$

Proof. *Consider the following Lyapunov function:*

$$V(t) = \tfrac{1}{2}\,s^T M\,s,\ \forall t \geq 0,\ V(0) \geq 0, \tag{27}$$

By taking the time derivative $\dot{V} = s^T M\,\dot{s} + \frac{1}{2}\,s^T \dot{M}\,s$ *along with the property 2 and (14), it comes*

$$\dot{V} = s^T(\bar{M}\,\ddot{q}_r + \bar{C}\,\dot{q}_r + \bar{H} - u + \xi). \tag{28}$$

Substituting the control law (25)-(26) into (28), yields to

$$\begin{aligned} \dot{V} &= s^T(\xi - \theta^T\phi\,\frac{s}{\|s\|}) \\ &\leq -(\theta^T\phi - \|\xi\|)\,\|s\|. \end{aligned} \tag{29}$$

[2] The one-norm of $x \in \mathbb{R}^n$ is defined by $\|x\|_1 = \sum_{i=1}^{n} |x_i|$.

From (17), it is easily verified that $(\theta^T\phi - \|\xi\|) \geq \kappa_0$*, then (29) gives*

$$\dot{V} \leq -\kappa_0 \|s\|. \tag{30}$$

From the property 1, the relation $2V = s^T M s \leq \mu_{2M}\|s\|^2$ *holds that results in*

$$\dot{V} \leq -\kappa_0 \sqrt{\frac{2}{\mu_{2M}}}\, V^{\frac{1}{2}}. \tag{31}$$

Using the lemma 1 and (31), the finite-time convergence $V(t) = 0$ *is achieved for* $t \geq t_r$ *such as*

$$t_r = \frac{\sqrt{2\,\mu_{2M}}}{\kappa_0}\, V^{\frac{1}{2}}(0), \tag{32}$$

with t_r *is a finite-time in the reaching phase.*

Therefore, if $t \geq t_r$ *then the controller (25) ensures* $s_i(t) = 0$ *for* $i = 1, \ldots, n$*. Consequently,* $\dot{s}_i(t) = 0$ *which allows to write the ith error dynamic*

$$\ddot{e}_i + \alpha_i\, \dot{e}_i^{\frac{q}{p}} + \beta_i\, e_i^{\frac{q}{2p-q}} = 0 \ \ for\ \ i = 1, ..., n. \tag{33}$$

Finally, according to the lemma 2 by taking $x_1 = e_i$ *and* $x_2 = \dot{e}_i$*, the ith sub-system (33) is globally finite-time-stable to the equilibrium point* $(\dot{e}_i, e_i) = (0, 0)$ *for* $i = 1, ..., n$*. The proposed controller ensures the exact tracking of the trajectory* $(q, \dot{q})$ *to the desired one* $(q_d, \dot{q}_d)$ *in finite-time. This completes the proof of the theorem 1.*

Remark 2. *From (10) and (32), in can be noted that the reaching time* t_r *is equal to zero if the vector* η *is chosen such as* $\eta = -\dot{e}(0)$*.*

3.3 Adaptive sliding mode controller

The proposed controller in the previous section guarantees the stability of the system if the elements $\bar{M}$, $\bar{C}$, $\bar{H}$ and θ are known. However, these elements are not easy to find in practice. Basing on a prior knowledge of these parameters is a strong hypothesis. In this section, in order to make the proposed controller less restrictive and make its implementation easy, a new adaptive law is presented without requiring any knowledge of the elements mentioned before. The following theorem points out the proposed adaptive control law:

Theorem 2. *Consider the system (1), the properties 1-3, the assumptions 1-5, and the switching law s given by (10), the estimation errors of* $\bar{M}$, $\bar{C}$, $\bar{H}$ *and* θ *are bounded and the trajectory* $(q, \dot{q})$ *tracks the desired one* $(q_d, \dot{q}_d)$ *under the adaptive controller*

$$u = \hat{M}\, \ddot{q}_r + \hat{C}\, \dot{q}_r + \hat{H} + v, \tag{34}$$

where v is an adaptive switching control term defined as

$$v = \hat{\theta}^T \phi\, \frac{s}{\|s\|}, \tag{35}$$

with $\hat{M}$, $\hat{C}$, $\hat{H}$ *and* $\hat{\theta}$ *are tuned by the following adaptation laws to estimate* $\bar{M}$, $\bar{C}$, $\bar{H}$ *and* θ *respectively:*

$$\begin{cases} \dot{\hat{M}} = \Gamma_M\, s\, \ddot{q}_r^T \\ \dot{\hat{C}} = \Gamma_C\, s\, \dot{q}_r^T \\ \dot{\hat{H}} = \Gamma_H\, s \\ \dot{\hat{\theta}} = \Gamma_\theta\, \phi\, \|s\| \end{cases} \tag{36}$$

where $\Gamma_M, \Gamma_C, \Gamma_H \in \mathbb{R}^{n\times n}$ *and* $\Gamma_\theta \in \mathbb{R}^{5\times 5}$ *are some symmetric positive-defined matrices.*

Proof. *Consider the following Lyapunov candidate function*

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad \forall t \geq 0,\ V(0) \geq 0, \tag{37}$$

with

$$\begin{cases} V_1 = \frac{1}{2}\, s^T M\, s \\ V_2 = \frac{1}{2}\, tr(\tilde{M}^T \Gamma_M^{-1} \tilde{M}) + \frac{1}{2}\, tr(\tilde{C}^T \Gamma_C^{-1} \tilde{C}) + \frac{1}{2}\, \tilde{H}^T \Gamma_H^{-1} \tilde{H} \\ V_3 = \frac{1}{2}\, \tilde{\theta}^T \Gamma_\theta^{-1} \tilde{\theta} \end{cases} \tag{38}$$

where $\tilde{M} = (\bar{M} - \hat{M})$, $\tilde{C} = (\bar{C} - \hat{C})$, $\tilde{H} = (\bar{H} - \hat{H})$ *and* $\tilde{\theta} = (\theta - \hat{\theta})$ *are the estimation errors.*

By taking the time derivative $\dot{V}_1 = s^T M\, \dot{s} + \frac{1}{2}\, s^T \dot{M}\, s$ *along the property 2, the dynamic equation (14), the control law (34)-(35), it comes*

$$\begin{aligned} \dot{V}_1 &= s^T(\bar{M}\, \ddot{q}_r + \bar{C}\, \dot{q}_r + \bar{H} - u + \xi) \\ &= s^T(\tilde{M}\, \ddot{q}_r + \tilde{C}\, \dot{q}_r + \tilde{H} - v + \xi) \\ &= s^T \tilde{M}\, \ddot{q}_r + s^T \tilde{C}\, \dot{q}_r + s^T \tilde{H} + s^T \xi - \hat{\theta}^T \phi\, \|s\|. \end{aligned} \tag{39}$$

Using the time derivative of V_2 *and the adaptation laws* $\dot{\hat{M}}$, $\dot{\hat{C}}$, $\dot{\hat{H}}$ *given in (36), the following development*[3] *is given:*

$$\begin{aligned} \dot{V}_2 &= -tr(\tilde{M}^T \Gamma_M^{-1} \dot{\hat{M}}) - tr(\tilde{C}^T \Gamma_C^{-1} \dot{\hat{C}}) - \tilde{H}^T \Gamma_H^{-1} \dot{\hat{H}} \\ &= -tr(\tilde{M}^T s\, \ddot{q}_r^T) - tr(\tilde{C}^T s\, \dot{q}_r^T) - \tilde{H}^T s \\ &= -\ddot{q}_r^T \tilde{M}^T s - \dot{q}_r^T \tilde{C}^T s - \tilde{H}^T s \\ &= -s^T \tilde{M}\, \ddot{q}_r - s^T \tilde{C}\, \dot{q}_r - s^T \tilde{H}. \end{aligned} \tag{40}$$

The time derivative of V_3 *is given by the following equation where the adaptation law of* $\dot{\hat{\theta}}$ *given in (36) is used:*

$$\begin{aligned} \dot{V}_3 &= -\tilde{\theta}^T \Gamma_\theta^{-1} \dot{\hat{\theta}} \\ &= -\tilde{\theta}^T \phi\, \|s\|. \end{aligned} \tag{41}$$

The addition of (39), (40) and (41) gives

$$\begin{aligned} \dot{V} &= s^T \xi - \hat{\theta}^T \phi\, \|s\| - \tilde{\theta}^T \phi\, \|s\| \\ &\leq -(\theta^T \phi - \|\xi\|)\, \|s\| \end{aligned} \tag{42}$$

The inequality $(\theta^T\phi - \|\xi\|) \geq \delta$ *holds according to (17), then*

$$\dot{V} \leq -\delta\, \|s\|. \tag{43}$$

It yields that $\dot{V} \leq 0$ *for* $\forall t$ *which means that the variables* s, $\tilde{M}$, $\tilde{C}$, $\tilde{H}$ *and* $\tilde{\theta}$ *are bounded. The assumption 5 allows to guarantee that* $\dot{e}$ *is uniformly continuous since* $\ddot{q}_d$ *and* $\ddot{q}$ *are bounded. As a result, the variable s given in (10) is also uniformly continuous. In addition, it can be verified from (43) that* $\lim_{t\to+\infty} \int_{t_0}^{t} \|s(\mu)\| d\mu \leq \frac{V(t_0)}{\delta}$*. Therefore, application of Barbalat's lemma (Barbalat, 1959) indicates that* $\lim_{t\to+\infty} \|s(t)\| = 0$*. According to the stability performances of the used switching variable s, it can be conclude that* $e \to 0$ *and* $\dot{e} \to 0$ *as* $t \to +\infty$*. Finally, the proposed adaptive controller guarantees a zero steady-state tracking error of the position and the velocity trajectories. This completes the proof of the theorem 2.*

[3] The equalities $x^T y = y^T x$ and $x^T y = \mathrm{tr}(y\, x^T)$ hold for $\forall x, y \in \mathbb{R}^n$.

4. EXPERIMENTAL RESULTS

To validate the effectiveness of the proposed adaptive controller, a real-time application with exoskeleton is performed by asking a healthy subject to apply alternatively flexion and extension movements. The used exoskeleton is ULEL (Upper Limb Exoskeleton of LISSI) which is developed by RB3D company especially for the LISSI laboratory (Laboratoire Images, Signaux et Systèmes Intelligents) of University Paris-Est Créteil (UPEC), France. This exoskeleton is designed to perform scientific researches on the rehabilitation of the human right arm.

The following modules of ULEL are used to carry out our validation tests (see the left side of Fig. 1):

- Shoulder module: connected to the frame module by a passive spherical joint used for the 3D orientation of the exoskeleton.
- Upper arm module: articulated to the shoulder module by an active rotational shoulder joint.
- Forearm module: articulated to the upper arm module by an active rotation elbow joint.

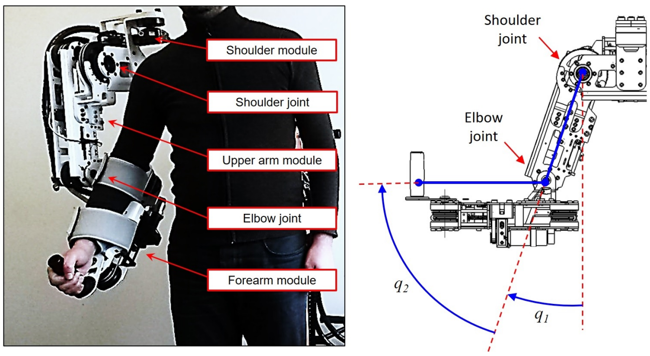

Fig. 1. Upper Limb Exoskeleton of LISSI (ULEL) on the left, and the kinematic diagram of the used modules of ULEL on the right.

The active joints of ULEL are actuated by powerful DC motors. An electrical control system is used to provide the regulation for motor currents. The applied torques at human joints are provided by a special screw and cable mechanical system. The joint positions are measured by incremental encoders with a good measurement accuracy. The controller is programmed on a PC equipped with a dSpace DS1103 PPC real-time controller card, using Matlab/Simulink and dSpace Control Desk software. The fourth-order Rung-Kutta's numerical solver with $0.001 sec$ sampling time are used to solve the nonlinear differential equations.

The experience is produced using the active shoulder and elbow joints of ULEL. The exoskeleton is worn by a healthy subject having 43 years old, measuring $1.69\,m$ and weighing $73\,kg$. For safety reason, the joints are constrained by safety ranges of motion and the electric motor currents are limited.

The used human-exoskeleton system is seen as a system decomposed into two subsystems numbered by the subscripts $i = 1$ and $i = 2$ for the shoulder and elbow joints respectively. The right side of Fig. 1 shows the kinematic diagram of the used modules of ULEL. The zero reference corresponds to the resting position toward the ground (all joints facing down). The desired reference trajectory $q_q(t)$ is chosen as a sine wave to ensure the existence and the boundness of the successive time derivations of the reference trajectories. This kind of periodic trajectories is often used in medical rehabilitation protocols. The parameters of the sliding surface function (10) are $p = 7$, $q = 9$, $\alpha = 3\,I$, $\beta = I$ and $\eta = -\dot{e}(0)$. The adaptation gains Γ_M, Γ_C, Γ_H and Γ_θ are chosen equal to $10\,I$. The initial values of the estimations $\hat{M}$, $\hat{C}$, $\hat{H}$ and $\hat{\theta}$ are equal to zero. The actuators of the used exoskeleton have delays and other practical imperfections. These constraints can lead to chattering and excitation of non-modeled dynamics when sliding mode control is applied. A boundary layer technique is used to avoid chattering excitation of the sliding mode control. The terms $\frac{s}{\|s\|+\epsilon}$ is introduced instead of $\frac{s}{\|s\|}$ in (35) where $\epsilon = 0.1$ is chosen heuristically.

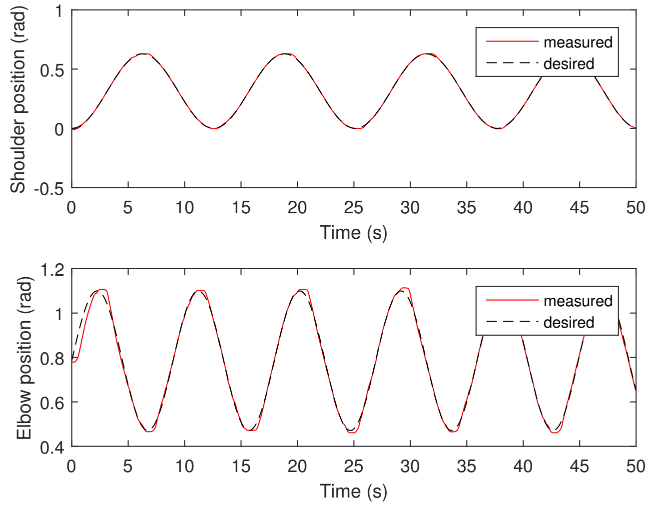

Fig. 2. Position trajectories $q(t)$ and $q_d(t)$.

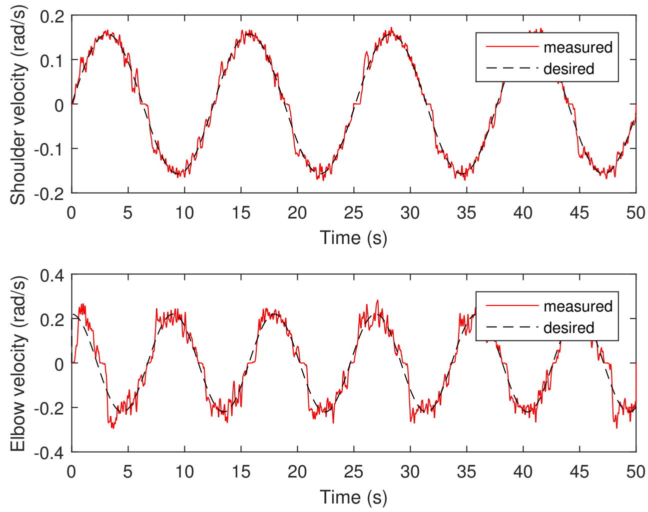

Fig. 3. Velocity trajectories $\dot{q}(t)$ and $\dot{q}_d(t)$.

Fig. 2 shows the desired and the measured positions for the shoulder and elbow joints, with good tracking of the desired trajectories. Fig. 3 presents the desired and the measured velocities. The small perturbations are due to the noisy measurements of the used instruments.

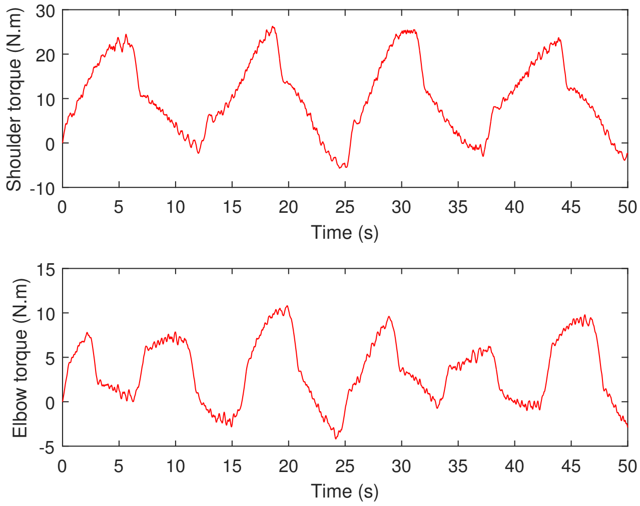

Fig. 4. Control inputs $u(t)$.

The control inputs $u(t)$ are shown in Fig. 4. In both cases of flexion and extension movements, the torque inputs of the exoskeleton are smooth. The torque increases if the flexion movement is needed and decreases otherwise. The high excitation of the power electrical system is discarded thanks to the used boundary layer technique. The applied torques meet the physical limits of the real system.

5. CONCLUSION

This paper proposes an adaptive integral-terminal-sliding-mode controller for exoskeletons ensuring trajectory tracking of the position and velocity. Only the structure of the dynamic model and some bounded properties were used for the controller design. The knowledge of the dynamic model and the uncertainties bounds are not required thanks to the proposed adaptation laws. The stability has been demonstrated by the Lyapunov theory. The robustness of the controller has been tested by asking a healthy subject to apply alternatively flexion and extension movements using an exoskeleton in real-time application. The experimental results show the effectiveness and the satisfactory performances of the proposed approach.

ACKNOWLEDGMENTS

This work is in part supported by the National Research Foundation (NRF) of South Africa (Grant Number: 90604). Opinions, findings and conclusions or recommendations expressed in any publication generated by the NRF supported research are those of the author(s) alone, and the NRF accepts no liability whatsoever in this regard.

REFERENCES

Barbalat, I. (1959). Systemes d'equations differentielles d'oscillations non lineaires. *Rev. Math. Pures Appl*, 4(2), 267–270.

Beckenbach, E.F. and Bellman, R. (1961). *Inequalities*. Springer-Verlag Berlin Heidelberg.

Bhat, S.P. and Bernstein, D.S. (2005). Geometric homogeneity with applications to finite-time stability. *Mathematics of Control, Signals and Systems*, 17(2), 101–127.

Burger, C.G. (2000). Development of robots for rehabilitation therapy : The palo alto va/stanford experience. *Journal of Rehabilitation Research and Development*, 37(6), 663–673.

Chen, Z., Li, Z., and Chen, C.P. (2017). Disturbance observer-based fuzzy control of uncertain mimo mechanical systems with input nonlinearities and its application to robotic exoskeleton. *IEEE transactions on cybernetics*, 47(4), 984–994.

Culmer, P.R., Jackson, A.E., Makower, S., Richardson, R., Cozens, J.A., Levesley, M.C., and Bhakta, B.B. (2010). A control strategy for upper limb robotic rehabilitation with a dual robot system. *IEEE/ASME Transactions on Mechatronics*, 15(4), 575–585.

Hale, J. (1969). *Ordinary Differential Equations*. Dover Books on Mathematics Series. Krieger, Huntington.

Khalil, W. and Dombre, E. (2002). *Modeling, Identification and Control of Robots*. Taylor & Francis, Inc., USA.

Li, Z., Liu, J., Huang, Z., Peng, Y., Pu, H., and Ding, L. (2017). Adaptive impedance control of human-robot cooperation using reinforcement learning. *IEEE Transactions on Industrial Electronics*.

Loconsole, C., Dettori, S., Frisoli, A., Avizzano, C.A., and Bergamasco, M. (2014). An emg-based approach for on-line predicted torque control in robotic-assisted rehabilitation. In *IEEE Haptics Symposium (HAPTICS), 2014*, 181–186. IEEE, Houston, Texas, USA.

Madani, T., Daachi, B., and Djouani, K. (2016). Non-singular terminal sliding mode controller: Application to an actuated exoskeleton. *Mechatronics*, 33, 136–145.

Madani, T., Daachi, B., and Djouani, K. (2014). Finite-time control of an actuated orthosis using fast terminal sliding mode. *IFAC Proceedings Volumes*, 47(3), 4607–4612.

Pehlivan, A.U., Losey, D.P., and O'Malley, M.K. (2016). Minimal assist-as-needed controller for upper limb robotic rehabilitation. *IEEE Transactions on Robotics*, 32(1), 113–124.

Perruquetti, W. and Barbot, J.P. (2002). *Sliding mode control in engineering*. CRC Press.

Proietti, T., Crocher, V., Roby-Brami, A., and Jarrassé, N. (2016). Upper-limb robotic exoskeletons for neurorehabilitation: a review on control strategies. *IEEE reviews in biomedical engineering*, 9, 4–14.

Riani, A., Madani, T., Benallegue, A., and Djouani, K. (2018). Adaptive integral terminal sliding mode control for upper-limb rehabilitation exoskeleton. *Control Engineering Practice*, 75, 108 – 117.

Yuri, S., Christopher, E., Leonid, F., and Arie, L. (2014). *Sliding mode control and observation*. Springer.

Zhihong, M., Paplinski, A.P., and Wu, H.R. (1994). A robust mimo terminal sliding mode control scheme for rigid robotic manipulators. *IEEE Transactions on Automatic Control*, 39(12), 2464–2469.