A Bilinear Approach to Model Predictive Control for Thermal Conditioning of Adaptive Buildings

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Abstract: The high resource and energy consumption of the building sector in both construction and operation is a growing problem worldwide. The largest contributor to operational energy consumption is thermal conditioning of the indoor space. In this context, inefficient control algorithms or parametrizations become a serious problem requiring thermal simulation models of buildings for system sizing and control parameter adjustments. However, the high complexity of the underlying dynamic models makes the design of model-based controllers difficult. Furthermore, typically used control schemes such as PI-control cannot incorporate all types of actuators that an adaptive building may provide.
In this work, we derive a bilinear thermal model for adaptive ultra-lightweight buildings from the linearized model output of the Modelica library BuildingSystems by incorporating environmental and internal disturbances as well as a number of possible actuators for an adaptive building into the model as time-varying bilinear inputs.
Based on the bilinear model, a model-predictive control algorithm is devised that incorporates disturbance forecasts. Exemplary simulations for a summer day show the efficacy of the control algorithm in employing indirect actuation.

Keywords: Adaptive buildings, model predictive control, bilinear model

1. INTRODUCTION

Resource and energy consumption have become one of the biggest global problems, leading to scarcity-induced conflicts and the current climate crisis. In addition, the rapidly growing world population creates a large demand for residential space. Today, the construction sector is already responsible for a major part of our energy and resource demand, about 42% of the total EU final energy consumption and more than 50% of extracted materials according to the European Commission (2011). A large part of this energy consumption is attributable to air conditioning of indoor spaces with the goal of creating productive and comfortable working and living spaces. Therefore, more efficient control approaches encompass a great energy saving potential.

One approach to mitigate the energy consumption problem is to improve the efficiency of conditioning systems by using building energy simulation software in the design stage of the building. With these tools, system sizing and HVAC (heating, ventilation and air conditioning) controller efficiency can be tuned more accurately, thus saving energy.

Another approach is to add new types of active components to buildings that can store and direct energy flows. This approach aims to increase the degrees of freedom for the algorithm to find an efficient strategy. This is one of the goals of the Collaborative Research Centre 1244 "Adaptive Skins and Structures for the Built Environment of Tomorrow" (Sobek and Sawodny, 2017). The project aims to reduce resource consumption of the building sector by employing adaptivity in the shells and structures to achieve ultra-lightweight buildings. To ensure user comfort in these buildings, new adaptive facade elements and matching control strategies are developed (Guenther et al., 2019; Harder et al., 2018). An overview over types of adaptivity is given by Aelenei et al. (2016) and Modin (2014). Notable examples in the context of this work include variable emissivity and absorptance structures (Athanasopoulos and Siakavellas, 2015; Bergeron et al., 2008) and variable insulation (Lopes Alves Homem, 2017).

Unfortunately, the most common tools in building energy simulation (namely TRNSYS and EnergyPlus) are very complex and not entirely suitable for the synthesis of new control algorithms, because the underlying differential equations cannot easily be accessed (or contain iterative algorithms) and adaptive components cannot be included without heavily modifying the source code. On the other hand, very simple RC chain models (Sturzenegger et al., 2014; Kircher and Zhang, 2015) can easily be used for controller synthesis, but lack precision as well as the ability to integrate many new actuator types. Furthermore, the model-free and common linear model based controller types are usually unable to integrate disturbance forecasts such as ambient temperature and solar irradiation.

To address these problems, we propose a novel way of modeling buildings for predictive control as bilinear systems by extracting a linearized model from a complex building simulation model and extending it with the nonlinear actuator dynamics. The complex model is developed with the Modelica BuildingSystems library (Nytsch-Geusen et al., 2013). Bilinear system models have been a topic of research for some time, also in the context of air conditioning (Kelman and Borrelli, 2011). Bilinear system models provide a more accurate description of nonlinear processes, yet are still mathematically more tractable than general nonlinear models which makes them suitable for the use in model predictive control (Yeo and Williams, 1987). For the considered adaptive actuators, the bilinear formulation is particularly well-suited due to the multiplicative coupling of the inputs and states. Some of the considered actuators are conventional, namely convective and radiant heating, shading and ventilation. Additionally, novel types of actuation such as variable surface absorptance, active thermal energy storage and variable insulation conductivity are integrated.

As an approach to integrate forecasts and enable predictive control, we integrate the developed model into an economic model predictive control (MPC) formulation that aims to ensure occupant comfort while minimizing energy consumption. An overview of other approaches to MPC for air conditioning in conventional buildings is given by Thieblemont et al. (2017). The special case of user comfort in MPC was also studied by Ascione et al. (2016).

Notable alternatives for building modeling and subsequent control design include a JModelica-based MPC tool by Jorissen et al. (2019) as well as the BLDG and BRCM toolboxes for Matlab for control-oriented modeling on an abstract level (Kircher and Zhang, 2015; Sturzenegger et al., 2014).

The main contributions of this work are:

- Creating a bilinear model of an adaptive building by extracting a linearized base model (Section 2.2) and subsequently extending it with bilinear time-varying control inputs in Section 2.3.
- Validation of the bilinear model in Section 3.1.
- Devising and testing a model predictive controller for the adaptive facade and conventional conditioning actuators in Section 3.2.

2. MODELING

This section introduces the building model as well as the methodical steps for the bilinear extension of the linearized base model, the linearized comfort model and control design using MPC.

2.1 Reference Building Model

The reference building model is created in Modelica with the BuildingSystems library. To simulate variable facade components in the reference model, the library is extended with variable conductivity and surface absorptance as well as a simple water thermal storage.

For modeling and simulation, a single office room with one external wall with a glazing fraction of 30% (all
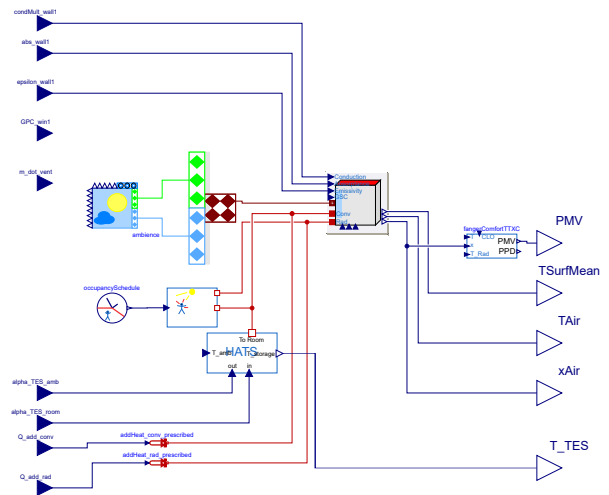

Fig. 1. Screenshot of a building model in Modelica.

others assumed to be adiabatic) is created. The facade has variable absorptance and variable conductivity of the insulation layer. An additional water storage tank with 1000 kg of water is added. This thermal storage can be thermally coupled to either the room air or the ambience and serves as an adaptive building mass.

For the simulations, typical weather data from Stuttgart, Germany is used. A screenshot of the building model is shown in Fig. 1.

2.2 Linearized Base Model

The thermal behavior of a building model is described by differential equations in the form of

$$0 = f(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d}, t) , \qquad (1)$$

where $\dot{\boldsymbol{x}}$ denotes the derivative of the variable $\boldsymbol{x}$ with respect to time t. The vector state variable $\boldsymbol{x}$ includes all the system states; in this case, these are internal energies or temperatures. The input vector $\boldsymbol{u}$ describes all manipulated variables, e.g. heating power, whereas $\boldsymbol{d}$ corresponds to the input disturbances such as solar irradiation. The (potentially time-varying) function f describes all the couplings and influences that the current state, manipulated variables and disturbances have on the rate of change of the state variables.

For control design and implementation it is desirable to have a model description that is as simple as possible while still capturing all the relevant thermal dynamics. Since most nonlinearities originate from the multiplication of inputs and states, a bilinear model structure provides a good representation of the complex building dynamics:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} + E\boldsymbol{d} + N\boldsymbol{x}\boldsymbol{u} . \qquad (2)$$

This form is achieved by modeling a reference building in Modelica, linearizing its core dynamics and extending with bilinear and time-varying inputs as described below.

Linearization Model Setup The base model contains all the internal thermal dynamics of the building, namely heat conduction, convection and radiation exchange.

Heat conduction within bodies of invariant thermal properties is a linear process and as such, no information is lost in the linearization step:
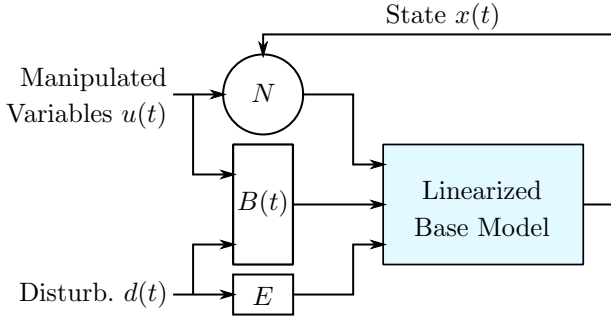
Fig. 2. Block diagram representation of the bilinear system.

$$\dot{Q}_{\text{cond}} = A \lambda \frac{1}{d} \cdot \Delta T \qquad (3)$$

with the constant conductivity λ, cross sectional area A and heat conduction distance d.

The convective heat exchange between a surface of temperature and the surrounding air (both room and ambient air) is

$$\dot{Q}_{\text{conv}} = \alpha \cdot \Delta T , \qquad (4)$$

where α is a variable coefficient that depends on the surface roughness and geometry as well as the air flow velocity and ΔT is the difference between surface and air temperatures. In the field of building models, heat exchange by natural convection is commonly expressed by empirical algorithms such as TARP Walton (1983). The nonlinear equations for α are evaluated at the operating point, thus reducing model fidelity if ceiling or floor heat fluxes are inverted.

The radiant portion of the heat exchange is also a nonlinear process w.r.t. the surface temperatures (given for gray surfaces of emissivity ε):

$$\dot{Q}_{\text{rad},ij} = A_i \cdot F_{ij} \cdot \varepsilon \cdot \sigma \cdot \left(T_i^4 - T_j^4 \right) . \qquad (5)$$

The view factor F_{ij} and surface area A_i are geometric constants, σ is the Stefan-Boltzmann constant. The linearization of this equation w.r.t. the temperatures around a point T_0 yields

$$\dot{Q}_{\text{rad,lin},ij} = A_i \cdot F_{ij} \cdot \varepsilon \cdot \sigma \cdot 4 \cdot T_0^3 \cdot (\Delta T_i - \Delta T_j) . \qquad (6)$$

The heat flow error for a deviation of $\Delta T_i = 20\,\text{K}$ from a linearization point $T_0 = 293.15\,\text{K}$ is about $2.2\,\%$, which is an acceptable value for the considered temperature range.

2.3 Bilinear Inputs

The linearized base model only comprises the internal dynamics of the building. Various system inputs need to be added to describe the interaction with the environment as well as control inputs. An overview of the whole bilinear system is given in Fig. 2. The new inputs are connected to heat flow inputs for the facade and window surface as well as radiant and convective heat flows into the room which were defined in the base model.

Facade Longwave Heat Exchange   The longwave heat exchange is a process that is nonlinear in the temperatures of the involved surfaces as given in (5), in this case the surface temperature of the facade and the sky temperature. The sky temperature can be measured in a weather station. The nonlinear impact is transformed to a linear disturbance input using (6).

Window Shading   Window shading prevents solar radiation from entering the building. The radiation that passes the window is added to the zone as a radiant heat flow. For an unshaded window it can directly be determined from measured solar irradiance, the geometrical conditions of window surface and solar position and the g-value of the window.

An approximation of shading with blinds is to assume a geometrical shading coefficient as the fraction of the window area which is obstructed for direct shortwave radiation. Diffuse radiation is always passing into the zone completely. The direct radiation entering the building is then

$$\dot{Q}_{\text{rad,win}} = (1 - GSC) \cdot I_{\text{irr,unshaded}} , \qquad (7)$$

with the irradiation entering the zone through the unshaded window given as $I_{\text{irr,unshaded}}(t)$.

It is apparent that the manipulated variable $(1 - GSC)$ is multiplied with a disturbance and thus not directly compatible with the bilinear formulation. To solve this conflict, the geometrical passing coefficient $GPC = (1 - GSC)$ is defined as the control input and $I_{\text{irr,unshaded}}(t)$ as a time-varying input coefficient, effectively turning the model into a bilinear time-varying system.

Variable Shortwave Absorptance   The surface absorptance $\alpha \in [0, 1]$ defines the fraction of the solar shortwave radiation that is absorbed by the facade surface. The absorbed radiation results in a heat flow into the facade surface:

$$\dot{Q}_{\text{abs,surf}} = \alpha \cdot A \cdot I_{\text{irr,surf}} . \qquad (8)$$

For most materials, the shortwave absorptance is almost constant. Adaptive facade structures allow for adjustable absorptance and thus provide a novel control input for the thermal conditioning of the indoor air.

Similarly to window shading, the upper limit on the absorbed radiation is the time-varying total incident irradiation of the surface $I_{\text{irr,surf}}(t)$ for $\alpha = 1$. The manipulated variable is chosen as α, entering the linear model through a time-varying input coefficient $I_{\text{irr,surf}}(t)$.

Thermal Energy Storage   Thermal mass is, together with insulation, the most important defining property of the thermal behavior of a building. To enable control of the effective mass of the building, a thermal energy storage (TES) is introduced. For the purpose of room temperature improvement in lightweight buildings, such systems have been investigated e.g. by Hoes (2014).

In this work, the TES is assumed to be a water tank of constant volume with pumps that can move the water through radiators either within the building or outdoors. The tank is otherwise assumed to be perfectly insulated. This virtually enables a deliberately controllable increase of the internal thermal mass of the building.

For the TES model, another system state for the tank water temperature T_{TES} has to be introduced. Additionally, there are 2 new manipulated variables α_{in} and α_{amb} for the

heat exchange coefficient between tank water and indoor or ambient air respectively.

The convective heat exchange between the TES water and the zone air is given by the following bilinear equation:
$$\dot{Q}_{\text{TES}\to\text{zone}} = \alpha_{\text{in}} \cdot (T_{\text{TES}} - T_{\text{zone}}) \ , \qquad (9)$$
with the zone and water temperatures being system states and the input α_{in}.

The heat exchange from TES to ambient air is given by
$$\dot{Q}_{\text{TES}\to\text{amb}} = \alpha_{\text{amb}} \cdot (T_{\text{TES}} - T_{\text{amb}}) \ . \qquad (10)$$
While the multiplication of the manipulated variable α_{amb} and the system state T_{TES} constitutes a bilinear input, the second part of the equation is a multiplication of manipulated variable and a disturbance T_{amb}. Again, this is modeled as a time-varying input gain.

Ventilation Air exchange with the environment has a large impact on both occupant well-being and the building's thermal behavior. For this reason, the supply rate of fresh outdoor air is an important control input.

The effect of an incoming airflow is described by the resulting enthalpy flows. All air is assumed to be of equal moisture content. The enthalpy of the ambient air flowing into the zone is
$$\dot{H}_{\text{in}} = \dot{m}_{\text{vent}} \cdot c_{\text{p}} \cdot T_{\text{amb}} \qquad (11)$$
with mass flow $\dot{m}_{\text{vent}}$ (the manipulated variable) and the constant isobaric thermal capacity of air c_{p}. As before, the multiplication of disturbance and manipulated variable is modeled as a time-varying input.

Assuming mass balance, the resulting outgoing enthalpy flow is bilinear:
$$\dot{H}_{\text{out}} = \dot{m}_{\text{vent}} \cdot c_{\text{p}} \cdot T_{\text{zone}} \ . \qquad (12)$$

Variable Thermal Conductivity A typical example for adaptivity of facades is variability of the insulation. Especially in the summer, low conductivity at daytime reduces the heat gain through the facade, whereas a high conductivity at night helps cool down the interior of the building.

In this work, the conductivity of one construction layer of the facade can be varied by multiplying (3) with a variable factor k_λ, yielding the inter-nodal heat flow
$$\dot{Q}_{\text{cond},\text{var},i\to j} = k_\lambda \cdot A\lambda \frac{1}{d} \cdot \Delta T_{i,j} \ . \qquad (13)$$
The principle is depicted in Fig. 3.

The wall node temperatures are states of the linear base system and are internally fed back as
$$\Delta T_{i,j}^* = k_\lambda \cdot (T_j - T_i) \ . \qquad (14)$$
The virtual temperature difference $\Delta T_{i,j}^*$ is then used as an input to the linear model to compute the resulting heat flows based on the original material properties. Using the temperature differences as inputs is equivalent to directly defining the insulation's internodal heat flow, but reduces the implementation effort. With the manipulated variable k_λ, variable conductivity is a bilinear input.

2.4 Linearized Comfort Model

To determine the comfort of building occupants in a certain environment, empirically derived comfort models
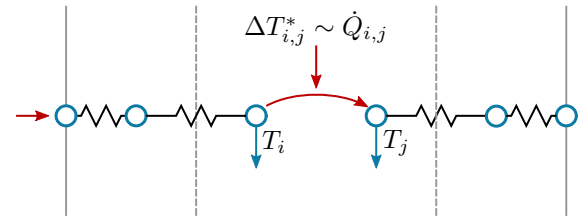

Fig. 3. In- and outputs for the variable conductivity.

for the average occupant are commonly used. The most prevalent model for human comfort is the so-called predicted mean vote (PMV) developed by Fanger (1970). A comfortable thermal state is defined as PMV = 0, with warm sensations for positive and cool sensations for negative values.

The algorithm used to calculate the PMV according to EN 16798 is nonlinear and contains an optimization routine, but exhibits mainly linear properties (Guenther and Sawodny, 2019). To add the PMV comfort model to the bilinear model, a linear approximation containing the variables clothing insulation (CLO), mean radiant temperature and air temperature is derived from random samples of the function on the intervals given in Table 1 by means of a least squares regression, yielding
$$\begin{aligned} \text{PMV}_{\text{lin}} = &- 7.45 + 0.14 \cdot T_{\text{air},\text{C}} \\ &+ 0.11 \cdot T_{\text{rad},\text{C}} + 1.84 \cdot \text{CLO} \ . \end{aligned} \qquad (15)$$

In the chosen region, the maximum absolute deviation of the linearized model from the PMV algorithm is $\hat{e}_{\text{PMV}} = 0.23$ which is deemed acceptable.

Table 1. The parameter ranges for the linearized PMV model.

Variable	Unit	Min.	Max.	Const.
Clothing insulation	CLO	0.5	1.0	-
Temperatures	°C	18	28	-
Relative humidity	%	-	-	50
Air velocity	m s^{-1}	-	-	0.1
Metabolic rate	MET	-	-	1.2

3. RESULTS

In this section, the validation results of the model are briefly summarized and an application example for a model predictive controller is shown.

3.1 Validation of the Bilinear Model

For the validation of the bilinear modeling approach against the nonlinear reference model, a set of test scenarios is devised. The room is equipped with a convective heating unit which keeps the temperature between a heating and cooling setpoint at 20 °C and 26 °C respectively.

In each test case, a single actuator is cycled between a low and high level for 3 h each. Table 2 lists the actuation variables and their respective minimum and maximum values. The relative error between the energy requirements of the bilinear model and the reference is recorded for one month and also given in the table. For the conductivity multiplier, the absolute energy consumption is low (1.18 kWh/d), which explains the high relative error.

The results show sufficient fidelity for the intended use as a predictor in MPC.

Table 2. Validation scenarios for the bilinear model.

Actuator	Unit	Low	High	Error (%)
Base test case	-	-	-	13.2
Radiant heating	W	−1000	1000	13.2
Ventilation	$\mathrm{kg\,s^{-1}}$	0	0.1	−6.6
TES $\alpha_{\mathrm{in/out}}$	$\mathrm{W\,K^{-1}}$	0	500	−4.4
Conductivity multiplier	-	1	10	23.1
Window shading	%	0	100	−0.8
Facade absorptance	-	0.1	1.0	11.5

3.2 Model Predictive Control

Given the time-varying constraints imposed by comfort and building physical requirements as well as available energy, the availability of forecasts for the disturbance inputs and the nonlinear nature of the system equations, MPC is a suitable strategy for the calculation of control inputs.

In MPC, control inputs are determined by repeatedly solving an optimization problem. Given an objective function and a set of constraints, the optimal state and input trajectory is calculated for a prediction horizon and then applied for a certain time before restarting the optimization. A thorough introduction into the topic is given by Rawlings et al. (2017).

The nonlinear optimization problem is solved with the multiple shooting method using the open-source toolbox CasADi for numerical optimization (Andersson et al., 2019). CasADi uses the interior point algorithm making use of symbolic derivatives for an efficient problem solution.

The result of the solver is then applied to the reference model through the Functional Mockup Interface (FMI) and simulated.

3.3 Simulation Results

The MPC prediction horizon is set to 12 h with a control interval of 1 h. The objective function penalizes the deviation from the comfortable conditions according to the PMV at any time and includes a regularization term for control effort and state deviation. To demonstrate the effectiveness of adaptive actuation, no conventional heating or cooling is present in the system. Transient terms from the initial conditions are avoided by running the simulation for multiple warm-up days.

The resulting state and actuation trajectories for one representative day in July are presented in Fig. 4. It can be seen that the control system predictively uses the available actuators to prevent overheating and manages to keep user comfort levels close to optimal. The slight deviation of the PMV from 0 is mainly due to the effect of air humidity. Both TES and the building structure are cooled down in periods of cool ambient air as evident by the TES actuation and the variable conductivity. Both window shading and facade absorptance are set to avoid any further heat gains and remain unchanged for periods without solar irradiation.

4. CONCLUSION

In this work, a bilinear building control model was developed based on the linearization result of a Modelica reference model of an adaptive building. The model was then used to create a model predictive controller for the reference model, using conventional actuation as well as adaptive properties.

The MPC was shown to perform well in simulation of the reference model, ensuring user comfort without the need for conventional heating or cooling equipment in a summer scenario.

4.1 Future Work

The developed model and controller will be used for an in-depth potential analysis of various facade adaptation methods. Based on the results, a direction for further research on the practical implementation of these abstract adaptations can be determined. The objective function of the optimization problem will subsequently be extended to incorporate the real energy consumption of the realized adaptive components. The simplified thermal notion of comfort should also be extended to include e.g. humidity and brightness.

The optimization problem may further be solved more efficiently by using the properties of the bilinear formulation instead of a generic nonlinear solver.

A demonstrative high rise building is constructed as part of the Collaborative Research Center 1244. When equipped with adaptive facade components, it will provide an opportunity to experimentally prove the developed concepts.

5. ACKNOWLEDGMENTS

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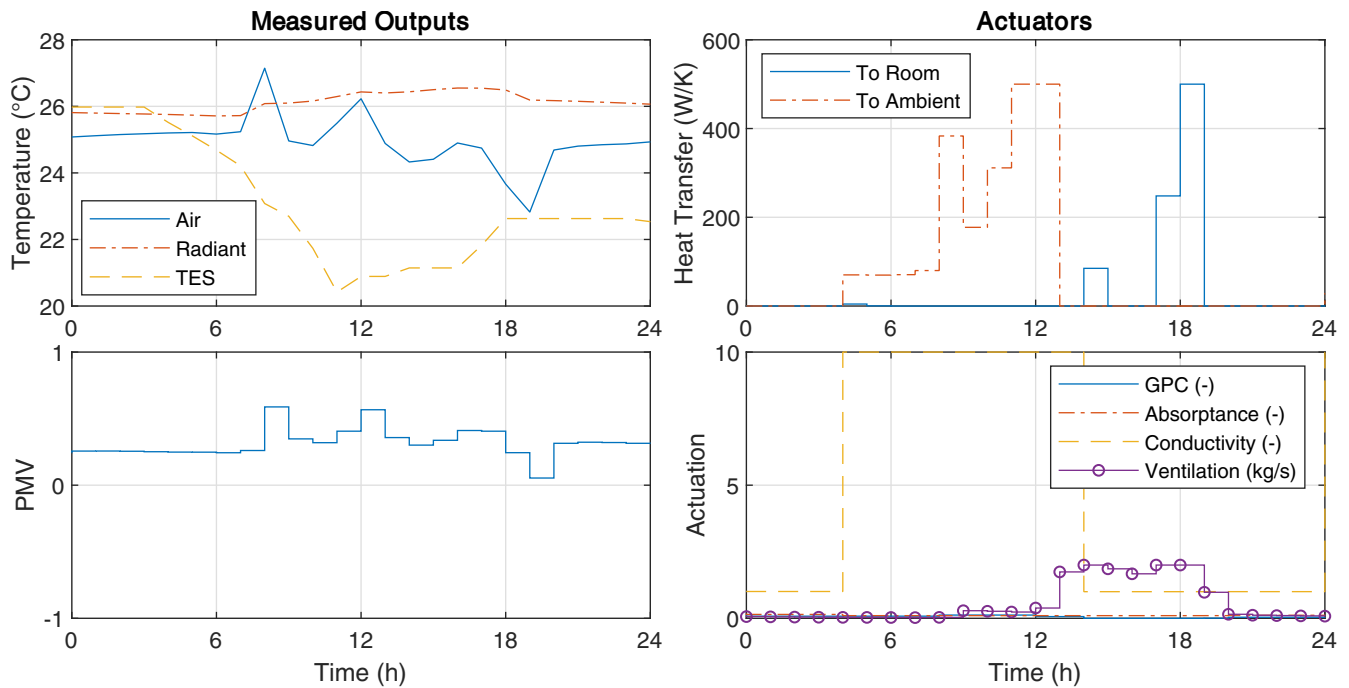
Fig. 4. Temperatures, comfort and actuation for one day of a summer test case.

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