Utilisation of the Controllable Inertial Morphing for Providing Spacecraft with Acrobatic Attitude Capabilities. *

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Abstract: The paper is exploring a new method of controlling of the attitude dynamics of the spacecraft with non-zero angular momentum, using deliberately applied changes to the spacecraft inertial properties, called inertial morphing (IM). This method does not employ classical gyroscopic devices, nevertheless it enables the spacecraft to perform various acrobatics manoeuvres, allowing interchanges between stable and unstable states. In one case scenario, it enables transformation of the stable spin into unstable flipping motion and establishment of the desired periods of the flips at various stages of the procedure. Special consideration is given to the selection of the controllable morphed parameters to impose the desired periods and patterns of the acrobatics. This paper exploits use of the unstable flipping motions of the systems and due to established mini-max relationships for the flipping periods, enables selection of the system parameters, maximizing or minimizing the values of the periods for faster (more agile) maneuvers. In the other scenario, IM is used to transfer the regular spin about one body axis into the regular spin about another nominated body axis. Numerous illustration cases are presented and application of the new enhanced capabilities are discussed in detail. For example, paper presents a scenario of the reconfiguration of the articulated spacecraft with its segments being inverted during the acrobatic procedure in the desired way, which may open new possibilities during the spacecraft operation, including re-boost and landing.

Keywords: Spacecraft, angular momentum, attitude dynamics, inertial morphing, regular spin, tumbling, flipping, period, control, acrobatics.

1. INTRODUCTION

A phenomenon of periodic flips of the rigid body, spun about its intermediate axis is attributed to the “Tennis Racquet Theorem” and is often called the “Dzhanibekov’s Effect”, named after V.A.Dzhanibekov, who discovered it in space in 1985 while observing flying spinning wing nuts. However, famous USA scientist-astronaut Owen Kay Garriott, even earlier, in 1973, has performed his experiments with boxed rigid objects and also observed the flipping patterns, associated with their spin, conceptually predicted by Beachley (1971). For more information, please, refer to Trivailo & Kojima (2019 RaES); Murakami (2016).

2. INTRODUCTION INTO INERTIAL MORPHING

It has been realised that if the spinning system is designed in a way, where the inertial properties could be changed in the desired way, using, for example, mechanical or electromagnetic means, than these controls could be used for manipulations with attitude dynamics of the system. For extensive discussions on the topic interested reader may refer to the following references: Trivailo & Kojima (2019 ISSFD); Trivailo & Kojima (2019 JSASS); Trivailo & Kojima (2018 IAC); Trivailo & Kojima (2017 IAC); Trivailo & Kojima (2017 ISSFD), Noack (2019).

These are some examples of the system, admitting IM: spacecraft, deploying solar arrays; system with linear motors to re-position concentrated masses; systems, releasing pre-constrained masses, etc. (Trivailo & Kojima (2019 ISSFD)).

One of the scenarios of application of the inertial morphing may involve stabilisation of the flipping motion: this task may have two conceptual solution, at the expense of gaining or loosing angular velocity of the predominant spin (see Trivailo & Kojima (2017 ISSFD)). In another case, IM can be used for initiation of the flipping motion on the system, being initially in the stable spin.

This paper presents two detailed cases on using controlled IM: (1) first example demonstrates controls, enabling spinning spacecraft to performing acrobatic 180° inversion and allowing to use the same thruster for boosting and braking stages; (2) second example shows 90° inversion, enabling transition of the spin about one body axis into the spin about another nominated orthogonal axis.
3. ANALYSIS OF THE PERIODIC FLIPPING MOTIONS

3.1 General Equations for calculation of the period T

The period of the unstable flipping motion can be calculated, using Eq. (37.12) in page 154 from the reference by Landau (1988):

\[ T = 4K \sqrt{\frac{I_{xx} I_{yy} I_{zz}}{(I_{zz} - I_{yy})(H^2 - 2K_0 I_{xx})}} \]  

(1)

where \( K \) is complete elliptic integral of the first kind

\[ K = \int_0^1 \frac{ds}{\sqrt{(1 - s^2)(1 - k^2 s^2)}} = \int_0^{\pi/2} \frac{du}{\sqrt{1 - k^2 \sin^2 u}} \]  

(2)

being a function of the parameter \( k^2 \)

\[ k^2 = \frac{(I_{yy} - I_{xx})(2K_0 I_{zz} - H^2)}{(I_{zz} - I_{yy})(H^2 - 2K_0 I_{xx})} \]  

(3)

It should be noted that Eqs. (1)-(3) correspond to the case of \( H^2 > 2K_0 I_{yy} \). If this condition is not observed, in Eqs. (1)-(3) subscripts “xx” and “zz” must be interchanged.

Eq.1 has been used (Trivailo & Kojima (2019 RaES)) to investigate selection of the value of the intermediate moment of inertia \( I_{yy} \) on the flipping period.

However, as the detailed analysis on the influence on the flipping period \( T \) of the angular velocity of the predominant rotation: \( \omega_y = 6 \) and \( \omega_y = 12 \) rad/s. The intersection lines of these level panes as a 3D surface plot, explaining influence on the predominant spinning angular velocity \( \omega_y \) and value of the intermediate moment of inertia \( I_{yy} \) for the system with \( I_{xx} = 3 \), \( I_{zz} = 3.5 \text{ kg}\cdot\text{m}^2 \) (i.e. \( \eta = 0.8571 \)).

3.2 Influence of the value of the angular velocity \( \omega_y \) of the predominant spin on the period T

In this paper we consider systems with non-zero initial angular momentum \( H_0 \). In case of the system with predominant spin about its intermediate axis, the major contributors to \( H_0 \) are the \( \omega_y \) and \( I_{yy} \).

Using Eq. 1, we can represent \( T \) as a 3D surface plot, explaining influence on the \( T(\omega_y, I_{yy}) \) function of its two argument. The resultant plot is shown in Fig. 1. It clearly reveals the tendency of the periods to become very large, when \( I_{yy} \) is approaching to the \( I_{xx} \) or \( I_{zz} \). However, the plot also reveals the ridge of high value of periods, being asymmetrically in between \( I_{xx} \) and \( I_{zz} \). As surface gradient is very high in vicinity of the ridge, it should be avoided for practical implementations, because in this area \( T \) would be very sensitive to small changes in \( I_{yy} \) which would make control of the system period impractical.

As an example, in Fig. 1 we also intersect the \( T \) surface with two illustrative level values of the period: \( T = 30 \) s and \( T = 50 \) s. The intersection lines show that for the desired value of the flipping period \( T \), there are multiple matching combinations of \( \omega_y \) and \( I_{yy} \), however, if the goal of the selection is to minimise the spin rate, then, there may be two local minimum specific values of \( I_{yy} \). Two contour lines for \( T = 30 \) s and \( T = 50 \) s are shown separately in Fig. 2. It shows, that, if, for example, the aim of the design process is to keep \( \omega_y \) low, for the \( T = 50 \) s, there are two solutions for \( I_{yy} \), approximately equal to \( I_{yy} = 3.335 \) and \( I_{yy} = 3.18 \).

3.3 Influence of the value of the period T on the angular velocity \( \omega_y \) of the predominant spin

Fig. 1. Variation of the period \( T(\omega_y, I_{yy}) \) of the flipping motion with the changes in the predominant spinning angular velocity \( \omega_y \) and value of the intermediate moment of inertia \( I_{yy} \) for the system with \( I_{xx} = 3 \), \( I_{zz} = 3.5 \text{ kg}\cdot\text{m}^2 \) (i.e. \( \eta = 0.8571 \)).

Results in Fig. 1 are presented for the \( \omega_y = \omega_y(T, I_{yy}) \), being a function of two arguments, \( T \) and \( I_{yy} \). This function is shown Fig. 3 as a 3D surface plot. For the illustration purposes, we assume interest in two special values of angular velocity of the predominant rotation: \( \omega_y = 6 \) and \( \omega_y = 12 \) rad/s. The intersection lines of these level panes...
Fig. 3. Variation of the initial spin rate $\omega_y(T, I_{yy})$ with the changes in the period of the flipping motion $T$ and value of the intermediate moment of inertia $I_{yy}$ for the system with $I_{xx}=3$, $I_{zz}=3.5 \text{ kg} \times \text{m}^2$ (i.e. $\eta=0.8571$).

Fig. 4. Contour lines for the $T(\omega_y, I_{yy})$ surface in Fig. 3 with in-between bands individually colored. Red band corresponds to the $\omega_y=4-6$ (rad/s) spin rate range; yellow band - to the $\omega_y=6-8$ range; green band - to the $\omega_y=8-10$; cyan band - to the $\omega_y=10-12$ and blue band - to the $\omega_y=12-14$ range and purple band - to the $\omega_y=14-16$ range.

and the $\omega_y$ surface, together with other contour curves, are given in Fig. 4.

3.4 Numerical Simulation of the Systems with IM

In order to be able to simulate systems with IM, we need to extend Euler’s equations, to allow variation in the moments of inertia of the system to be taken into consideration. These extended equations are:

\[
\begin{bmatrix}
\dot{I}_{xx} & 0 & 0 \\
0 & \dot{I}_{yy} & 0 \\
0 & 0 & \dot{I}_{zz}
\end{bmatrix}
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} +
\begin{bmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix} +
\begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}
\begin{bmatrix}
I_{xx} & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} (4)
\]

These equations can be solved numerically, using various methods. For details and examples of solutions, please, refer to Trivailo & Kojima (2017 ISSFD) and Trivailo & Kojima (2019 RaES).

3.5 Geometric Interpretation of Solutions

We employ graphical interpretation of the flipping motion, using angular momentum unit sphere (AMS) and kinetic energy ellipsoid (KEE), constructed in the non-dimensional coordinates of the normalised angular momentum $H$. The values of the semi-major axes of the ellipsoid are given with the following relationships:

\[
a_x = q\sqrt{I_{xx}(t)}; \quad a_y = q\sqrt{I_{yy}(t)}; \quad a_z = q\sqrt{I_{zz}(t)}; (5)
\]

where $q = \sqrt{K(t)/H(0)}$.

For the illustration system with $I_{xx} = 3$, $I_{yy} = 4$, and $I_{zz} = 5$, the AMS is shown with light blue color and KEE is shown with semi-transparent red ellipsoid. The line of intersection between both is known as polhoid and for the flipping case also corresponds to the separatrices, shown with thick black lines. Interestingly, that from Eq. 5 it follows that $a_x/a_z = \sqrt{I_{xx}/I_{zz}} = \sqrt{3/5} = 1.1547$, which explains the extent of the bulging of the part of the KEE over the AMS along the $\pm H_z$ direction.

In our methodology, in order to perform fast and agile attitude transformations of the systems, we use IM to transfer the system into the state, when its angular momentum vector is sliding along the separatrix, therefore, we call this method “insertion into separatrix”.

\[15107\]
Spacecraft is in stable $y$-axis spin: thruster is used for boost.

Morphing-1 is activated to initiate $y$ flips.

180 degrees $y$ flip is completed.

Morphing-2 is activated to stop $y$ flips.

Spacecraft is in stable $y$-axis spin: thruster is used for braking.

Fig. 6. Example of application of inertial morphing for inversion of the spacecraft, allowing to use the same thruster for the boosting and braking stages.

3.6 Application of Inertial Morphing: Detailed Example-1

Figure 6 is dedicated to one (out of many) possible applications of the IM. It illustrates inversion of the spacecraft, using two morphing procedures: the first is to activate unstable flipping motion and second is to stop flipping motion. This enables for a single thruster to be used for acceleration of the spacecraft (boosting stage, Fig. 6a) and also for its deceleration (braking stage, Fig. 6e).

To demonstrate the feasibility of the proposed application, let us consider six-masses model of the spacecraft with corresponding numerical parameters $m_x=4$ kg, $m_y=5$ kg; $m_z=1$ kg. The key requirement to this design would be ability of the system to re-position six paired masses (for example, via linear actuators), in accordance to the control considerations. Let us assume that the mission profile would enable installation of the spacecraft with initial predominant spin about $y$ axis (with angular velocity $\omega_y = 8.57$ and other components of $\omega$ being small: $\omega_x=\omega_z=0.01$, all in rad/s). If the masses are initially positioned at their locations, shown in Fig 7 with white spheres, with position radii equal to $r_{x1}=548$, $r_{y1}=510$, and $r_{z1}=447$ mm correspondingly, then the inertial properties of the spacecraft would be $I_{xx,0}=3; I_{yy,0}=2.8; I_{zz,0}=5$ (all in kg×m²). With these selected parameter, the $y$ spin of the system would be stable, as axis of rotation coincides with the minimal inertia axis. Let assume that at the instant $t=10$ s we wish to initiate flipping motion of the system. This can be achieved by applying inertial “Morphing-1”, during which moment of inertia $I_{yy}$ should become an intermediate axis. Aiming for the new moments of inertia to be $I_{xx,1}=3; I_{yy,1}=2.8; I_{zz,1}=5$ (kg×m²), we calculate new radii for the masses, using the following relationships:

$$
I_{xx} = 2 (m_y r_y^2 + m_z r_z^2); \\
I_{yy} = 2 (m_z r_z^2 + m_x r_x^2); \\
I_{zz} = 2 (m_x r_x^2 + m_y r_y^2).
$$

Resultant values of the new position radii, ensuring transition of the system from stable regular spin to the flipping unstable motion are: $r_{x1}=581$, $r_{y1}=480$ and $r_{z1}=592$ mm. Therefore, to trigger the spinning motion, it is just necessary to move paired masses from initial positions (shown in Fig. 7 with white spheres) to their new positions, shown with black spheres. For better perception of the 3D design, an imagined semi-transparent $xz$ plane is added to the figure. For the system, initially satisfying $I_{yy,0} < I_{xx,0} < I_{zz,0}$ condition, rapid assignment at $t=10$ s of the new moments of inertia, satisfying now $I_{xx,1} < I_{yy,1} < I_{zz,1}$ condition, transfers regular $y$ spin motion into unstable spin. Its period can be calculated using Eq. 1 for the corresponding regular spin conditions at $t=10$ s: $I_{xx,1}=3, I_{yy,1}=4.8, I_{zz,1}=5$, $\omega_x=0.2917$, $\omega_y=5.0284$, $\omega_z=-0.6714$. Calculations gives us $T=17.784$
This paper presents details of the new method of control of the attitude dynamics of the spacecraft, which does not require conventional gyroscopic devices, but instead of the spacecraft inertias $I_{xx}$, $I_{yy}$, $I_{zz}$; (b) time history of the angular velocities $\omega_x$, $\omega_y$, $\omega_z$.

(c) time history of the angular momentum $\mathbf{H}$ components.

Fig. 8. Realisation of the spacecraft “y”-axis inversion (shown in Fig.6) via two inertial morphings (as per Fig.7). s, therefore, the time to $180^\circ$ flip would be $T/2=8.8920$ s. At this very moment, rapid “Morphing-2” should be applied to stop flipping phase and stabilise the system. For this morphing, in accordance with Trivailo & Kojima (2017 ISSFD) any set of new moments of inertia can be selected, strictly satisfying any of the two conditions: $I_{yy,2} < I_{xx,2} < I_{zz,2}$ (“solution $I_{yy}$ min” strategy) or $I_{xx,2} < I_{zz,2} < I_{yy,2}$ (“solution $I_{yy}$ max” strategy). In one case scenario, return to the initial moments of inertia can be implemented. However, as an additional example, we illustrate implementation of the $I_{xx}=3$, $I_{yy}=5.2$, $I_{zz}=5$ (“solution-1” strategy) scenario. To achieve these new inertia characteristics, as per Eq. 6, spacecraft control masses should be rapidly moved to their final positions: $r_x=671$, $r_y=374$ and $r_z=894$ mm. The described morphings (achieved via controlled changes in $r_x(t)$, $r_y(t)$ and $r_z(t)$), time histories for the resulting moments of inertia, angular velocity and angular momentum components are presented in Fig. 8. Interestingly, that rapid changes in $\omega_y$ and $I_{yy}$ do not lead to similar changes in $H_y$. Also, as evidenced by Fig. 8c, the total angular momentum $|\mathbf{H}|$ is conserved during the flipping acrobatics.

The same principle can be used for complete reconfiguration (“re-packaging”) of the articulated compound spacecraft, consisting, for example, of three segments $A_1$-$B_1$, $B_2$-$C_2$, $C_3$-$D_3$. If all of these segments are un-docked, they can independently perform the same flipping maneuvre, as described above, then (after stabilisation) docked to a new configuration $B_1$-$A_1$, $C_2$-$B_2$, $D_3$-$C_3$. Note, that the re-configuration can be applied to the selected segments only and to the spacecraft with any number of segments. For example, if only central segment $B_2$-$C_2$ is inverted, the new configuration would be $A_1$-$B_1$, $C_2$-$B_2$, $C_3$-$D_3$.

3.7 Application of Inertial Morphing: Detailed Example-2

Figure 9 is dedicated to another possible applications of the IM. It illustrates $90^\circ$ change of axis of spin of the spacecraft, using three morphing procedures: the first is to activate unstable flipping motion; second is to switch to $x$ separatrix and the third is to stop tumbling motion with transfer of spacecraft spin from $y$ to a new nominated body axis, being $x$ in this illustration example. It should be noted, that in Fig. 10 initial and final spin axes are presented in body axis system. And in the global axis system, due to the law of conservation of angular momentum, both, initial and final spin orientations are aligned with the same direction of the angular momentum $|\mathbf{H}|$ vector.

Spacecraft morphing parameters, corresponding to Fig.9-10, are shown in Table 1. Initially system is in stable spin (shown as IM0 in table). IM1 was designed to insert the system into ”y” separatrix, similar to shown in Fig.5. This initiates a ”y” flip, during which IM2 is applied, forcing the system to transfer to ”x” separatrix. So, this acrobatics involves two “installments into separatrices”. Important control consideration is: IM2 is applied at the instant when in body axes vector $\mathbf{H}$ (to be displayed in boxy axes, as in Fig.5) passes intersection of ”x” and ”y” separatrices. IM3 is designed to stop ”x” flips in the same way as shown in Fig.8.

4. CONCLUSION
is using inertial morphing. Resulting in a lighter and simple hardware implementation (with simple mechanical, electrical and/or magnetic devices, capable of changing the mass distribution of the system), it may be used on the smaller spacecraft. The key of the proposed efficient control method is in geometric interpretation of the changes of the kinetic energy ellipsoid of the system during applied inertial morphing. This interpretation enables determination of the controls for the desired transformation of the initial attitude motion into another type of motion with desired characteristics. This method exploits unstable flipping motion of the system, which occurs when the system is provided with the predominant spin about its intermediate axis of inertia. For the desired acrobatics, the system is deliberately transferred into the flipping mode with the pertinent separatrices, and then is switched to the new inertial properties at the instants, corresponding to the intersection with the separatrices, typically corresponding to different body axis, pertinent to the new desired flipping direction. After the transfer of one unstable flipping into another flipping is achieved, the motion can be further stabilised when the angular momentum vector reaches one of two pole positions. The paper also presents a method of selection of the morphing parameters to secure the flips with desired periods. To illustrate the application of the presented methodology, an articulated spinning spacecraft is considered. During the demonstration, its segments were disconnected, enabling some segments to perform inversions via flips with further inertial morphing stabilisation and finally consolidation of the spacecraft inverted components. This can be, for example, useful at the landing phase of the mission, when the instruments from the central part of the spacecraft are repositioned to the front.

REFERENCES


