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Enhanced cooperative single-range underwater navigation based on optimal trajectories

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Abstract: This work addresses the observability analysis for a cooperative range-based navigation system based on the optimization of an index. A nonlinear model is first defined in order to describe the motion of the vehicle and a mobile beacon. Then, the Fisher Information Matrix is introduced to explain how it is related with the observability problem. A unconstrained optimization problem is formulated in order to find the best sequence of actions for the beacon to ensure observability in the system; the unconstrained problem does not take into account physical limitation of the vehicle and beacon. Then, four different scenarios are solved using different constraints; we show that, when the beacon is rotating with variable angular velocity we get a better strategy than rotating with constant velocity, despite that in both scenarios the system is observable. Finally, we show that increasing the energy provided to rotate the beacon does not improve further the observability of the system. These results are important from a theoretical and practical point of view, since they represent a strategy to plan the motion of the beacon to guarantee observability in the system.

Keywords: Marine system navigation, guidance and control; Acoustic-Based Networked Control and Navigation; Cooperative control.

1. INTRODUCTION

In recent years, cooperative robotics has become an interesting field for the underwater robotics community. Different task can be accomplished through the cooperation of robotic systems; for instance, manipulation and transportation of underwater objects (Casalino et al., 2015); underwater mapping (Djapic et al., 2018); environmental monitoring (Bayat et al., 2017), among others.

Particularly, cooperative robotics in the field of underwater navigation has managed to generate interest in the scientific community due to its advantages. For instance, Rui and Chitre (2010) presented a cooperative positioning system between two AUVs; the idea was to use one vehicle to localize, while the other one was executing a lawnmower path over the survey area. Fallon et al. (2010) developed an algorithm for cooperative AUV navigation with an autonomous surface craft; they developed a path planning algorithm for the surface vehicle, and the AUV was able to localize itself with respect to the surface vehicle. Webster et al. (2013) reported a decentralized extended information filter for single beacon cooperative navigation between vehicles; they used ranges and state information from a single reference, the other vehicles were able to improve their localization. Parlangeli and Indiveri (2015) described the single-range observability issues related to cooperative underactuated underwater vehicles; they described all possible unobservable motions for the vehicles given the initial conditions and the velocity commands. Tan et al. (2014) explored the use of a single beacon vehicle for range only localization to support other AUVs; they developed a cooperative path-planning algorithm for the beacon based on dynamic programming and Markov decision formulation. Mandić et al. (2016) developed a mobile beacon control algorithm that ensures observability for single range navigation using a cost function based on the rank condition; the goal of the algorithm was to reduce this cost as much as possible.

In the work of Rúa et al. (2019), a new underwater navigation system has been proposed and it is based on a beacon with circular motion installed on board a support platform (see Fig. 1); it was proved that under certain conditions the system becomes observable just by rotating the beacon. Motivated by this work, this paper addresses the observability problem of the same mechanism based on the Fisher Information Matrix (FIM). By using the

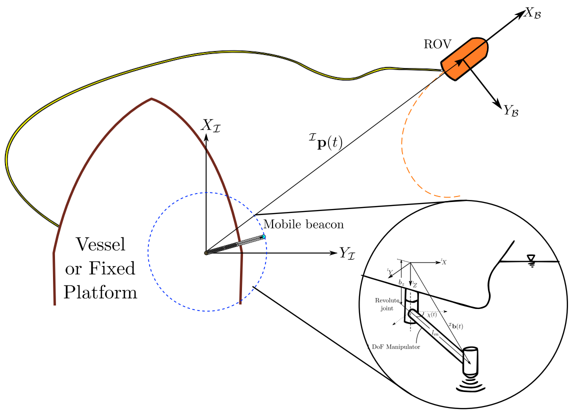


Fig. 1. Cooperative underwater system proposed in Rúa et al. (2019)

FIM, we find out the best sequence of actions for the beacon that improves accuracy on the estimation. All the analyses carried out in the work of Rúa et al. (2019) about the system observability had been made from a yes or no point of view. Here, we proved that the accuracy on the estimation can be improved by rotating the beacon with a sequence of actions, and those actions are better than just letting the beacon rotate with constant velocity.

Notation. The Euclidean norm in $\mathbb{R}^n$ is denoted by $||\cdot||$. Let $\mathbf{w}(\zeta) := [\cos(\zeta)\ \ \sin(\zeta)]^\top$ and $\mathbf{w}^\perp(\zeta) := [-\sin(\zeta)\ \ \cos(\zeta)]^\top$ be orthonormal vectors with $\zeta \in [0, 2\pi)$. The expected value of a random vector $\boldsymbol{X} \in \mathbb{R}^n$ is denoted as $\mathbb{E}\{\boldsymbol{X}\}$.

2. PROCESS MODELLING AND PROBLEM FORMULATION

To analyze the motion of the vehicle and the beacon, two coordinate frames are defined: an Inertial Earth-fixed frame $\{\mathcal{I}\}$ (this frame is considered as the North, East, Down frame NED); attached to a port facility or a stationary support vessel), where the motion of the vehicle is described, and a body-fixed frame $\{\mathcal{B}\}$, which is conveniently fixed to the vehicle and moves with it. Additionally, consider a beacon which is attached or deployed under the vessel or fixed support platform. The objective of this beacon is to be active in order to guarantee observability of the system. Then, the kinematics equations of the vehicle and beacon are given by

$$\left.\begin{aligned} {}^{\mathcal{I}}\dot{\mathbf{p}}(t) &= v(t)\mathbf{w}(\psi(t)) + {}^{\mathcal{I}}\mathbf{v}_c(t) \\ \dot{\psi}(t) &= r(t) \\ {}^{\mathcal{I}}\dot{\mathbf{v}}_c(t) &= 0 \\ {}^{\mathcal{I}}\dot{\mathbf{b}}(t) &= l_m\omega_m(t)\mathbf{w}^\perp(\chi(t)) \\ \dot{\chi}(t) &= \omega_m(t) \\ d(t) &= ||{}^{\mathcal{I}}\mathbf{b}(t) - {}^{\mathcal{I}}\mathbf{p}(t)|| \end{aligned}\right\}, \tag{1}$$

where $t \in [0, t_f)$ and $t_f > 0$, ${}^{\mathcal{I}}\mathbf{p} \in \mathbb{R}^2$ is the vehicle's position, $v : [0, t_f) \to \mathbb{R}$ is the vehicle's speed, $\psi : [0, t_f) \to [0, 2\pi)$ is the course angle of the vehicle, ${}^{\mathcal{I}}\mathbf{v}_c \in \mathbb{R}^2$ is the velocity of the current, $r : [0, t_f) \to \mathbb{R}$ is the course rate of the vehicle, ${}^{\mathcal{I}}\mathbf{b} \in \mathbb{R}^2$ is the beacon's position, l_m is the length of the beacon's manipulator, $\omega_m : [0, t_f) \to \mathbb{R}$ is the angular rate of the beacon, $\chi : [0, t_f) \to [0, 2\pi)$ is the angular position of the beacon, and $d \in \mathbb{R}$ is the distance or range between the vehicle and beacon. For more details on the system, you can refer to Rúa et al. (2019). In what follows, we assume that the beacon's positions are known. The solution of the system (1) at time t with initial condition $(\mathbf{p}_0, \psi_0, \mathbf{v}_{c_0}, \chi_0)$ is given by

$$\left.\begin{aligned} {}^{\mathcal{I}}\mathbf{p}(t) &= \mathbf{p}_0 + \int_0^t v(\tau)\mathbf{w}(\psi(\tau))\,d\tau + t\,{}^{\mathcal{I}}\mathbf{v}_{c_0} \\ \psi(t) &= \psi_0 + \int_0^t r(\tau)d\tau \\ {}^{\mathcal{I}}\mathbf{v}_c(t) &= \mathbf{v}_{c_0} \\ \chi(t) &= \chi_0 + \int_0^t \omega_m(\tau)d\tau \\ {}^{\mathcal{I}}\mathbf{b}(t) &= l_m\mathbf{w}(\chi(t)) \\ d(t) &= ||{}^{\mathcal{I}}\mathbf{b}(t) - {}^{\mathcal{I}}\mathbf{p}(t)|| \end{aligned}\right\}, \tag{2}$$

In order to avoid collision or entanglement, we assume that the distance between the vehicle and beacon have a safety guard given by

$$||{}^{\mathcal{I}}\mathbf{b}(t) - {}^{\mathcal{I}}\mathbf{p}(t)|| \geq \mathcal{R}$$

for all $t > 0$ and $\mathcal{R} > 0$.

Throughout this work, the speed and rotation of the vehicle, and the angular velocity of the manipulator are assumed as inputs to the system. These variables can be available from an Inertial Measurement Unit (IMU) for the vehicle's rotation, a Doppler Velocity Log (DVL) for the speed, and a Rotary Encoder (for the angular velocity of the manipulator).

Problem Statement: given the system described by the set of equations (2), find the best sequence of inputs for the system that improves the overall observability of the system.

3. FISHER INFORMATION MATRIX

The observability can be analyzed from a binary point of view, that is, the system is observable or not. Although, the above is important to design an observer, it is also important to find out which are the best trajectories relating observability in the system. One way to quantify this is by using the Fisher Information Matrix (FIM). The FIM give us a quantitative measure of how much information a random variable $\mathcal{X}$ carries about an unknown parameter $\boldsymbol{\theta}$. The problem for the Fisher Information Matrix can be written as follow: consider the problem of estimating an unknown parameter $\boldsymbol{\theta} \in \mathbb{R}^n$ from a set of measured data given by $\boldsymbol{y} \in \mathbb{R}^m$. Let $g(\boldsymbol{y}) : \mathbb{R}^n \to \mathbb{R}^m$ be an unbiased estimation of $\boldsymbol{\theta}$. Then, the error covariance of an unbiased estimator is bounded by

$$\mathbb{E}\left\{[g(\boldsymbol{y}) - \boldsymbol{\theta}][g(\boldsymbol{y}) - \boldsymbol{\theta}]^\top\right\} \geq FIM(\boldsymbol{\theta})^{-1}, \tag{3}$$

where

$$FIM(\boldsymbol{\theta}) = \mathbb{E}\left\{(\nabla_{\boldsymbol{\theta}} \ln f(\boldsymbol{y}|\boldsymbol{\theta}))(\nabla_{\boldsymbol{\theta}} \ln f(\boldsymbol{y}|\boldsymbol{\theta}))^\top\right\}, \tag{4}$$

and $f(\boldsymbol{y}|\boldsymbol{\theta})$ is the likelihood function. The result given by the equation (3) is called the Cramer-Rao bound. This result establishes a lower bound on the variance of an unbiased estimator. For our purpose, the objective is to minimize as much as possible the bound which ultimately translates in a better performance in the estimation. The unknown parameter for the problem at hand is $\boldsymbol{\theta} := [\mathbf{p}_0^\top, \mathbf{v}_{c_0}^\top]^\top$ and the measured data model is given by

$$\boldsymbol{y} = \boldsymbol{z} + \boldsymbol{\eta}, \tag{5}$$

where $\boldsymbol{y} := [y_0, y_1, ..., y_{m-1}]^\top$ is a vector containing m range measurements, $\boldsymbol{z} := [d_0, d_1, ..., d_{m-1}]^\top$ is the real or actual range measurement given by (2), and $\boldsymbol{\eta} := [\eta_1, \eta_2, ..., \eta_{m-1}]^\top$ is a vector containing the measurement noise with $\eta_k \sim \mathcal{N}(0, \sigma^2)$. Note that the FIM explicitly depends on the range between the vehicle and beacon, which in turns implies that is going to depend on the inputs of the system.

With this in mind, we first derive the FIM for the problem under consideration. Then, we solve the problem without taking into account any constraint (unconstrained optimization problem). Finally, we solve different numerical scenarios where we put constraints over the motion of the beacon and the vehicle. Also, we solve the problem when the vehicle is executing its mission, and the beacon is helping to improve the observability of the system.

4. UNCONSTRAINED TRAJECTORY OPTIMIZATION

Consider the system described by the equations (1). Let $m \in \mathbb{N}$ and consider a time sequence of length m, such as $0 = t_0 < t_1 < ... < t_{m-1} = t_f$, where t_k are sampling instants at which the range measure is acquired. For simplicity of the analysis, in what follows, we assume that the speed, course rate of the vehicle, and the angular velocity of the beacon are bounded piecewise constant functions of time, that is

$$v(t) = \bar{v}_k \in [\bar{v}_{\min}, \bar{v}_{\max}], t \in [t_k, t_{k+1}),$$
$$r(t) = \bar{r}_k \in [\bar{r}_{\min}, \bar{r}_{\max}], t \in [t_k, t_{k+1}),$$

and

$$\omega_m(t) = \bar{\omega}_{m_k} \in [\bar{\omega}_{m_{\min}}, \bar{\omega}_{m_{\max}}], t \in [t_k, t_{k+1}).$$

Based on this assumption, for all $t \in [t_k, t_{k+1})$, the model (2) can be written as

$$\left.\begin{aligned}
{}^{\mathcal{I}}\mathbf{p}(t) &= \begin{cases} {}^{\mathcal{I}}\mathbf{p}_k + \dfrac{\bar{v}_k}{\bar{r}_k}[-\mathbf{w}^\perp(\psi(t)) + \mathbf{w}(\psi_k)] + (t-t_k){}^{\mathcal{I}}\mathbf{v}_{c_0}, \\ \hfill \text{if } \bar{r}_k \neq 0 \\ {}^{\mathcal{I}}\mathbf{p}_k + (t-t_k)(\bar{v}_k\mathbf{w}(\psi_k) + {}^{\mathcal{I}}\mathbf{v}_{c_0}), \ \text{otherwise,} \end{cases} \\
\psi(t) &= \psi_k + (t-t_k)\bar{r}_k \\
{}^{\mathcal{I}}\mathbf{v}_c(t) &= \mathbf{v}_{c_0} \\
\chi(t) &= \chi_k + (t-t_k)\bar{\omega}_{m_k} \\
{}^{\mathcal{I}}\mathbf{b}(t) &= l_m\mathbf{w}(\chi(t))
\end{aligned}\right\} \tag{6}$$

Then, the model at time t_{k+1} and with constant sample time $T = t_{k-1} - t_k$, is given by

$$\left.\begin{aligned}
{}^{\mathcal{I}}\mathbf{p}_{k+1} &= \begin{cases} {}^{\mathcal{I}}\mathbf{p}_k + \dfrac{\bar{v}_k}{\bar{r}_k}[-\mathbf{w}^\perp(\psi_{k+1}) + \mathbf{w}(\psi_k)] + T{}^{\mathcal{I}}\mathbf{v}_{c_0}, \\ \hfill \text{if } \bar{r}_k \neq 0 \\ {}^{\mathcal{I}}\mathbf{p}_k + T(\bar{v}_k\mathbf{w}(\psi_k) + {}^{\mathcal{I}}\mathbf{v}_{c_0}), \ \text{otherwise,} \end{cases} \\
\psi_{k+1} &= \psi_k + T\ \bar{r}_k \\
{}^{\mathcal{I}}\mathbf{v}_{c_{k+1}} &= \mathbf{v}_{c_0} \\
\chi_{k+1} &= \chi_k + T\ \bar{\omega}_{m_k} \\
{}^{\mathcal{I}}\mathbf{b}_{k+1} &= l_m\mathbf{w}(\chi_{k+1})
\end{aligned}\right\} \tag{7}$$

Now that the model of the system has been described, it is possible to derive the particular FIM for the problem at hand. Recall that the FIM is given by (4) and the measured data vector is given by (5). Since the model of the measured data has a normal distribution, the likelihood function for the measurement vector $\boldsymbol{y}$ with respect to the unknown parameter $\boldsymbol{\theta} := [\mathbf{p}_0^\top, \mathbf{v}_{c_0}^\top]^\top$ is given by

$$f(\boldsymbol{y}|\boldsymbol{\theta}) = (2\pi)^{-m/2}|R|^{-1}\exp\left(-\frac{1}{2}(\boldsymbol{y}-\boldsymbol{z})^\top R^{-1}(\boldsymbol{y}-\boldsymbol{z})\right),$$

where $R = \sigma^2 I_m$ is the covariance matrix. Now, in order to obtain the FIM for our problem, we need the gradient of the logarithm, that is,

$$\nabla_{\boldsymbol{\theta}} \ln f(\boldsymbol{y}|\boldsymbol{\theta}) = (\nabla_{\boldsymbol{\theta}}\boldsymbol{z})^\top R^{-1}(\boldsymbol{y}-\boldsymbol{z})$$

Recall that the range vector $\boldsymbol{d}$ which is formed by stacking the range measurements, implicitly depends on the initial conditions of the system. A straightforward computation shows that the FIM for our problem is given by

$$FIM_{\mathbf{u}}(\boldsymbol{\theta}) = \sigma^{-2}(\nabla_{\boldsymbol{\theta}}\boldsymbol{z})^\top(\nabla_{\boldsymbol{\theta}}\boldsymbol{z}), \tag{8}$$

where

$$\nabla_\theta \boldsymbol{z} = \begin{bmatrix} -\frac{\mathbf{d}_0^\top}{d_0} & -t_0\frac{\mathbf{d}_0^\top}{d_0} \\ \vdots & \vdots \\ -\frac{\mathbf{d}_{m-1}^\top}{d_{m-1}} & -t_{m-1}\frac{\mathbf{d}_{m-1}^\top}{d_{m-1}} \end{bmatrix}_{m\times 4}$$

In the above, remember that $\mathbf{d}_k$ denotes the relative position vector at time t_k from the beacon with respect to the vehicle, that is, $\mathbf{d}_k = \mathbf{b}_k - \mathbf{p}_k$ and the norm is given by $d_k = ||\mathbf{d}_k||$. Additionally, the range vector depends on the inputs of the system, which makes the FIM dependent on the inputs. For the sake of simplicity, the following compact notation is used

$$\mathcal{D} := \begin{bmatrix} \frac{\mathbf{d}_0^\top}{d_0} \\ \vdots \\ \frac{\mathbf{d}_{m-1}^\top}{d_{m-1}} \end{bmatrix} \in \mathbb{R}^{m\times 2}$$

$$\mathcal{T} := \operatorname{diag}(t_0, t_1, ..., t_{m-1}) \in \mathbb{R}^{m\times m}.$$

Then, the Fisher information matrix is given by

$$FIM_{\mathbf{u}}(\boldsymbol{\theta}) = \sigma^{-2}\begin{bmatrix} \mathcal{D}^\top\mathcal{D} & \mathcal{D}^\top\mathcal{T}\mathcal{D} \\ \mathcal{D}^\top\mathcal{T}\mathcal{D} & \mathcal{D}^\top\mathcal{T}^2\mathcal{D} \end{bmatrix} \tag{9}$$

Remark 1. The Fisher information for our problem has the same structure that the one tackled in Crasta et al. (2016). Nevertheless, it is important to point out that the constraint in the motion for the beacon is different. Recall that the motion of the beacon is restricted to a small area given by the manipulator. At first glance it can be a disadvantage compared to using another vehicle as a beacon, since the manipulator will impose a smaller range operation of the system. However, the dynamics of the manipulator is much faster than another underwater vehicle, which allows the system to execute more excited maneuvers.

Following standard procedures for defining the optimization problem, we have

$$\max_{\mathbf{u}} \ \ln\det FIM_{\mathbf{u}}(\boldsymbol{\theta}). \tag{10}$$

Now, we have written the optimization problem to be solved.

Proposition 1. The optimal cost function for the unconstrained problem (10) is given by

$$J(\mathbf{u}^*) = \ln\left(\frac{T^4 m^4 (m^2-1)^2}{2304\sigma^8}\right) \tag{11}$$

A complete proof of this solution can be found in Crasta et al. (2016).

Remark 2. Notice that the optimal value for the cost function is directly proportional to the number of range measurements taken into account in the optimization problem. Additionally, if the variance of the sensor is to large, the accuracy of the system is going to decrease.

Remark 3. Although the optimization did not take into account any of the constraints for the vehicle and beacon, this value gives us an overview of the maximum value that we can achieve in our optimization problem with constraints.

5. CONSTRAINED TRAJECTORY OPTIMIZATION

In the previous section, we tried to solve the problem without taking into account any restriction for the inputs in the vehicle and the beacon. Basically, equation (11) gives us an intuition of the maximum value that we can achieve if the vehicle and beacon can perform any trajectory. Now, with the constraint on the type of movement for the vehicle and beacon given by (7) and bounds for the inputs, we try to solve the problem using numerical optimization methods. Four different optimization problems are proposed:

- **Problem 1**: both inputs from vehicle and beacon are optimization variables. That is $\mathbf{u} := [\bar{v}_k,\ \bar{r}_k,\ \bar{\omega}_{m_k}]^\top$
- **Problem 2**: the vehicle will be executing a particular mission, while the input of the beacon is the optimization variable. That is $\mathbf{u} := [\bar{\omega}_{m_k}]$.
- **Problem 3**: the vehicle will be executing a particular mission, and we want to find the optimal constant angular velocity for the beacon.
- **Problem 4**: a multi-objective optimization taking into account an energy cost function.

To solve all these problems, we resort to numerical methods to maximize the determinant of the FIM using the *Genetic Algorithm* toolbox from Matlab (MathWorks, 2019).

5.1 Problem 1 - Vehicle and Beacon help to improve observability

For the first problem, the inputs from the vehicle and the beacon are going to be used as optimization variables. This means that, both the vehicle and the beacon will help to maximize the FIM, which in turns means to improve the accuracy of the estimation. Additionally, we will impose upper and lower bounds for the vehicle's speed and course rate, as well as bounds for the angular velocity of the beacon. The optimization problem will be

$$\begin{aligned} \max_{\mathbf{u}} \quad & \ln\det FIM_{\mathbf{u}}(\boldsymbol{\theta}) \\ \text{s.t.} \quad & \text{Eq.}(7) \\ & 0 < \bar{v}_k < \bar{v}_{ub} \\ & -\bar{r}_{ub} < \bar{r}_k < \bar{r}_{ub} \\ & -\bar{\omega}_{m_{ub}} < \bar{\omega}_{m_k} < \bar{\omega}_{m_{ub}} \end{aligned} \tag{12}$$

where $\mathbf{u} := [\bar{v}_k,\ \bar{r}_k,\ \bar{\omega}_{m_k}]^\top$, $\boldsymbol{\theta} := [\mathbf{p}_0^\top, \mathbf{v}_{c_0}^\top]^\top$, $k \in \{1, 2, ..., m\}$ and $FIM_{\mathbf{u}}(\boldsymbol{\theta})$ is given by equation (9). For the first problem, the idea is to find the best sequence of actions for the vehicle and beacon that maximizes the FIM. We solve the problem for the following initial conditions: the initial beacon's position is given by $\mathbf{b}_0^\top = [1.4142, 1.4142]^\top$ m and the initial position and orientation of the vehicle are $\mathbf{p}_0^\top = [3, 4.5]^\top$ m and $\psi_0 = \pi/3$ rad, respectively. The ocean current is $\mathbf{v}_c^\top = [0.3, 0.1]^\top$ m/s. The bounds for the linear speed and angular rate of the vehicle are given by $\bar{v}_{ub} = 1.5$ m/s and $\bar{r}_{ub} = \pi/9$ rad/s. The bound for the angular rate of the beacon is $\bar{\omega}_{m_{ub}} = \pi$ rad/s. The variance of the sensor is 0.1 m, the sample time $T = 1$ s, and the number of measurements taken into account for the optimization problem is $m = 12$. The unconstrained optimal $J_u(\mathbf{u}^*)$ solution given by equation (11) is 30.54 and with constrained $J_c(\mathbf{u}^*)$ is 29.56. Figure 2, 3, and 4 show the vehicle's trajectory, beacon's trajectory and optimal input for the beacon, respectively.

5.2 Problem 2 - Beacon helps to improve observability

For the second problem, the inputs from the vehicle are given for a particular mission, which means that the vehicle is not going to help to maximize the FIM. Therefore, just the rotation of the beacon is used in the optimization process. Additionally, we impose upper and lower bounds for the beacon's rotation. The optimization problem is given by (12), where this time $\mathbf{u} := [\bar{\omega}_{m_k}]$. The unknown parameters $\boldsymbol{\theta}$ and k remain the same and the upper bound for the rotation speed of the beacon is given by $\bar{\omega}_{m_{ub}} = \pi/6$. For this particular problem, we solve the following scenario: the vehicle is moving in straight line, that is, $v(t) = 1.5$ m/s and $r = 0$ rad/s. The initial position of the beacon is given by $\mathbf{b}_0 = [1.4142, 1.4142]^\top$ [m] and the initial position and orientation of the vehicle are $\mathbf{p}_0 = [3, 3]^\top$ [m] and $\psi_0 = 0$ rad, respectively. The ocean current is $\mathbf{v}_{c_0} = [0, 0.3]^\top$ [m/s]. The sample time T and the number of samples m were set up to 1 s and 10, respectively. For these conditions, the unconstrained problem reaches its maximum at $J_u = 29.079$, while the solution of the problem at hand is $J_c = 24.254$. Although, only the beacon's rotation was involved in the optimization process, the optimal value is very close to the unconstrained solution. Figure 5, 6, and 7 show the

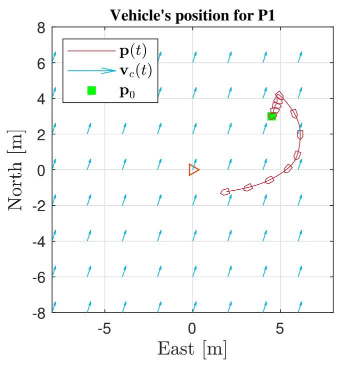


Fig. 2. Vehicle's trajectory optimal solution for problem one and all four scenarios. Even though, the beacon trajectory is not reflected in the figures, beacon is rotating around the origin.

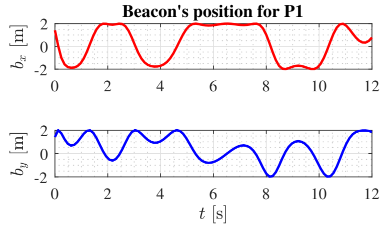


Fig. 3. Beacon's trajectory optimal solution for the first problem.

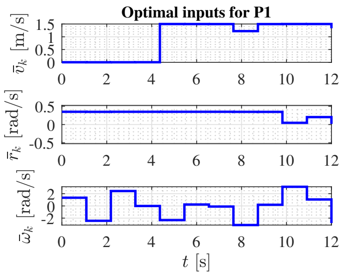


Fig. 4. Optimal solution for problem one.

vehicle's trajectory, beacon's trajectory and optimal input for the beacon, respectively.

5.3 Problem 3 - Best constant rotation for the beacon

For the third problem, the objective is to find the best optimal constant rotation of the beacon which maximizes the FIM. Therefore, just the rotation of the beacon is used in the optimization problem and it is constant all the time. This implies that $\mathbf{u} := \bar{\omega}_{m_k}$, with $\bar{\omega}_{m_k}$ being constant for all $k \in \{1, 2, ..., m\}$. The unknown parameter $\boldsymbol{\theta}$ remains the same. The same scenario from the previous problem is solved with these conditions. The vehicle's trajectory is the same as shown in Figure 5. The optimal angular rotation for the beacon is $\bar{\omega}_m = 0.5$ rad/s. The optimal cost function for this scenario is $J_c = 21.936$. It is important to highlight at this point, that although it was shown in Rúa et al. (2019) that the system is observable when the beacon rotates and the vehicle is going in straight line, it had not been concluded which would be the best rotational speed for it. Additionally, the observability could be improved if a sequence of actions for rotational speed was performed. Finally, since the optimization problem was transformed to the point of having only one input variable, we can plot

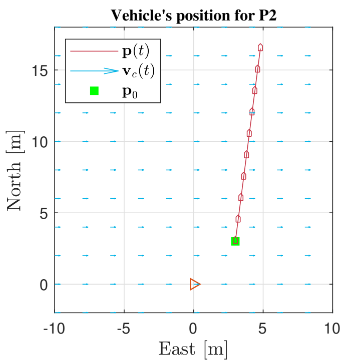


Fig. 5. Vehicle's trajectory for the second problem. The vehicle's trajectory is not generated as result of the optimization problem.

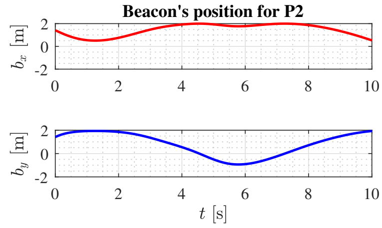


Fig. 6. Beacon's trajectory optimal solution for the second problem.

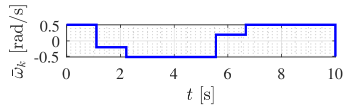


Fig. 7. Optimal solution for the second problem. Recall that just $\bar{\omega}_k$ was used in the optimization process.

the cost function for different values of beacon's angular velocity and speed for the vehicle (see Figure 8). Notice, that even for different vehicle's speed, the optimal input for the beacon remains approximately the same.

5.4 Problem 4 - Energy Cost Function

Up to this time, the optimization problem has been based in one goal: maximizing the Fisher Information Matrix in order to improve the accuracy on the estimation. We have found that different inputs achieve good performance in relation with the maximum optimal cost from the solution of the unconstrained optimization. This performance was achieved in most of the cases by accelerating the beacon from one direction to other (see Figure 6). Now, we are interested in the inclusion of a second cost function that relates the energy required for the beacon's movement. The above implies, that we want to maximize the FIM while at the same time minimizing the energy consumption of the beacon. Then, we have a multi-objective optimization problem that involves two cost functions: the FIM and the energy consumption.

The energy consumption of the beacon can be related mostly to the rotational kinetic energy, which is given by $K_e = \frac{1}{2} I \omega_m^2(t)$, where I is the moment of inertia and ω_m is the beacon's angular velocity. Since the moment of inertia is constant and is not within the optimization variables, the energy cost function can be written as

$$J_2 = \int_0^{t_f} \omega_m^2(\tau) d\tau, \tag{13}$$

and for a piecewise constant input, then

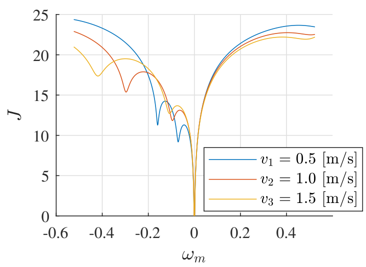
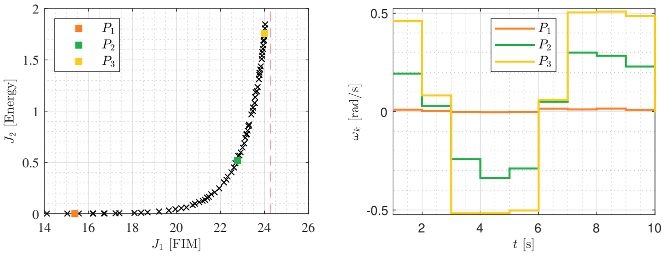


Fig. 8. Cost Function plot for constant beacon's angular velocity and different vehicle's speed.



(a) (b)

Fig. 9. (a) Pareto front for the multi-objective optimization problem. (b) Input selection for the beacon movement. Notice that even if we increased more the energy, the observability is not going to improve further.

$$J_2 = T \sum_{k=0}^{m-1} \bar{\omega}_{m_k}^2. \tag{14}$$

Now, the multiple optimization formulation is given by

$$\begin{aligned} \min_{\mathbf{u}} \quad & (J_1, J_2) \\ \text{s.t.} \quad & -\bar{\omega}_{m_{ub}} < \bar{\omega}_{m_k} < \bar{\omega}_{m_{ub}} \end{aligned} \tag{15}$$

where $J_1 = -\ln\det FIM_{\mathbf{u}}(\boldsymbol{\theta})$ and J_2 is given by (14). To solve this problem, we resort to numerical algorithms, particularly with the *Global Optimization Toolbox* from Matlab. For this problem, we tested just the scenario where the vehicle moves in straight lines like in Figure 5. The same parameters as in the second problem were set up for this case. Since we are solving a multi-objective optimization problem, Figure 9 shows the Pareto Front and some of the solution for the beacon's input. Notice that if we want to improve the FIM, then, we need to spend more energy, which means that there is a compromise between the both of them. Additionally, even if we continuously increase the energy of the system, we are not going to improve the navigation system.

6. CONCLUSIONS AND FUTURE WORK

We addressed the observability problem from a different perspective, instead of the classical yes or no point of view. We formulated an optimization problem for finding the best sequence of action for the system that improves the observability. To achieve this, a cost function using the Fisher Information Matrix was derived. Next, the problem was solved from an unconstrained and constrained perspective. For the first, an analytical solution was found, which give us the best FIM that the system can achieve without taking into account any constraint in the motion of the vehicle or beacon. For the constrained problem, we solved four different scenarios, where the motion of the vehicle and beacon were involved, as well as the energy consumption.

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REFERENCES

Bayat, B., Crasta, N., Crespi, A., Pascoal, A.M., and Ijspeert, A. (2017). Environmental monitoring using autonomous vehicles: a survey of recent searching techniques. *Current Opinion in Biotechnology*, 45, 76–84. doi:10.1016/j.copbio.2017.01.009.

Casalino, G., Simetti, E., Manerikar, N., Sperinde, A., Torelli, S., and Wanderlingh, F. (2015). Cooperative underwater manipulation systems: Control developments within the MARIS project. *IFAC-PapersOnLine*, 48(2), 1–7. doi:10.1016/j.ifacol.2015.06.001.

Crasta, N., Moreno-Salinas, D., Bayat, M., Pascoal, A., and Aranda, J. (2016). Optimal Motion Planning for Range-Based Marine Vehicle Positioning in the Presence of Unknown Currents. *IFAC-PapersOnLine*, 49(23), 41 – 47. doi:10.1016/j.ifacol.2016.10.319.

Djapic, V., Nad, D., Mandic, F., Miskovic, N., and Kenny, A. (2018). Navigational challenges in diver-AUV interaction for underwater mapping and intervention missions. *IFAC-PapersOnLine*, 51(29), 366–371. doi: 10.1016/j.ifacol.2018.09.496.

Fallon, M.F., Papadopoulos, G., Leonard, J.J., and Patrikalakis, N.M. (2010). Cooperative AUV navigation using a single maneuvering surface craft. *The International Journal of Robotics Research*, 29(12), 1461–1474. doi:10.1177/0278364910380760.

Mandić, F., Mišković, N., Palomeras, N., Carreras, M., and Vallicrosa, G. (2016). Mobile beacon control algorithm that ensures observability in single range navigation. *IFAC-PapersOnLine*, 49(23), 48–53. doi: 10.1016/j.ifacol.2016.10.320.

MathWorks, T. (2019). Global Optimization Toolbox: Genetic Algorithm. URL `https://www.mathworks.com`.

Parlangeli, G. and Indiveri, G. (2015). Single range observability for cooperative underactuated underwater vehicles. *Annual Reviews in Control*, 40, 129 – 141. doi: 10.1016/j.arcontrol.2015.09.008.

Rúa, S., Crasta, N., Vásquez, R.E., Betancur, M.J., and Pascoal, A.M. (2019). Cooperative range-based navigation using a beacon with circular motion installed on board the support platform. *IFAC-PapersOnLine*.

Rui, G. and Chitre, M. (2010). Cooperative positioning using range-only measurements between two AUVs. In *IEEE OCEANS2010*, 1–6. doi: 10.1109/OCEANSSYD.2010.5603615.

Tan, Y.T., Gao, R., and Chitre, M. (2014). Cooperative path planning for range-only localization using a single moving beacon. *IEEE Journal of Oceanic Engineering*, 39(2), 371–385. doi:10.1109/JOE.2013.2296361.

Webster, S.E., Walls, J.M., Whitcomb, L.L., and Eustice, R.M. (2013). Decentralized extended information filter for single-beacon cooperative acoustic navigation: Theory and experiments. *IEEE Transactions on Robotics*, 29(4), 957–974. doi:10.1109/TRO.2013.2252857.