Multi-objective control engineering benchmark ⋆

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Abstract: In this paper we are presenting the statement and evaluation guidelines of a control engineering benchmark, oriented for multi-objective optimisation design techniques. This is done with the aim of promoting new research on this field, by defining a benchmark to have reproducibility and comparability of the three steps involved in the multi-objective process: problem statement, optimisation process and multi-criteria decision making. The proposed benchmark is a single-input single-output process based on the Peltier effect. Rules and guidelines, merged with common practices in control systems engineering, are highlighted and disclosed in the multi-objective open invited track 2020.

Keywords: Multi-objective optimisation, Parametric optimisation, Evolutionary Algorithms, Intelligent Control, Process Control.

1. INTRODUCTION

Control engineering problems are generally multi-objective problems; this means that there are several specifications and requirements that must be fulfilled, often in conflict. A traditional approach for calculating a solution with a desired trade-off is defining an optimisation statement. Multi-objective optimisation techniques deal with such a problem from a particular perspective by searching for a set of potentially preferable solutions: the so-called Pareto set. The designer may then analyse the trade-off among solutions in this set, and select the most preferable alternative according to the problem at hand. Controller tuning can be considered as a multi-objective problem, given that a set of requirements and specifications must be fulfilled. In this sense, multi-objective optimisation techniques have shown to be valuable tools for this task (Meza et al., 2016).

Benchmarks allow researchers to have a higher degree of reproducibility and comparability of diverse control techniques (Kroll and Schulte, 2014). Even if it is possible to find literature on control engineering benchmarks (Dixon and Pike, 2006; Bejarano et al., 2017; Kroll and Schulte, 2014; Mercader et al., 2019; Romero and Sanchis, 2011; Fernández et al., 2011; Atanasijevic-Kunc et al., 2010; Eriksson et al., 2019) and solutions involving multi-objective techniques (Xue et al., 2010; Kagami et al., 2019), there is not a specific benchmark to test the specific steps involved in the multi-objective optimisation design process. That is, to test its problem statement, optimisation process and multi-criteria decision making stage.

Testing such scenarios is not trivial, given that the optimisation process requires a model, obtained by getting data from the process, to calculate its design objectives. This simple, but critical step, is important to be considered in:

- the problem statement, to include robustness objectives and/or constraints;
- the optimisation process, given that a model is used instead of the process;
- in the decision making process, where it is crucial that the Pareto front approximated with the model preserves the trade-off coherence when its design alternatives (controllers) are evaluated in the real process.

Therefore, proposing a multi-objective control engineering benchmark, taking into account such characteristics, could be an opportunity to promote work oriented to multi-objective optimisation for controller tuning and its viability to solve real-world problems. That is the aim of this paper. To define a control engineering benchmark for testing multi-objective optimisation procedures in an integral way.

The remainder of this works is organised as follows: in Section 2 a brief background on multi-objective optimisa-

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tion techniques is provided; in Section 3 the benchmark is described and in Section 4 an example on how to use and report results of this benchmark is commented. Finally, some conclusions are derived and future work commented.

2. THE MULTI-OBJECTIVE OPTIMISATION PROCESS

As referred in Miettinen (1999), a multi-objective problem (MOP) with m objectives [1], can be stated as follows:

$$\min_{\boldsymbol{x}} \boldsymbol{J}(\boldsymbol{x}) = [J_1(\boldsymbol{x}), \ldots, J_m(\boldsymbol{x})] \tag{1}$$

subject to:

$$\boldsymbol{K}(\boldsymbol{x}) \leq 0 \tag{2}$$

$$\boldsymbol{L}(\boldsymbol{x}) = 0 \tag{3}$$

$$\underline{x_i} \leq x_i \leq \overline{x_i}, i = [1, \ldots, n] \tag{4}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]$ is defined as the decision vector with $\dim(\boldsymbol{x}) = n$; $\boldsymbol{J}(\boldsymbol{x})$ as the objective vector and $\boldsymbol{K}(\boldsymbol{x})$, $\boldsymbol{L}(\boldsymbol{x})$ as the inequality and equality constraint vectors respectively; $\underline{x_i}, \overline{x_i}$ are the lower and the upper bounds in the decision space.

It has been noticed that there is not a single solution in MOPs, because there is not generally a better solution in all the objectives. Therefore, a set of solutions, the Pareto set $\boldsymbol{X_P}$, is defined. Each solution in the Pareto set defines an objective vector in the Pareto front $\boldsymbol{J_P}$ (See Figure 1). It is important to notice that most of the times we rely only in Pareto front and set approximations, $\boldsymbol{J_P^*}, \boldsymbol{X_P^*}$.

All the solutions in the Pareto front are a set of Pareto optimal and non-dominated solutions, where:

- Pareto optimality (Miettinen, 1999): An objective vector $\boldsymbol{J}(\boldsymbol{x}^1)$ is Pareto optimal if there is not another objective vector $\boldsymbol{J}(\boldsymbol{x}^2)$ such that $J_i(\boldsymbol{x}^2) \leq J_i(\boldsymbol{x}^1)$ for all $i \in [1, 2, \ldots, m]$ and $J_j(\boldsymbol{x}^2) < J_j(\boldsymbol{x}^1)$ for at least one j, $j \in [1, 2, \ldots, m]$.
- Dominance (Coello and Lamont, 2004): An objective vector $\boldsymbol{J}(\boldsymbol{x}^1)$ is dominated by another objective vector $\boldsymbol{J}(\boldsymbol{x}^2)$ if $J_i(\boldsymbol{x}^2) \leq J_i(\boldsymbol{x}^1)$ for all $i \in [1, 2, \ldots, m]$ and $J_j(\boldsymbol{x}^2) < J_j(\boldsymbol{x}^1)$ for at least one j, $j \in [1, 2, \ldots, m]$. This is denoted as $\boldsymbol{J(x_2)} \preceq \boldsymbol{J(x_1)}$.

To successfully implement the multi-objective optimisation approach, three fundamental steps are required: MOP statement, the multi-objective optimisation (MOO) process and the multi-criteria decision making (MCDM) stage. Such steps are detailed below.

- MOP statement: implies building a parametric model for optimisation; the design concept definition; design objectives of interest; constraints and decision variables; and finally the cost function for optimisation.
- MOO process: implies selecting or coding a desired algorithm to approximate the Pareto front. The output of the optimisation process is a Pareto front approximation.
- MCDM stage: implies analysing the approximated Pareto front, its design alternatives, to ponder their trade-offs for selecting the most suitable and preferable solution for the problem at hand. It usually requires methodologies or visualisation strategies for multi-criteria analysis.

Therefore, any benchmark that tests multi-objective procedures in control systems must allow to test individually or altogether those three steps. To this end, here we propose a benchmark and its guidelines for assessing such an evaluation.

3. BENCHMARK DESCRIPTION

This section details the proposed benchmark. First, the process is introduced, followed by the control task. Finally, the evaluation is presented along with some considerations.

3.1 Process

Th benchmark control problem is based on the Peltier cell model developed in Huilcapi et al. (2017), presented in the multi-objective open invited track in 2017. Matlab-Simulink© is used as the platform for this benchmark (See Figure 2). Files are available via the File Exchange Platform [2] and the ResearchGate site [3].

The thermal balance of the cold face in the Peltier thermoelectric module is represented by a set of differential equations. Such equations are used to simulate a single-input single-output (SISO) process, to control the temperature in the cold face, using as input the voltage applied to the Peltier cell within a range from 0[V] to 6[V] (see Figure 3).

Here-after such set of equations and their Simulink© implementation will be called (pseudo)-process, given that it will be considered as the real-process for evaluation purposes. That means that this (pseudo)-process cannot be used actively in the optimisation process (as a parametric model or cost function for optimisation). The optimisation must be carried out using an identified model from this (pseudo)-process.

3.2 Control task via multi-objective optimisation techniques

The ultimate control task is to tune a controller to achieve an overall better performance when compared with the one resulting of the Ziegler-Nichols (ZN) method (Ziegler and Nichols, 1942). In the multi-objective sense, we are looking for a controller which will dominate the ZN-controller in the (pseudo)-model. For this, we will define the identification experiment of Figure 4, where a model between the range $[-5, 5]^o C$ is approximated (See Equation (5)).

$$M = \frac{-6.75}{10s + 1} \exp(-0.5s) \tag{5}$$

This lead to the proportional-integral (PI) controller using the Ziegler-Nichols rule, via its critical gain of Equation 6.

[1] A maximisation problem can be converted to a minimisation problem. For each of the objectives that have to be maximised, the transformation: $\max J_i(\boldsymbol{x}) = -min(-J_i(\boldsymbol{x}))$ could be applied.

[2] `https://www.mathworks.com/matlabcentral/fileexchange/75408`

[3] `https://www.researchgate.net/project/Control-engineering-benchmarks-and-competitions`

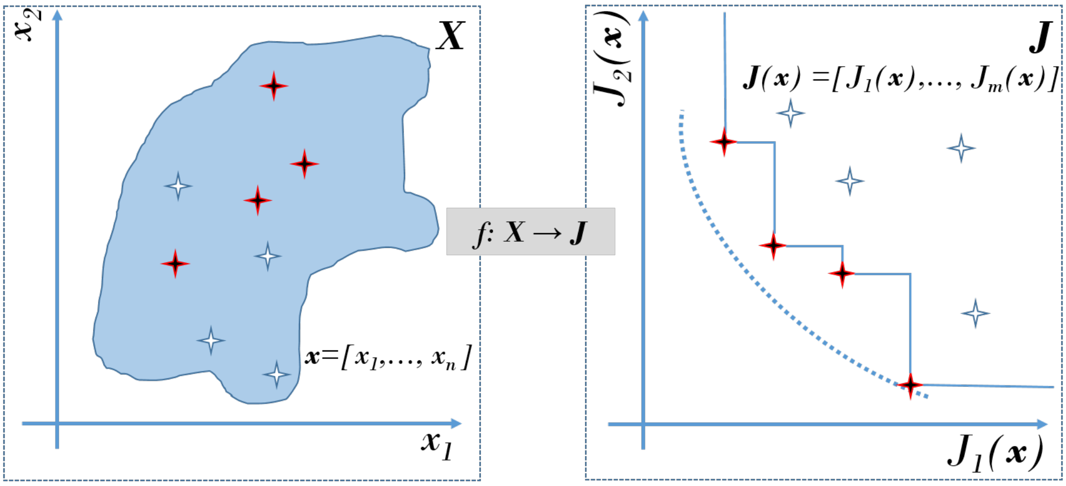


Fig. 1. Pareto optimality and dominance concepts for a min-min MOP. Dark solutions are non-dominated solutions which approximate (solid line) the Pareto front (dotted line) in the objective space $\boldsymbol{J}$.

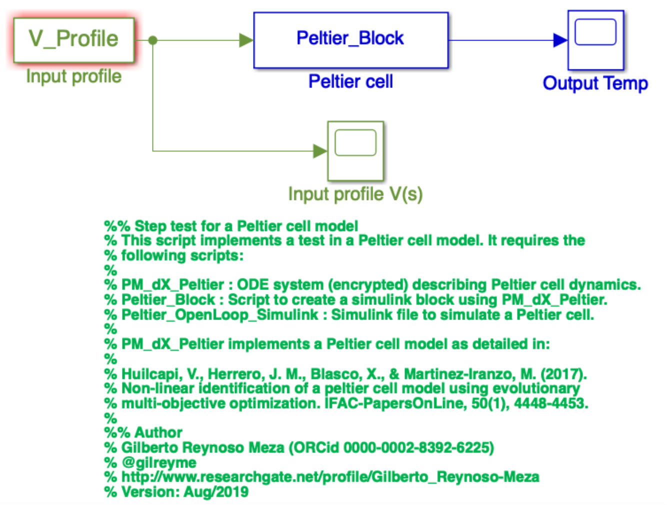


Fig. 2. Simulink© process

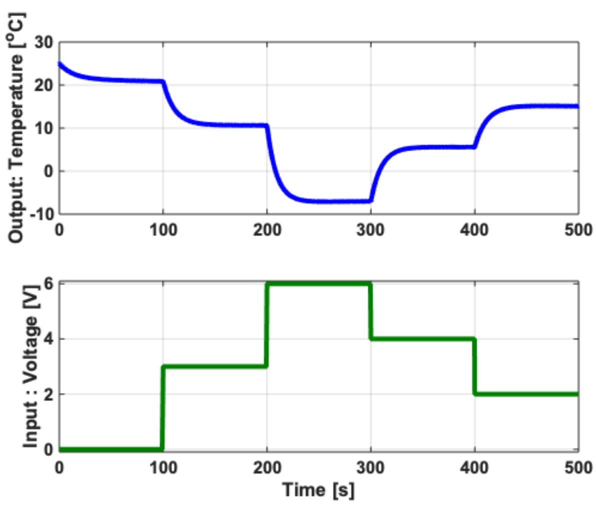


Fig. 3. Open loop test

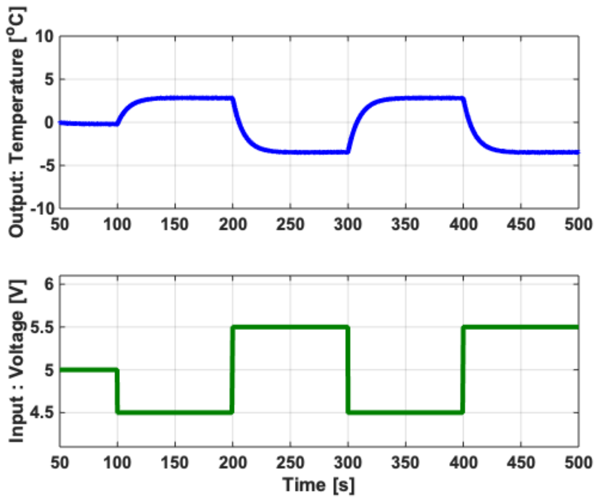


Fig. 4. Identification experiment.

$$C_{zn} = -2.3120 - \frac{1.4142}{s} \tag{6}$$

This controller with the test in the (pseudo)-process leads to the performance depicted in Figure 5.

The PI controller is used, given that it is the first control solution to implement, and any other control structure and tuning efforts must be capable of getting a better performance than the PI ZN-controller; furthermore, it should make a significant difference. Therefore, this controller will be suggested always as a reference controller. Besides it will be used to normalise the performance of any other controller. This will lead to an interpretable and meaningful measure of improvement in each one of the design objectives.

Roughly speaking, any control engineer will follow the next steps to tune a controller:

(1) Defining an experiment to get data from the (pseudo)-process P.
(2) Approximating a model M for tuning purposes.

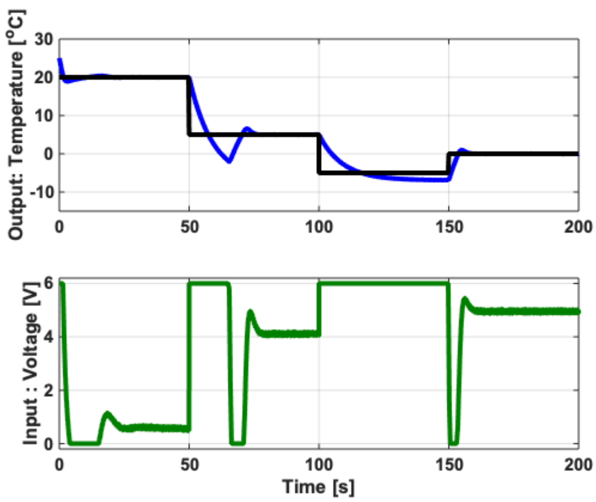


Fig. 5. Closed loop test with the pseudo-process, using the ZN-controller (Equation (6)).

(3) Tuning a given controller C using M.
(4) Testing C in P to validate the controller.

If multi-objective optimisation techniques are used, model M is used to approximate a Pareto front $\boldsymbol{J_P^*}$ for some design objectives. Normally, any evaluation on the performance of algorithms used are referred to the Pareto front approximation of the model M. Nevertheless at the end, in order to have success in this process, the final valuation must be performed in the (pseudo)-process P. Therefore, here we are proposing to evaluate the multi-objective process by:

- Checking the trade-off coherence of $\boldsymbol{J_P^*}$ in the process, by evaluating $\boldsymbol{X_P^*}$ in P to get $\boldsymbol{J_P^\star}$. This means that we are going to evaluate how many Pareto optimal solutions from $\boldsymbol{J_P^*}$ persist when evaluated in the process to approximate $\boldsymbol{J_P^\star}$.
- Verify the decision making policy, in order to check how many tests are required to get a Pareto-optimal solution in $\boldsymbol{J_P^\star}$, using information from $\boldsymbol{J_P^*}$.

The above commented leads to a careful choice of the model for the MOO process; of the MOP statement, for including robust objectives and constraints that guarantee internal coherence of the approximated Pareto front $\boldsymbol{J_P^*}$; and of the MCDM stage to get a Pareto optimal solution $\boldsymbol{J_P^\star}$ using $\boldsymbol{J_P^*}$.

At this point it is more important to guarantee trade-off preservation between a solution performance, when evaluated in the model and in the (pseudo)-process.

3.3 Evaluation and considerations

Any partial result of this process is valuable and it could be reported. Nevertheless, essentially, you must report:

(1) Evaluation in the (pseudo)-process of the statistically significant Pareto front approximation from the optimisation process. Independently from the design objectives used in the MOP, normalised integral value of the absolute error (IAE) and normalised total variation of control action (TV) must be reported. Test should be different from the one used in the optimisation process.
(2) Hypervolume of the statistically significant Pareto front approximation $\boldsymbol{J_P^\star}$ generated by the algorithm used in the (pseudo)-process, using the normalised IAE and TV.
(3) Ratio of non-dominated solutions versus design alternatives in the Pareto front in the (pseudo)-process.
(4) Rank, according to your decision making criteria, to select a Pareto optimal solution $\boldsymbol{J_P^\star}$ from $\boldsymbol{J_P^*}$.

4. BENCHMARK EXAMPLE

Here it follows an example on how to use the benchmark, by using a simple multi-objective process. All of the three stages are described: the MOP statement, MOO process and MCDM.

4.1 MOP statement

Design objectives are calculated via Matlab-Simulink©, by implementing a PI controller and an optimisation model, with a simple step input in the reference. Considering the control engineering benchmark here defined, a MOP statement is stated as follows:

- The Model M (Equation (5)) is selected as parametric model for optimisation.
- Design concept under consideration is a simple PI controller, having parameters $\boldsymbol{x} = [kp, ki]$ with the following structure:

$$C = kp + \frac{ki}{s} \tag{7}$$

- Multi-objective problem statement:

$$\min_{\boldsymbol{x}} \boldsymbol{J}(\boldsymbol{x}) = [\overline{J_{IAE}(\boldsymbol{x})}, \overline{J_{TV}(\boldsymbol{x})}] \tag{8}$$

where:

$$J_1(\boldsymbol{x}) = \overline{J_{IAE}(\boldsymbol{x})} = \frac{J_{IAE}(\boldsymbol{x})}{J_{IAE}(\boldsymbol{x_{ZN}})} \tag{9}$$

$$J_2(\boldsymbol{x}) = \overline{J_{TV}(\boldsymbol{x})} = \frac{J_{TV}(\boldsymbol{x})}{J_{TV}(\boldsymbol{x_{ZN}})} \tag{10}$$

and $\boldsymbol{x_{ZN}} = [-2.3120, -1.4142]$ is the ZN-controller tuned using the model M.
- Constraints are defined as follows:

$$\overline{J_{IAE}(\boldsymbol{x})} \leq 1 \tag{11}$$

$$\overline{J_{TV}(\boldsymbol{x})} \leq 1 \tag{12}$$

$$\Re\left(eig\left(\frac{M \cdot C}{1 + M \cdot C}\right)\right) < 0 \tag{13}$$

$$-5 \leq x_i \leq 0 \;,\; i = [1, 2] \tag{14}$$

Limits on kp,ki has been selected according to the feasible space of a PI controller for the Model which guarantees a closed loop stability. Limits on $\overline{J_{IAE}(\boldsymbol{x})}$ and $\overline{J_{TV}(\boldsymbol{x})}$ are imposed for pertinence purposes as well as to guarantee controller with better performance than the Ziegler-Nichols. The last constraint is used to guarantee stability in the closed loop. A basic penalty technique is employed as described in Reynoso-Meza et al. (2012).

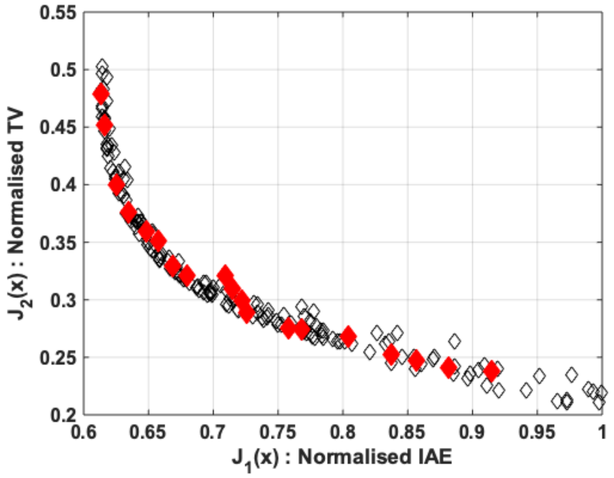


Fig. 6. Pareto front approximation after 11 runs (unfilled diamonds) and the one with the median value of hypervolume (filled diamonds).

4.2 MOO process

For the optimisation process, the spMODEx algorithm is used implemented in Matlab© and available at File exchange [4]. The algorithm is used with its standard parameters with a linear recombination. A total of 1000 function evaluations and a total of 11 runs are used. Optimisation process ran in a standard PC, running Windows© 10 and Matlab© 2016. After 11 runs, the Pareto front with the median value of hypervolume is selected (see Figure 6).

4.3 MCDM stage

In Table 1, design alternatives of the selected Pareto front are depicted. Such design alternatives are evaluated in the (pseudo)-process, resulting in the Pareto front approximation $\boldsymbol{J}_{\boldsymbol{P}}^{\star}$ in Figure 7. As it can be noticed, just two solutions are Pareto optimal in the (pseudo)-process. In the same table, last column specify the ranking of each solution, according to the TOPSIS criteria (Behzadian et al., 2012) as a decision making procedure. As it can be noticed, by following this criteria, Pareto-optimal solutions $\boldsymbol{J}_{\boldsymbol{P}}^{\star}$ are selected after 17 and 19 tests, from $\boldsymbol{J}_{\boldsymbol{P}}^{\star}$.

In Figure 8 the time performance of the design alternative (controller) 18 is shown. The final assessing of the whole process is depicted in Table 2.

5. CONCLUSION

In this paper a benchmark control problem for multi-objective controller tuning is proposed. It is based on a SISO non-linear model of a Peltier cell, where cold face temperature should be controlled by voltage input.

This benchmark for multi-objective optimisation considers common practices in the control engineering field, consisting in identifying a model from the process, for the tuning procedure. This means that the MOP statement should consider, from the beginning, discrepancies between

[4] https://www.mathworks.com/matlabcentral/fileexchange/65145

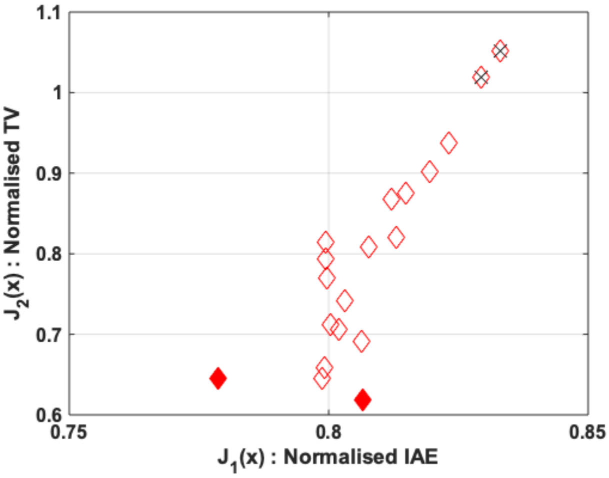
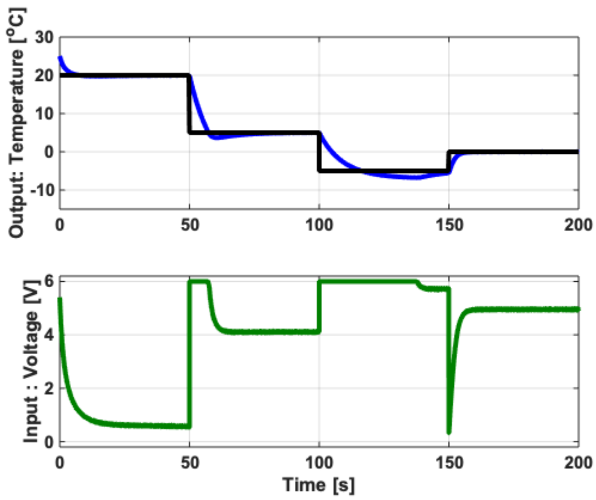


Fig. 7. Pareto front approximation $\boldsymbol{J}_{\boldsymbol{P}}^{\star}$ with the (pseudo)-process (filled diamonds) vs. non Pareto optimal solutions approximated in the $\boldsymbol{J}_{\boldsymbol{P}}^{*}$ (unfilled diamonds).



(a) DA18

Fig. 8. Closed loop test of the design alternative 18 in the (pseudo)-process.

process and model; the MOO optimisation process should consider that the Pareto front approximated in the model will be different from the one in the process; finally the MCDM stage must take into account that many Pareto-optimal solutions approximated in the model, will not keep their trade-off coherence when evaluated in the process.

According to the feedback on this benchmark by the interested community, more definitions and problems will be defined.

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Design alternative	J_1	J_2	kp	ki	$\overline{IAE}$	$\overline{TV}$	Dominated in (pseudo)-process	Ranking
1	0.6138	0.4786	-1.8606	-0.1865	0.8331	1.0515	-	18
2	0.6166	0.4523	-1.7938	-0.1782	0.8294	1.0189	-	16
3	0.6251	0.3999	-1.6485	-0.1643	0.8232	0.9381	Y	12
4	0.6345	0.3762	-1.5772	-0.1568	0.8195	0.9014	Y	11
5	0.6485	0.3599	-1.5268	-0.1495	0.8148	0.8754	Y	9
6	0.6578	0.3510	-1.4983	-0.1454	0.8120	0.8678	Y	6
7	0.6688	0.3292	-1.4223	-0.1434	0.8130	0.8206	Y	2
8	0.6798	0.3209	-1.3941	-0.1371	0.8076	0.8079	Y	1
9	0.7090	0.3208	-1.3964	-0.1297	0.7993	0.8148	Y	7
10	0.7149	0.3099	-1.3556	-0.1283	0.7994	0.7941	Y	5
11	0.7225	0.2995	-1.3151	-0.1269	0.7996	0.7706	Y	4
12	0.7256	0.2891	-1.2721	-0.1281	0.8032	0.7413	Y	3
13	0.7582	0.2757	-1.2169	-0.1234	0.8003	0.7126	Y	8
14	0.7679	0.2744	-1.2107	-0.1244	0.8020	0.7061	Y	10
15	0.8038	0.2680	-1.1819	-0.1266	0.8064	0.6912	Y	13
16	0.8370	0.2524	-1.1184	-0.1181	0.7992	0.6586	Y	14
17	0.8565	0.2470	-1.0953	-0.1166	0.7987	0.6459	Y	15
18	0.8813	0.2414	-1.0776	-0.1033	**0.7788**	**0.6454**	**N**	17
19	0.9149	0.2382	-1.0552	-0.1202	**0.8066**	**0.6188**	**N**	19

Table 1. Design alternatives for the selected Pareto front approximation $\boldsymbol{J_P^*}$. In bold, Pareto optimal solutions in $\boldsymbol{J_P^\star}$. $\overline{IAE}$ and $\overline{TV}$ stands for the IAE and TV, respectively, normalised according to the performance of the ZN controller in the (pseudo)-process experiment.

Hypervolume	Pareto optimality ratio	Performance MCDM
0.0836	2/19	17/20

Table 2. Performance indices comparing $\boldsymbol{J_P^*}$ and $\boldsymbol{J_P^\star}$.

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