

A Design Method of High Order Repetitive Controllers

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Abstract: It turns out that higher-order RC is useful for improving the robustness of the entire control system, and its importance is only increasing. In this paper, we propose a new high-order RC design method. The proposed high-order RC design method includes more design parameters than the same order conventional high-order RC, which increases design flexibility. Further improvement of robustness can be expected. It will also prove useful for improving time response. Further, it is possible to suppress a disturbance having a frequency different from the frequency of the target periodic signal. This is not possible with conventional high-order RC.

Keywords: Repetitive control, higher order repetitive control, digital control, sensitivity functions, disturbance rejection.

1. INTRODUCTION

It is well known that RC has a very high ability to follow a periodic signal. However, if there is a slight fluctuation in the frequency of the periodic signal, there will be a problem that the tracking performance will deteriorate. That means the lower robustness is a weak point. In order to improve its robustness, higher-order RCs were proposed. Higher order RCs make use of measured errors not only from the previous period, but also from one or more earlier periods. Lo and Longman (2005) and (2006) develop an understanding of how this approach can improve period robustness, from both a frequency response point of view and a root locus analysis approach. Many studies have shown that it has a certain effect Ahn et al. (2013a) and (2013b). As a remarkable effect of the higher order RCs, since the notch at the frequency of the periodic signal of sensitivity function can be widened, the sensitivity function can be kept low in the vicinity of the frequency of the periodic signal. Even if there is a slight fluctuation, the effect of it does not be so large Longman (2010). This is very important in the application. For example, in the case of constant speed rotation of a motor, the frequency of rotation is substantially constant, but there is unevenness in rotation, so there is a fluctuation in rotational speed even during constant speed rotation. Therefore, robustness in the vicinity of the rotation frequency are required and the importance of research on higher order RCs become more and more important. Sometimes the period fluctuates, and in this situation one wants an RC design that is robust to uncertainty in the disturbance period. It is this situation that motivated Steinbuch (2002) and Steinbuch et al. (2004). The approach there develops higher order RCs to create improved robustness to the disturbance period. The authors have proposed a new high-order RC design method Guo et al. (2014) from a unique point of view. Compared with the conventional high-order RC,

the higher order RC proposed by the authors can give a further degree of freedom while maintaining the characteristics of the conventional higher order RCs. In addition, it was clarified that it is useful not only for the sensitivity function but also for improving the time response. However, the authors' research so far is intended to clarify the difference from conventional high-order RC, and is a basic study and cannot be said to be a systematic study.

In this paper, we examine the design method of higher order RC proposed by the authors in a more general case. The purpose is to provide higher-order RC design guidelines through consideration of several cases.

2. PROPOSED METHOD

We consider the block diagram in Fig.1 as the repetitive control (RC) system, where $R(z)$ is the repetitive controller.

2.1 Problems with general high-order RC design methods

Suppose that the period of the command (or disturbance, or both when both are present) is pT , where T is the sample time interval, and p is the number of time steps per period.

The form for Nth order RC is

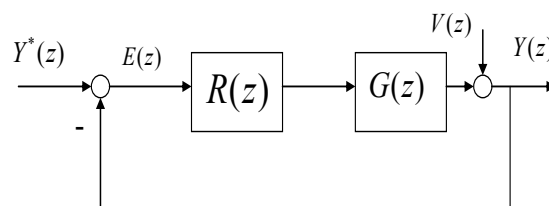


Fig. 1. Basic repetitive control systems

$$u(k) = \sum_{j=1}^N \alpha_j [u(k - jp + \varphi e(k - jp + 1))], \quad (1)$$

where N is the number of periods one wishes to include, and $\alpha_1, \alpha_2, \dots, \alpha_N$ are the weights that will be chosen by the designer. One may think of this as creating a weighted average, in which case each weight should be non-negative. However, as pointed out in Steinbuch (2002), there is no need to restrict the weights to be positive. In practice, it is convenient to design the RC law in the frequency domain include a compensator $\varphi(z)$, using the form

$$R_N(z) = \frac{\varphi(z) [\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \dots + \alpha_N]}{z^{Np} - [\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \dots + \alpha_N]} = \frac{N_r(z)}{D_r(z)}, \quad (2)$$

$$N_r(z) = \varphi(z) [\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \dots + \alpha_N],$$

$$D_r(z) = z^{Np} - [\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \dots + \alpha_N], \quad (3)$$

and in addition, it is necessary to restrict the choice of the α_j to satisfy $\alpha_1 + \alpha_2 + \dots + \alpha_N = 1$.

For first order RC, $N=1$, $\alpha_1 = 1$, we have

$$D_r(z) = z^p - 1.$$

Then the condition

$$D_r(z)[Y^*(z) - V(z)] = (z^p - 1)[Y^*(z) - V(z)] = 0 \quad (4)$$

Holed described at Longman (2010). Because it is the difference of the value of periodic functions at the present time and shifted one period ahead because p is the number of time steps per period.

How to determine the parameters $\alpha_1, \alpha_2, \dots, \alpha_N$, will be the designing freedom. A similar way as shown in the follows has been used in conventional studies

$$D_r(z) = z^{Np} - [\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \dots + \alpha_N] = (z^p - 1)^N \quad (5)$$

Such a high-order RC is simple in design and similar to the first-order RC. If the repeated design proposed by the authors Guo et al. (2014) is used, it is not difficult to determine design parameters $\alpha_1, \alpha_2, \dots, \alpha_N$ of the high-order RC. In addition, the notch at the sensitivity function in the vicinity of the frequency of the target periodic signal can be widened to increase the robustness. Compare with such design method, the authors proposed a design method Guo et al. (2014) that can include more design parameters as follows from a different viewpoint.

$$D_r(z) = z^{Np} - [\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \dots + \alpha_N] = (z^p - 1)^{n_0} (z^p - q_1)^{n_1} \dots (z^p - q_k)^{n_k} \quad (6)$$

where $n_0 + n_1 + \dots + n_k = N$. But that will be difficult to show how to determine all of the free parameters $\alpha_1, \alpha_2, \dots, \alpha_N$ and n_0, n_1, \dots, n_k in (6), we only show the case $N=3$, and $n_0 = n_1 = n_2 = 1$ at Guo et al. (2014) in order to show the new proposed design method of (6) have many prospect characteristics, and left so many objects for future studies.

However, the problem is how to design these parameters as the order increases. These design parameters are unlikely to be completely free and seem to have some limitations. It's not so easy to reveal the limitations. However, it cannot be said that it is an easy-to-use design method without clarifying such restrictions. To solve this problem, we propose a two-step method as follows.

2.2 Design method of second order and third order RC

In order to overcome the above problems, we realized that research on simple 2nd and 3rd order RC is important discussed in Guo et al. (2017). It is likely that some limitations on the design of high-order RC will be obtained from clarifying the limitations of such lower order RC. The results about second order and third order RC are summarized in Guo et al. (2017) and the main points are listed below.

For $N=2$, from (6) we have

$$D_r(z) = z^{2p} - [\alpha_1 z^p + \alpha_2] = (z^p - 1)(z^p - q_1). \quad (7)$$

Compare the last equation, then the parameters α_1, α_2 can be easily determined as follows

$$\begin{cases} \alpha_1 = q_1 + 1 \\ \alpha_2 = -q_1 \end{cases}. \quad (8)$$

Property 1: For the second order RC (7),

1) The free parameter q_1 must be a real number.

For $N=3$, we have

$$D_r(z) = z^{3p} - [\alpha_1 z^{2p} + \alpha_2 z^p + \alpha_3] = (z^p - 1)(z^p - q_1)(z^p - q_2). \quad (9)$$

The relations between $\alpha_1, \alpha_2, \alpha_3$ and q_1, q_2 can be directly calculated as follows and satisfy $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

$$\begin{aligned} \alpha_1 &= 1 + q_1 + q_2 \\ \alpha_2 &= -(q_1 q_2 + q_1 + q_2) \\ \alpha_3 &= q_1 q_2. \end{aligned} \quad (10)$$

Property 2: For the third order RC (9), only following cases are possible for a selection of the free parameter q_1, q_2

- 1) q_1, q_2 are real numbers.
- 2) q_1, q_2 are complex conjugate numbers.

The advantages of such a design method compared to conventional high-order RC was expressed in Guo et al. (2014) and (2017).

From these results, it is clear that there are limitations. For higher order RCs, these limitations are more complex and may be more difficult to design.

2.3 Proposed high-order RC design method

In order to overcome the difficulties to design the general high-order RC (6), the authors found that this problem can be overcome if higher order RC design can be designed by a combination of 2nd and 3rd order RC. Since there is a difference between even and odd orders of high-order RC, each example will be described in detail below.

2.3.1 6th order RC design

The 6th order RC controller is shown as follows.

$$R_6(z) = \frac{\varphi(z)[\alpha_1 z^{(6-1)p} + \alpha_2 z^{(6-2)p} + \dots + \alpha_6]}{z^{6p} - [\alpha_1 z^{(6-1)p} + \alpha_2 z^{(6-2)p} + \dots + \alpha_6]} = \frac{N_6(z)}{D_6(z)}, \quad (11)$$

$$N_6(z) = \varphi(z)[\alpha_1 z^{(6-1)p} + \alpha_2 z^{(6-2)p} + \dots + \alpha_6], \quad (12)$$

$$D_6(z) = z^{6p} - [\alpha_1 z^{(6-1)p} + \alpha_2 z^{(6-2)p} + \dots + \alpha_6] = (z^p - 1)(z^p - q_1) \dots (z^p - q_5) \quad (13)$$

The design parameters are $q_1, q_2 \dots q_5$, but these parameters must be determined under the following conditions.

- There must be real numbers because the control algorithm (1) must implement in the real world.
- $\alpha_1 + \alpha_2 + \dots + \alpha_N = 1$, in order to ensure the tracking error is zero.

Compare the left and right side of (13), the following relations are obtained.

$$\alpha_1 = 1 + q_1 + (q_2 + q_3) + (q_4 + q_5) \quad (14)$$

$$\alpha_2 = -\{q_1 + (q_2 + q_3) + (q_4 + q_5) + q_1(q_2 + q_3 + q_4 + q_5) + q_2 q_3 + (q_2 + q_3)(q_4 + q_5) + q_4 q_5\} \quad (15)$$

$$\alpha_3 = q_1(q_2 + q_3 + q_4 + q_5) + q_2 q_3 + (q_2 + q_3)(q_4 + q_5) + q_4 q_5 + q_1 q_2 q_3 + q_1(q_2 + q_3)(q_4 + q_5) + q_2 q_3(q_4 + q_5) + q_4 q_5(q_2 + q_3) \quad (16)$$

$$\alpha_4 = -(q_1 q_2 q_3 + q_1(q_2 + q_3)(q_4 + q_5) + q_2 q_3(q_4 + q_5) + q_4 q_5(q_2 + q_3) + q_1 q_2 q_3(q_4 + q_5) + q_1 q_4 q_5(q_2 + q_3) + q_2 q_3 q_4 q_5) \quad (17)$$

$$\alpha_5 = q_1 q_2 q_3(q_4 + q_5) + q_1 q_4 q_5(q_2 + q_3) + q_2 q_3 q_4 q_5 + q_1 q_2 q_3 q_4 q_5 \quad (18)$$

$$\alpha_6 = -q_1 q_2 q_3 q_4 q_5 \quad (19)$$

From above relations and considering the conditions a), b), the parameters $q_1, q_2 \dots q_5$ can be selected as follows.

Property 3: For the 6th order RC (11), following cases are possible for the selection of the free parameters $q_1, q_2 \dots q_5$.

- All parameters as real numbers.
- One must be a real number, the left 4 parameters must be tow pairs of complex conjugate numbers.

Proof: The case of 1) is obvious and trivial. The case 2) is clear from relations (14)-(19), if one selects a parameter as a real number and left 4 parameters as two pairs of complex conjugate numbers, then all of the parameters $\alpha_1, \alpha_2, \dots \alpha_6$ will satisfy the conditions a) and b).

That is a very interesting result. Because remember the section 2.2, we have the Property1 and 2. Then we can think of the result for 6th order RC can be believed as the combination of one second order RC and tow third order RCs. In addition, the role of such parameters can similarly refer from the role the parameters in the second order RC plus tow third order RCs, respectively. That will extremely simplify the difficulty of design the parameters, as well as very useful to provide who to use these parameters. That is why we say the fundamental characteristics of the second order RC and the third order RC are very important in Guo et al. (2017).

2.3.2 7th order RC design

Above subsection show the case of even order RCs. The odd case needs to consider.

The 7th order RC controller can be shown as follows.

$$R_7(z) = \frac{\varphi(z)[\alpha_1 z^{(7-1)p} + \alpha_2 z^{(7-2)p} + \dots + \alpha_7]}{z^{7p} - [\alpha_1 z^{(7-1)p} + \alpha_2 z^{(7-2)p} + \dots + \alpha_7]} = \frac{N_7(z)}{D_7(z)}, \quad (20)$$

$$N_7(z) = \varphi(z)[\alpha_1 z^{(7-1)p} + \alpha_2 z^{(7-2)p} + \dots + \alpha_7], \quad (21)$$

$$D_7(z) = z^{7p} - [\alpha_1 z^{(7-1)p} + \alpha_2 z^{(7-2)p} + \dots + \alpha_7] \quad (22)$$

$$= (z^p - 1)(z^p - q_1) \dots (z^p - q_6) \quad (23)$$

The design parameters are $q_1, q_2 \dots q_6$, also these parameters must be determined under the conditions a) and b).

Compare the left and right side of (23), the following relations are obtained.

$$\alpha_1 = 1 + (q_1 + q_2) + (q_3 + q_4) + (q_5 + q_6) \quad (24)$$

$$\alpha_2 = -\{(q_1 + q_2) + (q_3 + q_4) + (q_5 + q_6) + (q_1 + q_2)(q_2 + q_3 + q_4 + q_5) + q_3 q_4 + (q_3 + q_4)(q_5 + q_6) + q_5 q_6\} \quad (25)$$

$$\alpha_3 = (q_1 + q_2 + q_1 q_2)(q_3 + q_4 + q_5 + q_6) + q_1 q_2 + (q_3 + q_4)(q_5 + q_6) + q_3 q_4 + q_5 q_6 + (q_1 + q_2)(q_3 + q_4)(q_5 + q_6) + q_3 q_4(q_1 + q_2 + q_5 + q_6) + q_5 q_6(q_1 + q_2 + q_3 + q_4) \quad (26)$$

$$\alpha_4 = -\{(q_1 q_2(q_3 + q_4 + q_3 q_4 + q_5 + q_6 + q_5 q_6) + q_3 q_4(q_1 + q_2 + q_5 + q_6 + q_5 q_6) + (q_1 + q_2)(q_3 + q_4)(q_5 + q_6) + q_1 q_2(q_3 + q_4)(q_5 + q_6) + q_3 q_4(q_1 + q_2)(q_5 + q_6) + q_5 q_6(q_1 + q_2)(q_3 + q_4)\} \quad (27)$$

$$\alpha_5 = q_1 q_2 q_3 q_4 + q_1 q_2 q_4 q_5 + q_3 q_4 q_5 q_6 + q_1 q_2(q_3 + q_4)(q_5 + q_6) + q_3 q_4(q_1 + q_2)(q_5 + q_6) + q_5 q_6(q_3 + q_4)(q_1 + q_2) + q_1 q_2 q_3 q_4(q_5 + q_6) + q_1 q_2 q_3 q_4(q_5 + q_6) + q_1 q_2 q_5 q_6(q_3 + q_4)$$

$$+q_3q_4q_5q_6(q_1 + q_2) \quad (28)$$

$$\alpha_6 = -\{q_1q_2q_3q_4(q_5 + q_6) + q_1q_2q_5q_6(q_3 + q_4) + q_3q_4q_5q_6(q_1 + q_2) + q_1q_2q_3q_4q_5q_6\} \quad (29)$$

$$\alpha_7 = q_1q_2q_3q_4q_5q_6 \quad (30)$$

From above relations and considering the conditions a), b), the parameters $q_1, q_2 \dots q_6$ can be selected as follows.

Property 4: For the 7th order RC (23), following cases are possible for the selection of the free parameters $q_1, q_2 \dots q_6$.

- 1) All parameters as real numbers.
- 2) $2n$ ($n=1,2$) be real numbers, the left $6-2n$ parameters must be selected as pairs of complex conjugate numbers.

Proof: The case of 1) is obvious and trivial. The case 2) is clear from relations (24)-(30), if $2n(n=1,2)$ parameters select as real numbers and left $6-2n$ parameters must be selected as pairs of complex conjugate numbers, then all of the parameters $\alpha_1, \alpha_2, \dots \alpha_7$ will satisfy the condition a) and b).

This result is almost similar to the Property 3, the 7th order RC can be believed as the combination of $2n$ pairs of second-order RCs plus $6-2n$ pairs of third-order RCs.

3. SIMULATION RESULTS

There are many methods to design the compensator $\varphi(z)$, in order to focus our method, we assume the $G(z)$ is minimal phase and stable, then create a compensator that is equal to $\varphi(z) = 1/G(z)$ in order to show some law characteristics of the higher order RCs. There are at least two main approaches to dealing with when $G(z)$ is non-minimal phase. One is the compensator design optimization in the frequency domain due to Panomruttanarug and Longman discussed in Longman (2010). And the second is due to Tomizuka discussed in Tomizuka (1987) and Tomizuka et al. (1989). Here we consider the latter approach because it lends itself to making design tool plots analogous to those given above. Simulation tool, Matlab/Simulink is used for simulation.

3.1 6th order RC

We choose the sampling rate $f=100$ [Hz], and the reference signal is periodic with period $p = 10$ time steps. So the fundament frequency is 10 [Hz], the Nyquist frequency is 50 [Hz]. The sensitivity transfer function is defined by $S(z) = [1 + R_6(z)G(z)]^{-1}$. We chose $q_1 = 0.9$, $q_{2,3} = -0.309 \pm 0.9511i$ and $q_{4,5} = -0.4540 \pm 0.891i$. The root locus for the conventional method (5), just like choosing $q_1 = q_2 = q_3 = q_4 = q_5 = 1$, are plotted in Fig.2, and for the proposed method (11) are plotted in Fig.3. Fig.4 and Fig.5 are the enlarge figures of Fig.2 and Fig.3, respectively. The sensitivity transfer function for the conventional method (5) also plot in Fig.6 and for the proposed method (11) are plotted in Fig.7. Compare with Fig.4 and Fig. 5, the poles are overlapping at $z=1$ in conventional method (5), but the poles are distributed around $z = 1$ in our proposed method. The merit of distributing poles around $z=1$ can be seen from the results of the sensitivity functions Fig. 6 and Fig.7. The notch at fundament frequency 10 [Hz] are widened in Fig.7 comparing with Fig.6. That means the robustness near around fundament frequency is

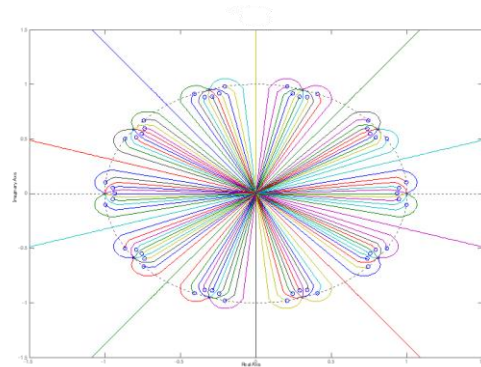


Fig.2. Root locus for the conventional method.

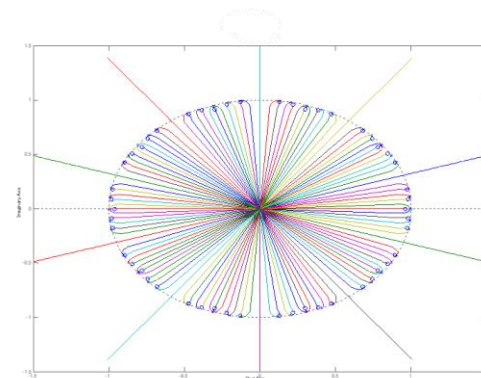


Fig.3 Root locus for the proposed method.

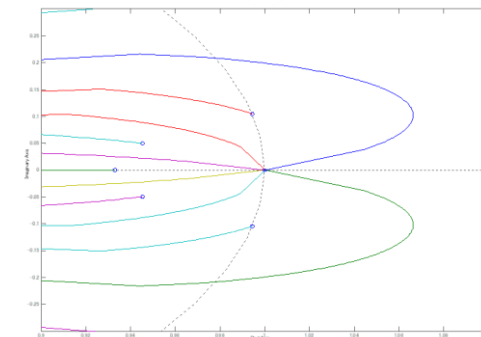


Fig.4. Enlarged part of Fig.2.

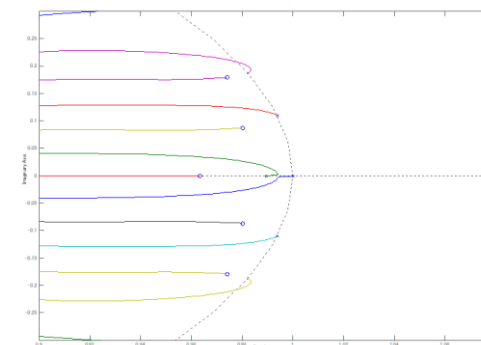


Fig.5. Enlarged part of Fig.3.

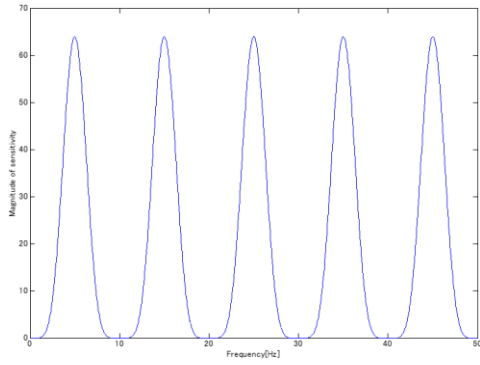


Fig.6. Sensitivity function of the conventional method.

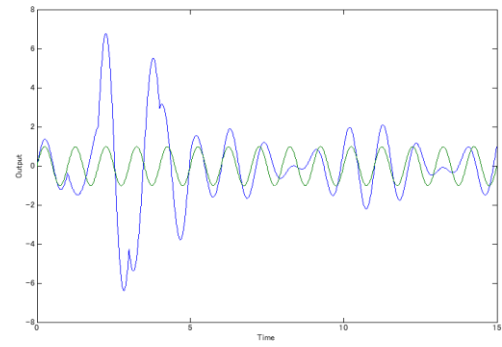


Fig.10. The conventional method (disturbance= $0.7\sin 2\pi$).

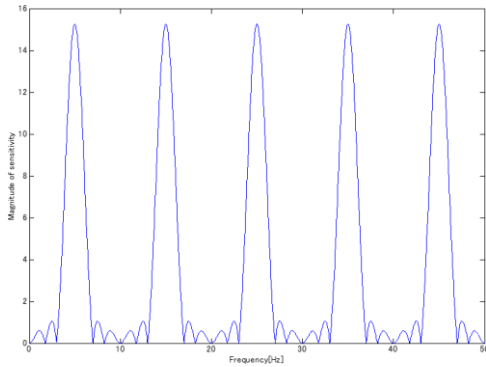


Fig.7. Sensitivity function of the proposed method.

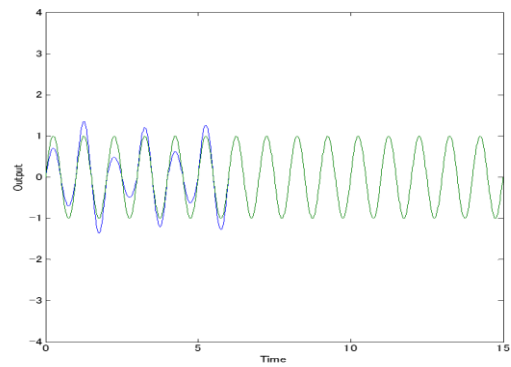


Fig.11. The proposed method (disturbance= $0.7\sin 2\pi$).

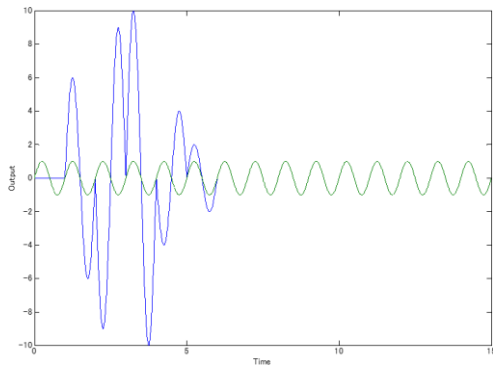


Fig.8. Time response of the conventional method.

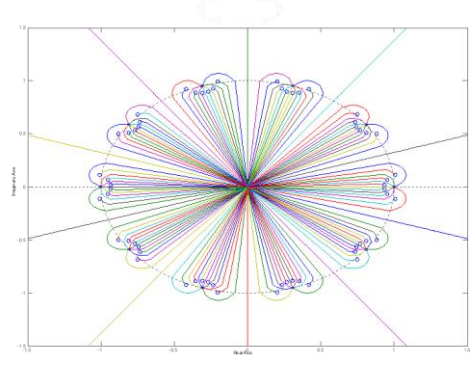


Fig.12. Root locus for the conventional method.

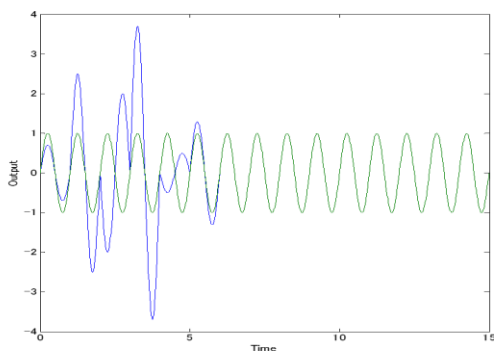


Fig.9. Time response of the proposed method.

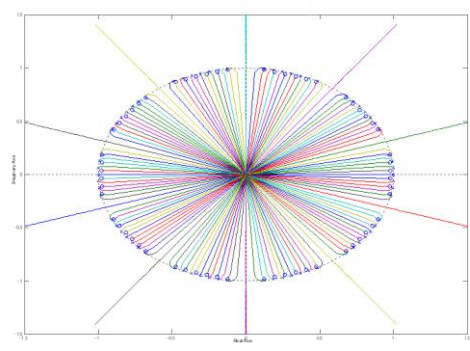


Fig.13 Root locus for the proposed method.

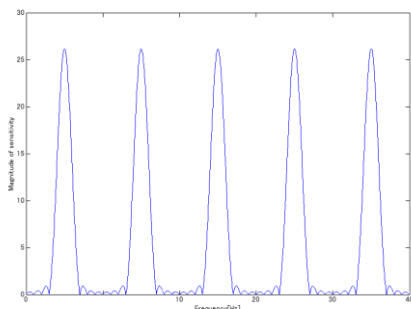


Fig.14. Sensitivity function of the proposed method.

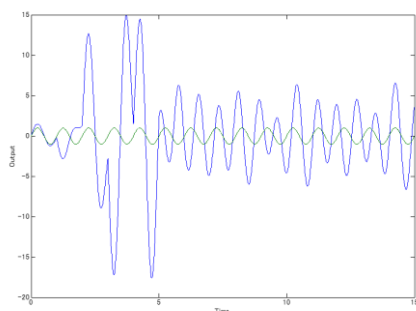


Fig.15. The conventional method (disturbance= $0.7\sin 2\pi$)

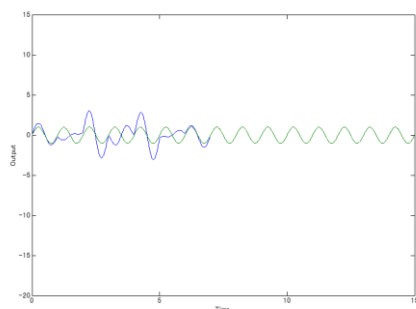


Fig.16. The proposed method (disturbance= $0.7\sin 2\pi$)

improved. It turns out that it is useful for reducing the effect when fundamental and disturbance period fluctuation. The time response of the closed-loop system in Fig. 1 is shown in Fig.8 for conventional and Fig.9 for proposed method, respectively. It is clear that the transient response can be greatly improved by our proposed method. In addition, Fig.10 and Fig.11 show the time response when the disturbance frequency is $0.7\sin 2\pi$ that is not the fundamental frequency 10[Hz]. Our proposed method Fig.11 can eliminate the disturbance but the conventional method Fig.10 can't. That is the very important point of our proposed method.

3.2 7th order RC

The simulation condition is as same as above section. We chose $q_{1,2} = 0.809 \pm 0.5878i$, $q_{3,4} = 0.309 \pm 0.9511i$, $q_{5,6} = -0.309 \pm 0.9511i$, The root locus for the conventional method (5) is plotted in Fig.12, and for the proposed method (11) is plotted in Fig.13. The sensitivity transfer function for the conventional method (5) is similar to Fig.6 and for the proposed method (11) are plotted in Fig.14.

Comparison results of Fig.15 and Fig.16 are the same as the previous subsection. That once again show that our proposed method has many advantages over conventional methods.

4. CONCLUSIONS

A new design method was proposed for the construction of the new high-order RC proposed by the authors. Compared to conventional high-order RC, the design parameters increased, and design guidelines for these parameters have been given. Through simulation, these parameters can not only widen the notch at the frequency of the periodic function of the sensitivity function, but can also completely suppress frequencies near the target periodic signal frequency. Thereby, it becomes possible to completely suppress a periodic disturbance signal different from the frequency of the target periodic signal. This is something that conventional high-order RC cannot do, and demonstrates the new capabilities of high-order RC. In the future, we will develop more systematic design methods.

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