Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Adaptive Distributed Control for Large-Scale Systems with Unknown Interconnection

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Abstract: This paper presents a distributed adaptive control law for large-scale systems with unknown interconnection parameters. An adaptive control law is designed to follow-up a model reference for a network through a controller that adjusts its parameters according to the dynamics of the reference, the neighborhood and the physical interconnection. This work presents a Model Reference Adaptive Control methodology for heterogeneous systems, such that the synchronization of the agents in the network is achieved even in the case where the interconnection is unknown. Stability properties of the proposed control law are validated via Lyapunov methods and boundedness of the synchronization errors is guaranteed. The authors propose a validation scheme of adaptive control for different references in a context of level control in tank networks and a synchronization analysis of the estimated constants.

Keywords: Interconnected systems, adaptive control, water networks, distributed control.

1. INTRODUCTION

Network systems have been extensivelystudied in a wide variety of applications and under different perspectives. In local and decentralized models, each system acts according to the available information, without considering the external dynamics of its environment (Tian and Liu (2008)). Centralized control strategies, widely used in systems with a leader-follower topology, share performance leader information based on their dynamics (Jadbabaie et al. (2003)). In a distributed network system, each agent acts according to the information shared with its neighbors, unlike centralized or local controllers, which do not share information through the entire network, losing online changes of the parameters of each agent. Networks that present physical communications are commonly known as large-scale systems, and it is a fairly common modelling strategy in power systems, air systems, hydraulic systems, car traffic among many other applications, as in Yucelen and Shamma (2014); Ghafoor et al. (2018); Mauser and Bach (2009); Suzumura and Kanezashi (2012).

Recently, a major challenge of interconnected systems is the synchronization of the network's agents to some reference model, under unknown interconnection parameters. The calculation of these constant gains under limited information (i.e. data is only available from an agent's own dynamics and its interactions with neighbors) is a nontrivial task. In Nguyen (2018), Model Reference Adaptive Control (MRAC) is presented as a useful tool, widely used in network systems and a wide variety different applications. This method allows starting from some matching conditions to replicate the dynamics of a system concerning with a reference (Jadbabaie et al. (2003)). The applications of this technology are diverse such as in transport, water, power systems, among others (Baldi et al. (2018); Saagi et al. (2016); Nguyen et al. (2018)).

An extension of the control theory derived from the MRAC is for distributed control, where not all the nodes of the system have direct communication with the reference model (Baldi et al. (2018)). In this case, it is necessary to estimate the reference as a *virtual reference* through communication between neighbors. The adaptive parameters are then estimated from the neighborhood for the synchronization of the entire network. This problem can be approached as a state regulation or output regulation type strategy, according to the characteristics of the network topology (Baldi and Frasca (2019)).

Some researches have focused on the development of adaptive control laws based on a reference model for large-scale systems with unknown communication parameters, but without the presence of uncommunicated agents with the reference (Lymperopoulos and Ioannou (2018), Yucelen et al. (2011)). Similarly, distributed adaptive control laws have been developed for unknown agent dynamics and without considering system interconnection terms associated with the network (Baldi et al. (2018); Jadbabaie et al. (2003)).

In this work, we analize the impact of the presence of physical interconnection on a case study of a tank network to develop a novel adaptive law to synchronize a network consisting of a layer of wireless and physical communication based on a reference model, where the reference can be isolated just with some system nodes. First, the basic concepts of MRAC are exposed in conjunction with the scenario of known interconnection parameters in the

⋆ Duvan Tellez-Castro is supported in part by Colciencias 727-2016.

system. Next, we consider the case where these parameters are unknown and should be estimated from an adaptive law. In both cases, a control law that guarantees network synchronization error boundedness is proposed and validated through Lyapunov Theory.

The paper is organized as follows. In Section 2 the formulation of the problem is presented. Section 3 presents the distributed control law for known interconnection parameters. In Section 4, we include unknown interconnection terms for the proposed control law. Section 5 describes the numerical example of an interconnected tank network, while in Section 6 conclusions and future work are outlined.

Notation. In this work, the notation used is fairly standard. $\mathbb{R}$ denotes the real number set. X and x denotes matrices and vectors respectively. For a matrix, the Euclidean norm of a signal is defined as $\|X\|^2 = \sum_{i=1}^{n} |x_i|^2$. $X^\top$, $x^\top$ and X^{-1} and x^{-1} describe the transpose and the inverse of a matrix or a vector respectively. The trace of a square matrix X is defined as $\mathrm{tr}(X)$.

To model the interconnections of the system, graph theory is adopted. A directed graph is defined as the pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the nodes set of the graph, and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is the communication edges set. The degree of each node depends on the number of neighbors it has. The adjacency matrix is defined as $A = [a_{ij}]$, where $a_{ii} = 0$ and $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, with $i \neq j$.

2. PROBLEM FORMULATION

This section contextualizes the problem of synchronizing a network of linear systems based on a reference model and in the presence of nodes that lack direct communication with the reference and have unknown interconnection parameters. Consider a network consisting of N nodes (subsystems) with dynamics:

$$\dot{x}_i = A_i x_i + b_i u_i + \sum_{j=1}^{N} A_{ij} x_j, \qquad i \in [1, ..., N], \tag{1}$$

where $x_i \in \mathbb{R}^n$ denotes the agent i states, A_i is an unknown matrix related to the agents states, b_i are known vectors with possibly heterogeneous agents ($A_i \neq A_j$ and $b_i \neq b_j$), $u_i \in \mathbb{R}^p$ represents the control input of the i agent. $x_j \in \mathbb{R}^n$ denotes the state of the neighboring agent and A_{ij} is the interconnection matrix. The reference model is described as

$$\dot{x}_0 = A_0 x_0 + b_0 r, \tag{2}$$

where $x_0 \in \mathbb{R}^n$ denotes the state, $r \in \mathbb{R}^p$ represents the reference, and A_0 and b_0 denotes the matrices of the reference model.

Problem. Consider N agents with dynamics (1), and reference model (2). Then, the objective of the control is to guarantee that all closed-loop synchronization error signals are bounded.

3. DISTRIBUTED MRAC WITH KNOWN INTERCONNECTION

In this section, we define the distributed control law for the problem formulated for nodes that communicate with the reference and for nodes that do not, in the case of known interconnection parameters. Initially, we validate the control strategy based on a reference model for one system. The following assumptions are standard in MRAC and are required in order to guarantee convergence the tracking errors in both the reference and its neighbors.

Assumption 1. The vector k^*_{mi} and the scalar k^*_{ri} exist and are defined as the solution to the following

$$\begin{aligned} A_0 &= A_i + b_i k^{*\top}_{mi}, \\ b_0 &= b_i k^*_{ri}. \end{aligned} \tag{3}$$

Constants in (3) are known as feedback matching conditions.

Assumption 2. The vector k^*_{mij} and the scalar k^*_{rij} exists and are defined such that

$$\begin{aligned} A_i &= A_j + b_j k^{*\top}_{mij}, \\ b_i &= b_j k^*_{rij}. \end{aligned} \tag{4}$$

Constants k^*_{mij} and k^*_{rij} in (4) are known as coupling matching conditions.

Assumption 3. The communication graph is acyclic and must contain at least one spanning tree where the leader is connected Baldi and Frasca (2019).

Proposition 1. Considering the system (1) without interconnection parameter based on model reference adaptive control methodology (Tao (2003)), that means $A_{ij} = 0 \forall j$, with Assumptions 1-3 verified, it is possible to synchronize node 1 to a reference model by the controller

$$u_1 = k^\top_{m1} x_1 + k_{r1} r,$$

and the adaptive laws

$$\begin{aligned} \dot{k}^\top_{m1} &= -\mathrm{sgn}\left(k_{r1}{}^*\right) \gamma\, b_0^\top P \left(x_1 - x_0\right) x_1^\top \\ \dot{k}_{r1} &= -\mathrm{sgn}\left(k_{r1}{}^*\right) \gamma\, b_0^\top P \left(x_1 - x_0\right) r, \end{aligned} \tag{5}$$

where the scalar $\gamma > 0$ is the adaptive gain, and P is a positive definite matrix satisfying

$$PA_0 + A_0^\top P = -Q, \qquad Q > 0.$$

Proof. It follows from Nguyen (2018).

For a large-scale system, the extension of the control theory based on a reference model is carried out.

Proposition 2. The synchronization of an agent that is connected to a reference model is done through the following control law

$$u_1 = k^\top_{m1} x_1 + k_{r1} r + \sum_{j=1}^{N} a_{1j} k^*_{1j} x_j,$$

with adaptive laws (5) and k_{1j} chosen based on Lemma 1 of Lymperopoulos and Ioannou (2016) to minimize the effect of closed-loop interconnections $\|A_1 - B_1 k_{1j}\|$.

Proof. It follows from Lymperopoulos and Ioannou (2018).

From these propositions, it is possible to extend the theory in a distributed way and including agents that lack direct communication with the leader.

Theorem 3. Consider N systems with dynamics (1), where only the node 1 has direct communication with the reference as in Proposition 2. The other systems employ the following control law

$$u_i = \alpha\left(\sum_{j=1}^{N} a_{ij}k_{mij}^{\top}x_j + k_{mi}\sum_{j=1}^{N} a_{ij}(x_i - x_j) + \ldots \right. \\ \left. \ldots + \sum_{j=1}^{N} a_{ij}k_{rij}u_j + \sum_{j=1}^{N} a_{ij}k_{ij}^{*}x_j\right), \quad (6)$$

with $\alpha = \frac{1}{\sum_{j=1}^{N} a_{ij}}$. The other terms are the result of the following adaptive laws

$$\dot{k}_{mij}^{\top} = -\operatorname{sgn}(k_{ri}^{*})\gamma\, b_0^{\top}P\left[\sum_{j=1}^{N} a_{ij}(x_i - x_j)\right]x_i^{\top},$$
$$\dot{k}_{mi}^{\top} = -\operatorname{sgn}(k_{ri}^{*})\gamma\, b_0^{\top}P\left[\sum_{j=1}^{N} a_{ij}(x_i - x_j)\right]\ldots \\ \ldots\left[\sum_{j=1}^{N} a_{ij}(x_i - x_j)\right]^{\top},$$
$$\dot{k}_{rij} = -\operatorname{sgn}(k_{ri}^{*})\gamma\, b_0^{\top}P\left[\sum_{j=1}^{N} a_{ij}(x_i - x_j)\right]u_j. \quad (7)$$

Then, the control law (6) guarantees that all synchronization errors are bounded.

Proof. The aim is to guarantee that the synchronization error between each node tends to zero, regardless of whether there is direct communication or not with the reference model, the following equation is defined

$$V(e_{ij}, \tilde{k}_{mi}, \tilde{k}_{ri}, \tilde{k}_{ij}, \tilde{k}_{rij}) = \sum_{i=1}^{N}\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)^{\top} P\ldots \\ \ldots\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right) + \sum_{j=1}^{N}\operatorname{tr}\left(\frac{\tilde{k}_{mi}^{\top}\tilde{k}_{mi}}{\gamma\,|k_{ri}^{*}|}\right) + \ldots \\ \ldots + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\operatorname{tr}\left(\frac{\tilde{k}_{mij}^{\top}\tilde{k}_{mij}}{\gamma\,|k_{ri}^{*}|}\right) + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\operatorname{tr}\frac{k_{ij}^{*\top}k_{ij}^{*}}{\gamma\,|k_{ri}^{*}|} + \ldots \\ \ldots + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\frac{\tilde{k}_{ri}^{2}}{\gamma\,|k_{r}^{*}|}, \quad (8)$$

where $j = 0$ is used as a representation of the reference. The error dynamics is represented as $e_i = x_i - x_0$ or $e_{ij} = x_i - x_j$ in short notation, and in long notation as

$$\dot{e}_{ij} = A_0(x_i - x_j) + b_j[u_j - k_{mij}^{*\top}x_i - k_{mi}^{*\top}e_{ij} - k_{rij}^{*}u_i + k_{ij}^{*}x_j],$$

$$\dot{e}_{ij} = A_0 e_{ij} + b_j[\tilde{k}_{mij}^{\top}x_i + \tilde{k}_{mi}^{\top}e_{ij} + \tilde{k}_{rij}^{\top}u_i + k_{ij}^{*}x_j], \quad (9)$$

with $\tilde{k}_{mij} = k_{mij} - k_{mij}^{*}$; $\tilde{k}_{mi} = k_{mi} - k_{mi}^{*}$; $\tilde{k}_{rij} = k_{rij} - k_{rij}^{*}$. The derivative of (8) along (9) can be obtained as

$$\dot{V} = \sum_{i=1}^{N}\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)^{\top}(PA_0 + A_0^{\top}P)\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right) + \ldots \\ \ldots + 2\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)^{\top} Pb_i\ldots \\ \ldots\left(\sum_{j=0}^{N} a_{ij}\tilde{k}_{mij}^{\top}x_i + \tilde{k}_{mi}^{\top}\sum_{i=1}^{N} a_{ij}e_{ij} + \sum_{i=1}^{N} a_{ij}\tilde{k}_{rij}u_i + \sum_{i=1}^{N} a_{ij}\tilde{k}_{ij}x_j\right) + \ldots \\ \ldots + \sum_{i=1}^{N}\operatorname{tr}\left(\frac{\tilde{k}_{mi}^{\top}\gamma^{-1}\dot{\tilde{k}}_{mi}}{|k_{ri}^{*}|}\right) + \sum_{i=1}^{N}\operatorname{tr}\left(\frac{\tilde{k}_{mij}^{\top}\gamma^{-1}\dot{\tilde{k}}_{mij}}{|k_{ri}^{*}|}\right) + \ldots \\ \ldots + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\frac{\tilde{k}_{rij}\gamma^{-1}\dot{\tilde{k}}_{rij}}{|k_{ri}^{*}|}, \quad (10)$$

and reducing the Lyapunov equation as

$$\dot{V} = -\sum_{i=1}^{N}\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)^{\top} Q\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right),$$

Then, (10) is bounded, which implies by Barbalat's Lemma that $\lim_{t\to\infty}\dot{V} = 0$ (Khalil (2002)). Thus, $\lim_{t\to\infty} e_{ij} = 0\ \forall i$, which we can proof that all the synchronization errors e_{ij} are bounded. ■

The proof of this theorem allows validating the distributed control strategy for interconnected systems with known parameters. However, in real cases not all of these parameters are available for the control design. In the following section the estimation used in case of lack of knowledge of the parameters is presented.

4. DISTRIBUTED MRAC WITH UNKNOWN INTERCONNECTION

In this section, the theory is again extended in a more realistic way, including unknown parameters of the interconnection, in this case the unknown parameters are those directly associated with the interconnection, and the matrix A of the reference model and and neighboring agents. The objective is to design a control law that guarantees the synchronization in a boundary way and estimates adequately those interconnection gains exposed in the previous section k_{ij}.

Theorem 4. The adaptive laws (7) are assigned in the same way for the synchronization of agents (1) with the reference model (2) and with its neighbors. Therefore, constant k_{ij} is a result of the adaptive law

$$\dot{k}_{ij}^{\top} = -\sum_{j=1}^{N}\operatorname{sgn}(k_{ri}^{*})\gamma\, b_0^{\top}P\left[a_{ij}(x_i - x_j)\right]x_j^{\top}, \quad (11)$$

then the control law (6) with adaptive laws (7) and (11) guarantees that all synchronization errors are bounded.

Proof. The objective of this test is to validate that Closed-loop error signals are bounded even in the presence of unknown parameters in the interconnection of the network, the following equation is taken

$$V(e_{ij},\tilde{k}_{mi},\tilde{k}_{ri},\tilde{k}_{ij},\tilde{k}_{rij}) = \sum_{i=1}^{N}\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)^{\top} P\ldots$$
$$\ldots\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right) + \sum_{j=1}^{N}\mathrm{tr}\left(\frac{\tilde{k}_{mi}^{\top}\tilde{k}_{mi}}{\gamma\,|k_{ri}^{*}|}\right)+\ldots$$
$$\ldots + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\,\mathrm{tr}\left(\frac{\tilde{k}_{ij}^{\top}\tilde{k}_{ij}}{\gamma\,|k_{ri}^{*}|}\right) + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\,\mathrm{tr}\left(\frac{\tilde{k}_{ij}^{\top}\tilde{k}_{ij}}{\gamma\,|k_{ri}^{*}|}\right)+\ldots$$
$$\ldots + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\frac{\tilde{k}_{ri}^{2}}{\gamma\,|k_{r}^{*}|}. \tag{12}$$

The error dynamics is represented equally as $e_i = x_i - x_0$ o $e_{ij} = x_i - x_j$ in short notation, and in long notation as

$$\dot{e}_{ij} = A_0(x_i - x_j) + b_j[u_j - k_{mij}^{*\top}x_i - k_{mi}^{*\top}e_{ij} - k_{rij}^{*}u_i + k_{ij}^{*}x_j],$$

$$\dot{e}_{ij} = A_0 e_{ij} + b_j[\tilde{k}_{mij}^{\top}x_i + \tilde{k}_{mi}^{\top}e_{ij} + \tilde{k}_{rij}^{\top}u_i + \tilde{k}_{ij}x_j], \tag{13}$$

with $\tilde{k}_{ij} = k_{ij} - k_{ij}^{*}$. The derivative of (12) along (13) can be obtained as

$$\dot{V} = \sum_{i=1}^{N}\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)^{\top}(PA_m + A_m^{\top}P)\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)+\ldots$$
$$\ldots + 2\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)^{\top} Pb_i\ldots$$
$$\ldots\left(\sum_{i=1}^{N} a_{ij}\tilde{k}_{mij}^{\top}x_i + \tilde{k}_{mi}^{\top}\sum_{i=1}^{N} a_{ij}e_{ij} + \sum_{i=1}^{N} a_{ij}\tilde{k}_{rij}u_i + \sum_{i=1}^{N} a_{ij}\tilde{k}_{ij}x_j\right)+\ldots$$
$$\ldots + \sum_{j=0}^{N}\mathrm{tr}\left(\frac{\tilde{k}_{mi}^{\top}\gamma^{-1}\dot{\tilde{k}}_{mi}}{|k_{ri}^{*}|}\right) + \sum_{i=1}^{N}\mathrm{tr}\left(\frac{\tilde{k}_{ij}^{\top}\gamma^{-1}\dot{\tilde{k}}_{ij}}{|k_{ri}^{*}|}\right)+\ldots$$
$$\ldots + \sum_{i=1}^{N}\mathrm{tr}\left(\frac{\tilde{k}_{mij}^{\top}\gamma^{-1}\dot{\tilde{k}}_{mij}}{|k_{ri}^{*}|}\right) + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\frac{\tilde{k}_{rij}\gamma^{-1}\dot{\tilde{k}}_{rij}}{|k_{ri}^{*}|}, \tag{14}$$

where reducing is found that

$$\dot{V} = -\sum_{i=1}^{N}\left(\sum_{j=0}^{N} a_{ij}e_{ij}\right)^{\top} Q\left[\sum_{j=0}^{N} a_{ij}e_{ij}\right],$$

then, (14) is bounded, this implies that $\dot{V} \leq 0$, and by Barbalat's Lemma we can proof that all the synchronization errors e_{ij} are bounded for $t \to \infty$ (Khalil (2002)). ■

Theorems 3 and 4 allow to validate the inclusion of an adaptive distributed control based on a reference model for network systems with known and unknown interconnection parameters. These strategies can be validated through a numerical example where the synchronization of a network is guaranteed and the estimation of the interconnection parameters in the controller is developed.

5. NUMERICAL EXAMPLE

In this section, simulation results are presented taking the theorems described in the previous sections to verify the effectiveness of the proposed control laws. Consider the system of couple water tanks where each agent consist of two tanks and the agent presents physical interconnection as it is shown in Fig. 1. It is common in the design of large-scale control water transportation networks the use of virtual tanks for oriented-control modeling by García et al.

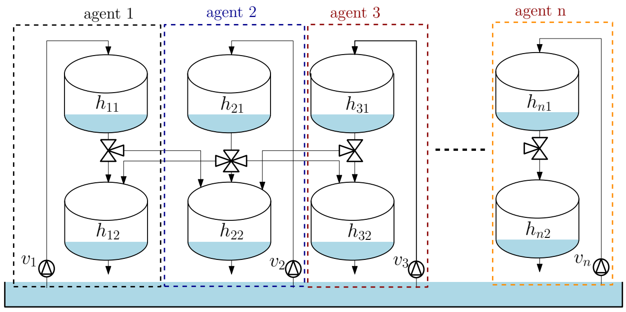

Fig. 1. Application case interconnected water network.

(2015). Based on Johansson (2000) Jalal and Rasmussen (2016), we can deduce the differential equation of each agent through Bernoulli's law and mass balances as

$$\dot{h}_{i1} = -\frac{q_{i1}(t)}{A_{i1}} + \frac{F_i}{A_{i1}},$$
$$\dot{h}_{i2} = -\frac{q_{i2}(t)}{A_{i2}} + \frac{1}{A_{i2}}\sum_{i=1}^{N}\sum_{j=1}^{N},\alpha_{ij}q_{i1}(t)$$

where $q_{is} = b_{is}(2gh_{is}(t))^{1/2}$ with $s \in \{1,2\}$ are the inlet and outlet flow rates, $F_i = k_i v_i$, correspond to inlet flow rates from pumps, $h_{is}(t)$, b_{is} , and A_{is} refer to the water level, cross-section of the outlet hole, and cross-sectional area of tank i, respectively. Each agent is linearized in a operation point with $x_i = [h_{i1} - h_{i1}^{0} \quad h_{i2} - h_{i2}^{0}]$ and $u_i = v_i - v_i^{0}$ the linear model is given by

$$\dot{x}_i = \begin{bmatrix} a_{1i} & 0 \\ 0 & a_{2i} \end{bmatrix} x_i + \begin{bmatrix} b_{1i} \\ 0 \end{bmatrix} u_i + \sum_{j=1}^{N} A_{ij}x_j,$$

where

$$a_{1i} = \frac{b_{i1}}{A_{i1}}\sqrt{g/2h_{i1}^{0}},\ \ a_{2i} = \frac{b_{i2}}{A_{i2}}\sqrt{g/2h_{i2}^{0}},\ \ b_{i1} = k_i/A_{i1},$$

with A_{ij} depending of the interconnections. We choose different scalar values for each system to considered heterogeneous nodes. The graph of the system is shown in Fig. 2, where the physical interconnection layer of blue color and the layer of the wireless communication graph of green color are observed.

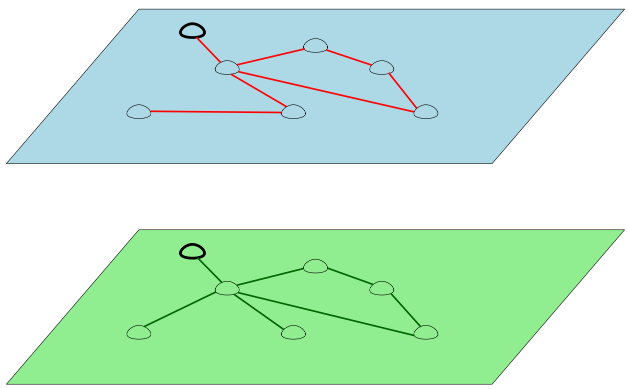

Fig. 2. Leader-follower physical and communication network graph.

The simulation parameters used are shown in Table 1. Note that these parameters are unknown and are used only for simulation, not for control design.

Table 1: Agents coefficients and initial conditions

	a_1	a_2	b_1	x_0
A_0	-0.25	-0.5	1	$[1\ -1]^\top$
A_1	-1.25	-1	0.5	$[1\ 0]^\top$
A_2	-0.5	-2.5	0.75	$[-1\ 0.5]^\top$
A_3	-0.75	-2	1.5	$[1\ 0]^\top$
A_4	-1.5	-2.5	1	$[-1\ 1]^\top$
A_5	-1	-2	1	$[-0.5\ 1]^\top$
A_6	-0.75	-1	0.5	$[0\ -1]^\top$

All agents dynamics are stable in open loop, including the reference model. For simulation purposes, the following additional parameters are necessary: $\gamma = 10$, $Q = \mathrm{diag}(100, 1)$. All the adaptive control laws are initialized to zero. From the validation of the Assumptions 1-3, in Fig 3 the result of the simulation is presented with known interconnection parameters of the form $k_{ij} = [k_{1i}\ k_{2i}]$. The interconnection matrices parameters A_{ij} of each system are described in the Table No.2 with the parameters defined as

$$A_{ij} = \begin{bmatrix} A_{ij1} & A_{ij2} \\ A_{ij3} & A_{ij4} \end{bmatrix}.$$

It is observed that along simulation time, the network converges to the reference model states, which leads to the synchronization error tending to zero satisfactorily.

Table 2: Coefficients system interconnection matrices

	A_{ij1}	A_{ij2}	A_{ij3}	A_{ij4}
A_{21}	1.05	3	3	2
A_{32}	0.35	1	1	1.85
A_{41}	1.85	1.5	1.7	0.9
A_{43}	0.5	0.4	0.3	1
A_{51}	1	1	0.4	0.45
A_{65}	2	1	0.8	2

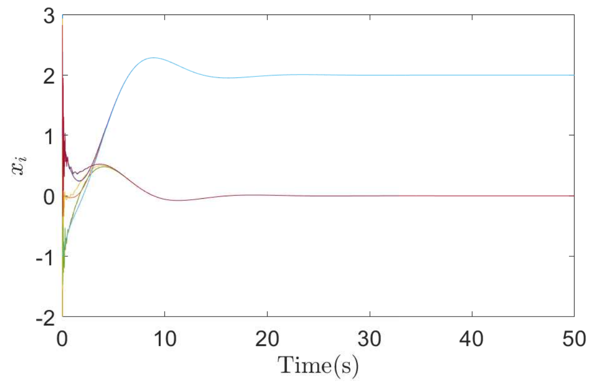

Fig. 3. Agents synchronization with known interconnection parameters.

In Fig. 4, the response of the same system with the unknown interconnection parameters is observed. The adaptive gain (11) is included and it is observed that, as in the previous case, the system presents a synchronization in the dynamics of its agents with respect to the reference model. In Fig. 5 the estimation error between the adaptive constants associated with the interconnection of the system and those obtained in Theorem 3 is presented. It is observed that all the constants tend to their estimate effectively.

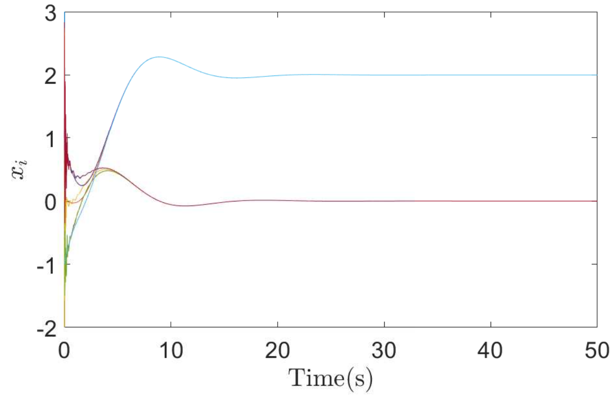

Fig. 4. Agents synchronization with unknown interconnection parameters.

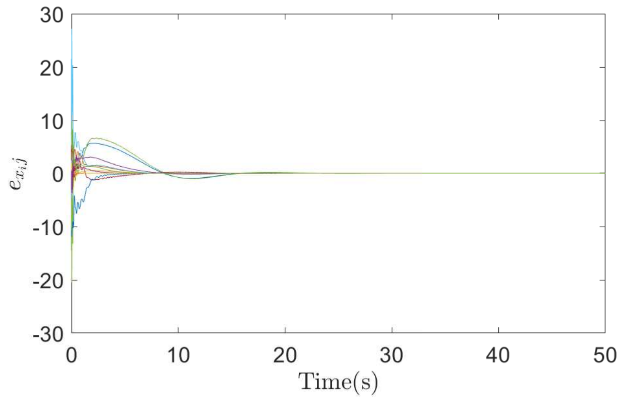

Fig. 5. Synchronization error system interconnection constants.

The reference of the model is modified to a sinusoidal signal, and in Fig. 6 its result is observed, as in the previous cases, the system achieves a convergence to the states of the reference model even with the change in the slope of control signal. The simulation results show

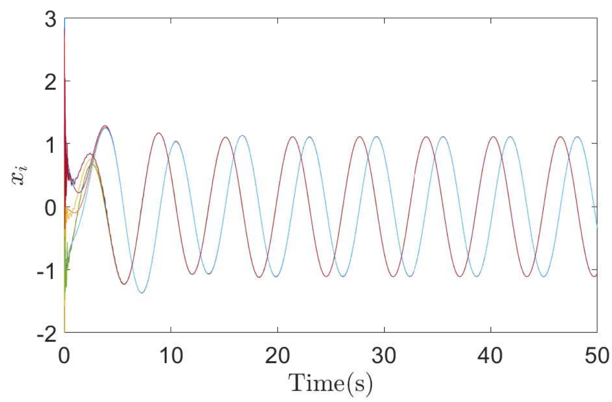

Fig. 6. Agents synchronization with unknown interconnection parameters in a sinusoidal reference.

a synchronization in the distributed network with known and unknown parameters, validating the convergence of the interconnection parameters and the estimation error with zero trend.

6. CONCLUSIONS AND FUTURE WORK

This work presents the development methodology of a distributed and adaptive controller for interconnected systems in the presence of unknown parameters. The problem contextualized to a tank networks is solved based on a distributed MRAC synchronization problem, where all the systems seek to have the same behavior from a reference model. Using adaptive control laws, it is possible to synchronize a system even in the presence of unknown interconnection parameters, making estimation and monitoring of the reference. The development of a control strategy based on a reference and distributed model allows extending the theory of interconnected systems for those networks where not all the agents present communication with a reference like industrial networks. This strategy proposes the synchronization through the estimation of network parameters through the information transmitted between neighbors. The estimation of parameters of interconnection in a network improves the operation of distributed systems by avoiding the faults present in the modeling of these parameters through adaptive control. More realistically the problem is addressed and allows a complete field of application in all interconnected systems.

For future work, we propose the extension of the theory to systems with nonlinear uncertainty, for its estimation through optimal control strategies or neural networks methodology, and likewise, the definition of this synchronization as an output regulation problem in similar contexts.

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