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Sliding Mode Observer Based Robust Fault Reconstruction for Modular Multilevel Converter with Actuator and Sensor Fault ⋆

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Abstract: In this paper, a design method of sliding mode observer (SMO) is proposed to solve the problem of robust fault reconstruction for modular multilevel converter (MMC) with actuator and sensor fault. A state space model of MMC system is established to consider simultaneously actuator fault, sensor fault and uncertainty. Based on the obtained system model, a SMO is introduced to reconstruct the fault and an augmented system is obtained. Especially, the fault can be detected and the fault dynamics can be reconstructed by controlling the sliding mode motion with the equivalent output. Moreover, the SMO gain can be designed via the semi-definite program method. Finally, the effectiveness and feasibility of this method can be verified by using a MMC simulation example.

Keywords: Fault reconstruction, Modular multilevel converter, Sliding mode observer, Linear matrix inequality.

1. INTRODUCTION

As power electronic technology advances, flexible DC transmission technology is developing towards higher voltage levels and greater transmission capacity, which is suitable to the future construction of transmission networks (Dekka et al. (2017)). The direct current transmission technology based on modular multilevel converter (MMC) has the advantages of low manufacturing difficulty, low loss and high waveform quality, and has become a hot spot for many scholars (Mo et al. (2019)).

MMC contains a large number of submodules (SM) and power switching devices, each of which is a potential point of fault. In order to ensure the security of MMC, it's not surprising that fault diagnosis of MMC has received a lot of attention (Li et al. (2019); Zhang et al. (2019)). Currently, the widely adopted fault diagnosis methods of MMC can be divided into three categories: data-driven (Li et al. (2019)), system model (Zhou et al. (2018); Zhang et al. (2020)) and hybrid method (Zhang et al. (2019)). Especially, model based method with observer attracts special interesting due to its clear mechanism explanation. Therefore, fault reconstruction plays an increasingly important role in modern industrial production.

On the other hand, fault reconstruction (Wang and Daley (1996)) directly provides fault information such as amplitude magnitude, fault type and fault evolution. Among the fault reconstruction, SMO has been proposed (Edwards et al. (2000)) to accurately construct a fault signal, detect and isolate the fault by using the equivalent output control concept. In Yan and Edwards (2008), a fault reconstruction scheme based on the inherent characteristics of SMO can be realized online. In the case of uncertain parameters, the proposed reconstruction signal can also approximate the fault signal to any desired accuracy. It is, therefore, the main purpose of this paper to address the fault reconstruction problem for MMC systems with actuator and sensor fault as well as model uncertainty.

Motivated by the above discussions, in this paper, we endeavor to design SMO by employing a SMO method for MMC subject to actuator and sensor fault as well as model uncertainty. The novelties of this paper lie in the following three aspect- s: 1) a new state space model is established to describe the MMC system actuator fault, sensor fault and systems uncertainty; 2) the SMO is introduced to realize the reconstruction of MMC fault; 3) the proposed SMO can be designed with the help of semi-definite program method.

2. MODEL DESCRIPTION OF MMC

2.1 State Space Model of MMC without Fault

Similar to Zhang et al. (2019)), the phase unit of MMC consists of two bridge arms, an upper arm and a lower arm, each of which has a series of identical SMs, a reactance L and a bridge arm equivalent resistance R connected in series. u_o and i_o are the output voltage and current of the AC side, u_u and i_u are the upper arm voltage

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and current, u_l and i_l are the lower arm voltage and current, respectively. i_c is the circulating current between the upper and lower arms, U_{dc} is the DC side voltage. The voltage-current equation of MMC operation is obtained by Kirchhoff's law:

$$\begin{cases} Ri_u + L\frac{di_u}{dt} = \frac{U_{dc}}{2} - u_u - u_o \\ Ri_l + L\frac{di_l}{dt} = \frac{U_{dc}}{2} - u_l + u_o \end{cases} \tag{1}$$

$$\begin{cases} i_o = i_u - i_l \\ i_c = \frac{i_u + i_l}{2} \end{cases} \tag{2}$$

Combine (1) and (2), the following MMC dynamic equations can be obtained

$$\begin{cases} L\frac{di_o}{dt} = -Ri_o - u_u + u_l - 2u_o \\ L\frac{di_c}{dt} = -Ri_c - \frac{u_u + u_l}{2} + \frac{U_{dc}}{2} \end{cases} \tag{3}$$

By sorting out (1) and (3), we establish the following equations

$$\begin{cases} \frac{di_u}{dt} = -\frac{R}{L}i_u - \frac{1}{L}u_u - \frac{1}{L}u_o + \frac{1}{2L}U_{dc} \\ \frac{di_l}{dt} = -\frac{R}{L}i_l - \frac{1}{L}u_l + \frac{1}{L}u_o + \frac{1}{2L}U_{dc} \\ \frac{di_o}{dt} = -\frac{R}{L}i_o - \frac{1}{L}u_u + \frac{1}{L}u_l - \frac{2}{L}u_o \\ \frac{di_c}{dt} = -\frac{R}{L}i_c - \frac{1}{2L}u_u - \frac{1}{2L}u_l + \frac{1}{2L}U_{dc} \end{cases} \tag{4}$$

Denote $x(t) = \mathrm{col}\{i_u, i_l, i_o, i_c\}$, $y(t) = \mathrm{col}\{i_u, i_l, i_c\}$, $u(t) = \mathrm{col}\{u_u, u_l\}$, $w(t) = \mathrm{col}\{u_o, U_{dc}\}$, equations (4) can be rewritten as the following state space model

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dw(t) \\ y(t) = Cx(t) \end{cases} \tag{5}$$

where

$$A = \mathrm{diag}\{-\frac{R}{L}, -\frac{R}{L}, -\frac{R}{L}, -\frac{R}{L}\},\ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} -\frac{1}{L} & 0 \\ 0 & -\frac{1}{L} \\ -\frac{1}{L} & \frac{1}{L} \\ -\frac{1}{2L} & -\frac{1}{2L} \end{bmatrix}, D = \begin{bmatrix} -\frac{1}{L} & \frac{1}{2L} \\ \frac{1}{L} & \frac{1}{2L} \\ -\frac{2}{L} & 0 \\ 0 & \frac{1}{2L} \end{bmatrix}.$$

2.2 MMC Model with Fault

Consider the following uncertain MMC systems with actuator and sensor faults:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dw(t) + Mf_a(t) + Q\xi(t,x,u) \\ y(t) = Cx(t) + Nf_s(t) \end{cases} \tag{6}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $w(t) \in \mathbb{R}^l$, $y(t) \in \mathbb{R}^p$ is the measurement output. $f_a(t)$ and $f_s(t)$ represent the norm-bound actuator faults and sensor fault, respectively, and satisfy $\| f_a(t) \| \leq \alpha_a$ and $\| f_s(t) \| \leq \alpha_s$ which α_a and α_s are positive scalars. $\xi(t,x,u) \in \mathbb{R}^h$ represents the unknown disturbance signal and satisfy $\| \xi(t,x,u) \| \leq \beta$ with scalar β is known. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $D \in \mathbb{R}^{n\times l}$, $C \in \mathbb{R}^{p\times n}$, $M \in \mathbb{R}^{n\times q}$, $N \in \mathbb{R}^{p\times r}$ and $Q \in \mathbb{R}^{n\times h}$ with $n > p > q$. Without loss of generality, we assume that the matrices C, M and N are full rank.

Similar to Brahim et al. (2013), an orthogonal transformation matrix $T_r \in \mathbb{R}^{p\times p}$ is introduced to convert sensor fault into invented actuator fault for system (6)

$$T_r y(t) = \begin{cases} y_1(t) = C_1 x(t) \\ y_2(t) = C_2 x(t) + N_2 f_s(t) \end{cases} \tag{7}$$

where $y_1(t) \in \mathbb{R}^{p-r}, y_2(t) \in \mathbb{R}^r$ and $N_2 \in \mathbb{R}^{r\times r}$ is a nonsingular matrix.

Next, new state $z_f \in \mathbb{R}^r$ that is a filtered version of y is introduced as

$$\dot{z}_f(t) = -A_f z_f(t) + A_f y_2(t) \tag{8}$$

where $-A_f \in \mathbb{R}^{r\times r}$ is a stable matrix.

By combining (6), (7) and (8), an augmented state space model with $n+r$ order can be described as

$$\begin{cases} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{z}_f(t) \end{bmatrix}}_{\dot{x}_a(t)} = \underbrace{\begin{bmatrix} A & 0 \\ A_f C_2 & -A_f \end{bmatrix}}_{A_a} \underbrace{\begin{bmatrix} x(t) \\ z_f(t) \end{bmatrix}}_{x_a(t)} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_a} u(t) + \underbrace{\begin{bmatrix} D \\ 0 \end{bmatrix}}_{D_a} w(t) \\ \qquad + \underbrace{\begin{bmatrix} M & 0 \\ 0 & A_f N_2 \end{bmatrix}}_{M_a} \underbrace{\begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix}}_{f(t)} + \underbrace{\begin{bmatrix} Q \\ 0 \end{bmatrix}}_{Q_a} \xi(t,x,u) \\ \underbrace{\begin{bmatrix} \dot{y}_1(t) \\ \dot{z}_f(t) \end{bmatrix}}_{y_a(t)} = \underbrace{\begin{bmatrix} C_1 & 0 \\ 0 & I_r \end{bmatrix}}_{C_a} \underbrace{\begin{bmatrix} x(t) \\ z_f(t) \end{bmatrix}}_{x_a(t)} \end{cases}$$

which is equivalent to

$$\begin{cases} \dot{x}_a(t) = A_a x_a(t) + B_a u(t) + D_a w(t) \\ \qquad + M_a f(t) + Q_a \xi(t,x,u) \\ y_a(t) = C_a x_a(t) \end{cases} \tag{9}$$

After stating the consequent conclusions, the following Assumptions and lemmas are needed.

Assumption 1. *rank* $(C_a M_a) =$ *rank* $(M_a) = q + r \leq p$.

Assumption 2. The invariant zeros of $(A_a, M_a, C_a)$are stable, there is *rank* $\begin{bmatrix} sI_{n+r} - A_a & M_a \\ C_a & 0 \end{bmatrix} = n + r + rank(M_a)$.

Lemma 1. (Edwards et al. (2000)) If A1 and A2 are satisfied, there is a nonsingular transformation matrix T_a, which makes the coordinate transformation $x_a \to T_a x_a$, the system (A_a, M_a, C_a) has the following structure:

$$A_a = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, M_a = \begin{bmatrix} 0 \\ M_2 \end{bmatrix}, C_a = [0 \quad T] \tag{10}$$

where $A_{11} \in \mathbb{R}^{(n+r-p)\times(n+r-p)}$, $A_{21} = [A_{211} \quad A_{212}]^T$, $A_{211} \in \mathbb{R}^{(p-q-r)\times(n+r-p)}$, $M_2 \in \mathbb{R}^{p\times(q+r)}$ with $M_2 = [0 \quad M_o]^T$, $M_o \in \mathbb{R}^{(q+r)\times(q+r)}$ is non-singular, $T \in \mathbb{R}^{p\times p}$ is an orthogonal matrix.

Lemma 2. (Edwards et al. (2000)) The pair (A_{11}, A_{211}) is detectable if and only if the invariant zeros of (A_a, M_a, C_a) are Hurwitz.

3. SLIDING MODE OBSERVER DESIGN

In this section, the SMO for the augmented uncertain system is designed with the help of the above Assumptions and Lemma. The SMO of augmented system (9) is given by

$$\begin{cases} \dot{\hat{x}}_a(t) = A_a\hat{x}_a(t) + B_a u(t) + D_a w(t) \\ \qquad - G_l e_y(t) + G_n v(t) \\ \hat{y}_a(t) = C_a \hat{x}_a(t) \end{cases} \tag{11}$$

where $\hat{x}_a(t)$ is the estimated state of $x_a(t)$, G_l and $G_n \in \mathbb{R}^{(n+r)\times p}$ are the linear feedback matrix and nonlinear feedback matrix of the SMO, respectively. The sliding mode strategy for optimal design $v(t)$ is defined as

$$v(t) = -\rho(t,x,u)\frac{P_o e_y}{\| P_o e_y \|} \; if \; e_y \; \neq 0 \tag{12}$$

where $e_y(t) = \hat{y}_a(t) - y_a(t)$ is the output estimation error, ρ is a known positive scalar determined by uncertainties, $P_o \in \mathbb{R}^{p\times p}$ is a symmetric positive definite matrix.

Define $e(t) = \hat{x}_a(t) - x_a(t)$, the estimation error systems can be obtained from (9) and (11)

$$\dot{e}(t) = A_o e(t) + G_n v - M_a f(t) - Q_a \xi(t,x,u) \tag{13}$$

where $A_o = A_a - G_l C_a$.

In lemma 1, let $G_n = \text{col}\{-LT^T, T^T\}$, where $L = [L^o \quad 0], L^o \in \mathbb{R}^{(n+r-p)\times(p-q-r)}$. The general structure of the interference distribution matrix can be expressed as $Q_a = \text{col}\{Q_1, Q_2\}$, where $Q_1 \in \mathbb{R}^{(n+r-p)\times h}$.

Lemma 3. (Tan and Edwards (2003)) If there exists a Lyapunov symmetric positive definite matrix P, that satisfies $PA_o + A_o^T P < 0$ with the structure

$$P = \begin{bmatrix} P_1 & P_1 L \\ L^T P_1 & T^T P_o T + L^T P_1 L \end{bmatrix} > 0 \tag{14}$$

where $P_1 \in \mathbb{R}^{(n+r-p)\times(n+r-p)}$, then $e(t)$ is asymptotically stable. Furthermore, sliding motion occurs on the surface $\mathcal{S} = \{e : e_y = 0\}$ in finite time governed by the system matrix $A_{11} + L^o A_{211}$.

By defining the following two positive scalars $\mu_0 = -\lambda_{max}(PA_o + A_o^T P)$, $\mu_1 =\| PQ_a \|$, we introduce the following lemma 4.

Lemma 4. (Tan and Edwards (2003)) If the positive scalar gain function ρ in (12) satisfies

$$\rho \geq \| P_o C_a M_a \| \alpha + \eta_o \tag{15}$$

where η_o is a small positive scalar, then $e(t)$ in (13) is ultimately bounded by the following set

$$\Psi_\varepsilon = \{e : \| e \| < \frac{2\mu_1\beta}{\mu_0} + \varepsilon\} \tag{16}$$

where ε is an arbitrarily small positive scalar.

It can be proved that a sliding motion is caused by using Lemma 4 and the sliding surface $\mathcal{S}$ with a suitable ρ . Introduce a new coordinate transformation related to the following non-singular matrix

$$T_L = \begin{bmatrix} I_{n+r-p} & L \\ 0 & T \end{bmatrix} \tag{17}$$

Applying T_L to the system (A_a, M_a, C_a) in lemma 1, we have

$$\mathcal{A}_a = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{bmatrix}, \mathcal{M}_a = \begin{bmatrix} 0 \\ \mathcal{M}_2 \end{bmatrix}, \mathcal{C}_a = [0 \;\; I_p] \tag{18}$$

where $\mathcal{A}_{11} = A_{11} + L^o A_{211}$, $\mathcal{A}_{21} = TA_{21}$, $\mathcal{M}_2 \in \mathbb{R}^{p\times(q+r)}$ with $\mathcal{M}_2 = TM_2$.

Since (A_{11}, A_{211}) is detectable, L^o can be chosen so that $\mathcal{A}_{11}$ is stable.

In this coordinate system, the nonlinear gain becomes $\mathcal{G}_n = \text{col}\{0, I_p\}$, on the other hand, P from (14) will have the block diagonal structure

$$\tilde{P} = (T_L^{-1})^T P T_L^{-1} = \begin{bmatrix} P_1 & 0 \\ 0 & P_o \end{bmatrix} \tag{19}$$

The uncertainty or disturbance distribution matrix have the following form

$$\mathcal{Q} = T_L Q_a = \begin{bmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \end{bmatrix} = \begin{bmatrix} Q_1 + LQ_2 \\ TQ_2 \end{bmatrix} \tag{20}$$

The state estimation error becomes

$$\dot{e}_L(t) = \mathcal{A}_o e_L(t) + \mathcal{G}_n v - \mathcal{M}_a f(t) - \mathcal{Q}_a \xi(t,x,u) \tag{21}$$

where $\mathcal{A}_o = \mathcal{A}_a - \mathcal{G}_l \mathcal{C}_a$.

Denote $e_L = [e_1 \quad e_y]^T$, block the state estimation error as follows

$$\begin{cases} \dot{e}_1(t) = \mathcal{A}_{11} e_1(t) + (\mathcal{A}_{12} - \mathcal{G}_{l,1}) e_y(t) - \mathcal{Q}_1 \xi(t,x,u) \\ \dot{e}_y(t) = \mathcal{A}_{21} e_1(t) + (\mathcal{A}_{22} - \mathcal{G}_{l,2}) e_y(t) + v(t) \\ \qquad - \mathcal{M}_2 f(t) - \mathcal{Q}_2 \xi(t,x,u) \end{cases} \tag{22}$$

where $\mathcal{G}_{l,1}$ and $\mathcal{G}_{l,2}$ represent appropriate partition of $\mathcal{G}_l$.

Theorem 1. If the SMO gain function ρ from (12) satisfies

$$\rho \geq 2 \| P_o \mathcal{A}_{21} \| \mu_1\beta/\mu_0 + \| P_o \mathcal{Q}_2 \| \beta \\ + \| P_o \mathcal{M}_2 \| \alpha + \eta_o \tag{23}$$

where η_o is a small positive scalar, then an ideal sliding motion takes place on the surface $\mathcal{S} = \{e : e_y = 0\}$ in finite time, gradually estimate the state of the system.

Proof. Choose Lyapunov function as $V_s(e_y) = e_y^T P_o e_y$, and the derivative along the trajectory is

$$\dot{V}_s = e_y^T (P_o(\mathcal{A}_{22} - \mathcal{G}_{l,2}) + (\mathcal{A}_{22} - \mathcal{G}_{l,2})^T P_o) e_y \\ + 2e_y^T P_o(\mathcal{A}_{21} e_1 - \mathcal{M}_2 f_o - \mathcal{Q}_2 \xi + v)$$

Due to $\tilde{P}$ from (19) is a block diagonal Lyapunov matrix for $(\mathcal{A} - \mathcal{G}_l \mathcal{C})$, then we have

$$P_o(\mathcal{A}_{22} - \mathcal{G}_{l,2}) + (\mathcal{A}_{22} - \mathcal{G}_{l,2})^T P_o < 0$$

Therefore, the following relation satisfies

$$\dot{V}_s \leq 2e_y^T (P_o(\mathcal{A}_{21} e_1 - \mathcal{M}_2 f_o - \mathcal{Q}_2 \xi - 2\rho \| P_o e_y \| \\ \leq -2 \| P_o e_y \| (\rho - \| P_o \mathcal{A}_{21} \| \| e_1 \| - \| P_o \mathcal{M}_2 \| \alpha \\ - \| P_o \mathcal{Q}_2 \| \beta)$$

On the other hand, $e(t) \in \Psi_\varepsilon$ implies $\| e \| < \frac{2\mu_1\beta}{\mu_0} + \varepsilon$ from lemma 4. By employing the definition of ρ in (23), we have

$$\dot{V}_s \leq -2\varepsilon_o \| P_o e_y \| \leq -2\varepsilon_o \varepsilon \sqrt{V_s}$$

where $\varepsilon = \sqrt{\lambda_{min}(P_o)}$. This proves that the output estimation error e_y will be reached zero infinite time, and sliding motion takes space.

4. ROBUST FAULT RECONSTRUCTION

In this section, we will analyze the ability of the SMO described in (11) and (12) to reconstruct the actuator and sensor fault simultaneously with the uncertainty $\xi(t, x, u)$. Since the SMO has been designed and the sliding motion has been reached, the output estimation error satisfies $e_y(t) = \dot{e}_y(t) = 0$, and the equation (22) becomes

$$\begin{cases} \dot{e}_1(t) = \mathcal{A}_{11}e_1(t) - \mathcal{Q}_1\xi(t, x, u) \\ 0 = \mathcal{A}_{21}e_1(t) + v_{eq} - \mathcal{M}_2 f(t) - \mathcal{Q}_2\xi(t, x, u) \end{cases} \tag{24}$$

where v_{eq} is the equivalent output error injection that represents the average characteristic of the sliding mode strategy and is necessary to maintain sliding motion.

$$v_{eq} = -\rho(t, x, u)\frac{P_o e_y}{\| P_o e_y \| + \delta} \; if \; e_y \neq 0 \tag{25}$$

where δ represents a positive scalar with a lesser degree of accuracy, which can reduce the chattering of sliding mode motion. The intention is to choose a scaling of the signal v_{eq} and the gain L^o to minimize the effect of the uncertainty $\xi(t, x, u)$ on the fault reconstruction.

Define $W_{as} = \begin{bmatrix} W_1 & M_o^{-1} \end{bmatrix}$, where $W_1 \in \mathbb{R}^{(q+r)\times(p-q-r)}$ represents the weighting matrix of design freedom and M_o has been described in lemma 1. The fault reconstruction form of the actuator and sensor can be expressed as

$$\hat{f}(t) = \begin{bmatrix} \hat{f}_a(t) \\ \hat{f}_s(t) \end{bmatrix} = W_{as}T^T v_{eq} \tag{26}$$

Rewriting (24) in terms of the coordinates of (18) and sorted out

$$\hat{f}(t) = -W_{as}A_{21}e_1(t) + W_{as}Q_2\xi(t, x, u) + f(t) \tag{27}$$

Thus the effect of $\xi(t, x, u)$ on $\hat{f}(t)$ is

$$\hat{f}(t) = f(t) + \hat{G}(s)\xi(t, x, u) \tag{28}$$

where the transfer function matrix

$$\hat{G}(s) = W_{as}A_{21}(sI - (A_{11} + LA_{21}))^{-1}(Q_1 + LQ_2) + W_{as}Q_2 \tag{29}$$

where s is the *Laplace* variable.

Select L^o and W_1 to minimise the effect of ξ on $\hat{f}(t)$. Applying bounded real lemma (Chilali and Gahinet (1996)), the gain of $G(s)$ from the uncertainty ξ to $\hat{f}(t)$ will not exceed $\gamma \in \mathbb{R}_+$.

$$\begin{bmatrix} \bar{P}(A_{11} + LA_{21}) + (A_{11} + LA_{21})^T\bar{P} & -\bar{P}(Q_1 + LQ_2) & -(W_{as}A_{21})^T \\ -(Q_1 + LQ_2)^T\bar{P} & -\gamma I_h & (W_{as}Q_2)^T \\ -W_{as}A_{21} & W_{as}Q_2 & -\gamma I_q \end{bmatrix} < 0 \tag{30}$$

where $\bar{P} \in \mathbb{R}^{(n-p)\times(n-p)}$ is symmetric positive definite.

The objective is to find $\bar{P}$, L and W_{as} to minimise γ subject to the inequality (30) and $\bar{P} > 0$. However, it is necessary to find the value of G_l and P having the equation (14) such that $P(A_a - G_lC_a) + (A_a - G_lC_a)^T P < 0$. Theorem 2 gives a design method for the SMO.

Theorem 2. Define $D_1 \in \mathbb{R}^{p\times p}, \gamma_o \in \mathbb{R}_+$ to be user-defined, and define the following matrices that have $p + h$ columns

$$\bar{Q} = [0 \quad Q], \; \bar{D} = [D_1 \quad 0], \; \bar{H} = [0 \quad W_{as}Q_2] \tag{31}$$

Assume there exists Lyapunov matrix P with the structure

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0, \; P_{12} = [P_{121} \quad 0] \tag{32}$$

where $P_{121} \in \mathbb{R}^{(n-p)\times(p-q)}$, W_1, E_2 and γ are satisfied

$$\begin{bmatrix} PA_a + A_a^T P - \gamma_o C_a(\bar{D}\bar{D}^T)^{-1}C_a & -P\bar{Q} & E^T \\ -\bar{Q}^T P & -\gamma_o I_{p+h} & \bar{H}^T \\ E & \bar{H} & -\gamma_o I_r \end{bmatrix} < 0 \tag{33}$$

$$\begin{bmatrix} P_{11}A_{11} + A_{11}^T P_{11} + P_{12}A_{21} + A_{21}^T P_{12}^T & * & * \\ -(P_{11}Q_1 + P_{12}Q_2)^T & -\gamma I_h & * \\ -W_{as}A_{21} & -W_{as}Q_2 & -\gamma I_r \end{bmatrix} < 0 \tag{34}$$

where $*$ makes (34) symmetric and

$$E = [-W_{as}A_{21} \quad E_2] \tag{35}$$

If the observer gains are chosen as

$$L^o = P_{11}^{-1}P_{121} \tag{36}$$

$$G_l = \gamma_o P^{-1}C_a^T(\bar{D}\bar{D}^T)^{-1} \tag{37}$$

then $P(A_a - G_lC_a) + (A_a - G_lC_a)^T P < 0$ is satisfied and $\| \hat{G}(s) \|_\infty < \gamma$.

Proof. Similar to Tan and Edwards (2003), the proof can be obtained.

By comparing P in (32) with (14), there is one-to-one correspondence between the variables $(P_{11}, P_{121}, P_{22})$ and (P_1, L^o, P_o), that is

$$P_1 = P_{11} \tag{38}$$

$$L^o = P_{11}^{-1}P_{121} \tag{39}$$

$$P_o = T(P_{22} - P_{12}^T P_{11}^{-1}P_{12})T^T \tag{40}$$

From what has been discussed above, the fault reconstruction scheme for MMC system can be handled by employing the following Algorithm 1.

Algorithm 1: Fault reconstruction with MMC.

Step 1. A system model (6) under fault is established by the state space model of (5), and an augmented state space model (9) is established which satisfy conditions (8).

Step 2. Design SMO forms $\dot{\hat{x}}_a$ and $\hat{y}_a$ and the control strategy $v(t)$ are designed to obtain the estimation error $\dot{e}(t)$.

Step 3. If sliding mode motion is reached, $e_y = \dot{e}_y = 0$, the equivalent output control item to maintain the sliding model performance is v_{eq}. Meanwhile, design the weighting matrix W_{as} and quasi-reconstructed signal to obtain the fault reconstruction form $\hat{f}(t) = f(t) + \hat{G}(s)\xi(\cdot)$, where $\hat{G}(s)$ from (29).

Step 4. Use YALMIP toolbox in MATLAB to solve the optimization problem to be sought: Minimize γ with regard to the variables $P_{11}, P_{121}, P_{22}, W_1$ and E_2 subject to (32), (33) and (34). First solve the weighted matrix W_{as}, P and γ, then calculate G_l from (37), P_o from (40), $G_n = \text{col}\{-LT^T, T^T\}$ and $L = [L^o \;\; 0]$.

5. SIMULATION ANALYSIS

An equivalent single-phase MMC prototype with six Submodules (Li et al. (2016)) is presented in this section to verify the effectiveness of robust reconstruction of simultaneous actuator and sensor failures, and the prototype parameters of involved system are listed in Table 1.

Table 1. Parameters of the MMC

Description	Param	Value
Arm inductance	L	5 mH
Arm resistance	R	0.3 Ω
DC link capacitor	U_{dc}	250 V
Number of SMs per arm	$N+M$	6
SM capacitor	C	1867 μF
Fundamental frequency	f_o	50Hz

In equation (6), the system matrix is

$$A=\begin{bmatrix}-60&0&0&0\\0&-60&0&0\\0&0&-60&0\\0&0&0&-60\end{bmatrix},B=\begin{bmatrix}-200&0\\0&-200\\-200&200\\-100&-100\end{bmatrix},$$

$$D=\begin{bmatrix}-200&100\\-200&100\\-400&0\\0&100\end{bmatrix},Q=\begin{bmatrix}0&0\\1&0\\0&1\\0&0\end{bmatrix},M=\begin{bmatrix}1\\1\\-1\\1\end{bmatrix},$$

$$N=\begin{bmatrix}0\\0\\1\end{bmatrix},C=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\end{bmatrix}.$$

When the filter matrix is selected as $A_f=0.5I$, the system matrix and the orthogonal matrix through singular value decomposition are

$$A_a=\begin{bmatrix}-60&0&0&0&0\\0&-60&0&0&0\\0&0&-60&0&0\\0&0&0.3536&-0.5&0.3536\\0&0&0&0&-60\end{bmatrix},D_a=\begin{bmatrix}-200&100\\-200&100\\-400&0\\0&100\\0&0\end{bmatrix},$$

$$B_a=\begin{bmatrix}-200&0\\0&-200\\-200&200\\-100&-100\\0&0\end{bmatrix},Q_a=\begin{bmatrix}0&0\\1&0\\0&1\\0&0\\0&0\end{bmatrix},M_a=\begin{bmatrix}1&0\\1&0\\-1&0\\1&0\\0&0.5\end{bmatrix},$$

$$C_a=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&0&0&1\end{bmatrix},T=\begin{bmatrix}-0.7071&0&0.7071\\-0.7071&0&-0.7071\\0&1&0\end{bmatrix}.$$

The suitable design parameters can be selected as $\gamma_o=1, D_1=I_3$, $\rho=30, \delta=10^{-5}$, I_3 is a third-order unit matrix. According to the proposed SMO design Steps, Theorem 2 is solved by using YALMIP toolbox (Lofberg (2004)), the optimization program assigns value $\gamma=0.2270$. The observer matrix can be obtained as

$$G_l=\begin{bmatrix}1.0184&0.9661&0.3606\\0.9831&0.9914&0.3609\\-1.4181&-1.3706&-1.6976\\0.0009&0.0007&-0.6416\\1.2729&1.2488&0.5114\end{bmatrix},$$

$$G_n=\begin{bmatrix}0.1416&0.1416&0\\0.1187&0&0.1187\\-0.7071&-0.7071&0\\0&0&-1\\0.7071&-0.7071&0\end{bmatrix},$$

$$P_o=\begin{bmatrix}3.1710&-1.8769&-2.3257\\-1.8769&3.3084&-2.4921\\-2.3257&-2.4921&7.8638\end{bmatrix},L=\begin{bmatrix}0.2002&0&0\\0.1679&0&0\end{bmatrix}.$$

The appropriate weighting matrix is set as

$$W_{as}=\begin{bmatrix}-0.7071&0&0.7071\\0&-2&0\end{bmatrix}$$

The unknown input disturbance $\xi(t,x,u)$ caused by the harmonic signal of the MMC is given by

$$\xi(t,x,u)=[0.2\sin(200\pi t)\quad 0.2\cos(300\pi t)]^T$$

Actuator fault and sensor fault are respectively set as

$$f_a(t)=\begin{cases}0.4\sin(\pi t), & 2\le t<16\\ 0, & otherwise\end{cases}$$

$$f_s(t)=\begin{cases}t-4, & 4\le t<8\\ -t+12, & 8\le t<12\\ 0, & otherwise\end{cases}$$

where sine signals are used to simulate the actuator intermittent fault f_a, ramp signal is used to describe the sensor abrupt fault f_s. The simulation results are listed as follows:

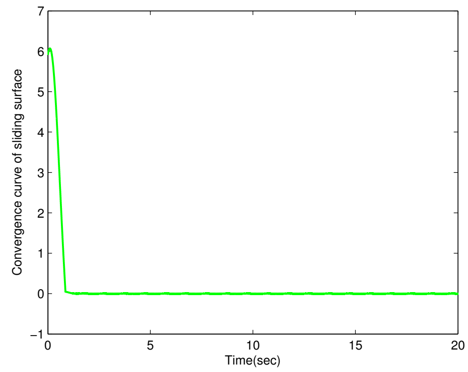

Fig. 1. The convergence curve of sliding surface S.

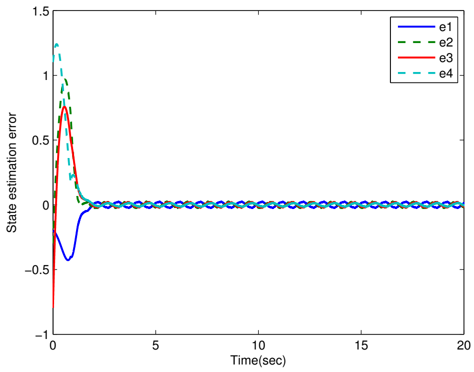

Fig. 2. The state estimation error $e(t)$.

It can be seen from Fig. 1 that the sliding mode motion reaches the sliding surface in less than $2s$ to ensure the accuracy of robust fault reconstruction. In Fig. 2, the estimated state is able to track the original state of the system for a limited time so that the state estimation error tends to be stable. In Fig. 3 and Fig. 4, the fault reconstruction method can reduce the influence of the

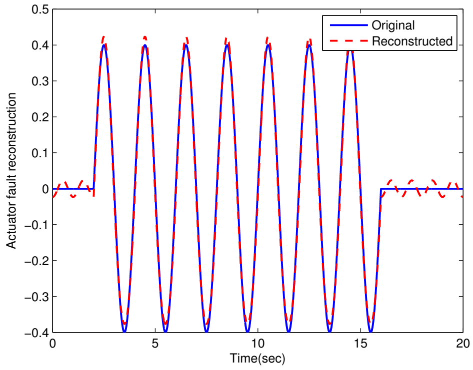

Fig. 3. Reconstruction of actuator fault for MMC.

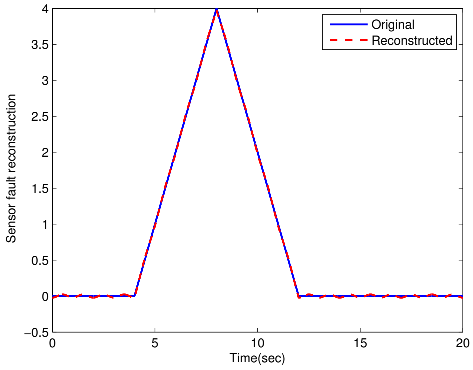

Fig. 4. Reconstruction of sensor fault for MMC.

uncertainty such as interference on the reconstruction result. Simultaneously, the proposed fault reconstruction algorithm can quickly realize online fault tracking and reconstruction for slow change actuator fault and abrupt change sensor fault.

6. CONCLUSION

This paper has presented a robust fault reconstruction strategy based on SMO for MMC with actuator fault and sensor fault. The transformation matrix is first introduced and a post filter is added, the augmented system converts the original system into a system that only contains actuator failures. By transforming the design of SMO gain into a optimization problem, and the observer gain can be obtained by using LMIs. At the same time, the equivalent output control method is used to directly obtain the fault information, and the fault reconstruction of the actuator and the sensor is realized. By applying it to the MMC model to obtain simulation results, the effectiveness of the proposed method is verified. Compared with other fault reconstruction methods, the robust fault reconstruction method based on SMO proposed in this paper has excellent robustness to external disturbances and other uncertainties of the system. Future research will focus on the fault tolerant control of MMC based on the result in fault reconstruction.

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