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Safety-extended Explicit MPC for Autonomous Truck Platooning on Varying Road Conditions

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Abstract:
Automotive platooning can significantly improve traffic safety and efficiency, but many control challenges need to be solved to function properly under realistic driving conditions. This paper proposes a novel multi-rate explicit model-predictive controller (eMPC) for safe autonomous distributed vehicle platooning in varying road friction conditions. A safety-augmented distributed predictive control formulation ensures safe vehicle spacing versus emergency braking of preceding vehicles given current friction estimates. This complex control problem is carefully formulated into an efficiently parametrized optimization problem realized as eMPC. The resulting platoon shows excellent performance in a complex vehicle dynamics co-simulation validation with low communication and computation demands.

Keywords: Nonlinear and optimal automotive control, Control architectures in automotive control, Vehicle dynamic systems, Distributed MPC, Vehicle formation

1. INTRODUCTION

Individual mobility, particularly automotive road transport, is globally present in our societies, with high impact on our economy, ecology, and safety. Compared to railway and air traffic, individual road traffic is the least safe transport mode by far. Many efforts to improve road safety have been done, with autonomous driving representing one important and effective future path (ETSC 2016).

Autonomous, closely-spaced vehicle platooning allows to simultaneously increase traffic flow, improve safety, and reduce fuel consumption due to aerodynamic drag reduction, especially for heavy-duty vehicles (Alam et al., 2015). The present work focuses on the longitudinal dynamic driving task in the context of cooperative vehicle platooning.

Employing model-predictive control (MPC) for platooning allows for smooth, efficient vehicle operation and directly incorporates system and safety constraints. It has been implemented for tasks ranging from simple tracking to elaborate trajectory optimization accounting for road topology, air drag, and drivetrain non-linearities (Turri et al., 2017). A strategy to attain distributed platoon situation-awareness with regard to traffic ahead is developed in Thormann et al. (2020) and extended by safety and efficiency measures. Safety and complexity requirements, however, pose significant challenges for MPC-based autonomous vehicle control, because reliable realtime optimization algorithms are needed, and elaborate fallback strategies have to be foreseen if the online optimizer fails to deliver a valid result in the given time slot.

One possible answer to these challenges is the explicit MPC (eMPC) method (Bemporad et al., 2002; Pistikopoulos et al., 2002). The basic idea is to solve the MPC optimization problem in a parametric manner a-priori. This solution is then represented in the form of a tabularized function — a piecewise-affine control law in the case of a quadratic-cost / linear-model / linear-constraints MPC formulation. One major advantage is the low computational requirements in the realtime evaluation of such a control law, allowing high sampling rates with simple hardware. Moreover, the fact that the explicit solution is already found beforehand yields guarantees on controller behaviour, and the controller's functionality can be formally verified much simpler than in an online MPC setting.

The main difficulty of the explicit MPC method, in turn, lies in its high complexity, or rather the excessive complexity growth with increasing problem/parameter dimensionality. This poses a significant obstacle in practice, and efficient low-dimensional problem formulations have to be formulated with great care to realize the richness of MPC design flexibility with managable complexity in the explicit MPC setting.

This paper focuses on the synthesis of such an explicit MPC for safe vehicle platooning in a distributed control setting. It maintains a safe distance between individual vehicles in a platoon, partially based on control structures shown in Thormann et al. (2020). Safety requirements are modeled to avoid, at any time, collisions with the preceding vehicle in unexpected emergency braking maneuvers with bounded deceleration. The controller implements velocity / inter-vehicle gap tracking and safety with respect to emergency braking. Also, it accepts a given estimate of the road friction coefficient to adjust its control authority and includes robustification measures versus model errors (unmodeled drivetrain dynamics).

The formulation of this predictive platoon controller is carefully optimized for a compact problem representation, exploiting non-uniform sampling along the horizon and a low-dimensional parametrization of the multitude of control goals outlined above. Eventually, an explicit MPC can be built which is tested in a highly realistic co-simulation of detailed vehicle dynamics based on the commercial simulator IPG TruckMaker®. Varying-friction platooning test cases demonstrate efficient and safe operation in acceleration, velocity-keeping, and emergency braking maneuvers in the platoon.

The paper is structured as follows: Section 2 introduces the vehicle dynamics design and validation models, Section 3 provides the background on the explicit MPC method. The MPC problem is developed in Section 4, the explicit MPC parametrization is discussed in Section 5, followed by the co-simulation-based validation results in Section 6.

2. SYSTEM MODEL

The model-predictive platoon controller is formulated with respect to a single "ego" vehicle (enumerated as i-th vehicle), see Figure 1. Thereby, p^i denotes the vehicle's absolute front bumper position, its length L^i, velocity v^i, acceleration a^i, and spatial gap d^i are defined. Hereforth, the focus is laid on the ego vehicle, and its index i, the predecessor vehicle's index $i{-}1$, and the follower vehicle's index $i{+}1$ are abbreviated as ego, pre, and fol, respectively. Whenever unambiguous, the ego vehicle's index i (or ego) is dropped to shorten notation.

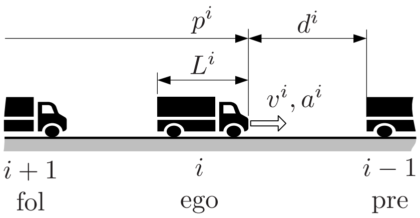

Fig. 1. Geometric specifications of a vehicle with a follower and a predecessor.

With state vector $\boldsymbol{x}(t) = [p(t)\ v(t)]^\intercal$ and control input $u(t) := a(t)$, the state-space model of the ego vehicle is given by

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\boldsymbol{x}(t) + \begin{bmatrix}0\\ 1\end{bmatrix}u(t). \tag{1}$$

To evaluate the state trajectories generated by (1) along a finite time horizon on a non-uniformly-sampled time grid, the discrete-time difference equation for sampling time $h_k > 0$ and zero-order hold of the input u reads

$$\underbrace{\begin{bmatrix}p_{k+1}\\ v_{k+1}\end{bmatrix}}_{\boldsymbol{x}_{k+1}} = \underbrace{\begin{bmatrix}1 & h_k\\ 0 & 1\end{bmatrix}}_{\boldsymbol{A}_k}\ \boldsymbol{x}_k + \underbrace{\begin{bmatrix}h_k^2/2\\ h_k\end{bmatrix}}_{\boldsymbol{b}_k}\ u_k\,, \tag{2}$$

where subscript k identifies sampling time instant $t_k = h_{k-1} + t_{k-1}$, $k \in \mathbb{Z}$ and $t_0 = 0$ is chosen to ease notation and refers to the current time, i.e., the start time of the considered finite time horizon. All quantities subscripted with an integer number $k \in \mathbb{Z}$ in this work refer to this discretization scheme.

3. EXPLICIT MPC METHODOLOGY

The explicit model predictive control is a well-known concept, popularized by Bemporad et al. (2002); Pistikopoulos et al. (2002), where the control action $u^\star$ is obtained by a mere evaluation of an affine function. The affine functions are obtained by obtaining a parametric solution to the original optimal control problem, i.e., to the model predictive control. The parametric solution can be obtained in a straight-forward fashion, if several assumptions hold, namely

(1) the objective function is convex linear or quadratic,
(2) constraints are convex linear.

Once the aforementioned assumptions hold, and the prediction horizon N together with the number of optimized control actions n_u is fairly small, the parametric solution can be obtained in a reasonable time using state-of-the-art explicit constructors. In this work, the MPC formulation is realized via the tools YALMIP (Löfberg, 2004), the Multi-Parametric Toolbox v3 (Herceg et al., 2013) and the solver of Gurobi® (Gurobi 2018).

In this work, we consider a standard formulation of the model predictive controller of the form

$$\min_{\boldsymbol{U}}\ \boldsymbol{U}^\intercal \boldsymbol{H}\boldsymbol{U} + \boldsymbol{\theta}^\intercal \boldsymbol{F}\boldsymbol{U} \tag{3a}$$

$$\text{s.t.}\ \ \boldsymbol{G}\boldsymbol{U} \le \boldsymbol{w} + \boldsymbol{S}\boldsymbol{\theta}, \tag{3b}$$

with convex quadratic objective function (3a) and linear inequality constraints (3b). The vector of parameters is denoted by $\boldsymbol{\theta}$ and the sequence of optimal control inputs by $\boldsymbol{U}$. The matrices $\boldsymbol{H}$, $\boldsymbol{F}$, $\boldsymbol{G}$, $\boldsymbol{S}$, and vector $\boldsymbol{w}$ can be derived by straightforward matrix manipulations from the original formulation of the MPC problem, as reported e.g. by Borrelli et al. (2017). For the problem (3), an explicit optimizer $u_0^\star = \kappa(\boldsymbol{\theta})$ is given by the piecewise affine function (PWA) defined as

$$\kappa(\boldsymbol{\theta}) = \begin{cases}\boldsymbol{\alpha}_1^\intercal\boldsymbol{\theta} + \beta_1 & \text{if } \boldsymbol{\theta} \in \mathcal{R}_1\\ \vdots & \\ \boldsymbol{\alpha}_{n_\mathrm{R}}^\intercal\boldsymbol{\theta} + \beta_{n_\mathrm{R}} & \text{if } \boldsymbol{\theta} \in \mathcal{R}_{n_\mathrm{R}}\end{cases}, \tag{4}$$

with n_R polyhedral regions

$$\mathcal{R}_i = \{\boldsymbol{\theta} \mid \boldsymbol{\Gamma}_i^\intercal\boldsymbol{\theta} \le \gamma_i\}, \quad i = 1, \ldots, n_\mathrm{R}. \tag{5}$$

4. SAFETY-AUGMENTED, FRICTION-AWARE MPC FORMULATION FOR PLATOONING

This chapter discusses the formulation of the optimal control problem (OCP), in the form of a quadratic MPC formulated on a finite horizon with a non-uniform time grid. This generalization enables an efficient problem formulation with fine granularity at the beginning of the horizon and large sampling times towards the end, allowing to cover long durations in the horizons with few time levels to be defined and hence few variables arising in the optimization problem. Furthermore, we present an explicit solution to the MPC to simplify the embedded implementation of the optimal control approach.

The OCP is formulated for each ego vehicle in the platoon, only requiring ego and predecessor information, thereby representing one possible distributed platoon control architecture.

4.1 Safety Extension of the OCP Formulation

We consider the safe platooning optimal control problem defined on a horizon of N samples with non-uniform sampling times h_k, $k \in \mathbb{N}_0^N$ where $\mathbb{N}_a^b$ stands for integers from the interval $[a, b]$. Two state trajectories are defined as follows,

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_k\, \boldsymbol{x}_k + \boldsymbol{b}_k\, u_k\,, \tag{6a}$$

$$\boldsymbol{x}^{\mathrm{fs}}_{k+1} = \boldsymbol{A}_k\, \boldsymbol{x}^{\mathrm{fs}}_k + \boldsymbol{b}_k\, u^{\mathrm{fs}}_k\,, \tag{6b}$$

where k denotes the time index within the horizon ($k = 0$ indicates the current time). Eq. (6a) refers to the "tracking" case, whereas (6b) models a "fail-safe" trajectory (superscript fs) which should remain feasible in case of emergency braking of the preceding vehicle. The basic idea of this approach is to include both trajectories in one OCP and enforce equality in both input sequences for the first N_{eq} time steps:

$$u^{\mathrm{fs}}_k = u_k, \quad k \in \mathbb{N}_0^{N_{\mathrm{eq}}} \tag{7}$$

$$\boldsymbol{x}_0 = \boldsymbol{x}^{\mathrm{fs}}_0. \tag{8}$$

The design and tuning for favorable tracking performance is thus separated from safety aspects treated via the fail-safe trajectory. In the fail-safe case, a safe emergency stop is to be executed within a given allowed braking distance d_{b}. The ego vehicle's fail-safe position trajectory should be bounded by d_{b} and scalar slack s (to provide reasonable behaviour in inherently unsafe situations):

$$p^{\mathrm{fs}}_k \leq d_{\mathrm{b}} + s,\ k \in \mathbb{N}_0^N, \tag{9}$$

$$s \geq 0. \tag{10}$$

Note that the entire braking maneuver to full stop needs to be covered (and constrained) in the fail-safe trajectory to ensure that the full stop is actually possible within the required braking distance, even if this pessimistic trajectory is usually not realized.

Remark: The key reason for choosing non-uniform sampling times is to allow for fine resolution at the start of the prediction window, but cover long horizons with reduced sample count (i.e., problem complexity). An optimal choice of the sampling times (such as based on the sensitivity of such choice on the control quality, possibly in an adaptive state-dependent fashion) is out of scope in this work. A simplified choice with two different sampling times will be taken here: a short sampling time is chosen initially for high control fidelity in the receding horizon realization, followed by a few longer sampling steps to cover the entire braking duration with few samples, see Table 1 below.

4.2 Input / Acceleration Constraints

The ego vehicle's acceleration is constrained by

$$-\mathrm{g}\ \hat{\mu} \leq u_k \leq \mathrm{g}\ \hat{\mu},\ k \in \mathbb{N}_0^N, \tag{11a}$$

$$-\mathrm{g}\ \hat{\mu} \leq u^{\mathrm{fs}}_k \leq \mathrm{g}\ \hat{\mu},\ k \in \mathbb{N}_0^N, \tag{11b}$$

$$u_{\min} \leq u_k \leq u_{\max},\ k \in \mathbb{N}_0^N, \tag{11c}$$

$$u_{\min} \leq u^{\mathrm{fs}}_k \leq u_{\max},\ k \in \mathbb{N}_0^N, \tag{11d}$$

where $\mathrm{g} = 9.81\,{}^{\mathrm{m}}/_{\mathrm{s}^2}$ is the gravitational constant, and $\hat{\mu}$ is the ego road friction coefficient estimate considered known in the scope of this work.

Also, dynamic limits on the acceleration are modeled assuming that acceleration is built up through PT1 dynamics with time constant τ,

$$\tau \dot{a}(t) + a(t) = a_{\mathrm{des}}(t), \tag{12}$$

and that the reference acceleration $a_{\mathrm{des}}(t)$ is bounded analogous to (11a–11b). Sampling eq. (12) yields

$$(1 - \alpha_{k-1})a_k + \alpha_{k-1}a_{k-1} = a_{\mathrm{des},k}, \tag{13}$$

with $\alpha_k = {}^{\tau}/_{h_k}$, yielding

$$-\mathrm{g}\ \hat{\mu} \leq (1 + \alpha_{k-1})u_k - \alpha_{k-1}u_{k-1} \leq \mathrm{g}\ \hat{\mu},\ k \in \mathbb{N}_0^N \tag{14a}$$

$$-\mathrm{g}\ \hat{\mu} \leq (1 + \alpha_{k-1})u^{\mathrm{fs}}_k - \alpha_{k-1}u^{\mathrm{fs}}_{k-1} \leq \mathrm{g}\ \hat{\mu},\ k \in \mathbb{N}_0^N. \tag{14b}$$

The dynamics (12) is considered uncertain and deliberately not included in the system model to avoid implicit dynamic inversion. However, the proposed constraint formulation (14) has been observed to robustify the OCP against lag model errors and will be used.

4.3 Objective Function Formulation

The performance criterion is chosen as

$$J = \sum_{k=0}^{N} h_k \cdot \left(\ell(\boldsymbol{x}_k, u_k, k) + \ell_{\mathrm{fs}}(\boldsymbol{x}^{\mathrm{fs}}_k, u^{\mathrm{fs}}_k, k)\right) + q_{\mathrm{s}} s \tag{15}$$

with the stage cost terms

$$\ell(\boldsymbol{x}_k, u_k, k) = \left\|\boldsymbol{x}_k - \boldsymbol{x}^{\mathrm{ref}}_k\right\|^2_{\boldsymbol{Q}_{\mathrm{x}}} + \|u_k\|^2_{\boldsymbol{Q}_{\mathrm{u}}} \tag{16a}$$

$$\ell^{\mathrm{fs}}(\boldsymbol{x}^{\mathrm{fs}}_k, u^{\mathrm{fs}}_k, k) = \left\|\boldsymbol{x}^{\mathrm{fs}}_k\right\|^2_{\boldsymbol{Q}^{\mathrm{fs}}_{\mathrm{x}}} + \left\|u^{\mathrm{fs}}_k\right\|^2_{\boldsymbol{Q}^{\mathrm{fs}}_{\mathrm{u}}}, \tag{16b}$$

as well as linear penalty on the slack s. The tracking reference trajectory chosen as

$$\boldsymbol{x}^{\mathrm{ref}}_k = \begin{bmatrix} t_k \cdot v_{\mathrm{ref}} \\ v_{\mathrm{ref}} \end{bmatrix}. \tag{17}$$

4.4 Assembling the Platooning MPC

Finally, the OCP is constructed as quadratic optimization problem given as

$$\min_{\boldsymbol{U}}\ J \tag{18a}$$

$$\text{s.t. Eqs. (6)}, (9-11, 14), \tag{18b}$$

$$u^{\mathrm{fs}}_k = u_k,\ k \in \mathbb{N}_0^{N_{\mathrm{eq}}}, \tag{18c}$$

$$\boldsymbol{x}_0 = \boldsymbol{x}^{\mathrm{fs}}_0, \tag{18d}$$

$$p_0 = 0, v_0 = v_{\mathrm{meas}}, \tag{18e}$$

with the stacked input sequence

$$\boldsymbol{U} = \left[u_0, \ldots, u_{N-1}, u^{\mathrm{fs}}_0, \ldots, u^{\mathrm{fs}}_{N-1}\right]^{\mathsf{T}}, \tag{19}$$

and coupling constraints (18c) and (18d). Eq. (18e) defines the position coordinate origin in the current OCP at the current ego position and thus simplifies notation without loss of generality. The OCP parameter vector reads

$$\boldsymbol{\theta} = [v_{\mathrm{meas}}\ v_{\mathrm{ref}}\ d_{\mathrm{b}}\ \hat{\mu}\ u_{-1}]^{\mathsf{T}}. \tag{20}$$

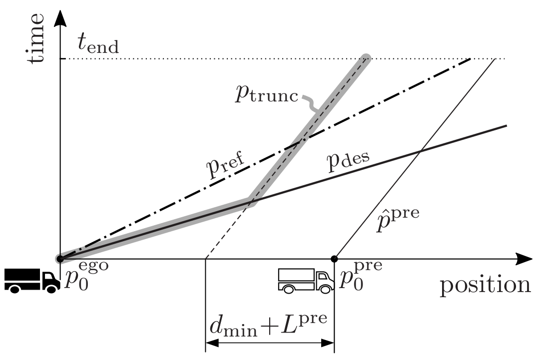

Fig. 2. Determining the scalar reference velocity v_{ref} from the predicted position trajectory at desired velocity v_{des}, clipped by predicted predecessor interference.

Therein, $v_{\mathrm{meas}} = v_0$ is the current, measured ego vehicle velocity, v_{ref} is the reference velocity, d_{b} the required braking distance, $\hat{\mu}$ the available ego vehicle's friction coefficient estimate, and u_{-1} the last applied input value. These parameters are determined below.

To improve the explicit MPC construction, the additional constraint

$$\boldsymbol{\theta}_{\min} \leq \boldsymbol{\theta} \leq \boldsymbol{\theta}_{\max}. \tag{21}$$

are added. The outlined OCP is then pre-solved via parametric optimization to obtain the eMPC law in an analytical fashion.

5. EXPLICIT MPC PARAMETRIZATION

5.1 Calculation of the Reference Velocity

To accomplish vehicle following in a local (ego) MPC setting, a prediction of the predecessor's motion is utilized to inform the ego MPC problem. In Thormann et al. (2020), the entire, high-dimensional predecessor's predicted trajectory is communicated to the ego vehicle when deemed necessary. In the present explicit MPC setting, however, the number of eMPC parameters (20) must be kept as small as possible to remain solvable. Hence, in this work, only a *scalar* reference velocity v_{ref} is utilized to inform the OCP (18) as efficiently as possible on the predecessor's actions. It needs to be determined in each time step which is done by constructing a "truncated" position trajectory p_{trunc} from a desired trajectory p_{des} and available predecessor prediction $\hat{p}^{\mathrm{pre}}$ as illustrated in Fig. 2. Finally, the reference velocity v_{ref} is determined from p_{ref}, which approximates p_{trunc}. This way, the main feature of situation awareness – namely, a predictive adaption of desired velocity based on a look-ahead horizon – can be realized with minimal parametrization complexity.

Truncating the linear desired position trajectory

$$p_{\mathrm{des}}(t) = p_0^{\mathrm{ego}} + v_{\mathrm{des}}\, t \tag{22}$$

by the constant-velocity prediction for the predecessor

$$\hat{p}^{\mathrm{pre}}(t) = p_0^{\mathrm{pre}} + v_0^{\mathrm{pre}}\, t\,, \tag{23}$$

from the measured predecessor state $(p_0^{\mathrm{pre}}, v_0^{\mathrm{pre}})$ yields the piecewise-linear function

$$p_{\mathrm{trunc}}(t) = \min(p_{\mathrm{des}}(t),\ \hat{p}^{\mathrm{pre}}(t) - L^{\mathrm{pre}} - d_{\min}) \tag{24}$$

which is consequently approximated with constant velocity v_{ref} in the horizon $t \in [0,\, t_{\mathrm{end}}]$:

$$v_{\mathrm{ref}} = \arg\min_{v} \int_0^{t_{\mathrm{end}}} \left(p_0^{\mathrm{ego}} + v\, t - p_{\mathrm{trunc}}(t)\right)^2 \mathrm{d}t. \tag{25}$$

Upon solving this regression, the numeric evaluation of (25) amounts to a simple weighted summation of the values of p_{trunc} across the prediction horizon which also reduces the effect of measurement noise in the predecessor's state on v_{ref}.

5.2 Required braking distance d_{b}

As a simplified anti-collision constraint even versus predecessor emergency braking, it is required that the ego vehicle be able to stop within the corresponding braking distance d_{b} (instantaneous distance to standstill).

This braking distance requirement is determined based on the current, known predecessor distance $p_0^{\mathrm{pre}} - L^{\mathrm{pre}}$, predecessor velocity v_0^{pre}, and the estimated predecessor's deceleration bound $\hat{a}_{\min}^{\mathrm{pre}} = -\mathrm{g}\ \hat{\mu}^{\mathrm{pre}} < 0$:

$$d_{\mathrm{b}} = p_0^{\mathrm{pre}} - L^{\mathrm{pre}} - \frac{(v_0^{\mathrm{pre}})^2}{2\ \hat{a}_{\min}^{\mathrm{pre}}}. \tag{26}$$

5.3 Friction estimation

In this work it is assumed that the effective coefficient of friction can be estimated for the ego vehicle in a sufficiently accurate and fast manner. One fundamental requirement is that sufficient and persistent excitation is present in the form of non-zero wheel slip or side-slip (in cornering maneuvers). Ref. Hsu et al. (2010) an others show suitable techniques to obtain such estimates.

Herein, $\hat{\mu}$ is considered available for the ego vehicle to parametrize the corresponding MPC constraints (11), (14). The friction coefficient of the preceding vehicle, $\hat{\mu}^{\mathrm{pre}}$, is also considered known to the ego vehicle (via V2V communication or through a conservative assumption) to determine the braking distance d_{b} via (26).

6. SIMULATION STUDIES

This section validates the safe explicit platooning MPC in a high-fidelity co-simulation with truck vehicle dynamics.

6.1 High-fidelity co-simulation setup

Validation is carried out on a detailed vehicle dynamics co-simulation illustrated in Fig. 3. One instance of the commercial simulation software IPG TruckMaker® per vehicle are co-simulated and coordinated by a central MATLAB® session via a custom-developed interface. Each individual truck is accessed in the co-simulation setting via a desired acceleration (produced by the proposed eMPC), which is then tracked via a lower level PI-acceleration controller in a basic, non-optimized parameter setting. The eMPC control actions (parameter calculation and explicit MPC evaluation) are carried out in MATLAB®. Real-time computation of the co-simulation is feasible for numerous trucks on common office computers.

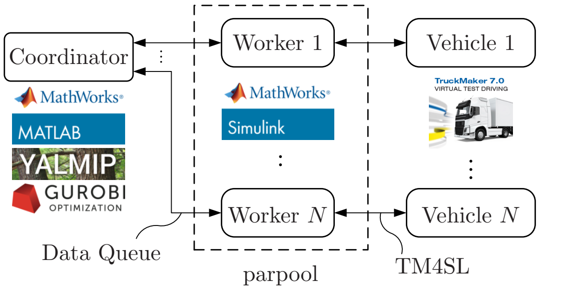

Fig. 3. Validation via vehicle dynamics co-simulation.

Table 1. MPC parameters

Parameter	Value
h_k	$0.1\,\mathrm{s}$ for $k < 3$; else $1\,\mathrm{s}$
N; t_N	$3 + 10 = 13$; $10.3\,\mathrm{s}$
$\boldsymbol{Q}_{\mathrm{x}}$	$\mathrm{diag}\left(\begin{bmatrix}1 & 10^{-5}\end{bmatrix}\right)$
$\boldsymbol{Q}_{\mathrm{x}}^{\mathrm{fs}}$	$\mathrm{diag}\left(\begin{bmatrix}10^{-4} & 10^{-5}\end{bmatrix}\right)$
$\boldsymbol{Q}_{\mathrm{u}}$; $\boldsymbol{Q}_{\mathrm{u}}^{\mathrm{fs}}$; q_{s}	10; 0.1; 4000
N_{eq}; τ	3; $0.4\,\mathrm{s}$
$u_{\min}$; $u_{\max}$	$-8\,\mathrm{m/s^2}$; $3\,\mathrm{m/s^2}$
$v_{\min}$; $v_{\max}$	$0\,\mathrm{m/s}$; $22.22\,\mathrm{m/s}$
$t_{\mathrm{gap}} = 0.3\,\mathrm{s}$ (time gap)	$d_{\min} = t_{\mathrm{gap}} \cdot v_0^{\mathrm{pre}}$

6.2 Closed-loop case studies

The MPC design choices and weight adjustments are carried out on relevant test cases of the OCP. Figure 4 shows the resulting eMPC law (18) with parameters as in Table 1. The resulting OCP is a convex quadratic program (QP) with 105 variables, 179 constraints and a mean runtime of 25 ms on an Intel Core i7-8565U CPU (2 GHz). The solution of the parametric QP is fairly complex but manageable, resulting in an explicit MPC with $n_{\mathrm{r}} = 35762$ regions, a binary image size of 27.2 MBytes and an evaluation time below 2 ms. To reduce this significant complexity of the explicit solution, further complexity reduction methods are subject of ongoing and future work.

The validation scenario comprises a platoon of three trucks (tractor units without trailers) which are parameterized identically in IPG TruckMaker®. The default model `Demo2AxleSemiTruck4x2_Volvo` is used with an 8-speed automatic gearbox. The platoon drives on a straight and flat road initially at standstill with inter-vehicle distances of 5 m. The vehicles accelerate up to a desired velocity of $v_{\mathrm{des}} = 50\,\mathrm{km/h}$ which is reached at 11 s. At time $t = 20\,\mathrm{s}$, the lead vehicle executes an emergency braking maneuver to standstill with maximum deceleration. Each

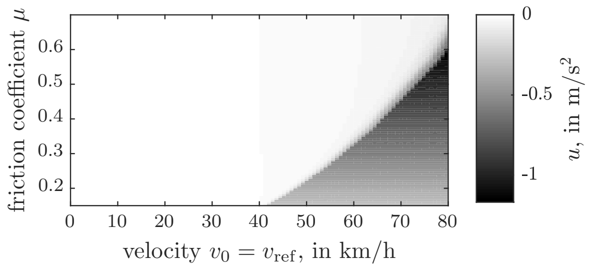

Fig. 4. Control law evaluated at $u_{-1} = 0$, $d_b = 50\,\mathrm{m}$ over μ and velocity

vehicle is controlled via the proposed safety-augmented explicit MPC, cf. Table 1.

Figure 5 demonstrates the basic functionality of the platoons controlled by the proposed safety-augmented explicit MPC. Dry road conditions are modeled ($\mu^{\mathrm{ego}} = 0.8$), and a conservative over-estimation of the predecessor's braking capabilities is utilized ($\hat{\mu}^{\mathrm{pre}} = 1.2\ \mu^{\mathrm{ego}}$). It is evident that all vehicles are operated safely at sufficient distances and safely manage to stop without collisions.

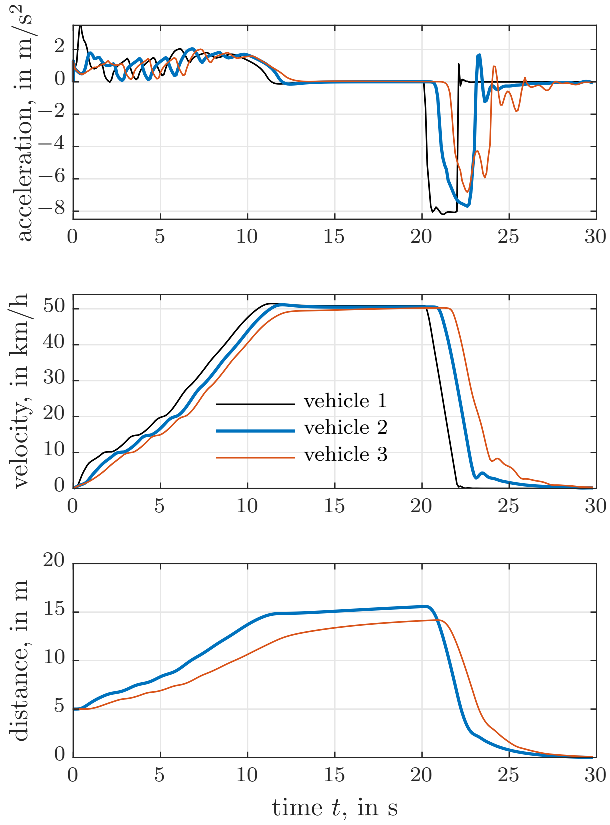

Fig. 5. Validation on dry road: $\hat{\mu} = 0.8$, $\hat{\mu}^{\mathrm{pre}} = 1.2\ \hat{\mu}^{\mathrm{ego}}$

Figure 6 shows the safe platoon behaviour for non-conservative bounds of the predecessor's braking ($\hat{\mu}^{\mathrm{pre}} = 1.0\ \hat{\mu}^{\mathrm{ego}}$) leading to slightly smaller inter-vehicle platoon distances. These results indicate that uncertainties in the grip estimates can be successfully dealt with by using conservative bounds in (26).

Figure 7 finally shows the platoon on varying road slip conditions. The road grip is reduced in the co-simulation ($\mu = 0.4$). The platoon vehicles initially assume $\hat{\mu} = 0.8$. At time $t = 10\,\mathrm{s}$ this estimate is changed instantaneously to $\hat{\mu} = 0.4$. Again, predecessor braking authority is over-estimated ($\hat{\mu}^{\mathrm{pre}} = 1.2\ \hat{\mu}^{\mathrm{ego}}$). Larger inter-vehicle distances are realized, and the platoon retains collision-safety under these varying-slip conditions as verified in the sudden braking maneuver at $t = 20\,\mathrm{s}$. Before that, the inter-vehicle distances are only moderately larger compared to the dry road case because both, ego and preceding vehicle's friction parameters, $\hat{\mu}$ and $\hat{\mu}^{\mathrm{pre}}$ respectively, are reduced, so both vehicles' braking distances enlarge, resulting in only a moderate net increase in required safety distance. The eMPC formulation thus automatically leads to safe platooning under the friction parameter assumptions.

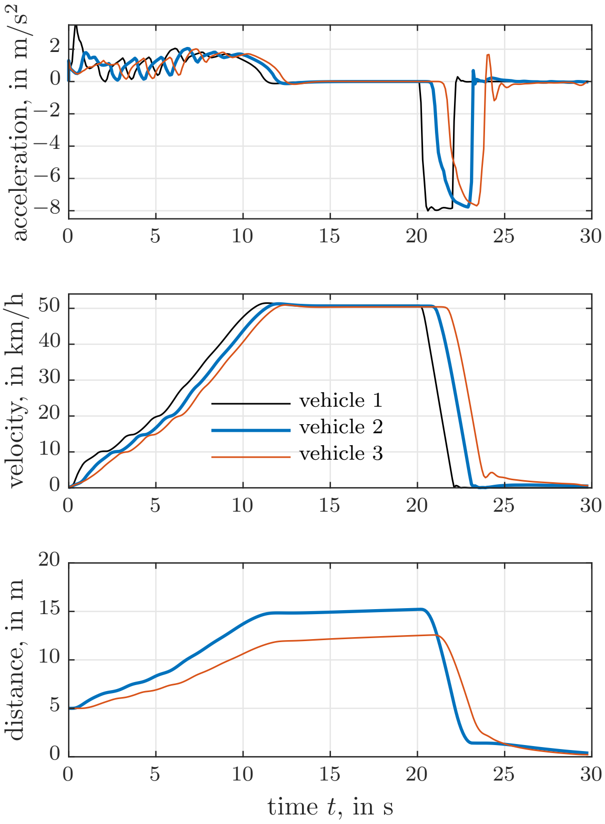

Fig. 6. Validation on dry road: $\hat{\mu} = 0.8$, $\hat{\mu}^{\mathrm{pre}} = 1.0\ \hat{\mu}^{\mathrm{ego}}$

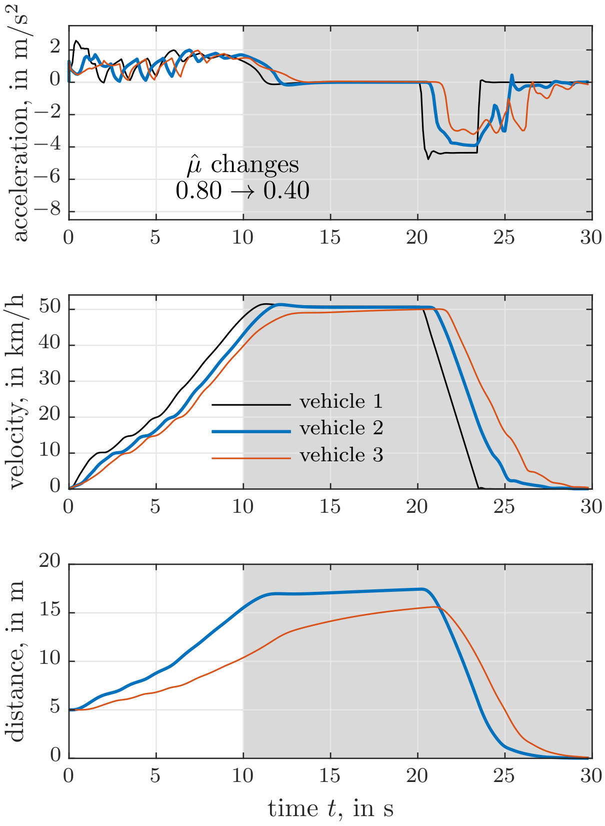

Fig. 7. Slippery road case: $\hat{\mu} = 0.8 \rightarrow 0.4$, $\hat{\mu}^{\mathrm{pre}} = 1.2\ \hat{\mu}^{\mathrm{ego}}$

7. CONCLUSIONS

An explicit distributed model-predictive controller has been developed to achieve safe automotive platooning behaviour for varying road slip conditions. A detailed control problem has been set up to ensure safety of the vehicle against predecessor emergency braking and efficiently track velocity and inter-vehicle gaps. To achieve a sufficiently low-dimensional parametrization, novel formulations have been devised, including a non-uniform sampling across the optimization horizon, a regression-based determination of the tracked reference velocity informed by the predecessor motion, and robustification measures. The controller also considers a current road friction coefficient estimate to yield corresponding, admissible control actions. In a detailed co-simulation validation, safe and efficient platoon behaviour under varying road conditions and emergency braking is demonstrated.

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