Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Position Control of a Planar Single-Link Flexible-Link Manipulator Based on Enhanced Dynamic Coupling Model ⋆

Qingxin Meng *,**,*** **Jundong Wu** *** **Ze Yan** *,** **Chun-Yi Su** ***

* *School of Automation, China University of Geosciences, Wuhan, Hubei 430074, China*
** *Hubei Key Laboratory of Advanced Control and Intelligent Automation for Complex Systems, Wuhan, Hubei 430074, China*
*** *Department of Mechanical, Industrial and Aerospace Engineering, Concordia University, Montreal, QC H3G 1M8, Canada (e-mail: cysu@alcor.concordia.ca)*

Abstract: Position control of a planar single-link flexible-link manipulator always faces huge challenges due to the underactuated characteristic of the system and the vibration of the flexible link. Since the system state variables related to the vibration is underactuated, this paper considers enhancing the dynamic coupling of this system. An enhanced dynamic coupling model is established by making equivalent transformation for the real dynamic model of the system. This enhanced dynamic coupling model makes the vibration variables appear as active variables in the proposed energy-based controller. Therefore, the proposed controller achieves good vibration suppression effect. The stability analysis is presented to prove that the proposed controller can effectively achieve the position control objective, and the simulation results further demonstrate the superiority of the proposed control method.

Keywords: Flexible-link manipulator, underactuated system, enhanced dynamic coupling, position control, vibration control.

1. INTRODUCTION

With the progress of science and the development of industrial production technology, the manipulators gradually replace human beings for various repetitive and heavy work (Paul, 1982). In the past forty years, the flexible-link manipulators (FLMs) attract a lot of attention from the researchers (Rahimi and Nazemizadeh, 2014; Gao et al., 2018). Compared with the rigid-link manipulator, the FLM has more slender structure and lighter mass. Therefore, it has the advantages of fast response, high-speed movement and large workspace (Dwivedy and Eberhard, 2006; He et al., 2019).

Although the research on the FLMs has been carried out for a long time, to this day, the control problem of these systems is still a challenging issue (He et al., 2019; Gao et al., 2018). The main reasons are two-folds: on the one hand, the FLM is a distributed parameter system and has infinite degree-of-freedom (DOF), which means this system has underactuated characteristic (Spong, 1998); on the other hand, the low rigidity link makes the manipulator easy to vibrate. How to realize the active vibration suppression is the key point in the position control of the FLMs.

As the simplest FLM, the planar single-link flexible-link (SLFL) manipulator is an ideal system for studying the position control of the FLMs. The position control objective of the SLFL manipulator can be divided into two parts. One is to move the flexible link to the target angle, and the other is to suppress the vibration of the flexible link. Till now, many control strategies have been proposed. The most common strategies are linear control (Robert H. Cannon and Schmitz, 1984), input shaping approach (Tzes and Yurkovich, 1993), sliding mode control (Qian and Ma, 1992). Furthermore, some adaptive strategies have been proposed, such as neural network strategy (Gao et al., 2018), fuzzy strategy (Sun et al., 2017), nonlinear adaptive approach (Yang et al., 1997), and etc.

Although the above strategies achieve the position control objective of the SLFL manipulator, they all depend on the system model with finite dimension. Since the system is a distributed parameter system, these strategies need to truncate the infinite dimension model to finite dimensional model by using the assumed modes method (AMM) or the finite element method (FEM). But unfortunately, the truncation might lead to the so-called spillover problems (Lochan et al., 2016). Shitole and Sumathi (2015) designed an auxiliary vibration mode estimator to solve the observation spillover, but this approach cannot solve the control

⋆ This work was supported by the National Natural Science Foundation of China under Grant 61773353, the Hubei Provincial Natural Science Foundation of China under Grant 2015CFA010, the 111 project under Grant B17040, and the Fundamental Research Funds for National Universities, China University of Geosciences (Wuhan) (Corresponding author: Chun-Yi Su).

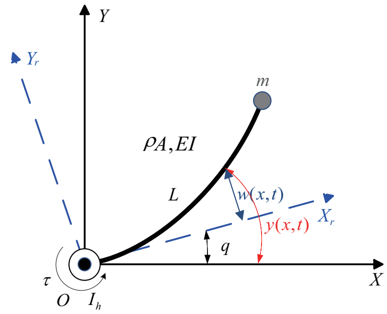

Fig. 1. Physical structure of the planar SLFL manipulator

spillover. Some boundary control approaches have been proposed based on the partial differential equations model of the system (Liu et al., 2016; Jiang et al., 2018). These approaches can avoid the spillover problems, but they realize the vibration control by adding a control torque at the tip of the flexible link, which requires an extra tip actuator.

The energy-based controller (Meng et al., 2018a) has the structure of the PD controller, which does not depend on the truncated system model (Yigit, 1994). It realizes the vibration suppression by making the system energy converge. Hence, this controller will not lead to the spillover problems. However, due to the lack of the underactuated variables in the controller, this controller cannot achieve good transient control performance according to the feedback vibration information. Sun et al. (2012) presented a method to enhance the dynamic coupling among the actuated variable and the underactuated variables.

Inspired by the above two methods, in this paper, we propose a method to enhance the dynamic coupling of a planar single-link flexible-link (SLFL) manipulator. Then, based on the enhanced dynamic coupling model, we design an energy-based controller to realize the position control of the SLFL manipulator. The stability of the control system and the realization of the position control objective are proved respectively. Finally, the effectiveness and superiority of the proposed controller are verified by simulation.

2. MODELING AND ANALYSIS

In this section, we will establish the dynamic model of this manipulator, and then analyze its characteristics.

In order to describe the formula derivation clearly and conveniently, we give the following definitions in this paper

$$\dot{\Diamond}=\frac{\partial \Diamond}{\partial t},\ \Diamond'=\frac{\partial \Diamond}{\partial x}. \tag{1}$$

The physical structure of the planar SLFL manipulator is shown in Fig. 1. XOY is the base coordinate and X_rOY_r is the rotation coordinate for the manipulator. ρA represents the mass per unit length of the flexible link. EI represents the bending stiffness of the material of the flexible link. m is the mass of the payload, and L is the length of the flexible link. q is the rotation angle of the flexible link. τ is the torque supplied by the joint motor. I_h is the inertia of the joint motor.

For a point at position x on the flexible link, its elastic deformation at time t is represented as $w(x,t)$, and its position with respect to the base coordinate XOY is represented as $y(x,t)$, which can be expressed as (Robert H. Cannon and Schmitz, 1984)

$$y(x,t) = xq + w(x,t). \tag{2}$$

The flexible link of the manipulator can be regarded as an Euler-Bernoulli beam (Book, 1990). One side of this beam is locked to the motor, the other side is connected to a payload and can vibrate freely. Thus, the boundary conditions of this beam can be expressed as (Sakawa et al., 1985)

$$\begin{aligned} &w(0,t) = 0, w'(0,t) = 0, w''(L,t) = 0, \\ &\rho A w'''(L,t) = -m w''''(L,t), \end{aligned} \tag{3}$$

and the vibration equation of this beam is

$$\rho A \ddot{w}(x,t) + EI w''''(x,t) + \rho A x \ddot{q} = 0. \tag{4}$$

Clearly, $w(x,t)$ and $y(x,t)$ contain two variables: x and t. For the convenience of modeling and model analysis, here, we use the AMM to separate the variables and describe $w(x,t)$ as

$$w(x,t) = \sum_{i=1}^{n} \phi_i(x) p_i(t), \tag{5}$$

where $\phi_i(x)$ is the ith assumed model function, which is related to the position x. $p_i(t)$ is the ith generalized coordinate, which is related to the time t. $n \in [1, \infty)$ is the number of the assumed modes which we truncate. The larger the value of n is, the more accurate the established model is.

According to the AMM, $\phi_i(x)$ has the following form (Sakawa et al., 1985)

$$\begin{aligned} \phi_i(x) = C_1 \cosh(\lambda_i x) + C_2 \sinh(\lambda_i x) + \\ C_3 \cos(\lambda_i x) + C_4 \sin(\lambda_i x), \end{aligned} \tag{6}$$

where λ_i is the ith eigenvalue of the system. C_1, C_2, C_3, C_4 are all constants, and they can be determined by solving (6) with the boundary conditions (3). After doing this, we rearrange (6) as

$$\begin{aligned} \phi_i(x) &= \frac{1}{\mho_i}\Big[\cosh(\lambda_i x) - \cos(\lambda_i x) - \\ &\quad \frac{\cosh(\lambda_i L) + \cos(\lambda_i L)}{\sinh(\lambda_i L) + \sin(\lambda_i L)} (\sinh(\lambda_i x) - \sin(\lambda_i x))\Big], \\ \mho_i &= \left[L + \frac{\rho A}{m\lambda_i^2}\left(\frac{1+\cosh(\lambda_i L)\cos(\lambda_i L)}{\sinh(\lambda_i L)\sin(\lambda_i L)}\right)^2\right]^{1/2}, \end{aligned} \tag{7}$$

and we obtain the eigenvalues equation of system as

$$\begin{aligned} \sinh(\lambda_i L)\cos(\lambda_i L) - \cosh(\lambda_i L)\sin(\lambda_i L) = \\ -\frac{\rho A}{m\lambda_i}\left[1 + \cosh(\lambda_i L)\cos(\lambda_i L)\right]. \end{aligned} \tag{8}$$

To obtain the dynamic model of the SLFL manipulator, we construct the Lagrange function as

$$L = E_k - E_p, \tag{9}$$

where E_k is the kinetic energy of the system, and E_p is the elastic potential energy of the system. E_k can be expressed as

$$E_k = \frac{1}{2} I_h \dot{q}^2 + \frac{1}{2} m [\dot{y}(L,t)]^2 + \frac{1}{2}\rho A \int_0^L [\dot{y}(x,t)]^2 \mathrm{d}x, \tag{10}$$

and E_p can be expressed as

$$E_p = \frac{1}{2} EI \int_0^L [y''(x,t)]^2 \mathrm{d}x. \tag{11}$$

By using the Euler-Lagrange approach (Ortega et al., 1998; Yan et al., 2019) and combining the orthogonality of the modal functions (Rahn, 2001)

$$\begin{cases} \int_0^L \rho A \phi_i(x)\phi_j(x)dx + m\phi_i(L)\phi_j(L) = 0 \\ \int_0^L EI{\phi_i}''(x){\phi_j}''(x)dx = 0 \end{cases} \quad i \neq j,$$
$$\begin{cases} \int_0^L \rho A \phi_i(x)\phi_j(x)dx + m\phi_i(L)\phi_j(L) = \psi_i \\ \int_0^L EI{\phi_i}''(x){\phi_j}''(x)dx = \omega_i^2 \psi_i \end{cases} \quad i = j, \tag{12}$$

where ψ_i is the generalized mass and $\omega_i = \lambda_i^2 \sqrt{EI/(\rho A)}$ is the natural frequency for the ith modal function, we obtain the dynamic model of the system as

$$M\ddot{\varsigma} + K\varsigma = U, \tag{13}$$

where $\varsigma = [q, p_1, p_2, \cdots, p_n]^T$ is the state variables vector of the system. The inertia matrix of the system, which is a positive-definite symmetric matrix, is given as

$$M = \begin{bmatrix} M_{qq} & M_{qp} \\ M_{pq} & M_{pp} \end{bmatrix} \in R^{(n+1)\times(n+1)}, \tag{14}$$

The elements of M are

$$\begin{aligned} M_{qq} &= I_h + mL^2 + \frac{1}{3}\rho A L^3, \\ M_{qp} &= M_{pq}^T = [\, \delta_1 \ \delta_2 \ \cdots \ \delta_n \,] \in R^{1\times n}, \\ M_{pp} &= \mathrm{diag}\, [\, \psi_1 \ \psi_2 \ \cdots \ \psi_n \,] \in R^{n\times n}, \end{aligned} \tag{15}$$

where $\delta_i = \rho A \int_0^L x\phi_i(x)dx + mL\phi_i(L)$. K is the elasticity matrix of the system, and it can be expressed as

$$K = \mathrm{diag}\left[\, 0 \ \omega_1^2\psi_1 \ \omega_2^2\psi_2 \ \cdots \ \omega_n^2\psi_n \,\right] \in R^{(n+1)\times(n+1)} \tag{16}$$

U is the control inputs matrix and has the following form

$$U = [\, \tau \ 0 \ 0 \ \cdots \ 0 \,]^T \in R^{(n+1)\times 1} \tag{17}$$

For the SLFL manipulator, the position control objective is to move the flexible link of this manipulator from its initial angle q_0 to a target angle q_d and suppress the vibration of the flexible link. That is, we should design a controller τ to make

$$\varsigma = [q, p_1, p_2, \cdots, p_n]^T \to [q_d, 0, 0, \cdots, 0]^T, \tag{18}$$

when $t \to \infty$.

Equation (13) shows that the system has $(n+1)$ DOF, but only has one control input. Therefore, this system is an underactuated system. In the state variables vector ς, q is the actuated variable and $p_1, p_2, \cdots, p_n$ are the underactuated variables. The total energy of this system is written as

$$\begin{aligned} E &= E_k + E_p \\ &= \frac{1}{2}\dot{\varsigma}^T M \dot{\varsigma} + \frac{1}{2}\varsigma^T K \varsigma. \end{aligned} \tag{19}$$

According to (14) and (15), it is clear that M is a constant matrix. Thus,

$$\begin{aligned} \dot{E} &= \dot{\varsigma}^T (M\ddot{\varsigma} + K\varsigma) + \frac{1}{2}\dot{\varsigma}^T \dot{M} \dot{\varsigma} \\ &= \dot{\varsigma}^T U = \dot{q}\tau. \end{aligned} \tag{20}$$

According to (20), we know that the system (13) is a passive system. That is, the time derivative of the system total energy only relates to the control input and the actuated variable. The underactuated variables are not included in $\dot{E}$. This characteristic tends to make the control methods based on the system energy cannot achieve good transient control performance (Sun et al., 2012). That is to say, for the position control of the SLFL manipulator, suppressing vibration by means of energy convergence cannot guarantee the efficiency of the vibration suppression.

To solve this problem, in next section, we artificially enhance the dynamic coupling among the actuated variable and the underactuated variables in dynamic model (13).

3. CONTROL SYSTEM DESIGN

In this section, the dynamic model of the system (13) is transformed into an enhanced dynamic coupling model. The transformed model is equivalent to the original model. A controller is designed for this enhanced dynamic coupling model, and then the stability of the controller is proven.

3.1 Enhanced dynamic coupling model

In order to enhance the dynamic coupling among the actuated variable and the underactuated variables, we introduce a compound state variable η as

$$\eta = q + \upsilon, \tag{21}$$

where υ is an undetermined variable, and it is related to the underactuated variables, i.e., p_i, $\dot{p}_i$, or $\ddot{p}_i$, $(i = 1, 2, \cdots, n)$. Thus

$$\dot{\eta} = \dot{q} + \dot{\upsilon}, \ \ddot{\eta} = \ddot{q} + \ddot{\upsilon}. \tag{22}$$

Accordingly, we change the state variables vector $\varsigma = [q, p_1, p_2, \cdots, p_n]^T$ to

$$\vartheta = [\eta, p_1, p_2, \cdots, p_n]^T. \tag{23}$$

Then, we can transform the dynamic model (13) into the following form

$$\begin{bmatrix} M_{qq} & \delta_1 & \delta_2 & \cdots & \delta_n \\ \delta_1 & \psi_1 & 0 & 0 & 0 \\ \delta_2 & 0 & \psi_2 & 0 & 0 \\ \vdots & 0 & 0 & \ddots & 0 \\ \delta_n & 0 & 0 & 0 & \psi_n \end{bmatrix} \ddot{\vartheta} + \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & \omega_1^2\psi_1 & 0 & 0 & 0 \\ 0 & 0 & \omega_2^2\psi_2 & 0 & 0 \\ \vdots & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \omega_n^2\psi_n \end{bmatrix} \vartheta = \left[\, \tau_u \ \delta_1\ddot{\upsilon} \ \delta_2\ddot{\upsilon} \ \cdots \ \delta_n\ddot{\upsilon} \,\right]^T, \tag{24}$$

where τ_u is a new control input. The relationship between τ_u and τ is

$$\tau_u = \tau + M_{qq}\ddot{v}. \tag{25}$$

It is not difficult to find that the enhanced dynamic coupling model (24) is equivalent to the dynamic model of the system (13). Referring to (19), we construct an energy function for the model (24) as

$$E_e = \frac{1}{2}\dot{\vartheta}^T M\dot{\vartheta} + \frac{1}{2}\vartheta^T K\vartheta \geq 0. \tag{26}$$

Combining (24) with (26), we obtain

$$\begin{aligned}\dot{E}_e &= \dot{\vartheta}^T\left(M\ddot{\vartheta} + K\vartheta\right) + \frac{1}{2}\dot{\vartheta}^T\dot{M}\dot{\vartheta} \\ &= \dot{\vartheta}^T\left[\,\tau_u\ \ \delta_1\ddot{v}\ \ \delta_2\ddot{v}\ \cdots\ \delta_n\ddot{v}\,\right]^T.\end{aligned} \tag{27}$$

Because the expression for $\dot{E}_e$ contains the actuated variable and the underactuated variables, the energy-based controller for the model (24) will achieve a good transient performance for the position control of the SLFL manipulator.

3.2 Controller design

Based on the enhanced dynamic coupling model (24), if we can design a controller to control the compound state variable η, we can directly control the angle and vibration of the flexible link. In this way, all the state variables of the SLFL manipulator become seemingly actuated.

According to the design process of the energy-based controller, we construct a Lyapunov function as

$$V = \frac{1}{2}K_p(\eta - q_d)^2 + E_e \geq 0, \tag{28}$$

where K_p is a positive design parameter. The derivative of this Lyapunov function with respect to t is

$$\begin{aligned}\dot{V} &= K_p\left(\eta - q_d\right)\dot{\eta} + \dot{E}_e \\ &= K_p\left(\eta - q_d\right)\dot{\eta} + \tau_u\dot{\eta} + \ddot{v}\delta_1\dot{p}_1 + \ddot{v}\delta_2\dot{p}_2 + \cdots + \ddot{v}\delta_n\dot{p}_n \\ &= \dot{\eta}\left[\tau_u + K_p\left(\eta - q_d\right)\right] + \ddot{v}\left(\delta_1\dot{p}_1 + \delta_2\dot{p}_2 + \cdots + \delta_n\dot{p}_n\right).\end{aligned} \tag{29}$$

We split the right side of (29) into two terms to design separately so as to ensure that $\dot{V}$ is negative. For the first term

$$\dot{V}_1 = \dot{\eta}\left[\tau_u + K_p\left(\eta - q_d\right)\right]. \tag{30}$$

To guarantee $\dot{V}_1$ is negative, we design the controller τ_u as

$$\tau_u = -K_p\left(\eta - q_d\right) - K_d\dot{\eta}, \tag{31}$$

where K_d is a positive design parameter. Therefore, $\dot{V}_1$ becomes

$$\dot{V}_1 = -K_d\dot{\eta}^2 \leq 0. \tag{32}$$

For the second term of the right side of (29), we have

$$\begin{aligned}\dot{V}_2 &= \ddot{v}\left(\delta_1\dot{p}_1 + \delta_2\dot{p}_2 + \cdots + \delta_n\dot{p}_n\right) \\ &= \ddot{v}\left[\rho A\int_0^L x\sum_{i=1}^{n}\left(\phi_i(x)\dot{p}_i\right)dx + mL\sum_{i=1}^{n}\left(\phi_i(L)\dot{p}_i\right)\right] \\ &= \ddot{v}\left[\rho A\int_0^L x\dot{w}(x,t)dx + mL\dot{w}(L,t)\right].\end{aligned} \tag{33}$$

According to the mean value theorems for definite integrals, we know that there is a point ε in the interval $[0, L]$ that can make the following equation hold

$$\rho A\int_0^L x\dot{w}(x,t)dx = \rho AL\varepsilon\dot{w}(\varepsilon,t). \tag{34}$$

Because the first natural frequency ω_1 is the dominant frequency in the flexible manipulator (Sun et al., 2017), the $\dot{w}(x,t)$ can be approximated as $\phi_1(x)\dot{p}_1(t)$. According to Lochan et al. (2016), we know that for the clamped-free cantilever beam, $\phi_1(x)$ is positive and monotonically increasing. Therefore, $\dot{w}(\varepsilon,t)$ and $\dot{w}(L,t)$ have same sign.

According to (33), we can determine the form of $\ddot{v}$ to make $\dot{V}_2$ negative. Thus, we construct $\ddot{v}$ as

$$\ddot{v} = -\sum_{i=1}^{n}\phi_i(L)\dot{p}_i(t) = -\dot{w}(L,t). \tag{35}$$

Then, we have

$$\begin{aligned}\dot{V}_2 &= \ddot{v}\left[\rho AL\varepsilon\dot{w}(\varepsilon,t) + mL\dot{w}(L,t)\right] \\ &= -\rho AL\varepsilon\dot{w}(L,t)\dot{w}(\varepsilon,t) - mL\dot{w}^2(L,t) \\ &\leq 0.\end{aligned} \tag{36}$$

Combining (29), (32), and (36), we have $\dot{V} \leq 0$. Thus, $V \in \mathcal{L}_\infty$ (i.e., V is bounded). Let Ξ be an invariant set of the system, which can be expressed as

$$\Xi = \left\{\left(\vartheta^T, \dot{\vartheta}^T\right)\middle|\,\dot{V} = 0\right\}. \tag{37}$$

When $\dot{V} \equiv 0$, according to (32) and (36), we can obtain that $\dot{V}_1 \equiv 0$, $\dot{V}_2 \equiv 0$, then we have $\dot{\eta} \equiv 0$, $\dot{w}(L,t) \equiv 0$ (i.e., $\dot{p}_1 = \dot{p}_2 = \cdots = \dot{p}_n \equiv 0$). Thus, $\ddot{\eta} \equiv 0$, $\ddot{p}_1 = \ddot{p}_2 = \cdots = \ddot{p}_n \equiv 0$ and η, p_1, p_2, $\cdots$, $p_n \in \mathcal{L}_C$, where $\mathcal{L}_C$ represents a set of all constants. Bringing all the above states into (24), we have $\tau_u = 0$. Hence $\eta = q_d$. Meantime, combining the vibration equation (4), we have the tip vibration equation for the beam,

$$\rho A\ddot{w}(L,t) + EIw''''(L,t) + \rho AL\left[\ddot{\eta} + \dot{w}(L,t)\right] = 0. \tag{38}$$

Because $EIw''''(L,t) = EI\lambda^4 w(L,t)$ (Sakawa et al., 1985), where λ is the eigenvalue of the system and it is approximately equal to λ_1 in (8), (38) is rewritten as

$$\rho A\ddot{w}(L,t) + EI\lambda^4 w(L,t) + \rho AL\left[\ddot{\eta} + \dot{w}(L,t)\right] = 0. \tag{39}$$

Submitting $\dot{p}_1 = \dot{p}_2 = \cdots = \dot{p}_n \equiv 0$, $\ddot{\eta} \equiv 0$ into (39), we have $w(L,t) \equiv 0$. Therefore, the largest invariant set of (37) is

$$\Gamma = \left\{\left(\vartheta^T, \dot{\vartheta}^T\right)\middle|\,\eta = q_d, p_1 = p_2 = \cdots = p_n = 0, \dot{\vartheta}^T = 0\right\}. \tag{40}$$

According to LaSalle's invariance theorem, it is not difficult to obtain the conclusion that the system state variables converge to Γ when $t \to \infty$.

From (25) and (31), we obtain the controller τ for the real dynamic model (13) as

$$\begin{aligned}\tau &= \tau_u - M_{qq}\ddot{v} \\ &= -K_p\left(\eta - q_d\right) - K_d\dot{\eta} + M_{qq}\dot{w}(L,t).\end{aligned} \tag{41}$$

Next, we discuss that under the controller (41), if $\eta \to q_d$, the control objective of the SLFL manipulator will be realized.

Theorem 1. When we make the compound state variable $\eta \to q_d$ as $t \to \infty$, the angle of the flexible link will converge to its target angle and the vibration of the flexible link will be suppressed (i.e., $q \to q_d$ and $p_1, p_2, \cdots, p_n \to 0$).

Proof. Based on (35), and $\dot{w}(L,0) = 0$, we can obtain

$$\eta = q - \int_0^t w(L,t)dt,\ \dot{\eta} = \dot{q} - w(L,t),\ \ddot{\eta} = \ddot{q} - \dot{w}(L,t). \quad (42)$$

According to (39), $w(L,t)$ can be written as

$$w(L,t) = -\frac{\rho A}{EI{\lambda_1}^4}\ddot{w}(L,t) - \frac{\rho AL}{EI{\lambda_1}^4}\ddot{\eta} - \frac{\rho AL}{EI{\lambda_1}^4}\dot{w}(L,t). \quad (43)$$

Therefore,

$$\int_0^t w(L,t)dt = -\frac{\rho A}{EI{\lambda_1}^4}\left[\dot{w}(L,t) - \dot{w}(L,0)\right] - \frac{\rho AL}{EI{\lambda_1}^4}\left[\dot{\eta}(t) - \dot{\eta}(0)\right] - \frac{\rho AL}{EI{\lambda_1}^4}\left[w(L,t) - w(L,0)\right]. \quad (44)$$

At the initial time of the position control, the manipulator has no vibration and angular velocity. Therefore, $w(L,0) = 0$, $\dot{w}(L,0) = 0$, $\dot{\eta}(0) = \dot{q}(0) - w(L,0) = 0$. Meantime, according to (40), we know

$$\lim_{t\to\infty} \dot{w}(L,t) = 0,\ \lim_{t\to\infty} \dot{\eta}(t) = 0,\ \lim_{t\to\infty} w(L,t) = 0. \quad (45)$$

Thus, $p_1, p_2, \cdots, p_n \to 0$, and the following equation holds

$$\lim_{t\to\infty} \int_0^t w(L,t)dt = 0. \quad (46)$$

Because

$$\lim_{t\to\infty} \eta(t) = \lim_{t\to\infty} q(t) - \lim_{t\to\infty} \int_0^t w(L,t)dt = q_d, \quad (47)$$

we obtain $\lim_{t\to\infty} q(t) = q_d$, i.e., $q \to q_d$. Then, Theorem 1 is proved.

Remark 1 *The controller (41) does not depend on the truncated system model. It is only related to the feedback information, including q, $\dot{q}$, $w(L,t)$, and $\dot{w}(L,t)$. Meantime, the feedback information does not involve the assumed mode variables, and they can be obtained by the sensors. Therefore, the controller (41) will not meet the spillover problems.*

4. SIMULATION

In this section, we perform simulations to demonstrate the effectiveness of the proposed controller (41). The MATLAB-Simulink simulation platform is used. Meanwhile, to express the superiority of the proposed controller, an energy-based PD controller (Yigit, 1994) and a sliding mode (SM) controller (Meng et al., 2018b) are also discussed for comparison with the proposed controller.

The form of the PD controller is shown as follow,

$$\tau = -k_p(q - q_d) - k_d\dot{q}, \quad (48)$$

where k_p and k_d are two control gains. The SM controller is given as

$$\tau = \frac{-\varpi S - \mu \mathrm{sgn}(S) - \beta\dot{q} - f_1}{g_1}, \quad (49)$$

where $S = \beta(q - q_d) + \dot{q}$ is the sliding mode surface, and ϖ, μ, β are the positive design parameters. The expressions of f_1 and g_1 can refer to Meng et al. (2018b).

The system parameters of the SLFL manipulator are shown in Table 1, and the initial angle of the manipulator is set to be $q_0 = 0$ rad. The target angle of the position control is set to be $q_d = 0.5$ rad. For the AMM, we set the number of the assumed modes n to be 3.

Table 1. Model parameters of the system

Parameters	Description	Value
L	Length of the link	1.5 m
EI	Bending stiffness	3.0 N·m^2
ρA	Mass per unit length	0.3 kg/m
I_h	Inertia of the motor	0.08 kg·m^2
m	Mass of the payload	3.0 kg

The simulation results are shown in Fig. 2. For the PD controller, its parameters are chosen as $k_p = 5$, $k_d = 10$. For the SM controller, its parameters are chosen as $\varpi = 1$, $\mu = 0.3$, $\beta = 1.2$. For the proposed controller, its parameters are chosen as $K_p = 5$, $K_d = 10$.

Based on the simulation results shown in Fig. 2, we know that the SM controller can move the link of the manipulator to the target angle, but it cannot suppress the vibration of the flexible link. Therefore, the SM controller cannot realize the position control objective of the SLFL manipulator. Fig. 2(c) shows that both the PD controller and the proposed controller (41) can suppress the vibration of the link. By contrast, the setting time of the proposed controller (about 14 s) is shorter than that of the PD controller (about 23 s). The proposed controller suppresses the vibration of the flexible link faster than the PD controller. Meantime, the maximum tip deflection of the flexible link caused by the controller (41) (about 0.1 m) is smaller than that of the PD controller (about 0.2 m). All these comparison results show that the proposed controller leads to an improved transient control performance.

5. CONCLUSION

In this paper, we propose a method to enhance the dynamic coupling among the actuated variable and the underactuated variables of the SLFL manipulator. An enhanced dynamic coupling model of this system is established. Then, a controller is developed based on this model. The proposed controller only requires the rotation information and the tip vibration information of the flexible link, which can be directly measured by the sensors. Thus, the proposed controller can be easily applied. The simulation results show that the proposed controller is effective and achieves good transient control performance. It is noteworthy that because the enhanced dynamic coupling model

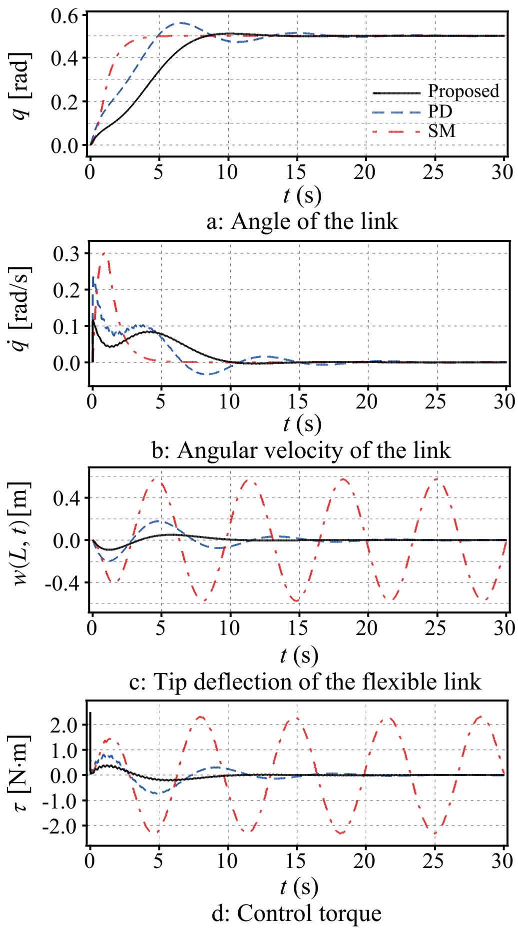

Fig. 2. Simulation results for the SLFL manipulator with three different controllers

can make the controller contain underactuated variables, it lays a good foundation for the adaptive control research of this system. In the future, we will use this approach to study the adaptive control methods for the flexible-link manipulator with mismatched uncertainty.

REFERENCES

Book, W.J. (1990). Modeling, design, and control of flexible manipulator arms: a tutorial review. In *Proc. IEEE Conf. Decision and Control*, 500–506 vol.2.

Dwivedy, S.K. and Eberhard, P. (2006). Dynamic analysis of flexible manipulators, a literature review. *Mech. Mach. Theory*, 41(7), 749–777.

Gao, H., He, W., Song, Y., Zhang, S., and Sun, C. (2018). Modeling and neural network control of a flexible beam with unknown spatiotemporally varying disturbance using assumed mode method. *Neurocomputing*, 314, 458–467.

He, W., He, X., Zou, M., and Li, H. (2019). Pde model-based boundary control design for a flexible robotic manipulator with input backlash. *IEEE Trans. Control Syst. Technol.*, 27(2), 790–797.

Jiang, T., Liu, J., and He, W. (2018). Boundary control for a flexible manipulator with a robust state observer. *J. Vib. Control*, 24(2), 260–271.

Liu, Z., Liu, J., and He, W. (2016). Adaptive boundary control of a flexible manipulator with input saturation. *Int. J. Control*, 89(6), 1191–1202.

Lochan, K., Roy, B., and Subudhi, B. (2016). A review on two-link flexible manipulators. *Annu. Rev. Control*, 42, 346–367.

Meng, Q., Lai, X., Wang, Y., and Wu, M. (2018a). A fast stable control strategy based on system energy for a planar single-link flexible manipulator. *Nonlinear Dyn.*, 94(1), 615–626.

Meng, Q., Lai, X., and Yan, Z. (2018b). A control strategy with zero residual vibration for a planar single-link flexible manipulator. In *Proc. 37th Chinese Control Conf.*, 344–347.

Ortega, R., Loría, A., Nicklasson, P.J., and Sira-Ramírez, H. (1998). *Passivity-based Control of Euler-Lagrange Systems*. Springer-Verlag, London, U.K.

Paul, R.P. (1982). *Robot Manipulators: Mathematics, Programming, and Control*. MIT Press, Cambridge, MA, USA, 1st edition.

Qian, W.T. and Ma, C.C.H. (1992). A new controller design for a flexible one-link manipulator. *IEEE Trans. Autom. Control*, 37(1), 133–137.

Rahimi, H. and Nazemizadeh, M. (2014). Dynamic analysis and intelligent control techniques for flexible manipulators: a review. *Adv. Robot.*, 28(2), 63–76.

Rahn, C.D. (2001). *Mechatronic Control of Distributed Noise and Vibration*. Springer-Verlag, Berlin, Germany.

Robert H. Cannon, J. and Schmitz, E. (1984). Initial experiments on the end-point control of a flexible one-link robot. *Int. J. Robot. Res.*, 3(3), 62–75.

Sakawa, Y., Matsuno, F., and Fukushima, S. (1985). Modeling and feedback control of a flexible arm. *J. Robot. Syst.*, 2(4), 453–472.

Shitole, C. and Sumathi, P. (2015). Sliding dft-based vibration mode estimator for single-link flexible manipulator. *IEEE/ASME Trans. Mechatronics*, 20(6), 3249–3256.

Spong, M.W. (1998). Underactuated mechanical systems. In B. Siciliano and K.P. Valavanis (eds.), *Control Problems in Robotics and Automation*, 135–150. Springer-Verlag, Berlin, Heidelberg.

Sun, C., He, W., and Hong, J. (2017). Neural network control of a flexible robotic manipulator using the lumped spring-mass model. *IEEE Trans. Syst., Man, Cybern., Syst.*, 47(8), 1863–1874.

Sun, N., Fang, Y., and Zhang, X. (2012). An increased coupling-based control method for underactuated crane systems: theoretical design and experimental implementation. *Nonlinear Dyn.*, 70(2), 1135–1146.

Tzes, A. and Yurkovich, S. (1993). An adaptive input shaping control scheme for vibration suppression in slewing flexible structures. *IEEE Trans. Control Syst. Technol.*, 1(2), 114–121.

Yan, Z., Lai, X., Meng, Q., and Wu, M. (2019). A novel robust control method for motion control of uncertain single-link flexible-joint manipulator. *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, 1–8. doi: 10.1109/TSMC.2019.2900502.

Yang, J.H., Lian, F.L., and Fu, L.C. (1997). Nonlinear adaptive control for flexible-link manipulators. *IEEE Trans. Robot. Autom.*, 13(1), 140–148.

Yigit, A. (1994). On the stability of pd control for a two-link rigid-flexible manipulator. *ASME J. Dyn. Systems Meas. Control*, 116, 208–215.