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Optimal Guaranteed Cost Event-triggered Control of Smart Grid Against Time Delay Switch Attack

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Abstract: This paper mainly studies the optimal robust guaranteed cost load frequency control (LFC) problem for a class of uncertain power system under time delay switch (TDS) attack. The closed-loop power system is modelled as time delay system when an event-triggered communication scheme is adopted to reduce bandwidth consumption. In order to obtain less conservative stability criteria of the system with additive time delays, a novel Lyapunov-Krasovskii (L-K) functional is proposed and some latest integral inequalities are applied as well. Then, an optimal guaranteed cost controller is designed to eliminate the system uncertainty and frequency fluctuation, and the minimum upper bound of the performance index can be obtained by solving a convex optimization problem.

Keywords: time delay switch attack, additive time delay, event-triggered scheme, optimal guaranteed cost control.

1. INTRODUCTION

Load frequency control (LFC) is adopted to maintain the frequency into its rating. As a typical cyber-physical system (CPS), smart grid is different from the traditional power system because of the introduction of cyber space Farhangi H (2010). However, serious accidents may be caused due to the openness of communication network. For example, in 2015, the Ukraine's power sector suffered from BlackEnergy attack, and this disaster made more than 230k residents fall into darkness for hours Tang Y (2016). Consequently, it is necessary to study the secure control problem of cyber attacked power system, and some valuable results have been reported such as Peng C (2019), Yang F (2019),Tan R (2017),Kurt M (2018). In Peng C (2019), the author summarized the existing defensive-measures such that the malignant impact caused by different attacks can be eliminated. As to the false data injection (FDI) attack, the impact on automatic generation control is mentioned, and an optimal attack strategy is derived from the attacker's point of view in Tan R (2017). When a malicious denial of service (DoS) attack is injected into power system, an LFC and event triggered scheme co-design approach is offered in Yang F (2019) to alleviate system performance degradation. Unfortunately, the TDS attack which may oscillate whole power system has not received much attention yet, only mentioned in few studies, such as Shafiqu M (2015). Modeling based on hybrid system and an augmented controller with a time delay estimator is offered in Sargolzaei A (2016).

Recently, event-triggered technique attracts much attention due to the lower bandwidth consumption under the premise of retaining the desired system performance when it is established Yue D (2013), Hu S (2012). Many scholars pay special attention to discrete event triggered scheme based on sampled data, and the modelling approach based on delay system is proposed in Yue D (2013), which facilitated the study of network induced constraints such as communication delay.

Motivated by existing weaknesses and strengths, we focus attention on the optimal guaranteed cost controller design of power system which takes the event triggered scheme and TDS attack into account. The main contributions can be listed as

i) The closed-loop power system is modeled as an additive time delay system when both event triggered scheme and TDS attack are considered, and the communication bandwidth consumption is drastically reduced.

ii) The optimal guaranteed cost controller designed can not only ensure the system asymptotically stable but also have certain robustness, and the triggering matrix and controller gain can be obtained simultaneously by solving a convex optimization problem with LMI constraints.

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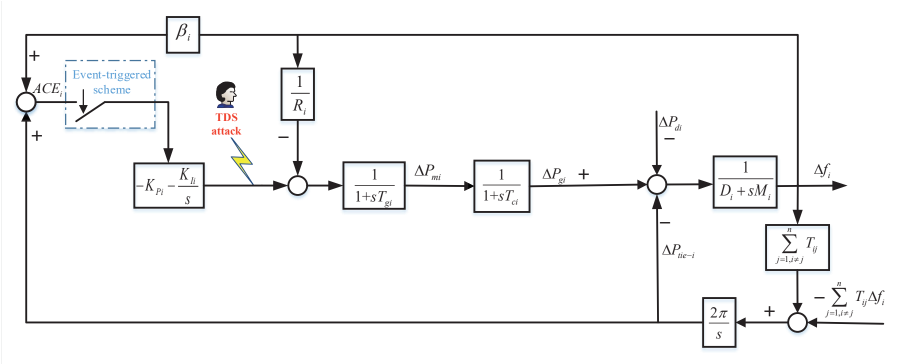


Fig. 1. Multi-area load frequency control power system with TDS attack

2. PROBLEM FORMULATION

2.1 Power System Model

In fact, some parameters in actual power system may be different from their nominal values, which means there exists system uncertainties. (Assuming that the time constants of governor and turbine deviate from the nominal value.) Then, combined with Fig. 1, the system dynamic model can be written like Yang F (2019):

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + Fw(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

with:

$$A_{ii} = \begin{bmatrix} -\frac{D_i}{M_i} & \frac{1}{M_i} & 0 & -\frac{1}{M_i} & 0 \\ 0 & -\frac{\lambda_1}{T_{ci}} & \frac{\lambda_1}{T_{ci}} & 0 & 0 \\ -\frac{\iota_1}{R_i T_{gi}} & 0 & -\frac{\iota_1}{T_{gi}} & 0 & 0 \\ 2\pi \sum\limits_{j\neq i} T_{ij} & 0 & 0 & 0 & 0 \\ \beta_i & 0 & 0 & 1 & 0 \end{bmatrix} B_{ii} = \begin{bmatrix} 0 & 0 & \frac{\lambda_2}{T_{gi}} & 0 & 0 \end{bmatrix}^T$$

The time-varying uncertainties are defined as below

$$[\Delta A \quad \Delta B] = DG(t)[E \quad \tilde{E}]$$

where $D, E, \tilde{E}$ are known constant matrices and uncertain time varying matrix $G(t)$ satisfies $G(t)^T G(t) \leq I$.

$$E = diag\{E_{11} \quad E_{22} \quad \cdots \quad E_{nn}\}$$

$$E_{ii} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\lambda_2}{T_{ci}} & \frac{\lambda_2}{T_{ci}} & 0 & 0 \\ -\frac{\iota_2}{R_i T_{gi}} & 0 & -\frac{\iota_2}{T_{gi}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{E} = diag\{\tilde{E}_{11} \quad \tilde{E}_{22} \quad \cdots \quad \tilde{E}_{nn}\}, \tilde{E}_{ii} = \begin{bmatrix} 0 & 0 & \frac{\lambda_2}{T_{gi}} & 0 & 0 \end{bmatrix}^T$$

$$\lambda_1 = \frac{1}{(1+\lambda)(1-\lambda)}, \quad \lambda_2 = \frac{\lambda}{(1+\lambda)(1-\lambda)}$$

$$\iota_1 = \frac{1}{(1+\iota)(1-\iota)}, \quad \iota_2 = \frac{\iota}{(1+\iota)(1-\iota)}$$

λ_i, ι_i, $i = 1, 2$ are the percentage of turbine and governor time constant deviating from nominal value respectively.

2.2 Event-triggered Communication Scheme

The triggered scheme can be obtained by substituting the output into the triggered condition given in Yue D (2013).

$$e(i_k h)^T C^T \Lambda C e(i_k h) < \rho x^T(t_k h) C^T \Lambda C x(t_k h) \tag{2}$$

where Λ is the triggered matrix, and $\rho \in (0, 1)$ is pre-set triggered threshold. $t_k h$ stands for the latest instant which the sampled data is triggered successfully. $i_k h = t_k h + rh$, $r = 1, \cdots, m_k$ is the unsuccessfully triggered sampled packet in interval $[t_k h, t_{k+1} h)$. m_k stands for the total sampled number which dissatisfies the triggered condition, and $e(i_k h) = x(i_k h) - x(t_k h)$.

Remark 1: It is obvious that the sampling data will be transmitted only when the triggered scheme (2) is violated,thus, the communication bandwidth can be saved.

The delay $d(t)$ is introduced and defined as:

$$d(t) = t - rh - t_k h, \quad t \in [t_k h + rh, t_k h + (r+1)h)$$

where $r = 0, 1, \cdots, m_k$. Then, $\forall t \in [t_k h, t_{k+1} h)$ define

$$e_k(t) = x(t_k h) - x(t_k h + rh) = x(t_k h) - x(t - d(t))$$

Then (1) is transformed into the following form:

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) - (B + \Delta B)KC(x(t - d(t) + e_k(t)) \\ \qquad\qquad + Fw(t) \\ y(t) = Cx(t) \end{cases} \tag{3}$$

2.3 Time Delay Switch Attack

As stated in Sargolzaei A (2018), TDS attack is a switch behavior where the delay $\tau(t)$ exists or not, and we study the case that TDS attack exists all the time. Here we emphasize that stability is a boundary of security.

Combined with the analysis in Shafiqu M (2015), then the system dynamic model (3) can be rewritten as follows.

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) - (B + \Delta B)KCx(t - d(t) - \tau(t)) \\ \qquad\qquad - (B + \Delta B)KCe_k(t) + Fw(t) \\ y(t) = Cx(t) \end{cases} \tag{4}$$

The delay functions are assumed to be continuous, differentiable, and satisfy the following constraints

$$\begin{aligned} 0 < d(t) \leq \bar{d}, \quad \dot{d}(t) = k_1 = 1 \\ 0 \leq \tau(t) \leq \bar{\tau}, \quad \dot{\tau}(t) \leq k_2 < 1 \\ 0 \leq \mu(t) = d(t) + \tau(t) \leq \bar{d} + \bar{\tau} = \bar{\mu}, \dot{\mu}(t) \leq 1 + k_2 = k \end{aligned} \tag{5}$$

The performance index is given as follows.

$$J = \int_0^\infty [x^T(t) M_1 x(t) + u^T(t) M_2 u(t)] dt \tag{6}$$

where M_1, M_2 are given symmetric positive definite matrices.

The main purpose of this paper is to study the attacked system stability under the premise of saving the limited network bandwidth while ensuring

1) The system (4) is asymptotically stable when $w(t) = 0$.

2) Under zero initial condition, for any nonzero $w(t) \in \Im_2[0; +\infty)$ and a prescribed $\gamma > 0$ is H_∞ performance index, the inequality $||y(t)||_2 \leq \gamma ||w(t)||_2$ holds;

3) There exists a controller u^* which is the optimal guaranteed cost controller and a positive scalar J^*, which is the minimum upper bound of J such that for all admissible uncertainties, system (4) is asymptotic stable.

3. MAIN RESULTS

3.1 Stability Analysis

Theorem 1: For given scalars $\bar{d}$, $\bar{\tau}$, $\bar{\mu}$, k_2, k and ρ, the system (4) is asymptotically stable if there exist positive define symmetric matrices $0 < P = P^T \in \mathbb{R}^{n\times n}$, $0 < Q_i \in \mathbb{R}^{n\times n}, i = 1\cdots 5$, $0 < R_j \in \mathbb{R}^{n\times n}, j = 1, 2$, $0 < Z_l \in \mathbb{R}^{n\times n}, l = 1, 2, 3$, and arbitrary matrices $S \in \mathbb{R}^{3n\times 3n}$ such that (7) holds.

$$\begin{bmatrix}
\Upsilon & * & * & * & * & * & * & * & * & * & * & * \\
F^T P e_1 & -\gamma^2 I & * & * & * & * & * & * & * & * & * & * \\
\bar{\mu} R_1 H & \bar{\mu} R_1 F & -R_1 & * & * & * & * & * & * & * & * & * \\
d_{12} R_2 H & d_{12} R_2 F & 0 & -R_2 & * & * & * & * & * & * & * & * \\
\frac{\bar{\mu}}{\sqrt{2}} Z_1 H & \frac{\bar{\mu}}{\sqrt{2}} Z_1 F & 0 & 0 & -Z_1 & * & * & * & * & * & * & * \\
\frac{\bar{d}}{\sqrt{2}} Z_2 H & \frac{\bar{d}}{\sqrt{2}} Z_2 F & 0 & 0 & 0 & -Z_2 & * & * & * & * & * & * \\
\frac{\bar{\tau}}{\sqrt{2}} Z_3 H & \frac{\bar{\tau}}{\sqrt{2}} Z_3 F & 0 & 0 & 0 & 0 & -Z_3 & * & * & * & * & * \\
C e_1 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * & * \\
M_1^{\frac{1}{2}} e_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * \\
M_2^{\frac{1}{2}} KC(e_2 + e_{23}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\
\sigma D^T P e_1 & 0 & \sigma \bar{\mu} D^T R_1 & \sigma d_{12} D^T R_2 & \sigma \frac{\bar{\mu}}{\sqrt{2}} D^T Z_1 & \sigma \frac{\bar{d}}{\sqrt{2}} D^T Z_2 & \sigma \frac{\bar{\tau}}{\sqrt{2}} D^T Z_3 & 0 & 0 & 0 & -\sigma I & * \\
\varpi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sigma I
\end{bmatrix} < 0 \quad (7)$$

where:

$$\begin{aligned}
\Upsilon &= Sym\{e_1^T P H\} + e_1^T (Q_1 + Q_2 + Q_3 + Q_4) e_1 - e_5^T Q_4 e_5 \\
&\quad + d_{12} e_6^T Q_5 e_6 - d_{12} e_7^T Q_7 e_7 + k_2 e_2^T Q_1 e_2 - (1 - k_2) e_4^T Q_3 e_4 \\
&\quad - \Pi_1^T \Psi_1 \Pi_1 - \Omega^T \tilde{R}_2 \Omega - \Xi_1^T Z_{d1} \Xi_1 - \Xi_2^T Z_{d2} \Xi_2 - \Xi_3^T Z_{d3} \Xi_3 \\
&\quad - e_{23}^T C^T \Lambda C e_{23} + \rho (e_2 + e_{23})^T C^T \Lambda C (e_2 + e_{23})
\end{aligned}$$

$$H = A e_1 - BKC e_2 - BKC e_{23}$$

$$H_1 = A e_1 - BKC e_2 - BKC e_{23} + F e_{24}, \quad d_{12} = \bar{\tau} - \bar{d}$$

$$\varpi = E e_1 - \tilde{E} KC e_2 - \tilde{E} KC e_{23}$$

$$H_2 = DG(t) E e_1 - DG(t) \tilde{E} KC e_2 - DG(t) \tilde{E} KC e_{23}$$

$$\tilde{R}_i = diag\{R_i, 3R_i, 5R_i\}, (i = 1, 2)$$

$$Z_{di} = diag\{2Z_i, 4Z_i, 6Z_i\}, (i = 1, 2, 3)$$

$$\Psi_1 = \begin{bmatrix} (2 - \alpha) \tilde{R}_1 + (1 - \alpha) T_1 & S \\ * & (1 + \alpha) \tilde{R}_1 + \alpha T_2 \end{bmatrix}$$

$$\alpha = \frac{\mu(t)}{\bar{\mu}} \quad T_1 = -S \tilde{R}_1^{-1} S^T \quad T_2 = -S^T \tilde{R}_1^{-1} S$$

$$\Pi_1 = \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_{10} \\ e_1 - e_2 - 6e_{11} \\ e_2 - e_5 \\ e_2 + e_5 - 2e_{12} \\ e_2 - e_5 - 6e_{13} \end{bmatrix}, \Xi_1 = \begin{bmatrix} e_1 - e_{14} \\ e_1 - e_{14} - 3e_{15} \\ e_1 - 6e_{14} - 3e_{15} - 30e_{16} \end{bmatrix}$$

$$\Xi_2 = \begin{bmatrix} e_1 - e_{17} \\ e_1 - e_{17} - 3e_{18} \\ e_1 - 6e_{17} - 3e_{18} - 30e_{19} \end{bmatrix}, \Omega = \begin{bmatrix} e_6 - e_7 \\ e_6 + e_7 - 2e_8 \\ e_6 - e_7 - 6e_9 \end{bmatrix}$$

$$\Xi_3 = \begin{bmatrix} e_1 - e_{20} \\ e_1 - e_{20} - 3e_{21} \\ e_1 - 6e_{20} - 3e_{21} - 30e_{22} \end{bmatrix}$$

and the upper bound of the quadratic performance index is given

$$\begin{aligned}
\hat{J} &= \gamma^2 ||w(t)||_2^2 + x^T(0) P x(0) + \int_{-\bar{\mu}}^{0} x^T(s) Q_4 x(s) ds \\
&+ d_{12} \int_{-\bar{\tau}}^{-\bar{d}} x^T(s) Q_5 x(s) ds + \bar{\mu} \int_{-\bar{\mu}}^{0} \int_{\beta}^{0} \dot{x}^T(s) R_1 \dot{x}(s) ds d\beta \\
&+ d_{12} \int_{-\bar{\tau}}^{-\bar{d}} \int_{\beta}^{0} \dot{x}^T(s) R_2 \dot{x}(s) ds d\beta \\
&+ \int_{-\bar{\mu}}^{0} \int_{\beta}^{0} \int_{\theta}^{0} \dot{x}^T(s) Z_1 \dot{x}(s) ds d\theta d\beta \\
&+ \int_{-\bar{d}}^{0} \int_{\beta}^{0} \int_{\theta}^{0} \dot{x}^T(s) Z_2 \dot{x}(s) ds d\theta d\beta \\
&+ \int_{-\bar{\tau}}^{0} \int_{\beta}^{0} \int_{\theta}^{0} \dot{x}^T(s) Z_3 \dot{x}(s) ds d\theta d\beta
\end{aligned} \quad (8)$$

Proof: Firstly, the L-K functional candicate is given as:

$$\begin{aligned}
V(t) &= \textstyle\sum_{i=1}^{4} V_i(t) \qquad V_1(t) = x^T(t) P x(t) \\
V_2(t) &= \textstyle\int_{t-\mu(t)}^{t} x^T(s) Q_1 x(s) ds + \int_{t-d(t)}^{t} x^T(s) Q_2 x(s) ds \\
&\quad + \textstyle\int_{t-\tau(t)}^{t} x^T(s) Q_3 x(s) ds + \int_{t-\bar{\mu}}^{t} x^T(s) Q_4 x(s) ds \\
&\quad + (\bar{\tau} - \bar{d}) \textstyle\int_{t-\bar{\tau}}^{t-\bar{d}} x^T(s) Q_5 x(s) ds \\
V_3(t) &= \bar{\mu} \textstyle\int_{-\bar{\mu}}^{0} \int_{t+\beta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds d\beta \\
&\quad + (\bar{\tau} - \bar{d}) \textstyle\int_{-\bar{\tau}}^{-\bar{d}} \int_{t+\beta}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds d\beta \\
V_4(t) &= \textstyle\int_{-\bar{\mu}}^{0} \int_{\beta}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z_1 \dot{x}(s) ds d\theta d\beta \\
&\quad + \textstyle\int_{-\bar{d}}^{0} \int_{\beta}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z_2 \dot{x}(s) ds d\theta d\beta \\
&\quad + \textstyle\int_{-\bar{\tau}}^{0} \int_{\beta}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z_3 \dot{x}(s) ds d\theta d\beta
\end{aligned}$$

where $P, Q_i, (i = 1\cdots 5), R_j, (j = 1, 2), Z_l, (l = 1, 2, 3)$ are matrices to be determined. For simplicity of vector and matrix representations, we define a column vector $\varsigma(t)$ as

$$\varsigma(t) = col\{x(t),\ x(t - \mu(t)),\ x(t - d(t)),\ x(t - \tau(t)),$$

$$x(t - \bar{\mu}),\ x(t - \bar{d}),\ x(t - \bar{\tau}),\ \tfrac{1}{\bar{\tau} - \bar{d}} \textstyle\int_{t-\bar{\tau}}^{t-\bar{d}} x(s) ds$$

$\frac{1}{\bar{\tau}-\bar{d}}\int_{t-\bar{\tau}}^{t-\bar{d}}\delta_{t-\bar{\tau},t-\bar{d}}x(s)ds,\ \frac{1}{\mu(t)}\int_{t-\mu(t)}^{t}x(s)ds,$

$\frac{1}{\mu(t)}\int_{t-\mu(t)}^{t}\delta_{t-\mu(t),t}x(s)ds,\ \frac{1}{\bar{\mu}-\mu(t)}\int_{t-\bar{\mu}}^{t-\mu(t)}x(s)ds,$

$\frac{1}{\bar{\mu}-\mu(t)}\int_{t-\bar{\mu}}^{t-\mu(t)}\delta_{t-\bar{\mu},t-\mu(t)}x(s)ds,\ \frac{1}{\bar{\mu}}\int_{-\bar{\mu}}^{0}x(t+s)ds$

$\frac{1}{\bar{\mu}}\int_{-\bar{\mu}}^{0}\delta_{-\bar{\mu},0}x(t+s)ds,\ \frac{1}{\bar{\mu}^2}\int_{-\bar{\mu}}^{0}\int_{\beta}^{0}\delta_{-\bar{\mu},0}x(t+s)dsd\beta,$

$\frac{1}{\bar{d}}\int_{-\bar{d}}^{0}x(t+s)ds,\ \ \frac{1}{\bar{d}}\int_{-\bar{d}}^{0}\delta_{-\bar{d},0}x(t+s)ds,$

$\frac{1}{\bar{d}^2}\int_{-\bar{d}}^{0}\int_{\beta}^{0}\delta_{-\bar{d},0}x(t+s)dsd\beta,\ \ \frac{1}{\bar{\tau}}\int_{-\bar{\tau}}^{0}x(t+s)ds,$

$\frac{1}{\bar{\tau}}\int_{-\bar{\tau}}^{0}\delta_{-\bar{\tau},0}x(t+s)ds,\ \frac{1}{\bar{\tau}^2}\int_{-\bar{\tau}}^{0}\int_{\beta}^{0}\delta_{-\bar{\tau},0}x(t+s)dsd\beta,$

$e_k(t),\ w(t)\}$

$e_i = \left[0_{n\times(i-1)n}\ I_n\ 0_{n\times(23-i)n}\right], i = 1,\cdots 23.$

Considering the event-triggered scheme and TDS attack, combined with previous description, the time derivative along the state trajectories of system (4) can be obtained:

$$\begin{aligned}\dot{V}(t) \le\ & \varsigma^T(t)\Delta\varsigma(t) - y^T(t)y(t) + \gamma^2 w^T(t)w(t) \\ & - x^T(t)M_1x(t) - \{[x(t-\mu(t)) + e_k(t)]^T(KC)^T \\ & M_2KC[x(t-\mu(t)) + e_k(t)]\}\end{aligned} \tag{9}$$

and Δ can be written as (10):

$$\begin{bmatrix} \Upsilon & * & * & * & * & * & * & * & * & * \\ F^TPe_1 & -\gamma^2 & * & * & * & * & * & * & * & * \\ \bar{\mu}\hat{H} & 0 & -R_1^{-1} & * & * & * & * & * & * & * \\ d_{12}\hat{H} & 0 & 0 & -R_2^{-1} & * & * & * & * & * & * \\ \frac{\bar{\mu}}{\sqrt{2}}\hat{H} & 0 & 0 & 0 & -Z_1^{-1} & * & * & * & * & * \\ \frac{\bar{d}}{\sqrt{2}}\hat{H} & 0 & 0 & 0 & 0 & -Z_2^{-1} & * & * & * & * \\ \frac{\bar{\tau}}{\sqrt{2}}\hat{H} & 0 & 0 & 0 & 0 & 0 & -Z_3^{-1} & * & * & * \\ Ce_1 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ M_1^{\frac{1}{2}}e_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * \\ M_2^{\frac{1}{2}}KC\Gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} \tag{10}$$

where $\Gamma = (e_2 + e_{23})$, $\hat{H} = H_1 + H_2$.

Some transformation should be made for solving the uncertain term in (10) according to the existing Lemma 2.4 in Xie L (1992). Then, combined with (7), we have

$$\begin{aligned}\dot{V}(t) \le & -y^T(t)y(t) + \gamma^2 w^T(t)w(t) - x^T(t)M_1x(t) \\ & -[x(t-\mu(t)) + e_k(t)]^T(KC)^TM_2KC[x(t-\mu(t)) + e_k(t)]\end{aligned} \tag{11}$$

Since $x(t)$ and $\dot{V}(t)$ are all continuous in t, taking integral to t from 0 to ∞ on both sides of (11), we can get
$V(\infty) - V(0) \le \int_0^{\infty}\{\gamma^2w^T(t)w(t) - y^T(t)y(t) - x^T(t)M_1x(t)$
$-[x(t-\mu(t))+e_k(t)]^T(KC)^TM_2(KC)[x(t-\mu(t))+e_k(t)]\}dt$
Hence under zero initial conditions, the following holds

$$\int_0^{\infty}[-y^T(t)y(t) + \gamma^2w^T(t)w(t)]dt \ge 0$$

The upper bound of the performance index can be obtained that

$$\begin{aligned}J = & \int_0^{\infty}\{x^T(t)M_1x(t) + [x(t-\mu(t)) + e_k(t)]^T(KC)^TM_2 \\ & KC[x(t-\mu(t)) + e_k(t)]\}dt \\ \le & V(0) + \int_0^{\infty}[-y^T(t)y(t) + \gamma^2w^T(t)w(t)]dt \\ \le & V(0) + \gamma^2||w(t)||_2^2 = \hat{J}\end{aligned}$$

This completes the proof.

3.2 Optimal Guaranteed Cost Load Frequency Control

Theorem 2: For given positive scalars ρ, $\bar{d}, \bar{\tau}$, $\bar{\mu}$, and k_2, if there are positive definite matrices X, $\bar{R}_j, j = (1,2), \bar{Q}_i, i = (1\cdots5), \bar{Z}_l, l = (1,2,3)$ such that the following LMI holds, we can conclude that the system with feedback gain K and weighted cost performance index (6) is asymptotically stable.

$$\begin{bmatrix} \bar{\Upsilon} & * & * & * & * & * & * & * & * & * & * & * \\ F^Te_1 & -\gamma^2I & * & * & * & * & * & * & * & * & * & * \\ \bar{\mu}\bar{H} & \bar{\mu}F & \bar{R}_1-2X & * & * & * & * & * & * & * & * & * \\ d_{12}\bar{H} & d_{12}F & 0 & \bar{R}_2-2X & * & * & * & * & * & * & * & * \\ \frac{\bar{\mu}}{\sqrt{2}}\bar{H} & \frac{\bar{\mu}}{\sqrt{2}}F & 0 & 0 & \bar{Z}_1-2X & * & * & * & * & * & * & * \\ \frac{\bar{d}}{\sqrt{2}}\bar{H} & \frac{\bar{d}}{\sqrt{2}}F & 0 & 0 & 0 & \bar{Z}_2-2X & * & * & * & * & * & * \\ \frac{\bar{\tau}}{\sqrt{2}}\bar{H} & \frac{\bar{\tau}}{\sqrt{2}}F & 0 & 0 & 0 & 0 & \bar{Z}_3-2X & * & * & * & * & * \\ CXe_1 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * & * \\ M_1^{\frac{1}{2}}Xe_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * \\ M_2^{\frac{1}{2}}KCX(e_2+e_{23}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ \sigma D^Te_1 & 0 & \sigma\bar{\mu}D^T & \sigma d_{12}D^T & \sigma\frac{\bar{\mu}}{\sqrt{2}}D^T & \sigma\frac{\bar{d}}{\sqrt{2}}D^T & \sigma\frac{\bar{\tau}}{\sqrt{2}}D^T & 0 & 0 & 0 & -\sigma I & * \\ \bar{\varpi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sigma I \end{bmatrix} < 0 \tag{12}$$

where

$$\begin{aligned}\bar{\Upsilon} = & Sym\{e_1^T\bar{H}\} + e_1^T(\bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 + \bar{Q}_4)e_1 - e_5^T\bar{Q}_4e_5 \\ & + d_{12}e_6^T\bar{Q}_5e_6 - d_{12}e_7^T\bar{Q}_5e_7 + k_2e_2^T\bar{Q}_1e_2 - (1-k_2)e_4^T\bar{Q}_3e_4 \\ & - \Pi_1^T\bar{\Psi}_1\Pi_1 - \Omega^T\hat{R}_2\Omega - \Xi_1^T\bar{Z}_{d1}\Xi_1 - \Xi_2^T\bar{Z}_{d2}\Xi_2 \\ & - \Xi_3^T\bar{Z}_{d3}\Xi_3 - e_{23}^T\bar{\Lambda}e_{23} + \rho(e_2+e_{23})^T\bar{\Lambda}(e_2+e_{23})\end{aligned}$$

$$\bar{H} = AXe_1 - BKCXe_2 - BKCXe_{23}$$

$$\bar{\varpi} = EXe_1 - \tilde{E}KCXe_2 - \tilde{E}KCXe_{23}$$

$$\hat{R}_i = diag\{\bar{R}_i, 3\bar{R}_i, 5\bar{R}_i\}$$

$$\bar{Z}_{di} = diag\{2\bar{Z}_i, 4\bar{Z}_i, 6\bar{Z}_i\}, \quad \bar{\Lambda} = XC^T\Lambda CX$$

$$\bar{\Psi}_1 = \begin{bmatrix}(2-\alpha)\hat{R}_1 + (1-\alpha)\hat{T}_1 & \bar{S} \\ * & (1+\alpha)\hat{R}_1 + \alpha\hat{T}_2\end{bmatrix}$$

Proof: Define $X = P^{-1}, X_1 = KCX, \bar{Q}_i = XQ_iX, \bar{R}_j = XR_jX, \bar{Z}_l = XZ_kX$. Then, multiply by $diag\{X, I, R_1^{-1}, R_2^{-1}, Z_1^{-1}, Z_2^{-1}, Z_3^{-1}, I, I, I, I, I\}$ on both sides of (8), and using Schur complement and Lemma 3.1 in Xie L (1992), (12) is derived.

Firstly, define matrices as follows.

$$\Theta_1 = \int_{-\bar{\mu}}^{0} x(s)x^T(s)ds \qquad \Theta_2 = d_{12}\int_{-\bar{\tau}}^{-\bar{d}} x(s)x^T(s)ds$$
$$\Theta_3 = \bar{\mu}\int_{-\bar{\mu}}^{0}\int_{\beta}^{0} \dot{x}(s)\dot{x}^T(s)dsd\beta$$
$$\Theta_4 = d_{12}\int_{-\bar{\tau}}^{-\bar{d}}\int_{\beta}^{0} \dot{x}(s)\dot{x}^T(s)dsd\beta$$
$$\Theta_5 = \int_{-\bar{\mu}}^{0}\int_{\beta}^{0}\int_{\theta}^{0} \dot{x}(s)\dot{x}^T(s)dsd\theta d\beta$$
$$\Theta_6 = \int_{-\bar{d}}^{0}\int_{\beta}^{0}\int_{\theta}^{0} \dot{x}(s)\dot{x}^T(s)dsd\theta d\beta$$
$$\Theta_7 = \int_{-\bar{\tau}}^{0}\int_{\beta}^{0}\int_{\theta}^{0} \dot{x}(s)\dot{x}^T(s)dsd\theta d\beta$$
$$\Theta_8 = \int_{-\bar{\mu}}^{0}\int_{\beta}^{0}\int_{\theta}^{0}\int_{\eta}^{0} \dot{x}(s)\dot{x}^T(s)dsd\eta d\theta d\beta$$

Assume that there exists positive scalar ϱ satisfying $x^T(0)X^{-1}x(0) < \varrho$, hence, we can obtain: $\begin{bmatrix} -\varrho & * \\ x(0) & -X \end{bmatrix} < 0$.

Introducing symmetric matrix ψ_1 satisfies $\psi_1 > \Theta_1^{\frac{1}{2}}X^{-1}\bar{Q}_4X^{-1}\Theta_1^{\frac{1}{2}}$, then we can obtain

$$\begin{bmatrix} -\psi_1 & \Theta_1^{\frac{1}{2}} \\ * & -X\bar{Q}_4^{-1}X \end{bmatrix} < 0$$

and

$$\int_{-\mu}^{0} x^T(s)X^{-1}\bar{Q}_4X^{-1}x(s)ds \le tr(\psi_1)$$

Similarly, the following inequalities are derived:

$$d_{12}\int_{-\bar{\tau}}^{-\bar{d}} x^T(s)X^{-1}\bar{Q}_5X^{-1}x(s)ds \le tr(\psi_2)$$
$$\bar{\mu}\int_{-\bar{\mu}}^{0}\int_{\beta}^{0} \dot{x}^T(s)X^{-1}\bar{R}_1X^{-1}\dot{x}(s)dsd\beta \le tr(\psi_3)$$
$$d_{12}\int_{-\bar{\tau}}^{-\bar{d}}\int_{\beta}^{0} \dot{x}^T(s)X^{-1}\bar{R}_2X^{-1}\dot{x}(s)dsd\beta \le tr(\psi_4)$$
$$\int_{-\bar{\mu}}^{0}\int_{\beta}^{0}\int_{\theta}^{0} \dot{x}^T(s)X^{-1}\bar{Z}_1\dot{x}(s)dsd\theta d\beta \le tr(\psi_5)$$
$$\int_{-\bar{d}}^{0}\int_{\beta}^{0}\int_{\theta}^{0} \dot{x}^T(s)X^{-1}\bar{Z}_2\dot{x}(s)dsd\theta d\beta \le tr(\psi_6)$$
$$\int_{-\bar{\tau}}^{0}\int_{\beta}^{0}\int_{\theta}^{0} \dot{x}(s)^TX^{-1}\bar{Z}_3\dot{x}(s)dsd\theta d\beta \le tr(\psi_7)$$

To sum up above,

$$\hat{J} \le \varrho + tr(\psi 1) + tr(\psi_2) + tr(\psi_3) + tr(\psi 4) + tr(\psi_5) + tr(\psi_6) + tr(\psi_7) + \gamma^2||w(t)||_2^2 = J^* \quad (13)$$

Hence, the minimization problem can be shown as follows.

$$\min_{\Xi} \; J^*$$

$$s.t.(1) \;\; (12), \qquad (2)\begin{bmatrix} -\varrho & * \\ x(0) & -X \end{bmatrix} < 0$$

$$(3)\begin{bmatrix} -\psi_1 & \Theta_1^{\frac{1}{2}} \\ * & \bar{Q}_4 - 2X \end{bmatrix} < 0, (4)\begin{bmatrix} -\psi_2 & \Theta_2^{\frac{1}{2}} \\ * & \bar{Q}_5 - 2X \end{bmatrix} < 0$$

$$(5)\begin{bmatrix} -\psi_3 & \Theta_3^{\frac{1}{2}} \\ * & \bar{R}_1 - 2X \end{bmatrix} < 0, (6)\begin{bmatrix} -\psi_4 & \Theta_4^{\frac{1}{2}} \\ * & \bar{R}_2 - 2X \end{bmatrix} < 0$$

$$(7)\begin{bmatrix} -\psi_5 & \Theta_5^{\frac{1}{2}} \\ * & \bar{Z}_1 - 2X \end{bmatrix} < 0, (8)\begin{bmatrix} -\psi_6 & \Theta_6^{\frac{1}{2}} \\ * & \bar{Z}_2 - 2X \end{bmatrix} < 0$$

$$(9)\begin{bmatrix} -\psi_7 & \Theta_7^{\frac{1}{2}} \\ * & \bar{Z}_3 - 2X \end{bmatrix} < 0$$

where Ξ indicates the constraints that for given positive scalars ρ, γ, $\bar{d}, \bar{\tau}, \bar{\mu}, k_2$, there exist positive definite matrices $X, \bar{R}_j, \bar{Q}_i, \bar{Z}_l, \psi_i$, and positive scalar ϱ such that the minimization problem is solvable.

Then the minimum upper bound of the performance index, the optimal guaranteed cost controller u^*, and the triggered matrix can be obtained simultaneously. The control gain can be expressed as: $K = X_1(CX)^+$.

Remark 3: It should be noted that only conservative sufficient conditions can be obtained in this paper, thus, the value of the calculated robust performance index is greater than its real value.

4. CASE STUDY

In this section, we aim to verify the controller designed by Theorem 2 has certain robustness for external disturbances. For the single area system, the nominal system parameters are given as follows: $T_{ch} = 0.3; T_g = 0.1; R = 0.05; D_i = 1; M_i = 5; \beta = 21; \lambda = \iota = 0.1; D = 0.5I_5; G(t) = diag\{0, sin(t), sin(t), 0, 0\}$, $M_1 = I_5, M_2 = 1$, and for $\forall t \in [-\bar{\mu}, 0]$, $x(t) = [0 \;\; e^t \;\; 0 \;\; e^{0.5t} \;\; 0]$.

Choose $\tau(t) = 0.25|sin(t)|$, $\bar{d} = 0.15, \bar{\tau} = 0.25, \rho = 0.01$, $\gamma = 10; \sigma = 0.06$. Supposed that an external disturbance occurs at $t = 2s$ and hold on about $2s$, the system state response is shown as Fig 2(a), we can easily conclude that the system is robust asymptotic stability. Then the controller gain matrix K and triggered matrix Λ are obtained as follows.

$$\Lambda = \begin{bmatrix} 0.3202 & -0.4619 \\ -0.4619 & 85.2596 \end{bmatrix}, K = [-0.4737 \quad 0.0018]$$

The minimum upper bound of the performance index $J^* = 1.9368$.

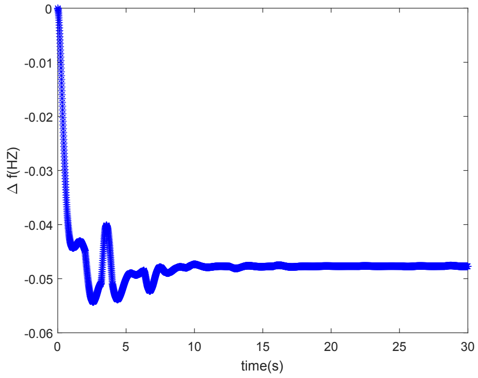
As to the three-area LFC power system, parameters in different regions are various. Choose $\gamma = 15; \sigma = 0.01$, and system initial value $x(t) = [0 \;\; 1 \;\; 0 \;\; 1 \;\; 0\,], t \in [-\bar{\mu}, 0]$. Then, the system gain matrix K and triggered matrixΛ are computed as follows:

$$\Lambda = diag\{\Lambda_1 \;\; \Lambda_2 \;\; \Lambda_3\}, \Lambda_1 = \begin{bmatrix} 0.1242 & -0.0034 \\ -0.0034 & 0.3309 \end{bmatrix}$$

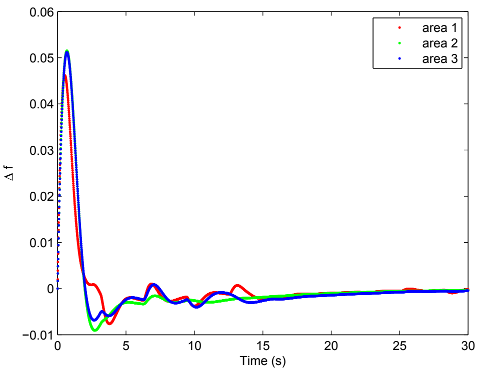
$$\Lambda_2 = \begin{bmatrix} 0.1242 & -0.0038 \\ -0.0038 & 0.3121 \end{bmatrix}, \Lambda_3 = \begin{bmatrix} 0.1181 & -0.0031 \\ -0.0031 & 0.3386 \end{bmatrix}$$

$$K = \begin{bmatrix} -0.2278 \; 0.0002 & & \\ & -0.0983 \; 0.0022 & \\ & & -0.0811 \; 0.0011 \end{bmatrix}$$

And the minimum upper bound for this three area LFC power system is $J^* = 2.2362$. Fig.2(b) shows the system state response when the external disturbance are injected into the power system at $t = 5s$, and hold on about $5s$. It is obvious that the three area system with controller K achieves robust asymptotic stability.

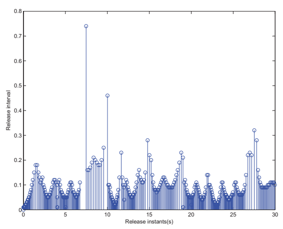


(a)

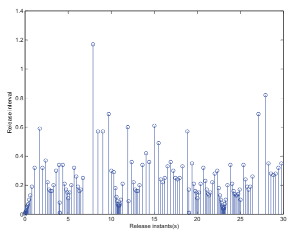


(b)

Fig. 2. (a)System frequency deviation response for single area power system. (b)System frequency deviation response for multi area power system



(a)



(b)

Fig. 3. (a)Release instants and intervals with $\rho = 0.01$. (b)Release instants and intervals with $\rho = 0.1$

Fig 3(a) and Fig 3(b) show the release instants and intervals by different event-triggered parameter ρ.

5. CONCLUSION

This paper mainly studies the optimal guaranteed cost control of power system with system uncertainties. Firstly, an additive time delay closed-loop system model is given when event-triggered communication scheme and TDS attack are taken into account. Then, less conservative stability criteria are derived based on an improved L-K functional and some latest inequalities such as truncated Bessel-Legendre inequality and improved extended reciprocal convex approach. Finally, the optimal robust guaranteed cost controller, triggered matrix and minimum upper bound of performance index are obtained simultaneously by solving a convex optimization problem.

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