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Simultaneous distributed localization, mapping and formation control of mobile robots based on local relative measurements

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Abstract: This paper investigates the problem of localizing a team of robots in an indoor environment while simultaneously keeping a robust formation and performing group motion. A distributed observer is proposed to estimate the positions of mobile robots as well as the landmarks under a common global frame. Every robot uses its available local relative measurements, as well as the estimated relative measurements to its neighbors in order to keep a robust formation. Simultaneously, each robot estimates the positions of all the landmarks based on the available on-board relative measurements but also based on the estimated positions from its neighbors. We provide the L^2-stability analysis of the closed-loop system where the group is also allowed to maneuver in the unknown environment. Simulation results are also given to show the efficacy of the method.

Keywords: Distributed SLAM, formation control, distributed observer, multi-agent systems

1. INTRODUCTION

Multi-robotic systems have been researched, developed and deployed in recent years as they can offer modularity, flexibility, fault tolerance and other advantages, compared to the use of a single robot. A number of industrial applications concerning multi-robotic systems are discussed in (Spletzer et al. (2001)).

The problem of localizing a group of robots, as well as an individual robot in a distributed way remains one of fundamental issues in the literature of multi-agent systems. Usually GPS signals, beacons or camera can be used to provide global position information for each agent to keep their formation and to perform group maneuvers. These external signals have been used in many recent deployments of swarm drones, such as the 2018 Winter Olympics in South Korea. However, this solution requires additional infrastructure for providing such external positioning and localization signals which may not readily be available. For example, the use of multi-robots in an indoor or warehouse environment can not make use of the GPS signals and may be lacking an infrastructure of cameras and beacons. In this case, the problem of localizing a group of robots in such an environment using only local measurements is not trivial.

Simultaneous localization and mapping (SLAM) is a well-developed method that is used to localize mobile robots in an unknown environment. A mobile robot can correct its estimated position by observing landmarks consistently. In our problem setting as illustrated in Figure 1, a team of robots maintains formation and maneuvers in an environment that contains multiple landmarks. When the robotic team encounters an obstacle which the robot A can directly measure/detect while the other robots (B, C and D) can not, the local collision avoidance algorithm in Robot A will influence the behavior of the group, in particular, the formation will be distorted. The capability of measuring such landmarks/obstacles by the other robots can circumvent this problem because they can proactively perform the same collision avoidance maneuver.

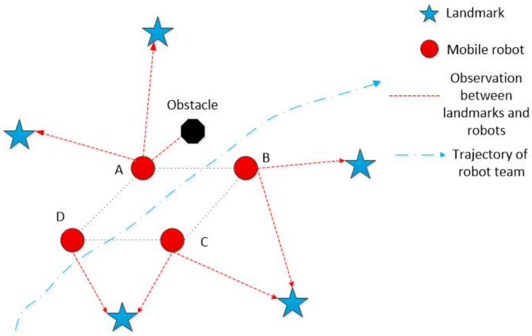

Fig. 1. Localizing a team of robots in an unknown environment

In this paper, as one of our contributions, the distributed simultaneous localization and mapping (distributed SLAM) is proposed to provide such capability for every robot. In particular, each robot uses its own local measurements to estimate the positions of objects (mobile robots and landmarks) in their vicinity that can directly be measured/detected, and estimates the positions of unobservable objects by communicating with their neighbors. As shown in Figure 1, each robot can not get all the position information with regard to the other robots as well as the landmarks due to its observation limitations. By communicating with their adjacent neighbors, the robots can get a consistent map of the environment which can be used to maintain formation and maneuver as well as perform obstacle avoidance.

For the past decade, the distributed estimation problem has been studied based on the use of distributed observer and on the use of the distributed Kalman filter ((Kim et al. (2016), Olfati-Saber (2005))). In (Kim et al. (2016)), the authors proposed a distributed observer which assumes an undirected connected graph and in (Han et al. (2018)) a distributed observer with a directed connected graph is proposed. Other works that investigate the problem of localizing multi-robots are, for instance, (Roumeliotis and Bekey (2000)) and (Ugrinovskii (2013)). In the former work, the authors proposed a distributed Kalman filter to localize a group of robots. In the latter, the authors proposed a distributed estimator for a SLAM system. In both works, the maintaining of formation and group maneuver are not part of the problem formulation. For only maintaining the formation and maneuvering, there are a number of distributed control laws that have been proposed in the literature. For instance, in (De Marina et al. (2016)), the authors proposed a distributed motion controller for rigid formation. In (Zhao and Zelazo (2015)), a distributed proportional-integral control law has been proposed to deal with the problem of bearing based formation maneuver problem. As another contribution of this paper, we combine and analyse the use of distributed SLAM based on the use of the distributed observer and the use of distributed formation and maneuver control law to maintain formation and steer the whole group.

The rest of this paper is organized as follows. In Section 2, we provide some preliminaries on graph theory and formulate our problem. The dynamic model of our system which consists of robot formation and landmarks is illustrated in Section 3. In Section 4, we give the specific structure of distributed observer for our system and provide the observation error dynamics. In Section 5, we present the L^2-stability analysis of the simultaneous distributed observer-based SLAM, formation and group motion controller. Numerical simulations are presented in Section 6. Finally, Section 7 concludes this paper.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Preliminaries and Notations

The information exchange among agents can be modeled by directed or undirected graphs. A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a set of vertices $\mathcal{V} = (1, 2, \ldots, n)$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Each edge $a_{ij} = (i, j)$ indicates the information flow from vertex j to vertex i. An undirected path connecting nodes i_0 and i_n is a sequence of undirected edges of the form (i_{k-1}, i_k), $k = 1, \ldots, n$. An undirected graph $\mathcal{G}$ is connected if there is an undirected path between any pair of distinct nodes. Denote $\mathcal{W} \in \mathbb{R}^{n\times n}$ as the weighted adjacency matrix, whose elements indicate whether pairs of vertices are adjacent or not in $\mathcal{G}$. Specifically, the (i, j) entry, denoted by w_{ij}, is strictly positive if the edge $a_{ij} = (i, j) \in \mathcal{E}$, and $w_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} \in \mathbb{R}^{n\times n}$ is defined as

$$\mathcal{L} = \mathcal{D} - \mathcal{W}.$$

where $\mathcal{D}$ is a diagonal matrix whose elements are chosen such that each row sum of $\mathcal{L}$ is zero. In an undirected graph, $w_{ij} = w_{ji}$, which implies that $\mathcal{L}$ is symmetric.

Throughout this paper, $\otimes$ represents Kronecker product. $\mathbb{1}_n$ denotes an n dimensional column vector of all ones. For vector x and matrix P, $||x||$, $||P||$ are Euclidean norm and matrix 2-norm respectively. Let $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ denote the set of real numbers, the set of n dimensional vector and the set of $m \times n$ matrix.

2.2 Problem formulation

In our case, a team of robots must maintain a stable formation while maneuvering in an unknown environment. Each local observer on every individual robot is used for estimating the positions of landmarks as well as the positions of mobile robots. The dynamics of mobile robots and static landmarks can be described as:

$$\dot{p}_i = u_i^1 + u_i^2 \quad (i = 1, \ldots, n) \tag{1}$$

$$\dot{m}_l = 0 \quad (l = 1, 2, \ldots, m), \tag{2}$$

where $p_i \in \mathbb{R}^2$ denotes the position of i-th mobile robots and $m_l \in \mathbb{R}^2$ is the position of l-th landmark. Let k_1 and k_2 denote the control gain of maneuvering and keeping formation respectively. The control input u_i^1 which is used for dealing with maneuvering is given by

$$u_i^1 = \dot{p}_c^* - k_1(p_c - p_c^*), \tag{3}$$

where $p_c := \frac{1}{n}\sum_{i=1}^{n} p_i \in \mathbb{R}^2$ is the position of centroid and $p_c^* \in \mathbb{R}^2$ is the desired position of centroid. The control input $u_i^2 \in \mathbb{R}^2$ is used for keeping a stable formation, which can be described as

$$\begin{aligned} u_i^2 = &-k_2 \sum_{j\in\mathcal{N}_i^+} a_{ij}(p_i - p_j - (p_i^* - p_j^*)) \\ &- k_2 \sum_{l\in\mathcal{N}_i^-} a_{il}(\hat{p}_i^i - \hat{p}_l^i - (p_i^* - p_l^*)), \end{aligned} \tag{4}$$

where $\hat{p}_i^i$ and $\hat{p}_l^i$ are the positions of i-th agent and l-th agent estimated by agent i. The term $p_i^* - p_j^*$ is the desired relative position between i-th agent and j-th agent. $\mathcal{N}_i^+$ is the set of indices of mobile robots measured directly by i-th agent to keep formation and $\mathcal{N}_i^-$ is the set of indices of mobile robots that can not be measured directly but are estimated by i-th agent using its own local observer to keep formation. Here we can see that the formation graph $\mathcal{G}_f$ can be divided into measurement graph $\mathcal{G}^+$ and estimation graph $\mathcal{G}^-$. The corresponding Laplacian matrices of graphs are denoted by $\mathcal{L}_f$, $\mathcal{L}^+$ and $\mathcal{L}^-$ respectively, which should satisfy

$$\mathcal{L}^+ + \mathcal{L}^- = \mathcal{L}_f.$$

Taking the illustration in Figure 1 as an example, the relationship between graphs can be shown in Figure 2.

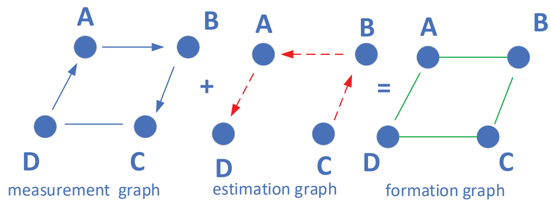

Fig. 2. The relationship between measurement graph $\mathcal{G}^+$, estimation graph $\mathcal{G}^-$ and formation graph $\mathcal{G}_f$.

Note that the measurement graph and formation graph should satisfy the following assumption:

Assumption 1. The measurement graph $\mathcal{G}^+$ should be strongly connected, and the formation graph is an undirected connected graph which is denoted by $\mathcal{G}_f$.

The measurements of i-th agent, which are comprised of the measurements between i-th agent and landmarks as well as the measurements between i-th agent and its neighbors, can be described by:

$$y_i = H_i \begin{bmatrix} p_v \\ p_L \end{bmatrix} (i = 1, 2, \ldots, n) \tag{5}$$

where y_i is the measurement of i-th mobile robot and the matrix H_i provides the local relative measurement that is available to the i-th agent. p_v is the collective position of all mobile robots and p_L is the collective position of all the static landmarks. (1) and (2) can be written collectively as

$$\begin{bmatrix} \dot{p_v} \\ \dot{p_L} \end{bmatrix} = \begin{bmatrix} U_1(p_c, p_c^*, \dot{p}_c^*) + U_2(\hat{x}_1, \ldots, \hat{x}_n, y_1, \ldots, y_n, p_1^*, \ldots, p_n^*) \\ 0 \end{bmatrix}, \tag{6}$$

where $\hat{x}_i$, which consists of estimated positions of agents and landmarks, is the state of whole system estimated by i-th agent. $U_1 := [u_1^1 \quad u_2^1 \ldots u_i^1 \ldots u_n^1]^T$ and $U_2 := [u_1^2 \quad u_2^2 \ldots u_i^2 \ldots u_n^2]^T$ are collective control inputs which deal with maneuvering and keeping formation respectively. The measurements can be also written collectively as

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} \begin{bmatrix} p_v \\ p_L \end{bmatrix}. \tag{7}$$

When disturbances coming from control inputs and measurements are taken into consideration, (6) and (7) can be described in the following compact form

$$\dot{x} = Ax + Bu + G\omega, \tag{8}$$

$$y = Hx + v = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} x + \begin{bmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_n \end{bmatrix} \tag{9}$$

where $\omega, \nu \in L^2$ are the disturbances and $x = [p_v^T \quad p_L^T]^T \in \mathbb{R}^{2(m+n)}$ is the state vector of overall system.

Based on the above system's description, the following distributed observer is introduced to provide real-time estimation on the positions of landmarks and mobile robots for every mobile robot:

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu + K_i(y_i - H_i\hat{x}_i) + \gamma M_i \sum_{j \in \mathcal{N}_i^c} a_{ij}(\hat{x}_i - \hat{x}_j) \tag{10}$$

where $\hat{x}_i \in \mathbb{R}^{2(m+n)}$ is the state of the system estimated by the i-th agent. $\mathcal{N}_i^c$ is the set of indices of the neighbors that i-th agent receives information from. The matrix K_i is the observer gain, γ and matrix M_i are coupling gain and coupling matrix to be determined.

Based on the above formulation, our simultaneous distributed localization, mapping and formation control problem is given as follows: For every agent i,

1. design a distributed observer that provides the estimated state $\hat{x}_i$ such that the map $\left[\begin{smallmatrix} \omega \\ \nu \end{smallmatrix}\right] \mapsto (\hat{x}_i - x)$ is L^2-stable; and
2. design a distributed control law $u_i = u_i^1 + u_i^2$ such that the maps $\left[\begin{smallmatrix} \omega \\ \nu \end{smallmatrix}\right] \mapsto p_c - p_c^*$ and $\left[\begin{smallmatrix} \omega \\ \nu \end{smallmatrix}\right] \mapsto (p_i - p_j) - p_i^* - p_j^*$ are L^2-stable for all neighboring agent j.

3. SYSTEM MODELING

In this section, we will present the process model and observation model for multi-robots which will be used in our distributed simultaneous localization and mapping algorithm to locate the robotic team in the indoor environment. From this point onward, we consider the setting where we have n mobile robots and m static landmarks.

3.1 Process model

The position of the i -th robot with respect to its own local frame is denoted by ${}^ip_i := [{}^ix_i \quad {}^iy_i]^T \in \mathbb{R}^2, i = 1, 2, \ldots, n$. The position of the l-th landmark with regard to the local frame of robot i is denoted by ${}^im_l := [{}^ix_{ml} \quad {}^iy_{ml}]^T \in \mathbb{R}^2, l = 1, 2, \ldots, m, i = 1, 2, \ldots, n$.

Assumption 2. The orientation of each robot has been aligned and the relative positions T_i^g between the initial positions of each agent i and the origin of global frame are known.

Assumption 3. The global information of the centroid's position p_c is available to all agents.

Using the mobile robots dynamics in (1) with the distributed formation and maneuver control laws in (3) and (4), the closed-loop robot dynamics is given by

$$\dot{p}_i = \dot{p}_c^* - k_1(p_c - p_c^*) - k_2 \sum_{j \in \mathcal{N}_i^+} a_{ij}(p_i - p_j - (p_i^* - p_j^*))$$
$$- k_2 \sum_{l \in \mathcal{N}_i^-} a_{il}(\hat{p}_i^i - \hat{p}_l^i - (p_i^* - p_l^*)) \quad (i = 1, 2, \ldots, n), \tag{11}$$

where $p_c^*, \mathcal{N}_i^+, \mathcal{N}_i^-$ have been described in (3) and (4). The dynamics which is described by (11) can compactly be written as

$$\dot{p}_v = \left[\mathbb{1}_n \otimes (\frac{-k_1}{n}(\mathbb{1}_n^T \otimes I_2)) - k_2(\mathcal{L}_f \otimes I_2) \right] p_v + \mathbb{1}_n \otimes$$
$$(\dot{p}_c^* + k_1 p_c^*) + k_2(\mathcal{L}_f \otimes I_2)P^* - k_2 DE \tag{12}$$

with

$$P^* = [(p_1^*)^T \quad (p_2^*)^T \dots (p_n^*)^T]^T \in \mathbb{R}^{2n\times 1},$$

$$\Gamma_i = [0 \dots \underbrace{1}_{i-\text{th}} \dots 0] \in \mathbb{R}^{1\times n}, D = \text{diag}\left(\begin{bmatrix} \Gamma_i \mathcal{L}^- & 0_{1\times m}\end{bmatrix} \otimes I_2\right).$$

The notation $\mathbb{1}_n$ is an $n \times 1$ column vector with all ones. Note that the description on the collective estimation error $E = \left[(\hat{x}_1 - x)^T \quad (\hat{x}_2 - x)^T \dots (\hat{x}_n - x)^T\right]^T$. The last term of (12) can be regarded as the disturbance of the system.

The collective dynamics of all the landmarks can be described by:

$$\dot{p}_L = 0, \tag{13}$$

where $p_L = [m_1^T \quad m_2^T \dots m_m^T]^T \in \mathbb{R}^{2m}$ is the collective position of all the landmarks. Since our system consists of landmarks and robots, the complete state of our system is given by $x = [(p_1)^T \dots (p_n)^T \quad (m_1)^T \dots (m_m)^T]^T = [p_v^T \quad p_L^T]^T \in \mathbb{R}^{2(m+n)}$ and the disturbance $\omega \in L^2$ is taken into consideration, the state equation can be given by

$$\dot{x} = Ax + Bu + d + G\omega, \tag{14}$$

where $A = \begin{bmatrix} \mathbb{1}_n \otimes (\frac{-k_1}{n}(\mathbb{1}_n^T \otimes I_2)) - k_2(\mathcal{L}_f \otimes I_2) & 0_{2n\times 2m} \\ 0_{2m\times 2n} & 0_{2m\times 2m}\end{bmatrix}$,

$u = \mathbb{1}_n \otimes (\dot{p}_c^* + k_1 p_c^*) + k_2(\mathcal{L}_f \otimes I_2)P^*, B = \begin{bmatrix} I_{2n\times 2n} \\ 0_{2m\times 2n}\end{bmatrix}$,

$d = \begin{bmatrix} -k_2 DE \\ 0_{2m\times 1}\end{bmatrix} \in \mathbb{R}^{2m+2n}, G = \begin{bmatrix} I_{2n} \\ 0_{2m\times 2n}\end{bmatrix}$ and $\omega \in L^2$.

3.2 Observation model

During the exploration process, each robot corrects the estimated position of mobile robots as well as landmarks by observing locally the landmarks. Let $\mathcal{M}_i \subset \{1, \dots, m\}$ be a fixed subset of indices of landmarks that are observed by the i-th mobile agent and we assume that the union of these sets satisfies $\cup_i \mathcal{M}_i = \{1, \dots, m\}$.

When the disturbance coming from measurement is considered, the observation model of our system can be described by

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_n\end{bmatrix} = \begin{bmatrix} H_1 \\ \vdots \\ H_n\end{bmatrix} x + \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_n\end{bmatrix}, \tag{15}$$

where y is the collective measurement and y_i is the measurement of i-th mobile. Signals $\nu_1, \nu_2, \dots, \nu_n \in L^2$ are the disturbances on the measurement. In (Andrade-Cetto and Sanfeliu, 2005), the authors pointed out that SLAM system is a partially observable system and the full observability can be obtained by enabling robot to observe the origin of the global frame constantly in an environment where the GPS signal is denied. The i-th agent observes the landmark l where $l \in \mathcal{M}_i$ and it also observes the neighboring agent j where $j \in \mathcal{N}_i^+$. Without loss of generality, the matrix H_i can be given by

$$H_i = \begin{bmatrix} 0_{2\times 2(i-1)} \; I_2 \; 0_{2\times 2(n+m-i)} \\ H_{i,\text{robot}} \quad 0_{2a_i\times 2m} \\ H_{i,\text{left}} \quad H_{i,\text{right}}\end{bmatrix} \tag{16}$$

where the first row assumes that each robot observe the global origin (for the full observability of the multi-agent systems later), $\text{a}_i = \text{card}(\mathcal{N}_i^+)$ and

$$H_{i,\text{robot}} = \begin{bmatrix} \bar{h}_{i1} \\ \bar{h}_{i2} \\ \vdots \\ \bar{h}_{ia_i}\end{bmatrix} \in \mathbb{R}^{2a_i\times 2n}$$

with $\bar{h}_{il} \in \mathbb{R}^{2\times 2n}$ given by

$$\begin{cases} \begin{bmatrix} 0_{2\times 2(i-1)} & I_2 & 0_{2\times 2({}^l\mathcal{N}_i^+ - 1 - i)} & -I_2 & 0_{2\times(2n - 2\,{}^l\mathcal{N}_i^+)}\end{bmatrix} \\ \quad (\text{if } {}^l\mathcal{N}_i^+ > i) \\ \begin{bmatrix} 0_{2\times 2({}^l\mathcal{N}_i^+ - 1)} & -I_2 & 0_{2\times(2i - 2 - 2\,{}^l\mathcal{N}_i^+)} & I_2 & 0_{2\times(n-i)}\end{bmatrix} \\ \quad (\text{if } {}^l\mathcal{N}_i^+ < i)\end{cases} \tag{17}$$

, where ${}^l\mathcal{N}_i^+$ is the l-th element of $\mathcal{N}_i^+$,

$$H_{i,\text{left}} = \begin{bmatrix} 0_{2\times 2(i-1)} & I_2 & 0_{2\times 2(n-i)} \\ \vdots & \vdots & \vdots \\ 0_{2\times 2(i-1)} & I_2 & 0_{2\times 2(n-i)}\end{bmatrix} \in \mathbb{R}^{2c_i\times 2n}$$

with $\text{c}_i = \text{card}(\mathcal{M}_i)$, and

$$H_{i,\text{right}} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_c\end{bmatrix} \in \mathbb{R}^{2c_i\times 2m}$$

with

$$h_j = \begin{bmatrix} 0_{2\times 2(\mathcal{M}_i^j - 1)} & -I_2 & 0_{2\times 2(m - \mathcal{M}_i^j)}\end{bmatrix} \in \mathbb{R}^{2\times 2m}$$

where $\mathcal{M}_i^j$ is the j-th element of $\mathcal{M}_i$.

It can be shown that the observation of the origin by every agent coupled with the assumption on $\cup_i \mathcal{M}_i = \{1, \dots, m\}$ implies that the observability grammian of the collective system has full rank. In other words, (14) and (15) is observable. This fact will be used later in our main result below.

4. DISTRIBUTED OBSERVER AND ERROR DYNAMICS

In this section, we will propose a distributed observer for the aforementioned distributed SLAM system and investigate its error dynamics which will be used for stability analysis.

4.1 Distributed Observer

In this subsection, we will present the specific structure of distributed observer for our system described in (14) and (15). For the system description given before, we can define a permutation matrix, which depends on the environment the i-th agent can observe, that can decompose the i-th agent dynamics into the observable and unobservable part. The structure of the permutation matrix takes the following form

$$T_i = [\, Z_i \;\; W_i \,] = \begin{bmatrix} I_{2n\times 2n} & 0_{2n\times 2m} \\ 0_{2m\times 2n} & T_{\text{tr}}\end{bmatrix} \in \mathbb{R}^{2(n+m)\times 2(n+m)}$$

such that

$$\begin{bmatrix} Z_i^T \\ W_i^T\end{bmatrix} A \,[Z_i \quad W_i] = \begin{bmatrix} \bar{A}_i^{11} & 0 \\ \bar{A}_i^{21} & \bar{A}_i^{22}\end{bmatrix}, H_i\,[Z_i \quad W_i] = \begin{bmatrix} \bar{H}_i & 0\end{bmatrix}, \tag{18}$$

where

$$\bar{A}_i^{11} = \begin{bmatrix} \mathbb{1}_n \otimes (\frac{-k_1}{n}(\mathbb{1}_n^T \otimes I_2)) - k_2(\mathcal{L}_f \otimes I_2) & 0_{2n\times 2c_i} \\ 0_{2c_i\times 2n} & 0_{2c_i\times 2c_i} \end{bmatrix},$$

$$\bar{A}_i^{12} = 0_{(2n+2c_i)\times(2m-2c_i)}, \bar{A}_i^{21} = 0_{(2m-2c_i)\times(2n+2c_i)},$$

$$\bar{A}_i^{22} = 0_{(2m-2c_i)\times(2m-2c_i)},$$

and $\bar{H}_i = \begin{bmatrix} 0_{2\times 2(i-1)} \ I_2 \ 0_{2\times(2n-2i)} & 0_{2\times 2c_i} \\ H_{i,\text{robot}} & 0_{2a_i\times 2c_i} \\ H_{i,\text{left}} & \bar{H}_{i,\text{right}} \end{bmatrix}$ with $\bar{H}_{i,\text{right}} = -I_{2c_i} \in \mathbb{R}^{2c_i\times 2c_i}$ and $(\bar{A}_i^{11}, \bar{H}_i)$ is observable.

Since the pair (A, H_i) is not necessarily observable, each agent gets the information concerning the unobservable state by communicating their estimated state with the neighboring agents. We assume the following for the communication graph $\mathcal{G}_c$ of the distributed observer.

Assumption 4. The communication graph $\mathcal{G}_c$ is an undirected connected graph.

Based on the aforementioned Kalman decomposition and assumption, the proposed distributed observer for the i-th agent is given by

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu + K_i(y_i - H_i\hat{x}_i) + \gamma W_i W_i^T \sum_{j\in\mathcal{N}_i^c} \alpha_{ij}(\hat{x}_j - \hat{x}_i)$$

$$K_i = Z_i\bar{P}_i^\infty \bar{H}_i^T R_i^{-1} \tag{19}$$

where $\hat{x}_i \in \mathbb{R}^{2(n+m)}$ is the estimated state of collective system calculated by the i-th agent, the parameter γ is the coupling gain to be chosen and $\bar{H}_i = H_i Z_i$. Matrix $\bar{P}_i^\infty$ is obtained by solving the following algebraic Riccati equation

$$\bar{A}_i^{11}\bar{P}_i + \bar{P}_i(\bar{A}_i^{11})^T + \bar{G}_i Q\bar{G}_i^T - \bar{P}_i\bar{H}_i^T R_i^{-1}\bar{H}_i\bar{P}_i = 0 \tag{20}$$

where $Q = Q^T > qI$ with $q > 0$ and $R_i = R_i^T > 0$ are design parameters and $\bar{G}_i = Z_i^T G_i$. As presented in (Bucy and Joseph (2005)), if $(\bar{A}_i^{11}, \bar{H}_i)$ is observable then the Riccati equation described in (20) admits the solution $\bar{P}_i^\infty > 0$.

4.2 Error dynamics

The estimation error of the i-th local observer is denoted by $e_i := \hat{x}_i - x$ and its error dynamics is given by

$$\dot{e}_i = (A - K_i H_i)e_i + \gamma W_i W_i^T \sum_{j\in\mathcal{N}_i^c} \alpha_{ij}(e_j - e_i) - d + K_i\nu_i - G\omega, \tag{21}$$

where d, ν_i, ω are as in (14) and (15). Using the same permutation matrix T_i as before, the estimation error of each local observer can be decomposed into the observable and unobservable part

$$e_{oi} = Z_i^T e_i \qquad \text{and } e_{\bar{o}i} = W_i^T e_i.$$

By direct substitution, the dynamics of observable and unobservable estimation error is given by

$$\dot{e}_{oi} = (\bar{A}_i^{11} - \bar{K}_i\bar{H}_i)e_{oi} - Z_i^T d + \bar{K}_i\nu_i - \bar{G}_i\omega$$

and

$$\dot{e}_{\bar{o}i} = \bar{A}_i^{21}e_{oi} + \bar{A}_i^{22}e_{\bar{o}i} + \gamma W_i^T \sum_{j\in\mathcal{N}_i^c}(Z_j e_{oj} - Z_i e_{oi}) + \gamma W_i^T \sum_{j\in\mathcal{N}_i^c}(W_j e_{\bar{o}j} - W_i e_{\bar{o}i}) - W_i^T d - \widetilde{G}_i\omega$$

respectively, where $\bar{K}_i = Z_i^T K_i, \widetilde{G}_i = W_i^T G_i$. The estimation error of local observer in (21) can collectively be written as

$$\dot{E} = \text{diag}(A - K_i H_i)E - \gamma\text{diag}(W_i W_i^T)(\mathcal{L}_c \otimes I_{2(m+n)})E - \mathbb{1}_n \otimes d + \text{diag}(K_i)\nu - (\mathbb{1}_n \otimes G\omega),$$

where $E := \begin{bmatrix} e_1^T & e_2^T & \dots & e_n^T \end{bmatrix}^T$ is the collective estimation error which can also be decomposed into observable part $E_o := \begin{bmatrix} e_{o1}^T & e_{o2}^T \dots e_{on}^T \end{bmatrix}^T$ and unobservable part $E_{\bar{o}} := \begin{bmatrix} e_{\bar{o}1}^T & e_{\bar{o}2}^T \dots e_{\bar{o}n}^T \end{bmatrix}^T$ whose time-derivative satisfies

$$\dot{E}_o = \text{diag}(\bar{A}_i^{11} - \bar{K}_i\bar{H}_i)E_o + k_2\text{diag}(Z_i^T)\bar{D}(\text{diag}(Z_i)E_o + \text{diag}(W_i)E_{\bar{o}}) + \text{diag}(\bar{K}_i)\nu - \text{diag}(\bar{G}_i)(\mathbb{1}_n \otimes \omega) \tag{22}$$

and

$$\dot{E}_{\bar{o}} = -\gamma\text{diag}(W_i^T)(\mathcal{L}_c \otimes I_{2(n+m)})\text{diag}(Z_i)E_o - \gamma\text{diag}(W_i^T)(\mathcal{L}_c \otimes I_{2(n+m)})\text{diag}(W_i)E_{\bar{o}} + k_2\text{diag}(W_i^T)\bar{D}(\text{diag}(Z_i)E_o + \text{diag}(W_i)E_{\bar{o}}) - \text{diag}(\widetilde{G}_i)(\mathbb{1}_n \otimes \omega) \tag{23}$$

with $\bar{D} = \mathbb{1}_n \otimes \begin{bmatrix} D \\ 0_{2m\times 2n(m+n)} \end{bmatrix}$ and $\mathcal{L}_c$ be the Laplacian matrix of communication graph $\mathcal{G}_c$.

It can be shown that $\text{diag}(Z_i^T)\bar{D}\text{diag}(W_i)E_{\bar{o}} = 0_{\bar{q}}$ and $\text{diag}(W_i^T)\bar{D}(\text{diag}(Z_i)E_o + \text{diag}(W_i)E_{\bar{o}}) = 0_{2n(m+n)-\bar{q}}$, where $\bar{q}$ is the sum of dimensions of all the agent's observable part. Thus, the error dynamics, which are described by (22) and (23), can be written as

$$\dot{E}_o = \text{diag}(\bar{A}_i^{11} - \bar{K}_i\bar{H}_i)E_o + k_2\text{diag}(Z_i^T)\bar{D}\text{diag}(Z_i)E_o + \text{diag}(\bar{K}_i)\nu - \text{diag}(\bar{G}_i)(\mathbb{1}_n \otimes \omega)$$

and

$$\dot{E}_{\bar{o}} = -\gamma\text{diag}(W_i^T)(\mathcal{L}_c \otimes I_{2(n+m)})\text{diag}(Z_i)E_o - \gamma\,\text{diag}(W_i^T)(\mathcal{L}_c \otimes I_{2(n+m)})\text{diag}(W_i)E_{\bar{o}} - \text{diag}(\widetilde{G}_i)(\mathbb{1}_n \otimes \omega)$$

5. MAIN RESULT

Theorem 1. Let us consider the system as in (14) and (15) satisfying Assumption 1 – 4. Assume that the distributed control gains k_1, k_2 and the coupling gain of distributed observer γ satisfy the following condition

$$\Omega = \text{diag}((\bar{P}_i^\infty)^{-1}\left(\frac{1}{\delta}I + \bar{G}_i Q\bar{G}_i^T\right)(\bar{P}_i^\infty)^{-1} + \bar{H}_i^T R_i^{-1}\bar{H}_i) - (\Psi + \Psi^T) > 0 \tag{24}$$

where $\bar{P}_i^\infty$ is the solution to the Riccati equation (20) with $Q = Q^T > qI$ for some positive $q, \delta > 0$,

$$\zeta_{\min}\left(\gamma\lambda_{\min} - \frac{1}{2\tau}\right) - \frac{\gamma^2\xi^2}{4} > 0 \tag{25}$$

for some $\tau > 0$ and

$$\Psi = k_2\text{diag}(\bar{P}_i^\infty)^{-1}\text{diag}(Z_i^T)\bar{D}\text{diag}(Z_i)$$
$$\lambda_{\min} = \sigma_{\min}\big(\text{diag}(W_i^T)(\mathcal{L}_c \otimes I_{(2n+2m)})\text{diag}(W_i)\big)$$
$$\xi = \|\text{diag}(W_i^T)(\mathcal{L}_c \otimes I_{(2n+2m)})\text{diag}(Z_i)\|$$
$$\zeta_{\min} = \sigma_{\min}\big(\Omega\big).$$

Then for every agent i, the mappings $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right] \mapsto (\hat{x}_i - x)$, $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right] \mapsto p_c - p_c^*$ and $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right] \mapsto (p_i - p_j) - p_i^* - p_j^*$ are L^2-stable for every neighboring agent j.

Proof. Let us first show the L^2-stability of the state observation error by using the following Lyapunov candidate for the distributed observer

$$V_o = \underbrace{\sum_{i=1}^{n} e_{oi}^T (\bar{P}_i^\infty)^{-1} e_{oi}}_{V_{\text{ob}}} + \underbrace{\frac{1}{2}\sum_{i=1}^{n} e_{\bar{o}i}^T e_{\bar{o}i}}_{V_{\text{un}}},$$

where V_{ob} and V_{un} are the terms concerning the estimation error of observable part and unobservable part respectively. The time-derivative of V_{ob} satisfies

$$\begin{aligned}\dot{V}_{\text{ob}} &= 2\sum_{i=1}^{n} e_{oi}^T (\bar{P}_i^\infty)^{-1}\dot{e}_{oi} = 2E_o^T \text{diag}(\bar{P}_i^\infty)^{-1}\dot{E}_o \\ &= 2E_o^T\text{diag}(\bar{P}_i^\infty)^{-1}(\text{diag}(\bar{A}_i^{11} - \bar{K}_i\bar{H}_i)E_o + k_2\text{diag}(Z_i^T) \\ &\quad \bar{D}\text{diag}(Z_i)E_o + \text{diag}(\bar{K}_i)\nu - \text{diag}(\bar{G}_i)(\mathbb{1}_n\otimes\omega)) \\ &< -E_o^T\Omega E_o + \delta\|\text{diag}(\bar{K}_i)\nu - \text{diag}(\bar{G}_i)(\mathbb{1}_n\otimes\omega)\|^2 \quad (26)\end{aligned}$$

where the algebraic Riccati equation (20) is applied to the second equality above and

$$\begin{aligned}\Omega &= \text{diag}((\bar{P}_i^\infty)^{-1}(\bar{G}_iQ\bar{G}_i^T + \frac{1}{\delta})(\bar{P}_i^\infty)^{-1} + \bar{H}_i^TR_i^{-1}\bar{H}_i) \\ &\quad - (\Psi + \Psi^T)\end{aligned}$$

with $\Psi = k_2\text{diag}(\bar{P}_i^\infty)^{-1}\text{diag}(Z_i^T)\bar{D}\text{diag}(Z_i)$. By the hypothesis of the theorem in (24), we have that $\Omega = \Omega^T > \zeta_{\min}$. Hence (26) implies that the map $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right] \mapsto (\hat{x}_i - x)$ is L^2-stable.

On the other hand, the computation on V_{un} gives us that

$$\begin{aligned}\dot{V}_{\text{un}} &= \sum_{i=1}^{n} e_{oi}^T\dot{e}_{oi} = E_{\bar{o}}^T\dot{E}_{\bar{o}} \\ &= E_{\bar{o}}^T\Big(- \gamma\text{diag}(W_i^T)(\mathcal{L}_c\otimes I_{(2n+2m)})\text{diag}(Z_i)E_o \\ &\quad - \gamma\text{diag}(W_i^T)(\mathcal{L}_c\otimes I_{(2n+2m)})\text{diag}(W_i)E_{\bar{o}} \\ &\quad - \text{diag}(\widetilde{G}_i)(\mathbb{1}_n\otimes\omega)\Big).\end{aligned}$$

As is shown in (Lee and Shim (2020)), $\text{diag}(W_i^T)(\mathcal{L}_c\otimes I_{(2n+2m)})\text{diag}(W_i)$ is positive definite. Thus, the inequality

$$\begin{aligned}\dot{V}_{\text{un}} &< -(\gamma\lambda_{\min} - \frac{1}{2\tau})\|E_{\bar{o}}\|^2 + \gamma\xi\|E_o\|\|E_{\bar{o}}\| + \frac{\tau}{2} \\ &\quad \times \|\text{diag}(\widetilde{G}_i)(\mathbb{1}_n\otimes\omega)\|^2\end{aligned}$$

holds, where $\lambda_{\min}$ is the minimum eigenvalue of $\text{diag}(W_i^T)(\mathcal{L}_c\otimes I_{(2n+2m)})\text{diag}(W_i)$ and $\xi = \|\text{diag}(W_i^T)(\mathcal{L}_c\otimes I_{(2n+2m)})\text{diag}(Z_i)\|$ and $\tau > 0$ (by use of Young's inequality).

Based on the above computation, the time-derivative of V_o takes the following form

$$\begin{aligned}\dot{V}_o &= \dot{V}_{\text{ob}} + \dot{V}_{\text{un}} \\ &< -\zeta_{\min}\|E_o\|^2 - (\gamma\lambda_{\min} - \frac{1}{2\tau})\|E_{\bar{o}}\|^2 + \gamma\xi\|E_o\|\|E_{\bar{o}}\| \\ &\quad + \delta\|\text{diag}(\bar{K}_i)\nu - \text{diag}(\bar{G}_i)(\mathbb{1}_n\otimes\omega)\|^2 \\ &\quad + \frac{\tau}{2}\left\|\text{diag}(\widetilde{G}_i)(\mathbb{1}_n\otimes\omega)\right\|^2 \\ &= -\left[\,\|E_o\|\;\|E_{\bar{o}}\|\,\right]\underbrace{\begin{bmatrix}\zeta_{\min} & \frac{-\gamma\xi}{2} \\ \frac{-\gamma\xi}{2} & \gamma\lambda_{\min} - \frac{1}{2\tau}\end{bmatrix}}_{\mathbb{M}}\begin{bmatrix}\|E_o\| \\ \|E_{\bar{o}}\|\end{bmatrix} \\ &\quad + \delta\|\text{diag}(\bar{K}_i)\nu - \text{diag}(\bar{G}_i)(\mathbb{1}_n\otimes\omega)\|^2 \\ &\quad + \frac{\tau}{2}\|\text{diag}(\widetilde{G}_i)(\mathbb{1}_n\otimes\omega)\|^2. \quad (27)\end{aligned}$$

By the hypothesis of the theorem in (24) and (25), we have that $\mathbb{M} = \mathbb{M}^T > 0$. Hence (27) implies that the map $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right] \mapsto (\hat{x}_i - x)$ is L^2-stable.

We will now show the L^2-stability of the formation error which is influenced by the external noise $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right]$ and the state observation error $\hat{x}_i - x$ which are both L^2 as established above. The formation error between the i-th and j-th agent is denoted by $e_{ij} := p_i - p_j - (p_i^* - p_j^*)$. The corresponding Lyapunov function for the formation is

$$V_f = \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} e_{ij}^Te_{ij} = \frac{1}{2}\mathbb{P}^T(\mathcal{L}_f\otimes I_2)\mathbb{P},$$

where $\mathcal{N}_i$ is the set of indices of mobile robots that i-th agent should maintain a relative position with, and $\mathbb{P} = \begin{bmatrix}(p_1 - p_1^*)^T & \ldots & (p_n - p_n^*)^T\end{bmatrix}^T$. The Lyapunov function V_f is bounded by

$$0 < c_1\left\|(\mathcal{L}_f\otimes I_2)\mathbb{P}\right\|^2 \le V_f \le c_2\left\|(\mathcal{L}_f\otimes I_2)\mathbb{P}\right\|^2.$$

The time-derivative of V_f is given by

$$\begin{aligned}\dot{V}_f &= -k_2\mathbb{P}^T(\mathcal{L}_f\otimes I_2)(\mathcal{L}_f\otimes I_2)\mathbb{P} - k_2\mathbb{P}^T(\mathcal{L}_f\otimes I_2)DE \\ &\quad + \mathbb{P}(L_f\otimes I_2)\omega \\ &\le -k_2\left\|(\mathcal{L}_f\otimes I_2)\mathbb{P}\right\|^2 + \frac{k_2}{2\epsilon}\left\|(\mathcal{L}_f\otimes I_2)\mathbb{P}\right\|^2 \\ &\quad + \frac{k_2\epsilon}{2}\left\|DE\right\|^2 + \frac{\|\mathbb{P}(L_f\otimes I_2)\|^2}{2\epsilon} + \frac{\epsilon\|\omega\|^2}{2} \\ &= -\left(k_2 - \frac{k_2}{2\epsilon} - \frac{1}{2\epsilon}\right)\left\|(\mathcal{L}_f\otimes I_2)\mathbb{P}\right\|^2 + \frac{k_2\epsilon}{2}\left\|DE\right\|^2 \\ &\quad + \frac{\epsilon\|\omega\|^2}{2} \\ &\le -\frac{1}{c_2}\left(k_2 - \frac{k_2}{2\epsilon} - \frac{1}{2\epsilon}\right)V_f + \frac{\epsilon}{2}\left\|DE\right\|^2 + \frac{\epsilon\|\omega\|^2}{2}. \quad (28)\end{aligned}$$

By taking $\epsilon > 0$ such that $k_2 - \frac{k_2}{2\epsilon} - \frac{1}{2\epsilon} > 0$ and since we have established that $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right] \mapsto E$ is L^2-stable,it follows from (28) that the map $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right] \mapsto p_i - p_j - (p_i^* - p_j^*)$ is L^2-stable.

Finally, we will show that L^2-stability of the centroid tracking error with respect to $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right]$. Let us denote the centroid tracking error by $e_c = p_c - p_c^*$ and we consider the following Lyapunov function

$$V_m = \frac{1}{2}e_c^Te_c.$$

Direct computation shows that

$$\begin{aligned}\dot{V}_m &= -k_1e_c^Te_c - \frac{k_2}{n}e_c^T\sum_{i=1}^{n}((\Gamma_i\otimes I_2)(\mathcal{L}_f\otimes I_2)\mathbb{P}) - \frac{k_2}{n}e_c^T \\ &\quad \sum_{i=1}^{n}(\Gamma_i\otimes I_2)DE + e_c^T(\frac{1}{n}(\mathbb{1}_n^T\otimes I_2)\omega) \\ &\le -(k_1 - \frac{k_2}{n\psi} - \frac{1}{2n\psi})\|e_c\|^2 \\ &\quad + \frac{k_2\psi}{2n}\|\sum_{i=1}^{n}(\Gamma_i\otimes I_2)(\mathcal{L}_f\otimes I_2)\mathbb{P}\|^2\end{aligned}$$

$$+\frac{k_2\psi}{2n}\|\sum_{i=1}^{n}(\Gamma_i\otimes I_2)DE\|^2+\frac{\psi\|(\mathbb{1}_n^T\otimes I_2)\omega\|^2}{2n}$$
$$=-2(k_1-\frac{k_2}{n\psi}-\frac{1}{2n\psi})V_m+\frac{k_2\psi}{2n}\|\sum_{i=1}^{n}(\Gamma_i\otimes I_2)DE\|^2$$
$$+\frac{k_2\psi}{2n}\|\sum_{i=1}^{n}(\Gamma_i\otimes I_2)(\mathcal{L}_f\otimes I_2)\mathbb{P}\|^2+\frac{\psi\|(\mathbb{1}_n^T\otimes I_2)\omega\|^2}{2n} \tag{29}$$

where $\Gamma_i=[0\dots\underbrace{1}_{i-\text{th}}\dots0]\in\mathbb{R}^{1\times n}$. By taking $\psi>0$ such that $k_1-\frac{k_2}{n\psi}-\frac{1}{2n\psi}>0$ and using the fact that the maps $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right]\mapsto E$ and $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right]\mapsto(\mathcal{L}_f\otimes I_2)\mathbb{P}$ are L^2-stable, (29) implies that the map $\left[\begin{smallmatrix}\omega\\ \nu\end{smallmatrix}\right]\mapsto p_c-p_c^*$ is L^2-stable

6. SIMULATION

Let us consider the case where three mobile robots keep a stable formation while moving in an environment which consists of five static landmarks. The initial positions of robots are $p_1=[0\quad 0]^T$, $p_2=[1\quad 0]^T$ and $p_3=[0\quad 1]^T$. The initial position of centroid is $p_c^*(0)=[\frac{1}{3}\quad\frac{1}{3}]^T$ and the velocity of centroid is defined as $\dot{p}_c^*=[v_{cx}\quad v_{cy}]^T$, where $v_{cx}=1$ m/s is the x-axis velocity of desired centroid and $v_{cy}=1$m/s is y-axis velocity of the desired centroid. $m_1(0)=[3\quad 5]^T$, $m_2(0)=[2\quad 6]^T$, $m_3(0)=[4\quad 7]^T$, $m_4(0)=[1\quad 4]^T$ and $m_5(0)=[5\quad 2]^T$ are the initial positions of landmarks. The desired relative positions are: $p_1^*-p_2^*=[-1\quad 0]^T$, $p_2^*-p_3^*=[1\quad -1]^T$, $p_3^*-p_1^*=[0\quad 1]^T$. The Laplacian matrices of the undirected connected communication graph and directed strongly connected measurement graph are given by:

$$\mathcal{L}=\begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{bmatrix},\mathcal{L}^+=\begin{bmatrix}1&-1&0\\0&1&-1\\-1&0&1\end{bmatrix}$$

respectively. The set of landmarks which robot can observe are given by $\mathcal{M}_1=\{1,2,5\}$, $\mathcal{M}_2=\{2,3\}$, $\mathcal{M}_3=\{4,5\}$.

By choosing a proper control gains such as $k_1=1, k_2=1, \gamma=2$ and setting a proper lower bound which satisfy $Q>qI=2I$, the matrix Ω described in (24) will be positive definite. The simulation results are shown in Fig. 3 where the agents can maintain a robust formation and track the position of centroid.

7. CONCLUSION

We propose the use of a distributed observer for localizing a team of robots while simultaneously maintaining formation distributedly and maneuvering. We prove the L^2-stability of the closed-loop system with respect to external disturbances and show its efficacy in a simulation result.

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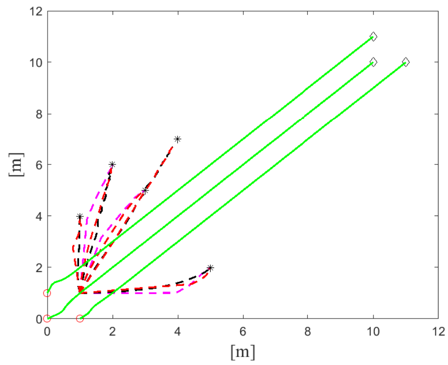

Fig. 3. Robot team maneuvers around the environment. Red circles and black diamonds represent the initial positions and destination of the mobile robots respectively. The green solid lines are the trajectories of mobile robots. The dash lines denote the trajectories of positions of landmarks estimated by each agent.

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