Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Algorithms for Integrated Processing of Marine Gravimeter Data and GNSS Measurements

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Abstract: Efficiency of using global navigation satellite system (GNSS) measurements for determining gravity anomalies (GA) at sea by solving filtering and smoothing problems based on GNSS and gravimeter data is studied. The GA, ship heaving, errors of GNSS and gravimeter measurements are presented as stochastic processes. The analysis is based on the standard deviations of the GA estimation errors, calculated at different heaving parameters and in different modes of GNSS data processing.

Keywords: filtering and smoothing, nonlinear system identification, software for system identification, Bayesian methods, global navigation satellite system, gravimetry.

1. INTRODUCTION

Currently, the results of gravimetric surveys conducted from aircraft or ships are widely used to solve the problems of mineral exploration, refinement of the Earth's model, and map-aided navigation (Zeng *et al.*, 2002; Forsberg, Olesen and Einarsson, 2015; Lu *et al.*, 2017; Peshekhonov *et al.*, 2017; Wang *et al.*, 2017; Forsberg *et al.*, 2018; Zhou *et al.*, 2018).

When determining GA from moving objects, it is necessary to compensate for the effect of vertical accelerations on the gravimeter. Currently, this is provided by different approaches used in airborne and marine gravimetry. In the case of airborne gravimetry, accelerations are so high and their power spectral densities (PSD) often overlap GA PSD, so they cannot be compensated for without additional navigation or velocity information (Stepanov and Koshaev, 2010; Krasnov and Sokolov, 2015; Becker *et al.*, 2016; Golovan *et al.*, 2018; Wang *et al.*, 2018). With this aim in view, precision navigation solutions obtained from GNSS phase measurements in the differential mode or velocity values obtained from these measurements are used. For marine vessels PSD of vertical accelerations is shifted in the frequency domain relative to the PSD of the GA. Taking this fact into consideration, it is possible to obtain GA estimates using filtering and smoothing algorithms based on the models of vessel heaving and without additional information on vertical accelerations (Bolotin and Yurist, 2011; Guo *et al.*, 2013; Koshaev, Motorin and Stepanov, 2019; Sokolov, Krasnov and Zheleznyak, 2019). Thus, the GNSS measurements are usually not used in GA estimation algorithms during marine surveys. Such an approach is effective if specified requirements for the sea state and the velocity of the vessel are met. However nothing prevents us from combining the two approaches, "airborne" and "marine", to determining GA, that is, to use filtering and smoothing algorithms to process gravimeter readings and precision navigation solutions from the GNSS taking into account the models of the vessel heaving and that of the GA. Preliminary simulation of the problem of determining GA in marine conditions (Koshaev, Motorin, and Stepanov, 2019) have shown that we can have certain benefits: first of all, relax the requirements for the survey conditions.

This aim of the paper is to estimate the potential of such a combined approach using data from a real gravimetric survey carried out from a light boat. The problem of integrated processing of marine gravimeter data and GNSS measurements is considered in the framework of the stochastic approach in the state space. It is shown that different GA estimation problems can be treated as a part of this general framework and can be solved using standard filtering and smoothing techniques. The unknown parameters for the algorithms are identified using real data by implementing Rao-Blackwellised filter.

The paper is structured as follows. Section 2 gives the general statement of the GA estimation problem and discusses different processing variants in the general framework. Section 3 considers the results of the identification of GA and heaving error model parameters based on the data obtained in a real gravimetric survey. Section 4 presents the analysis of the GA potential accuracy determined by simulation. The results of the GA estimation obtained by using real data are discussed in Section 5.

2. OPTIMAL ALGORITHMS FOR INTEGRATED PROCESSING OF GNSS MEASUREMENTS AND GRAVIMETER DATA

Based on the results obtained in the previous research (Koshaev, Motorin and Stepanov, 2019), we formulate the statement of the problem of GA estimation from a moving vessel within the framework of a stochastic approach in the state space and introduce all necessary models.

We use the Jordan model (Jordan, 1972) to describe the GA. According to this model, when the marine vessel is moving along a straight trajectory (tack) with velocity V, the observed GA can be described by a shaping filter in the form:

$$\begin{aligned} \dot{x}_1 &= -\beta x_1 + x_2, \\ \dot{x}_2 &= -\beta x_2 + x_3, \\ \dot{x}_3 &= -\beta x_3 + w_g, \\ \tilde{g} &= -\beta\zeta x_1 + x_2, \end{aligned} \tag{1}$$

where $\beta = V\sigma_{\partial\tilde{g}/\partial l} / \sqrt{2}\sigma_{\tilde{g}}$, $\sigma_{\tilde{g}}$ is a GA standard deviation (SD); $\sigma_{\partial\tilde{g}/\partial l}$ is the SD of the GA gradient in (mGal/km); w_g is a system white noise with power spectral density (PSD) $10\beta^3\sigma_{\tilde{g}}^2$, $\zeta = (\sqrt{5}-1)/\sqrt{5}$. The PSD for the Jordan model can be written as:

$$S_{\tilde{g}}(\tilde{\omega}) = 2\alpha^3\sigma_{\tilde{g}}^2 \frac{5\tilde{\omega}^2 + \alpha^2}{\left(\tilde{\omega}^2 + \alpha^2\right)^3} \tag{2}$$

where $\tilde{\omega} = \omega / V$, ω is the angular frequency, $\alpha = \sigma_{\partial\tilde{g}/\partial l} / \sqrt{2}\sigma_{\tilde{g}}$.

Depending on the type of marine moving vessel and external, for example, weather, conditions, the heaving model can be described in different ways. In this paper, we use the model described by the following shaping filter (Rivkin, 1980):

$$\begin{aligned} \dot{x}_4 &= x_5, \\ \dot{x}_5 &= x_6, \\ \dot{x}_6 &= -a_3 x_4 - a_2 x_5 - a_1 x_6 + w_{\Delta h}, \end{aligned} \tag{3}$$

where $\Delta h = x_4$ is the vertical displacement above an averaged sea level in the survey area with SD $\sigma_{\Delta h}$; $\dot{h} = -x_5$, $\ddot{h} = -x_6$ are the vertical velocity and acceleration. The coefficients in (3) are defined as $a_3 = \left(\lambda^2 + \mu^2\right)\gamma$, $a_2 = \lambda^2 + \mu^2 + 2\mu\gamma$, $a_1 = 2\mu + \gamma$, where $\lambda = 2\pi / T$, T is a prevailing heaving period; $\mu = 1/\tau$, τ is a correlation interval; γ is the specified coefficient. The PSD of system white noise $w_{\Delta h}$ is $2\sigma_{\Delta h}^2 a_3 \left(a_1 a_2 - a_3\right)/a_1$. The SD of vertical displacement $\sigma_{\Delta h}$ and SD of vertical accelerations $\sigma_{\ddot{h}}$ are related by equation $\sigma_{\ddot{h}} = \sigma_{\Delta h}\sqrt{(a_2 a_3)/a_1}$. Further, we assume that $\tau = 3T$ and $\gamma = 0.1\text{s}^{-1}$. We chose model (3) for this problem because it can correctly represent the essential properties of Δh, which is a stationary narrowband process with a finite second-order derivative. The PSD for vertical accelerations can be written as

$$S(\omega) = \frac{1}{\pi} \frac{C^2\omega^4}{\omega^6 + \left(a_1^2 - 2a_2\right)\omega^4 + \left(a_2^2 - 2a_3 a_1\right)\omega^2 + a_3^2}. \tag{4}$$

The gravimeter readings can be defined as

$$y_g = \tilde{g} + \ddot{h} + v_g, \tag{5}$$

where $\tilde{g}$ is a GA, $\ddot{h}$ is a vertical acceleration, and v_g is a gravimeter white-noise error with PSD R_g. It is assumed that measurements (5) are generated without delay, the normal component of gravity acceleration has already been excluded, and the Eotvos effects and the gravimeter drift have been compensated for. Taking into account models (1), (3), measurements (5) can be rewritten as

$$y_g = -\beta\zeta x_1 + x_2 - x_6 + v_g. \tag{6}$$

The measurements of height h above the reference ellipsoid and the vertical velocity $\dot{h}$ obtained from the GNSS data are written as

$$\begin{aligned} y_h &= h + \delta h + v_h = h^* - \Delta h + \delta h + v_h, \\ y_{\dot{h}} &= \dot{h} + v_{\dot{h}}, \end{aligned} \tag{7}$$

where h^* is an averaged sea level in the survey area above the reference ellipsoid, δh is a slowly varying error of the height determined by the GNSS, $v_h, v_{\dot{h}}$ are white noise GNSS errors with PSDs R_h, $R_{\dot{h}}$. The GNSS measurements (7) are recalculated from the phase center of antenna to the gravimeter location which is not shown in expression (7) for simplification.

The value of h^* can be unknown; moreover, it can vary depending on tides. Slowly varying error δh of height determined by the GNSS can be represented as a stationary first-order Gauss-Markov process. Taking this into account, we introduce additional components into the state vector $x_7 = h^*$, $x_8 = \delta h$ in the form:

$$\begin{aligned} \dot{x}_7 &= 0, \\ \dot{x}_8 &= -a_m x_8 + \sigma_m\sqrt{2a_m}w_m, \end{aligned} \tag{8}$$

where $a_m = 1/\tau_m$ is the inverse correlation interval, σ_m is the error SD, w_m is the white noise of unit PSD. Thus, measurements (7) can be represented as

$$\begin{aligned} y_h &= -x_4 + x_7 + x_8 + v_h, \\ y_{\dot{h}} &= -x_5 + v_{\dot{h}}. \end{aligned} \tag{9}$$

From the preceding it is inferred that the problem of integrated processing of the gravimeter data and GNSS measurements, which is aimed to obtain GA values, can be formulated within the framework of the stochastic approach in the state space as follows. Find the optimal, in the mean-square sense, estimate of state vector $x = \left[x_1 \ldots x_8\right]^T$ described by equations (1), (3), (8) using measurements (6) and (9).

The formulated problem statement is the most complete from the standpoint of the data used and will be further referred to as **statement 1**. Common practice of marine and airborne gravimetry is to solve truncated, in terms of the data involved, variants of the problem statement.

While processing the results of marine gravimetric surveys, only gravimeter data with GA and heaving models are used (Peshekhonov *et al.*, 2017; Golovan *et al.*, 2018). This is explained by the fact that under certain conditions, the PSD of the vessel's vertical accelerations (3) and GA (1) are spaced in the frequency domain, as noted in the introduction. It is this circumstance that makes it possible to smooth out the measurement errors without distorting the GA. It is easy to see that within the framework of this problem statement for the integrated data processing, the marine gravimetric survey data processing corresponds to estimation of a six-dimensional vector described by equations (1), (3) using measurements (6). Hereinafter it will be referred to as **statement 2.**

In airborne gravimetry practice, the aircraft vertical accelerations are usually compensated by using GNSS data. Thus, the statement becomes invariant with respect to the aircraft dynamics. This is provided with the use of differential measurements in the form (Stepanov and Koshaev, 2010):

$$\begin{aligned} z_h &= y_h - \eta, \\ z_{\dot h} &= y_{\dot h} - \vartheta, \end{aligned} \tag{10}$$

where $\vartheta = \int_{t_0}^{t} y_g(\tau)d\tau$, $\eta = \int_{t_0}^{t} \vartheta(\tau)d\tau$ are the increments of velocity and height obtained by integrating the gravimeter data (5); t_0 is the initial moment of the problem solution; t is the time to which the GNSS measurements $y_h, y_{\dot h}$ apply. Let us complement the GA model (1) with states $x_4 = \eta, x_5 = \vartheta$ and add a state for the slowly varying error of the height determined by the GNSS. In this case state vector can be written as

$$\begin{aligned} \dot x_4 &= x_5, \\ \dot x_5 &= \tilde g + v_g = -\beta\zeta x_1 + x_2 + v_g, \\ \dot x_6 &= -a_m x_6 + \sigma_m \sqrt{2a_m} w_m, \end{aligned} \tag{11}$$

where x_1, x_2 are taken from model (1) and $x_6 = \delta h$ describes the slowly varying error of the height determined by the GNSS, like x_8 in (8). Considering introduced notation and equation (7), we can write measurements z_h, $z_{\dot h}$ as

$$\begin{aligned} z_h &= -x_4 + x_6 + v_h, \\ z_{\dot h} &= -x_5 + v_{\dot h}, \end{aligned} \tag{12}$$

where x_4, x_5, x_6 correspond to the shaping filter (11). Thus, within the framework of this problem statement for the integrated data processing, the airborne gravimetric survey data processing corresponds to the estimation of the state vector described by equations (1), (11) using measurements (12). Hereinafter this statement will be referred to as **statement 3.**

Each of the three problem statements for GA estimation can be considered as a linear filtering or smoothing problem that can be solved with the use of the well-known optimal, in the mean-square sense, algorithms (Gelb, 1974; Sarkka, 2013). At the same time, recall that in the filtering mode, we use only "past" measurements, those preceding the current estimation point, while in the smoothing mode, both the "past" and "future" measurements are used. Thus, filtered estimates can be obtained online, while smoothed estimates can be obtained only in postprocessing.

3. IDENTIFICATION OF MODEL PARAMETERS

To implement optimal filtering and smoothing algorithms, we need to have information about all the parameters introduced in the models above. However, some model parameters are not always known in advance. In particular, this concerns the SD of the GA derivative, correlation intervals, and SD of vertical accelerations and GNSS errors. To identify these parameters, we use the previously developed algorithms described in (Motorin, Stepanov and Koshaev, 2015; Stepanov, Koshaev and Motorin, 2015; Stepanov et al., 2016). In addition to x described above, we introduced vector θ, which includes unknown constant parameters of the models. The algorithms for joint estimation of θ and x are based on the methods of multiple model filtering and Rao-Blackwellization procedure. As part of the research, we developed MatLab-based software to implement these algorithms, making it possible to identify the necessary parameters with the use of real gravimeter measurements and GNSS data.

We used the data from the Chekan-AM gravimeter (Krasnov, Sokolov and Elinson, 2014) and a NovAtel GNSS receiver, which were obtained in the Lake Ladoga water area near St. Petersburg. The equipment was installed on a small boat. The data of the vertical displacements and GNSS velocities were recalculated to the installation site of the gravimeter using the data on the motion angles obtained from the Chekan gyro platform.

In this research, we considered three possible modes of GNSS operation: normal (standalone) mode with the SD of the white-noise measurement error in height of 2 m, code mode or DGLONASS/DGPS with the SD of 0.35 m, and differential modes of phase measurement processing with resolution of their ambiguity—Real Time Kinematic (RTK) in the filtering problem and Post Processing Kinematic (PPK) in the smoothing problem (Leick, Rapoport and Tatarnikov, 2015). The base station was located at a distance of 80 km from the location of the survey. In the experiments, we used the so-called PPK solutions. GNSS data were received at a frequency of 10 Hz.

During the studies, we identified the following parameters: SD of the GA derivative $\sigma_{\partial\tilde g/\partial l}$ for (1), the prevailing period T and the SD of vertical accelerations for (3), the correlation interval τ_m and SD σ_m of the slowly varying error δh for (7).

The parameters of GA model (1) and the error in height δh determined by the GNSS were estimated in accordance with **statement 3.** In this case, the vector θ of unknown model parameters was determined as $\theta^1 = \left[\sigma_{\partial\tilde{g}/\partial l}, \tau_m, \sigma_m\right]^T$. The estimates of the SD of the GA derivative confirmed the known data for the studied area at a level of 1–3 mGal/km. The estimates of the correlation interval of the slowly varying GNSS error τ_m were 8–13 minutes, and the SD of the error σ_m was 4–6 cm in the RTK/PPK modes. As will be shown below, the application of the standalone mode and the DGLONASS/DGPS mode is inefficient. Therefore, the SD and the correlation interval of the slowly varying error in height measurements of the GNSS were identified only for the RTK/PPK mode.

The parameters of the vessel vertical accelerations $\sigma_{\ddot{h}}$, T were estimated in accordance with **statement 2.** In this case, vector θ of unknown filter parameters was determined as $\theta^2 = \left[\sigma_{\ddot{h}}, T\right]^T$. The estimates of the prevailing period of vertical accelerations T were 2 s in all cases, which is explained by the small size of the boat from which the survey was performed. The estimates of the SD of vertical accelerations $\sigma_{\ddot{h}}$ vary significantly depending on the tack, ranging from 20 to 150 Gal. These are typical period and SD values for following real surveys and different directions are not considered. They can also be explained by the small size of the boat, as a result of which, a significant difference in vertical accelerations manifested itself depending on the direction of the waves. As a result, during the simulation, we considered two different values of the SD of vertical accelerations.

4. ANALYSIS OF THE GA ACCURACY ESTIMATION WITH THE USE OF GNSS MEASUREMENTS

Using the parameters calculated in Section III, let us analyze the expected gain in the GA accuracy estimation due to GNSS measurements on a marine vessel by simulation. This is done by calculation of the SD for GA estimation errors of optimal filtering and smoothing algorithms corresponding to the three above statements. To perform these calculations, we obtained recursive equations, tolerant to computational errors, for the covariance matrices of filtering and smoothing errors and used them as a basis for our software. This allowed us to obtain the desired results without using numerical statistical testing procedures, and, as a consequence, increase the efficiency of estimation accuracy analysis. The developed software is an extended version of the software presented in (Stepanov and Koshaev, 2011), which is upgraded to solve smoothing problems.

The model parameters were chosen as follows. SD of the GA were set equal to $\sigma_{\tilde{g}} = 10$ mGal, based on a priori knowledge of the field in this area. For the SD of the GA derivative, we used the average tack value of 2 mGal/km. For the correlation interval of the slowly varying GNSS error τ_m, we used the average tack value of 10 min; for the SD σ_m of the error, a priori values of 2 m, 0.35 m, and 0.02 m were used, depending on the mode. The SD of the white-noise GNSS error was set equal to the SD of the slowly varying GNSS error. For vertical accelerations, we chose an average period T=2 s and considered two SD values of 20 Gal and 150 Gal. The SD of the gravimeter noise v_g averaged over 1 s was 1 mGal. For the vessel velocity V, the values of 10 knots corresponding to the average velocity were set. The obtained steady-state values of the SD of the GA estimation errors are given in Table 1. The time of the transient process was approximately 800 s. The SD of the GA estimation errors for the case when the heaving model and the GNSS measurements are used together **(statement 1)** are outlined by a dashed line.

The bottom line of Table 1 shows the SD of GA errors obtained without using GNSS data (**statement 2**), corresponding to the algorithms for processing marine survey data. The last column of Table 1 contains the SD for the problem solved without taking into account the heaving model (**statement 3**), corresponding to the algorithms of the airborne gravimetric survey.

Table 1. SD of the GA estimation errors for filtering (F) and smoothing (S) based on simulation results (mGal)

GNSS mode	Heaving model parameters				W/O heaving model	
	$\sigma_{\ddot{h}}$=20 Gal		$\sigma_{\ddot{h}}$=150 Gal			
	F	S	F	S	F	S
PPK	0.45	0.12	0.46	0.12	0.46	0.12
DGPS	0.78	0.15	1.27	0.28	1.40	0.36
Standalone	0.79	0.15	1.45	0.29	1.87	0.53
Not used	0.80	0.15	1.61	0.30		

From the results presented in Table 1, it follows that as it was noted earlier, the accuracy of GA estimation significantly increased (by a factor of 3–6) in the smoothing mode as compared to the filtering mode (Stepanov et al., 2016). This explains the fact that only the smoothing mode is used during the survey.

From Table 1 it also follows that to increase the accuracy of GA estimates using GNSS measurements is possible only in the case when GNSS precision data are processed in the RTK/PPK modes. The reason for that can be in the “leakage” of slowly varying GNSS error into GA estimates. For the statements considered, the use of RTK/PPK leads to about twofold increase in the accuracy, under significant vertical accelerations which relaxes the requirements for the level of vertical displacements when determining GA. Another conclusion is that it does not make much sense to take into account the heaving model when processing GNSS data in RTK/PPK modes since SD of the GA estimation error are approximately the same both with and without account for the model.

5. THE RESULTS OF REAL DATA PROCESSING

In order to verify the effectiveness of using GNSS measurements in marine surveys and the possibility of approaching the SD values obtained by the simulation under idealized conditions, we solved the GA smoothing problem using the test data described in Section III. During testing, the tacks were passed in two (forward and backward) directions, which provided the possibility of obtaining SD of the GA estimation errors by comparing the GA estimates obtained on opposite directions. For each of the four straight tacks passed forward and backward on the speed about 10 knots we set the parameters $\sigma_{\partial\tilde{g}/\partial l}$, a_m, σ_m, $\sigma_{\ddot{h}}$, T that were determined on the survey according to the identification results in Section III. The rest of the parameters were set in accordance with the parameters used in the simulation in Section IV. Based on the results of predictive simulation, we considered the GA estimates in the smoothing mode using GNSS measurements in the PPK mode as the most accurate ones. Thus, the problem was solved in the formulation corresponding to **statements 3 and 2**. Examples of the obtained GA estimates are shown in Figs. 1–2.

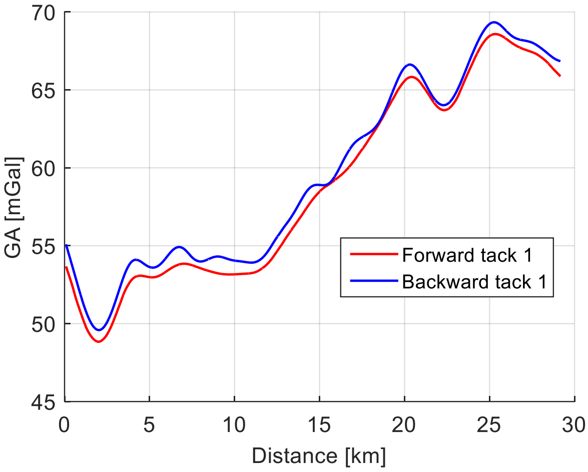

Fig. 1. Smoothed GA estimates obtained using PPK solutions on forward and backward directions of tack 1.

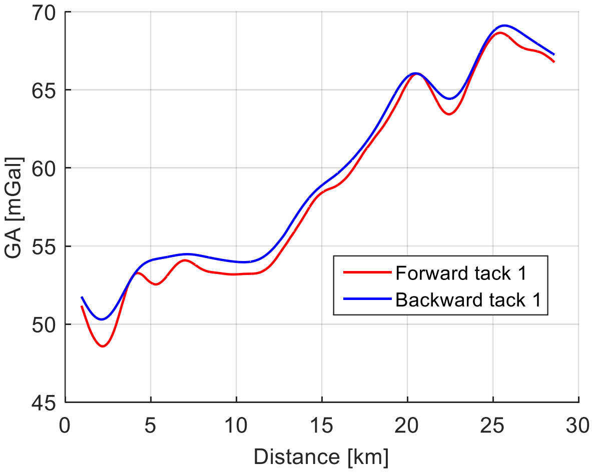

Fig. 2. Smoothed GA estimates obtained without GNSS data on forward and backward directions of tack 1.

To compare real data processing results with simulation one we estimate the SD of the errors in determining GA using the differences in the GA estimates obtained on opposite directions. Table 2 presents these estimated SD in comparison with the calculated SD, derived from the data of the smoothing algorithm covariance matrices. Note that the SD in table 2 are in a good match. The SD obtained by processing real data are also in good agreement with the GA estimation accuracy predicted by simulation: at a level of 0.1–0.3 mGal.

Table 2. SD of GA estimation errors for smoothing based on the real data processing results (mGal)

		Tack 1	Tack 2	Tack 3	Tack 4
With GNSS	Calculated SD	0.21 0.18	0.12 0.15	0.24 0.25	0.14 0.17
	Estimated SD	0.20	0.11	0.22	0.11
Without GNSS	Calculated SD	0.15 0.30	0.17 0.22	0.17 0.26	0.22 0.25
	Estimated SD	0.26	0.12	0.31	0.09

The results in table 2 are provided only for insignificant vertical acceleration. However, we do not see a twofold gain in accuracy in the processing using GNSS data, even on the tacks where significant vertical accelerations were observed. Incomplete correspondence of the results of real data processing and simulation is explained by some idealization of the latter, which we have already mentioned above.

6. CONCLUSIONS

The problem of marine GA estimation on a moving vessel has been formulated as a problem of integrated processing of gravimeter data and GNSS measurements in the framework of the stochastic approach. Three statements have been formulated, including the general statement, and the other two corresponding to the traditional algorithms used for processing marine and airborne surveys.

The software implementing nonlinear algorithms has been developed to identify the model parameters, including those defining the GNSS solution errors, vertical accelerations, and GA. The model identifications were carried out with real measurements.

Dedicated software has been developed on the basis of computational error-resistant recursive equations for the covariance matrices of filtering and smoothing errors. It was used to analyze the expected accuracy of GA estimation with and without GNSS data. The results have shown that significant increase in the accuracy of GA determination is possible only with the use of GNSS phase measurements in the RTK/PPK modes. In this case, the heaving model may not be taken into consideration. The standalone mode and code differential modes of GNSS data processing are significantly inferior to RTK/PPK in terms of efficiency and practically do not give any noticeable benefits in GA estimation using GNSS measurements.

The results of real gravimetric survey data processing have shown the possibility of approaching the potential accuracy of the GA estimation. But at the same time, it has become clear that we need to take into account errors in determining the Eotvos corrections and the errors in the recalculation of GNSS solutions from the antenna phase center to the gravimeter, which is the subject-matter of the future work.

ACKNOWLEDGMENT

This work was supported by the Russian Science Foundation, project no. 18-19-00627.

The results of Section 3 were obtained with support of the Russian Foundation for Basic Research, project no. 18-08-01101.

REFERENCES

Becker, D. *et al.* (2016) 'Latest Results in Strapdown Airborne Gravimetry Using an iMAR RQH Unit', in *Proceedings of 4th IAG Symposium on Terrestrial Gravimetry: Static and Mobile Measurements (TG-SMM 2016)*. St. Petersburg, pp. 19–25.

Bolotin, Y. V and Yurist, S. S. (2011) 'Suboptimal smoothing filter for the marine gravimeter GT-2M', *Gyroscopy and Navigation*, (2), pp. 152–158.

Forsberg, R. *et al.* (2018) 'Exploring the Recovery Lakes region and interior Dronning Maud Land, East Antarctica, with airborne gravity, magnetic and radar measurements', *Geological Society, London, Special Publications*, 461(1), pp. 23–34.

Forsberg, R., Olesen, A. V. and Einarsson, I. (2015) 'Airborne gravimetry for geoid determination with Lacoste Romberg and Chekan gravimeters', *Gyroscopy and Navigation*, 6(4), pp. 265–270.

Gelb, A. (1974) *Applied optimal estimation*. Cambridge, England: M.I.T. Press.

Golovan, A. A. *et al.* (2018) 'Application of GT-2A Gravimetric Complex in the Problems of Airborne Gravimetry', *Izvestiya, Physics of the Solid Earth*, 54(4), pp. 658–664.

Guo, L. *et al.* (2013) 'Preferential filtering for gravity anomaly separation', *Computers & Geosciences*, 51, pp. 247–254.

Jordan, S. K. (1972) 'Self-consistent Statistical Models for Gravity Anomaly and Undulation of the Geoid', *Journal of Geophysical Research*, 77(20), pp. 2156–2202.

Koshaev, D. A., Motorin, A. V. and Stepanov, O. A. (2019) 'Efficiency of using Satellite Measurements for Marine Gravimetry', in *2019 26th Saint Petersburg International Conference on Integrated Navigation Systems (ICINS)*. IEEE, pp. 1–5.

Krasnov, A. A. and Sokolov, A. V. (2015) 'A modern software system of a mobile Chekan-AM gravimeter', *Gyroscopy and Navigation*, 6(4), pp. 278–287.

Krasnov, A. A., Sokolov, A. V and Elinson, L. S. (2014) 'A new air-sea gravimeter of the Chekan series', *Gyroscopy and Navigation*, 5(3), pp. 181–185.

Leick, A., Rapoport, L. and Tatarnikov, D. (2015) *GPS Satellite Surveying*. Hoboken, NJ, USA: John Wiley & Sons, Inc.

Lu, B. *et al.* (2017) 'Airborne Gravimetry of GEOHALO Mission: Data Processing and Gravity Field Modeling', *Journal of Geophysical Research: Solid Earth*, 122(12), pp. 10586-10604.

Motorin, A. V., Stepanov, O. A. and Koshaev, D. A. (2015) 'Identification of gravity anomaly model parameters in airborne gravimetry problems using nonlinear filtering methods', *Gyroscopy and Navigation*, 6(4), pp. 318–323.

Peshekhonov, V. G. *et al.* (2017) *Sovremennyye metody i sredstva izmereniya parametrov gravitatsionnogo polya Zemli (Modern methods and equipment for Earth gravity field investigation) (in Russian)*. Edited by V. G. Peshekhonov and O. A. Stepanov. Saint-Petersburg, Russia: CSRI Elektropribor.

Rivkin, S. S. (1980) *Opredeleniye lineynykh skorostey i uskoreniy kachki korablya inertsial'nym metodom (Inertial method for estimation the linear velocities and accelerations of ship) (In Russian)*. Lenningrad: CSRI Rumb.

Sarkka, S. (2013) *Bayesian Filtering and Smoothing*. Cambridge University Press.

Sokolov, A. V., Krasnov, A. A. and Zheleznyak, L. K. (2019) 'Improving the Accuracy of Marine Gravimeters', *Gyroscopy and Navigation*, 10(3), pp. 155–160.

Stepanov, O. A. *et al.* (2016) 'Comparison of Stationary and Nonstationary Adaptive Filtering and Smoothing Algorithms for Gravity Anomaly Estimation on Board the Aircraft', in *Proc. IAG Symposium on Terrestrial Gravimetry 'Static and Mobile Measurements'*. St. Petersburg: CSRI Elektropribor, pp. 53–60.

Stepanov, O. A. and Koshaev, D. A. (2011) 'A Program for Designing Linear Filtering Algorithms for Integrated Navigation Systems', *IFAC Proceedings Volumes*. Elsevier, 44(1), pp. 4256–4259.

Stepanov, O. A. and Koshaev, D. A. (2010) 'Analysis of filtering and smoothing techniques as applied to aerogravimetry', *Gyroscopy and Navigation*, 1(1), pp. 19–25.

Stepanov, O. A., Koshaev, D. A. and Motorin, A. V. (2015) 'Designing models for signals and errors of sensors in airborne gravimetry using nonlinear filtering methods', in *Proc. of ION International technical meeting, ITM*. Dana Point, CA: Institute of Navigation, pp. 279–284.

Wang, H. *et al.* (2017) 'Location Accuracy of INS/Gravity-Integrated Navigation System on the Basis of Ocean Experiment and Simulation', *Sensors*, 17(12), p. 2961.

Wang, M. *et al.* (2018) 'Improving the Strapdown Airborne Vector Gravimetry by a Backward Inertial Navigation Algorithm', *Sensors*, 18(12), p. 4432.

Zeng, H. *et al.* (2002) 'Detection of reservoirs from normalized full gradient of gravity anomalies and its application to Shengli oil field, east China', *GEOPHYSICS*, 67(4), pp. 1138–1147.

Zhou, X. *et al.* (2018) 'A Combined Gravity Compensation Method for INS Using the Simplified Gravity Model and Gravity Database', *Sensors*, 18(5), p. 1552.