Higher Order PD and iPD controller tuning

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Abstract: This paper deals with tuning of proportional-derivative (PD) and "intelligent" proportional-derivative (iPD) controller for the double integrator plus dead time (DIPDT) system. iPD corresponds to a PD controller augmented with a disturbance-observer based integral (I) action based on finite impulse response (FIR) filter. Since noise reduction requires working with as short sampling period as possible, the design of controllers is appropriate to implement in the continuous-time domain with a suitable discretization. In this way, when the noise attenuation filters are approximated by an equivalent dead time added to the plant dead time, stabilizing controllers with higher order output derivatives may be introduced simply and without excessive noise-induced control effort. Together with filtration of the reconstructed disturbance it significantly improves the overall loop performance.

Keywords: iPD control, disturbance observer, optimal tuning, performance portrait method.

1. INTRODUCTION

Disturbance reconstruction and compensation, together with uncertainty rejection, represent key concepts in control design (Chen et al., 2016). Finally, all the available method lead to design of appropriate filters. Among them, especially from the programming point of view, special features may be found in the so called finite impulse response (FIR) filters and in their application to nonlinear systems. The resulting structures are used under denotation "model free", or "intelligent PID" control D'Andrea-Novel et al. (2010); Fliess and Join (2013, 2014). This somehow "exotic" title corresponds to use of the simplest possible integral linear models with "ultra local" properties based on the 'flatness" theory, which are given in form

$$y^{(v)} = F + \alpha u \tag{1}$$

where $y^{(v)}$ with $v \ge 1$ represents an output derivative (mostly it is enough to work with v = 1 and v = 2), $\alpha \in \mathbb{R}$ is a constant parameter, frequently without a physical interpretation, here denoted as a gain of the integral model (1) K_s and F is a continuously updated process parameter merging impact of a possibly nonlinear and uncertain internal plant dynamics and external disturbances. By denoting models (1) as integral ones, we may come to similar conclusions as in feedback linearization (Isidori, 1995), which is transforming a broad class of nonlinear systems to control of integrator chains. This paper continues in exploring application of FIR filters in different control tasks (Huba and Bisták, 2017; Belai and Huba, 2017; Huba and Belai, 2017; Huba and Huba, 2018).

2. PROBLEM FORMULATION

When wishing to extend ultra-local integral models to more realistic situations, they have to include some dead time appearing always in transport of mass and information. It may also embrace equivalent dead time used for approximating more general delays, for example several shorter time constants of noise attenuation filters (e.g. by using the "half-rule" method Skogestad (2003)). As stressed in Fliess and Join (2014), problems with delays and corrupting noise which "remain one of the most irritating questions in the model-free setting, do necessitate further investigations". In this paper we will deal with optimal tuning of the so called iPD (intelligent PD) controller and its modifications with higher-order derivatives in application to a double integrator with a gain $K_s \equiv \alpha$ with an input disturbance d_i , a control signal $u(t - T_d)$ delayed by a dead time T_d and a plant output y

$$\ddot{y}(t) = K_s \left[u(t - T_d) + d_i(t - T_d) \right]$$
 (2)

To keep the description compatible with other papers on PID and disturbance observer (DO) based control, it considers $F \equiv K_s d_i$, where, similarly as F, also d_i is continuously updated and merging impact of a possibly uncertain internal feedback dynamics and external disturbances. Since it is possible to manipulate just the plant input, it is more straightforward to use d_i instead of F.

2.1 DO with integral filters

Determination of an estimate of a piecewise constant unknown disturbance d_i from a noisy measured output yand u is based on modification of (2)

$$\hat{l}_i(t - \overline{T}_d) = \ddot{y}(t)/\overline{K}_s - u(t - \overline{T}_d)$$
(3)

With \overline{K}_s and \overline{T}_d representing model parameter estimates equal ideally to K_s and T_d , by a double integration of a constant \hat{d}_i over a time interval of a length L yielding

$$\overline{d}_i = L^2 \hat{d}_i (t - T_d) = \frac{y(t) - 2y(t - L) + y(t - 2L)}{K_s} - \int_{t-L}^t u(\tau - T_d) d\tau^2$$
(4)

an improved noise attenuation in \overline{d}_i may be achieved. Since the integration represents a marginally stable operation,



Fig. 1. Simulink scheme of the iPD controller for $k = T_d/T = 11$, N = L/T = 50, $KT = K_s T^2/2$

it should be replaced by a discrete-time approximation calculated by means of always stable FIR filters. Each FIR filter is accomplishing summation on last N = L/T samples, T being the sampling period. In Simulink (Fig. 1) the required integration may be modeled by series of two FIR filters, each of them with N coefficients 1/N

$$Q_d(z) = \frac{1}{N} \sum_{i=1}^N z^{-i} = \frac{1}{N z^N} \sum_{i=0}^{N-1} z^i ; \quad N = IP(\frac{L}{T}) \quad (5)$$

 z^{-1} represent a shift operator and IP an integer part. When implemented by embedded control, such a summation of arbitrarily long sequence requires manipulation limited just to the first and last sequence samples. In controller design, $T \ll L$, or $N \gg 1$ significantly influence speed and precision of the setpoint tracking and input disturbance estimation. The question is, how they should be chosen with respect to required setpoint and input disturbance responses and noise attenuation. However, the loop performance also depends on the stabilizing control.

2.2 Stabilizing PD controller

In the simplest case, the DIPDT system may be stabilized by an ideal PD controller

$$R(s) = K_p + K_d s \tag{6}$$

Since the basic problem of continuous-time PD control is associated with obtaining the derivative of the output variable, it would seem preferable to use a discrete controller derived by replacing the output derivative of by difference of its two subsequent values. When considering a sampling period T, such a discrete time PD controller is

$$R(z) = K_P + K_D(1 - z^{-1}) = K_P + K_D \frac{z - 1}{z}$$
(7)

After expressing the derivative operator as

$$s \approx \frac{1 - z^{-1}}{T} = \frac{z - 1}{zT} \tag{8}$$

it seems that it should be possible to compare (at least for short sampling periods) the gains K_p , K_P , K_d/T and K_D .

2.3 Performance measures

For a quantitative evaluation of the speed of responses, the IAE (Integral of Absolute Error) will be used defined as

$$IAE = \int_0^\infty |e(t)| dt \; ; \; e = w - y \; ; \; w = setpoint \qquad (9)$$

With IAE_s and IAE_i corresponding to unit setpoint and input disturbance steps, the loop optimization will consider the cost function

$$IAE_{\Sigma} = IAE_s + IAE_i \tag{10}$$

Thereby, for user specified tolerable deviations from ideal shapes of the setpoint and disturbance step responses at the plant input and output $\epsilon_{ys}, \epsilon_{yi}, \epsilon_{us}, \epsilon_{ui}$ (Huba, 2013a,c) the optimization constraints will be formulated as

$$TV_0(y_s) \le \epsilon_{ys}; \ TV_1(y_i) \le \epsilon_{yi} TV_2(u_s) \le \epsilon_{us}; \ TV_2(u_d) \le \epsilon_{ui}$$
(11)

Consider a setpoint step response y_s at the output with an initial value y_{0s} and with a final value $y_{\infty s}$. The TV_0 performance measure

$$TV_0(y_s) \approx \sum_i |y_{i+1s} - y_{is}| - |y_{\infty s} - y_{0s}|$$
 (12)

is used for quantifying deviations from monotonicity. $TV_0(y_s) = 0$ for strictly monotonic (MO) response, else $TV_0(y_s) > 0$. An inversion of the required monotonic output (Huba, 2013b) leads to necessity to deal with two-pulse (2P) input shapes. These ideally consist of three MO intervals separated by two extreme points. The required 2P input with an initial value u_0 and final value u_∞ consists of three MO intervals separated by $u_{m1} = max\{u\} \notin (u_0, u_\infty), u_{m2} = min\{u\} \notin (u_0, u_\infty), (u_{m1} - u_\infty)(u_{m2} - u_\infty) < 0$. An application of $TV_0(u)$ to such intervals of responses u_s , or u_i yields

$$TV_2(u) = \sum_i |u_{i+1} - u_i| - |2u_{m1} - 2u_{m2} + u_\infty - u_0|$$
(13)

 $TV_2(u) = 0$ just for strictly 2P response, else $TV_2(u) > 0$. For the sake of simplicity we will denote as MO output disturbance responses those characterized by $TV_1(y_2) = 0$, i.e. responses returning from an initial deviation caused by a delayed feedback monotonically to zero (Huba, 2013c). Introduction of the shape related constraints (11) is important for a consideration of technological requirements as e.g. reduction of oscillations, generated noise, dissipated heat, superfluous control effort, actuator wear, etc. (Huba, 2013c). However, at the same time, by systematically building on a basic mathematical concept of a monotonicity it consequently results in improved robust control design methods. Thereby, when wishing to define an *optimal* control as something unique, it should be characterized by an ideal situation with no deviations, i.e. with

$$\epsilon = \epsilon_{ys} = \epsilon_{yi} = \epsilon_{us} = \epsilon_{ui} \to 0 \tag{14}$$

But, with respect to the always limited precision of control and computer simulations, some "sufficiently" small $\epsilon = \epsilon_{ys} = \epsilon_{yi} = \epsilon_{us} = \epsilon_{ui} = 0.001$ will be chosen instead.

3. CONTROLLER TUNING

3.1 Continuous-time-domain based PD tuning

Continuous PD control yields a setpoint response

$$F_w(s) = \frac{Y(s)}{W(s)} = \frac{K_s(K_d s + K_p)}{e^{T_d s} s^2 + K_s(K_p + K_d s)}$$
(15)

with the characteristic polynomial

$$A_{PD}(s) = e^{T_d s} s^2 + K_s (K_p + K_d s)$$
(16)

The fastest possible non-oscillatory transients correspond to a triple-real-dominant-pole (TRDP) tuning fulfilling

$$\left[A_{PD}(s); \ \frac{dA_{PD}(s)}{ds}; \ \frac{dA_{PD}^2(s)}{ds^2}\right]_{s=s_o} = \mathbf{0}$$
(17)

from $dA_{PD}^2(s)/ds^2 = 0$ it follows

$$s_o = E/T_d; \ E = \sqrt{2} - 2$$
 (18)
 $e^E(10\sqrt{2} - 14) \quad 0.079$

$$K_{p0} = \frac{(E_{0} - E_{1})}{K_{s} T_{d}^{2}} = \frac{1}{K_{s} T_{d}^{2}}$$

$$K_{d0} = -\frac{E e^{E}}{K_{s} T_{d}} = \frac{0.461}{K_{s} T_{d}}$$
(19)

When expressed in terms of dimensionless coefficients

$$\kappa_0 = K_{p0} K_s T_d^2 = 0.079; \ \delta_0 = K_{d0} K_s T_d = 0.461$$
 (20)

3.2 Simplified continuous-time iPD tuning

A continuous time iPD tuning may be derived by considering PD (6) extended by DO based on integral filters

$$F_I(s) = (1 - e^{-sL})/(sL)$$
 (21)

yielding a closed-loop transfer functions R(s)F(s)S(s)

$$F_w(s) = \frac{R(s)F_u(s)S(s)}{1 + R(s)F_u(s)S(s)(1 + \frac{1 - 2e^{-sL} + e^{-2sL}}{K_sL^2R(s)})}$$
$$F_i(s) = \frac{S(s)}{1 + R(s)F_u(s)S(s)(1 + \frac{1 - 2e^{-sL} + e^{-2sL}}{K_sL^2R(s)})}$$
(22)

with

$$F_u = \frac{1}{1 - F_I^2(s)e^{-T_d s}} = \frac{s^2 L^2 e^{T_d s}}{s^2 L^2 e^{T_d s} - 1 + 2e^{-sL} - e^{-2sL}}$$
(23)

After some manipulation

$$F_w(s) = \frac{R(s)F_u(s)S(s)}{1 + R(s)F_u(s)S(s)(1 + \frac{1 - 2e^{-sL} + e^{-2sL}}{K_sL^2R(s)})}$$
$$F_i(s) = \frac{S(s)}{1 + R(s)F_u(s)S(s)(1 + \frac{1 - 2e^{-sL} + e^{-2sL}}{K_sL^2R(s)})}$$
(24)

Finally, we get $F_w(s)$ (15) and

$$F_{i}(s) = \frac{S(s)}{1 + R(s)F_{u}(s)S(s)(1 + \frac{1 - 2e^{-sL} + e^{-2sL}}{K_{s}L^{2}R(s)})} = \frac{K_{s}e^{-T_{d}s}(s^{2}L^{2}e^{T_{d}s} - 1 + 2e^{-sL} - e^{-2sL})}{s^{2}L^{2}(e^{T_{d}s}s^{2} + K_{s}(K_{p} + K_{d}s))}$$
(25)

By considering

$$e^{-sL} \approx 1 - sL + \frac{s^2L^2}{2}; \ e^{-T_ds} \approx 1 - T_ds;$$

 $e^{-2sL} \approx 1 - 2sL + \frac{4s^2L^2}{2}$ (26)

$$F_i(s) \approx \frac{K_s T_d s}{e^{T_d s} s^2 + K_s (K_p + K_d s)}$$
(27)

It means that the "optimal" tuning (19) dominates also in the iPD loop with DO based on integral low-pass filters.

4. DISCRETE-TIME PD CONTROLLER TUNING

For the zero-order-hold equivalent discrete-time plant

$$S(z) = \frac{K_s T^2}{2} \frac{z+1}{(z-1)^2}$$
(28)

it may again be useful to introduced normed coefficients $\kappa = K_P K_s T^2/2; \ \delta = K_D K_s T^2/2$ (29)

The corresponding closed loop transfer function is

$$F_w(z) = \frac{\kappa z + \delta(z-1)}{z^{k+1}(z-1)^2 + (\kappa z + \delta(z-1))(z+1)}$$
(30)

The first problem is that for the discrete-time system it is not possible to express the optimal values of the controller parameters analytically (e.g. by the multiple real dominant pole method). Such a solution exists just for a discretetime state controller (Huba et al., 1998), which again needs measurement, or reconstruction of the output derivative. Under assumption of mutual convergence of continuous and discrete time controllers for short sampling periods the proportional gains expressed as $K_p = \kappa_0/(K_s k^2 T^2)$, or $K_P = 2\kappa/(K_s T^2)$ and the derivative gains K_d/T and K_D yield equivalences of optimal tunings

$$\kappa = \kappa_0 / (2k^2); \ \delta = \delta_0 / (2k) \tag{31}$$

4.1 Optimal loop tuning by the PPM

Similarly as in Huba (2013c,a,d), in absence of analytical approaches, optimal controller tuning may be determined by the performance portrait method (PPM). It is based on mapping the loop performance for all relevant loop parameter values and then looking for an inverse relation between the required loop performance and the corresponding loop parameters. In a general case, as for example, robustness analysis of systems with uncertain interval parameters(Huba, 2013c), a performance portrait (PP) has to be generated by a loop simulation with some plant parameters (e.g. $T = 1, K_s = 1$ and an identified loop delay $T_d = kT$) and a disturbance observer FIR filter parameter N. After being evaluated in terms of the performance measures from Section 2.3, the results may be stored in a 4D matrix of dimensionless loop parameters

$$\kappa = K_P K_s T^2 / 2 \in \mathcal{K} = [\kappa_{min}, \kappa_{max}]$$

$$\delta = K_D K_s T^2 / 2 \in \mathcal{D} = [\delta_{min}, \delta_{max}]$$

$$N = L/T \in \mathcal{N} = [N_{min}, N_{max}]$$

$$k = T_d / T \in \mathcal{T} = [k_{min}, k_{max}]$$
(32)

and used to tune controllers with a specified performance. To analyze optimal discrete-time PD control (7), it is enough to use a 2D PP generated for a chosen $T_d = kT$ with two variable parameters κ and δ (Fig. 2). Comparison of the tuning (31) resulting from discretization of the continuous-time control according to (8) shows that the experimentally found optimal tuning does not converge to the equivalence-based values even for $T << T_d$. It may also be documented by the setpoint step responses in Fig. 3: after the discretization (31), the corresponding transients (TRDP-d) show significant overshooting, whereas the original continuous-time output responses (TRDP-c), or the discrete-time solutions derived in the state-space in Huba et al. (1998) (TRDP-ss) are monotonic, but much slower than those corresponding to optimal PPM based tuning.

4.2 Equable control signal shapes

Step responses in Fig. 3 document also another important feature of the discrete-time PD control with output derivative replaced by difference: after a short and high initial pulse of the control signal it continues with much



Fig. 2. 2D performance portrait for $T_d = k = 20$ (T = 1) with an analytically derived optimal working point TRDP-d (31)and working point derived by PPM for $\epsilon = 0.001$

lower amplitudes during the rest of the transient at the output. This could lead to problems with control constraints. Elimination of peaks from the control signal and acceleration of the transients can be achieved by using regulators with higher derivatives. For example, for the proportional-derivative-accelerative (PDA) controller

$$R_{PDA}(s) = K_p + K_d s + K_{d2} s^2$$
(33)

the multiple real dominant pole method yields

$$\kappa_0 = K_{p0} K_s T_d^2 = 0.210; \ \delta_0 = K_{d0} K_s T_d = 0.784; \alpha_0 = K_{d20} K_s = 0.206$$
(34)

A controller with 3rd order output derivative denoted as

$$R^{3}(s) = K_{p} + K_{d}s + K_{d2}s^{2} + K_{d3}s^{3}$$
(35)

introducing also feedback from jerk yields optimal values

$$\kappa_0 = K_{p0}K_sT_d^2 = 0.361; \ \delta_0 = K_{d0}K_sT_d = 1.083; \alpha_0 = K_{d20}K_s = 0.406; \ \gamma_0 = K_{d30}K_s/T_d = 0.045$$
(36)

Comparison with PD control (20) shows a significant increase of K_p with increasing m and decrease of the peaks in u(t) with more even coupling of signals at the input and output of the system. Whereas the PDA controller may yet use signals measured on the plant (or its model), for achieving the 3rd output derivative \ddot{y} it is already always necessary to propose an appropriate reconstruction.

4.3 Filtration aspects

Above analysis has shown that the frequently applied controller discretization by replacing output derivative according to its mathematical definition by the output difference ideally leads to decreased IAE values. However, we do not know to express the optimal controller tuning analytically and the course of the control action is very uneven. Therefore, to avoid these handicaps and to in-



Fig. 3. Transients corresponding to analytically derived optimal continuous time control TRDP-c (19) with a measured output derivative, to discrete-time control with tuning TRDP-d (31), to discrete-time control derived in the state space TRDP-ss, to controller derived by PPM for $\epsilon = 0.001$, to PDA controller with measured output, output velocity and acceleration signals and to PD₄³ control ($T_e = nT_f$) with reconstruction of first, second and third order derivatives.





troduce noise filtration for all possible output derivatives (including the 0th one), the loop implementation will be based on augmenting the ideal PD controller (6) (Fig. 4), or controllers with mth order derivatives, with filters

 $Q_n^m(s) = s^m/(1 + T_f s)^n; n \ge m, m = 0, 1, 2, ...$ (37) Their dynamics may be considered by specifying an equivalent delay T_e corresponding to their common part¹

$$Q_n(s) = 1/(1 + T_f s)^n; \ n \ge 1$$
(38)

 $^{^1~}$ by an appropriate implementation, all the required output derivatives may be achieved by a single filter with $P_n=(1+T_fs)^n$



Fig. 5. Transients corresponding to PD control TRDPc (19) with filtered output and its derivative, to discrete-time control with tuning TRDP-d (31), to discrete-time control derived in the state space TRDP-ss, to non-filtered PD control tuned by PPM for $\epsilon = 0.001$, to PDA controller with filtered output, its derivative and 2nd derivative and PD₄³ control $(T_e = nT_f)$ with filtered output and its first, second and third order derivatives.

and by adding this delay to the identified plant model delay T_m , which constitutes the total loop delay T_d (additivity of several dead time elements) as

$$T_d = T_m + T_e \tag{39}$$

 T_e may be approximated by a modified "half rule" (Skogestad, 2003)

$$T_e = n\tau; \ \tau \in [T_f/2, T_f] \tag{40}$$

(with $\tau = T_f/2$ applicable roughly for $nT_f < T_m/2$), or by equivalent delays introduced in Huba (2013b). Transients in Fig. 5 demonstrate that use of PD_4^3 control based on 4th order filters with the time constant $T_f = 4T$ (included into the design by means of the equivalent delay (39), (40) with $\tau = T_f$) enables to speed up the transients with respect to the PD control based on measured output derivative and still to keep a moderate impact of the measurement noise simulated in Matlab/Simulink by Uniform Random Number block with amplitude 1% of w. PPM based discrete-time PD control not only shows high initial pulse with non-homogenous shape of transients, but also very high noise impact. Therefore, in evaluation of complete controller with disturbance reconstruction and compensation we will continue just with quasi-continuous controllers derived by the multiple real dominant pole approach and output derivatives according to (37).

4.4 Discussion

Setpoint and disturbance step responses of three modifications of iPD control with different derivative degrees in



Fig. 6. Setpoint and disturbance step responses corresponding to iPD control with tuning TRDP-c (19), to iPDA control (33) and iPD₄³ control (35), all with output and its derivatives filtered according to (37), $n = 4, T_f = 4T, T_e = nT_f, T_d = 20, L = 30, T = 1$

Figs 6-7 illustrate impact of the integration length L = NTon loop performance: shorter N = 10 corresponds to faster disturbance response than for N = 30, however, on cost of an increased noise impact. Thereby, the setpoint responses do not change more significantly and, with respect to the derivative degree m, the noise impact is not straightforward: for iPDA control is not so high as for iPD, or iPD₄³. Furthermore, use of controllers with higher order derivatives decreases peaks in the control signal responses



Fig. 7. Setpoint and disturbance step responses corresponding to iPD control with tuning TRDP-c (19), to iPDA control (33) and iPD₄³ control (35), all with output and its derivatives filtered according to (37), $n = 4, T_f = 4T, T_e = nT_f, T_d = 20, L = 10, T = 1$

and the control signal is distributed more evenly over the duration of the transients at the output.

5. CONCLUSIONS

The paper discussed basic problems in designing iPD controllers with integral action based on use of FIR filters for time delayed double integrators under noise impact. Thereby, it established a framework for their more detailed investigation. In the next step, several available solutions to disturbance reconstruction and rejection will be compared in terms of speed of transients, noise attenuation, robustness and control constraints - aspects, which are important for majority of mechatronic applications.

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