Min-max Operation Optimization of Renewable Energy Combined Cooling, Heating, and Power Systems Based on Model Predictive Control

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Abstract: The renewable energy combined cooling, heating, and power (CCHP) systems can effectively utilize the residual heat generated by the system to provide thermal energy or cooling energy for users, which can highly improve the utilization efficiency of primary energy. However, the prediction error of renewable energy sources (RES) and load can affect the optimal operation of the system. This paper considers the prediction error of RES and load. According to the dynamic characteristics of energy storage unit, an energy optimization model with prediction error is proposed. On this basis, a min-max optimization operation strategy based on model predictive control (MPC) is proposed. The operating cost of the system is taken as the objective function. By optimizing the output of each device in the system, the operating cost of the system is minimized under the maximum prediction error.

Keywords: Renewable energy CCHP systems, Prediction error, Min-max optimization, Model predictive control

1. INTRODUCTION

Energy shortages, low energy efficiency, and serious emissions pollution are energy-related problems that need to be solved at this stage. The renewable energy combined cooling, heating, and power (CCHP) systems provide a viable solution to solving energy problems which as the name states, can provide cooling, heating and power for users at the same time (Wu et al. 2006). The systems can realize the waste heat recovery of the system, improve energy utilization, and has been widely implemented in hospitals, supermarkets and schools (Arcuri, P. et al. 2007). It has gradually become a research hotspot.

There has been much research on optimizing operation of renewable energy CCHP systems using different strategies and optimization methods. (Fang, F. et al. 2012) proposed an optimal operational strategy depending on an integrated performance criterion (IPC) to improve the comprehensive operational performance. The optimal operational strategy based on two typical operating modes, i.e., following the electric load (FEL) and following the thermal load (FTL). (GUO, L. et al. 2009) considered the influence on economic operation of price of electricity from public grid, price of fuel and price of the electricity sold to grid, a new energy optimization and management model of combined cooling and power (CCP) distributed energy supply system is proposed. (WANG, Y.W. et al. 2012) establishes multi-objective optimization model with primary energy saving ratio, net present value and carbon dioxide emission as objective function, main components capacities as decision variables. The optimal system schemes are gained by real-coded genetic algorithm under following electricity and following thermal operation strategy separately. A single objective function with the lowest operating cost and a multi-objective function with the largest energy saving rate and carbon dioxide emission reduction rate are established to research system optimization operation mode (DAI X.Y. et al. 2017).

In order to get a better operation strategy, the model predictive control has been successfully applied to the combined heating, and power (CHP) system and the CCHP micro grid system, and has achieved certain results (Houwing, M. et al. 2011, G. K. H. Larsen. et al. 2014). Model Predictive Control technique has been applied to take system uncertainties into account and the micro grid operations optimization problem has been formulated using Mixed-Integer Linear Programming (Gambino, G. et al. 2014). (Luo, Z. et al. 2017) considers the volatility of renewable energy and various load demands, proposes a two-stage coordinated control strategy divided into economic dispatching stage and real-time adjusting stage. In economic dispatching stage, it utilizes a model predictive control incorporating piecewise linear efficiency curves to schedule the operation based on the forecast information. In real-time adjusting stage to tackle the power fluctuations based on the real-time information. (Hou, W.M. et al 2009) propose a decentralised controller based on a model predictive control (MPC) strategy and the performance of the MPC-controlled PEMFC system is illustrated under different conditions regarding energy pricing, domestic energy demand, and system configuration.

In the above research, the application of MPC reduces the impact of the prediction error of renewable energy and load on the optimal operation of the system, but the prediction error of renewable energy and load still exists. In order to improve the robustness of the system, this paper proposes the min-max optimization strategy based on model predictive control, the aim is to minimize the cost of the system with the maximum error.
The rest of the paper is organized as follows. Section II describes the system in detail. Section III introduces the min-max optimization of the system based on MPC. Section IV briefly introduces how to solve the min-max optimization problem. Section V gives the case studies and simulation results. We end the paper with some conclusions.

2. SYSTEM DESCRIPTION

In this section, a detailed description of the renewable energy CCHP systems structure is given. An energy optimization model with errors is established based on the energy flow of the system. Because of the existence of energy storage unit in the system, the dynamic characteristics of the energy storage unit are also analyzed.

2.1 Renewable energy CCHP Systems

The renewable energy CCHP systems in this paper include renewable energy sources (RES), power generation unit (PGU), waste heat recovery device, gas boiler, electric chiller and absorption chiller. In order to reduce energy waste, some energy storage equipment such as battery and thermal storage tank is also added. The renewable energy CCHP systems that can provide three form of energies (electric energy, thermal energy and cooling energy) simultaneously. Fig. 1 shows the structure of the renewable energy CCHP systems.

Fig. 1. Structure of the renewable energy CCHP systems

2.2 Energy flow analysis

In order to research the optimal operation strategy, the energy flow of the renewable energy CCHP systems is analyzed first. In the source and load there is an error between the predicted and actual values, this paper establishes the energy model of the system with the error. The electricity energy balance in the system at time \( t \) can be expressed as:

\[
E_{PGU}(t) + E_{grid}(t) + E_{sG}(t) + E_{ew}(t) + E^2_{ew}(t) = E_{ew}(t) + E_L(t) + E^1_{ew}(t)
\]  

where \( E_{PGU}(t) \), \( E_{grid}(t) \), \( E_{sG}(t) \), \( E_{ew}(t) \) and \( E_L(t) \) represent the electricity generated by the PGU, electricity purchased from the grid, the power generation of the battery, the power generation of photovoltaic and wind power, electricity consumed by the electric chiller and electricity load of buildings at time \( t \), respectively. \( E^2_{ew}(t) \) and \( E^1_{ew}(t) \) represent the prediction error of renewable energy source and the prediction error of electricity load, respectively. \( E_{PGU}(t) \) can be represented as:

\[
E_{PGU}(t) = G_{PGU}(t) \times \eta_{el} \times \eta_{th}
\]

where the \( G_{PGU}(t) \), is the natural gas consumption of the PGU at time \( t \), \( \eta_{el} \) and \( \eta_{th} \) represent the electrical efficiency and thermal efficiency of the PGU, respectively.

The thermal energy balance in the renewable energy CCHP systems at time \( t \) can be expressed as:

\[
H_{ew}(t) + H_b(t) + H_{st}(t) = H_{ab}(t) + H_L(t) + H_{ew}(t)
\]

where \( H_{ew}(t) \), \( H_b(t) \), \( H_{st}(t) \), \( H_{ab}(t) \) and \( H_L(t) \) represent heat from waste heat recovery unit, heat generated by the boiler, heat from thermal storage tank, heat consumed by absorption chiller and hot load at time \( t \), respectively. \( H_{ew}(t) \) represent the prediction error of hot load. \( H_{ew}(t) \) can be expressed as:

\[
H_{ew}(t) = \left[H_f(t) + H_s(t)\right] \times \eta_{che}
\]

where \( H_f(t) \) and \( H_s(t) \) represent waste heat of jacket water and waste heat of exhaust at time \( t \), respectively. \( \eta_{che} \) represent the conversion efficiency of the heat recovery device. \( H_f(t) \) and \( H_s(t) \) can be expressed as:

\[
H_f(t) = G_{PGU}(t) \times (1 - \eta_{el} \eta_{th}) \times f_j
\]

\[
H_s(t) = G_{PGU}(t) \times (1 - \eta_{el} \eta_{th}) \times f_e
\]

where \( f_j \) and \( f_e \) represent the ratio of the heat of the jacket water and the heat of the exhaust to the total waste heat of the PGU.

The heat generated by the boiler \( H_b(t) \) can be expressed as:

\[
H_b(t) = G_b(t) \times \eta_b
\]

where \( \eta_b \) represent the thermal efficiency of the gas boiler, \( G_b(t) \) represent the natural gas consumption of the gas boiler.

The cooling energy balance at time \( t \) can be expressed as:
\[ C_{ec}(t) + C_{ab}(t) = C_L(t) + C_{err}(t) \] (8)

where \( C_{ec}(t) \), \( C_{ab}(t) \) and \( C_L(t) \) respectively represent cooling energy generated by electric chiller, cooling energy generated by absorption chiller and the cold load at time \( t \). \( C_{err}(t) \) represent the prediction error of the cooling energy. \( C_{ec}(t) \) and \( C_{ab}(t) \) can be expressed as:

\[ C_{ec}(t) = E_{ec}(t) \times COP_{ec} \] (9)
\[ C_{ab}(t) = H_{ab}(t) \times COP_{ab} \] (10)

where \( E_{ec}(t) \) and \( H_{ab}(t) \) respectively represent power consumption of the electric chiller and heat consumption of the absorption chiller at time \( t \). \( COP_{ec} \) and \( COP_{ab} \) are the coefficient of performance of the electric chiller and absorption chiller, respectively and are fixed in this study.

### 2.3 Storage dynamics

The energy storage unit is an important component of the renewable energy CCHP systems. The energy storage unit can regulate intermittent renewable energy source and peak power loads and it is especially helpful in matching supply and demand over a 24-hour period of time. The energy storage unit in this renewable energy CCHP systems mainly include battery and thermal energy tank. In what follows a storage dynamics model for electrical storage systems and thermal energy tank is described.

The electric energy of the battery at time \( (t+1) \) can be expressed as:

\[ x_e(t+1) = (1-\tau) \times x_e(t) - \eta_{se} \times E_{se}(t) \] (11)

where \( x_e(t) \) represent electric energy of the battery at time \( t \). \( \tau \) is the self-discharge coefficient of the battery, \( \eta_{se} \) is the charge/discharge rate. The level of the electric energy of the battery at time \( t \) should meet the following constraints

\[ x_e(t) \leq x_{max}^e(t) \] (12)
\[ E_{min}^{se} \leq E_{se}(t) \leq E_{max}^{se} \] (13)

where \( x_{max}^e(t) \) is the maximum capacity of the battery. \( E_{min}^{se} \) and \( E_{max}^{se} \) are upper and lower limits of battery charge and discharge power.

The dynamic model of the thermal storage tank is similar to that of the battery, that can be formulated as follows:

\[ x_h(t+1) = (1-\mu) \times x_h(t) - \eta \times H_{sh}(t) \] (14)
\[ x_h(t) \leq x_{max}^h(t) \] (15)

where \( x_h(t) \) is the thermal energy storage of the thermal storage tank. \( \mu \) and \( \eta \) represent the self-discharge coefficient and charge/discharge rate of the thermal storage tank, respectively. \( x_{max}^h(t) \) is the maximum capacity of the thermal storage tank. \( H_{shmin} \) and \( H_{shmax} \) are the constraints of thermal storage power.

### 3. MIN-MAX OPERATION OPTIMIZATION BASED ON MPC

Model predictive control is an optimal control method that converts an infinite long optimization into a limited long optimization at each sample time, using the receding horizon strategy and considering the dynamic performance of the system, the control objectives and the constraints. In this section, MPC is presented to deal with the operation optimization of the renewable energy CCHP systems. Because there is an error between the prediction data (include prediction of renewable energy source and prediction of load) and the actual data. An operation optimization model with prediction error is established, the operating cost of the system is taken as the objective function, and the MPC-based min-max optimization method is used to determine the output of each device. The optimal control makes the system still have the minimum cost in the case of maximum prediction error. The optimization block diagram is shown in Fig. 2.

![Optimization block diagram](image)

**Fig. 2 Optimization block diagram**

### 3.1 Determination of Prediction Error Range

The above optimization model includes the prediction error of renewable energy and load. The min-max optimization method needs to first determine the range of error (include the prediction error of renewable energy \( E^2_{err} \), the prediction error of electric load \( E_{err}^l \), the prediction error of thermal load \( H_{err} \) and the prediction error of cooling load \( C_{err} \). The
method of determination is as follows:

The ARIMA model is used to predict the data of renewable energy. The predicted value and actual value of the historical data are used to obtain the error range of the ARIMA model for renewable energy prediction. The predicted value of the load is obtained by the grey model. The error range of the grey model for the load forecast is also obtained by comparing the predicted value with the actual value of the historical data.

3.2 Rolling Optimization

Rolling optimization is the main feature of model predictive control. In the rolling part, prediction and optimization are conducted in each time interval. The aim of the optimization is to minimize the total operation costs in the case of maximum prediction error over a fixed future time horizon. Before the \( t + 1 \) time interval, the optimization problem is solved based on predictions of RES and load over the future time horizon (from the \( t + 1 \) time to the \( t + P \) time interval), but only the instruction of the \( t + 1 \) time is implemented. In the next interval, the time horizon moves forward by one-time interval as shown in Fig. 3.

Fig. 3 Rolling optimization time window

- **Objective function:** The operating cost of the renewable energy CCHP systems is taken as the objective function. The objective function can be expressed as follows:

\[
\text{MIN MAX } J = \sum_{t=0}^{kM} \left[ P_{\text{gas}} (G_{\text{PGU}}(t) + G_{\text{J}}(t)) + P_{\text{grid}} (t) E_{\text{grid}} (t) \right] \tag{17}
\]

where \( P_{\text{gas}} \) represent gas price and \( P_{\text{grid}} \) represent time-of-use electricity price. \( M \) is the optimization horizon.

- **constraints:** In addition to satisfying the constraints of (1) - (16), the optimization objectives must satisfy the following inequality constraints in consideration of the actual operation of the system:

\[
0 \leq E_{\text{PGU}} (t) \leq E_{\text{PGU max}} \tag{18}
\]

\[
0 \leq E_{\text{gr}} (t) \leq E_{\text{gr max}} \tag{19}
\]

\[
0 \leq H_{\text{h}} (t) \leq H_{\text{h max}} \tag{20}
\]

\[
0 \leq C_{ab} (t) \leq C_{ab max} (t) \tag{21}
\]

\[
0 \leq C_{ac} (t) \leq C_{ac max} (t) \tag{22}
\]

4. SOLUTION TO THE MIN-MAX OPTIMIZATION PROBLEM

4.1 Processing method of min-max problem

Considering the following simple min-max problem. The system is described as:

\[
x_{k+1} = x_k + u_k + w_k \tag{23}
\]

The optimized objective function is expressed as:

\[
\text{MIN MAX } J_k = \max \left( x_{j,k} , u_{j,k} , \ldots , x_{k+N-\overline{1},k} , u_{k+N-\overline{1},k} \right) \tag{24}
\]

\[
\text{s.t. } x_{j,k} \in X , u_{j,k} \in U , w_{j,k} \in W \tag{25}
\]

where \( u \), and \( x \) represent the optimized input and the state, respectively. \( w \) is the disturbances. \( X \) and \( U \) are the constraints of the \( x \) and \( u \). \( l \) represents the objective function of \( x \), \( u \) and \( w \).

Define \( t \) as the upper bound of the objective function \( J_k \), for \( \forall w \in W \) make \( J_k \leq t \) established, so the optimization problem can be expressed as:

\[
\text{MIN } t
\]

\[
\text{s.t. } l (x_{j,k}^{(i)} , u_{j,k}^{(i)} , \ldots , x_{k+N-\overline{1},k}^{(i)} , u_{k+N-\overline{1},k}^{(i)}) \leq t , \forall w \in W
\]

\[
u_{k+j,k}^{(i)} \in U
\]

\[
x_{j,k}^{(i)} \in X
\]

From (Scokaert, P.O.M. et al. 1998) , if the function \( J_k \) is linear or convex about \( w \), the simple min-max problem can be solved by the following ways:

Select the vertices of \( W \) and substitute it into the objective function \( J_k \) to get the maximum value \( \text{MAX } J_k \) in the presence of disturbance. Then get the optimal value of \( t \) by optimizing the input \( u \).

4.2 Conversion of objective function

In this paper, considered the objective function (17), and the energy model, we can get the following conversion formula:

\[
J(t) = P_{\text{gas}} (t) \left( \frac{E_{\text{PGU}} (t)}{\eta_{in} \eta_{th}} \right) + \frac{H_{h} (t)}{\eta_{h}} + P_{\text{grid}} (t) E_{\text{grid}} (t) \tag{26}
\]
where the \( E_{PGU}(t) \) and \( H_b(t) \) can be expressed as follows:

\[
E_{PGU}(t) = \frac{C_{ec}}{\eta_{P}} + E_l(t) - E_{grid}(t) - E_{SE}(t)
\]

\[
H_b(t) = \frac{C_{bL}(t) - C_{ec}(t) + C_{err}(t)}{\eta_{P}} + H_l(t) + H_{err}(t) - H_{ext}(t) - H_{SH}(t)
\]

\( H_{ext}(t) \) can be expressed as:

\[
H_{ext}(t) = \frac{E_{PGU}(t)}{\eta_{P}} (1 - \eta_{P} \eta_{SH})(f_j + f_c) \eta_{char}
\]

\( C_{ec}(t) \), \( E_{grid}(t) \), \( H_{SH}(t) \) and \( E_{SE}(t) \) are selected as optimization variables, and the min-max optimization problem can be coded by YALMIP.

4.3 Multiple Time Horizon Model of Storage Unit

The energy storage equipment in the system have the dynamic characteristics and according to the formula (11), (14), the model of storage unit at the prediction horizon \( P \) and the optimization horizon \( M \) can be expressed as follows:

The model of the battery:

\[
\begin{align*}
x_{e}(t+1) &= ax_{e}(t) - \eta_{ba}E_{SE}(t), \\
x_{e}(t+2) &= a^2x_{e}(t) - a\eta_{ba}E_{SE}(t) - \eta_{ba}E_{SE}(t+1), \\
&\quad \vdots \\
x_{e}(t+P) &= a^P x_{e}(t) - a^{P-1}\eta_{ba}E_{SE}(t) - \ldots \\
&\quad - \left( a^{P-M}\eta_{ba} + \ldots + \eta_{ba}\right)E_{SE}(t+M-1)
\end{align*}
\]

Expressed:

\[
X_{e}(t) = Ax_{e}(t) - G_{e}E_{SE}(t)
\]

where \( X_{e}(t) = \begin{bmatrix} x_{e}(t+1) \\ x_{e}(t+2) \\ \vdots \\ x_{e}(t+P) \end{bmatrix}, E_{SE}(t) = \begin{bmatrix} E_{SE}(t) \\ \vdots \\ E_{SE}(t+M-1) \end{bmatrix} \)

\[
A = \begin{bmatrix} a \\ \vdots \\ a^P \end{bmatrix}, \quad G_{e} = \begin{bmatrix} \eta_{ba} & 0 & 0 \\ \vdots & \vdots & \vdots \\ a^{P-1}\eta_{ba} & \ldots & \eta_{ba} \\ a^{P-M}\eta_{ba} & \ldots & \sum_{i=0}^{P-M} a^{i}\eta_{ba} \end{bmatrix}
\]

where \( a = 1 - \tau \). The model of the thermal storage tank is similar to the battery model and will not be described here.

5. CASE STUDY

To verify the performance of the proposed algorithm, a hypothetical CCHP building in Jinan, China, was used for case studies. The equipment capacity of renewable energy CCHP systems is shown in Table 1. The parameters such as COP of chiller is shown in Table 2. The natural gas price and time-of-use electricity price are given in Table 3. For MPC the rolling time is 15min and the optimization time is 1h.

### Table 1. Equipment capacity

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Value (KW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGU</td>
<td>200</td>
</tr>
<tr>
<td>Gas boiler</td>
<td>50</td>
</tr>
<tr>
<td>Absorption chiller</td>
<td>300</td>
</tr>
<tr>
<td>Electric chiller</td>
<td>250</td>
</tr>
<tr>
<td>Wind power</td>
<td>55</td>
</tr>
<tr>
<td>Battery</td>
<td>( x_e :300, E_{SE}^{max} :240(kw/h) )</td>
</tr>
<tr>
<td>Thermal tank</td>
<td>( x_b :120, E_{SE}^{max} :120(kw/h) )</td>
</tr>
</tbody>
</table>

### Table 2. Equipment performance parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>COP of Absorption chiller</td>
<td>0.7</td>
</tr>
<tr>
<td>electrical efficiency of the PGU</td>
<td>0.88</td>
</tr>
<tr>
<td>thermal efficiency of the PGU</td>
<td>0.3</td>
</tr>
<tr>
<td>COP of Electric chiller</td>
<td>4</td>
</tr>
<tr>
<td>Efficiency of gas boiler</td>
<td>0.82</td>
</tr>
<tr>
<td>Efficiency of heat recovery</td>
<td>0.8</td>
</tr>
</tbody>
</table>

### Table 3. Gas price and time-of-use electricity price

<table>
<thead>
<tr>
<th>Time</th>
<th>Electricity price (¥/kWh)</th>
<th>Gas price (¥/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00-14:00</td>
<td>1.070</td>
<td>0.315</td>
</tr>
<tr>
<td>14:00-18:00</td>
<td>0.687</td>
<td></td>
</tr>
<tr>
<td>23:00-7:00</td>
<td>0.260</td>
<td></td>
</tr>
</tbody>
</table>

In this case study, the error range of the renewable energy and load forecasting models obtained by statistics is shown in Table 4.

### Table 4 Error range of predicted model

<table>
<thead>
<tr>
<th>Items</th>
<th>Range(kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renewable energy / ( E_{err}^{2} )</td>
<td>[-2.2]</td>
</tr>
<tr>
<td>Electricity load / ( E_{err}^{1} )</td>
<td>[-10,10]</td>
</tr>
<tr>
<td>Cooling load / ( C_{err}^{1} )</td>
<td>[-12,12]</td>
</tr>
<tr>
<td>Thermal load / ( H_{err} )</td>
<td>[-2.2]</td>
</tr>
</tbody>
</table>
The optimization results of the case study are shown in Fig. 4-6.

Fig. 4 Electrical optimization result

Fig. 5 Cooling optimization result

Fig. 6 Thermal optimization result

6. CONCLUSIONS

In this paper, a min-max optimal operation of renewable energy CCHP systems based on MPC is proposed. The operating cost of the system is taken as the objective function and the dynamic characteristics of energy storage unit are considered. By optimizing the output of each device in the system, the operating cost of the system is minimized under the maximum prediction error.

A hypothetical renewable energy CCHP building in Jinan, China, was used for simulation. This optimization strategy can effectively improve the robustness of the system and reduce the impact of uncertainties in renewable energy and load on the system.

ACKNOWLEDGMENT

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