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Skyhook Controller Design using Bilinear Matrix Inequalities

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Abstract: This paper focuses on the design of the so called sky-hook controller, which is used to isolate vibrations on suspension systems. The design of the sky-hook controller is posed as a single input-single output static output feedback control problem. The design of the sky-hook controller is posed using the generalized plant with the sparse structure of the sky-hook controller. It is shown that, the root-locus plot for visualization and some of the w4ell known stability analysis methods can be utilized to acquire a stability interval for the sky-hook controller. By gridding the stability interval, it has been shown that there may exist convex sub intervals, and posing a BMI problem with the corresponding D region, it is possible to solve the skyhook design problem, with regard to $\mathscr{H}_2$ or $\mathscr{H}_\infty$ optimality. The sky-hook design is simulated using two different suspension systems and an experiment is carried out on a system for three different road profiles. It has been sown that using a sky-hook controller instead of an LQR controller is plausible, since the number of required sensors is reduced, therefore the cost is reduced and the performance is almost equal for both controllers.

Keywords: Disturbance rejection(linear case), Robust Control, Output feedback(linear case), Sky-hook control, $\mathscr{H}_2$ optimal control, $\mathscr{H}_\infty$ optimal control, Active suspension systems, Quarter car model

1. INTRODUCTION

Many industrial applications benefit from the advantages of robust control techniques, especially where sustaining the performance of the control system is important under disturbances and changes in physical parameters, due to wearing, temperature changes, pressure changes, etc. For linear robust control applications, a good benchmark problem is the control of an active suspension system using a quarter car model, where the aim is to minimize the effect of the road disturbance to the passenger. This benchmark problem is well studied in the literature, e.g. with LQR control Choi et al. (1998), Sam et al. (2000), with robust control Ma and Chen (2006), Guo and Zhang (2012), Wang and Wilson (2001), with fuzzy logic Li et al. (2012), Li et al. (2011), with PID control Erol and Delibaşı (2018). Most robust control techniques provide full state feedback controllers or high order controllers, but there are also reduced order controller design approaches and low level controller design methods. The complexity and the realizability of the controller is important for a quarter car model, since in the industrial application, for the full car model, the cost of the design can grow fourfold. A good candidate for a low cost and applicable control method is the so called sky-hook strategy, which was published by Karnopp et al. (1974). This approach proposes a single gain feedback using the vertical velocity of the body mass and aims to simulate an imaginary damping element Karnopp (1995) which is connected between the body mass and a "hook in the sky". This imaginary element is used to isolate the body mass from the ground, aiming to minimize the body acceleration and the suspension travel. Some work related with the sky-hook approach is as follows. A non jerk sky-hook controller is proposed by Ahmadian et al. (2004), some sky-hook designs are studied in Priyandoko et al. (2009), Li and Goodall (1999) and Fallah et al. (2011), and a comparison of $\mathscr{H}_\infty$ optimal control and sky-hook control is given in Sammier et al. (2003).

In general, the robust sky-hook control problem is a static output feedback control problem, which is an open problem. Actually, it is not possible to define the optimization problem using LMIs. In fact the constraints arise as Bilinear Matrix Inequalities(BMIs). BMIs are NP hard problems to solve as mentioned in Van Antwerp and Braatz (1999) and, in general, it is not possible to find the global optimum.

In this paper, the existence of the sky-hook controller has been examined and a gridding based sky-hook controller design has been proposed. Due to the possibly large feasible interval of the sky-hook controller and to design an applicable sky-hook controller the design problem is also stated using BMIs. A $\mathbb{D}$ region based convex interval is used to design the sky-hook controller. It has been shown that, a sky-hook controller designed using the BMI problem posed in this paper with the appropriate $\mathbb{D}$ region, performs almost as good as the LQR controller. It has been shown that the sky-hook controller produces a smaller control signal and it is stated that the sky-hook controller could be more cost efficient, since the sky-hook needs only one sensor measurement to operate.

The rest of this paper is organized as follows: In the following section, the mathematical model is provided. The controller design approach is presented in section 3. Some results of simulations and experiments are discussed in sections 4 and 5, respectively. The conclusions and possible further studies are given in the last section.

2. MODEL

The dynamics of a quarter car model has a state space realization as follows,

$$\begin{aligned}
\dot{x}_1 &= x_2 - x_4 \\
\dot{x}_2 &= -\frac{K_s}{M_s}x_1 - \frac{B_s}{M_s}x_2 + \frac{B_s}{M_s}x_4 + \frac{1}{M_s}u \\
\dot{x}_3 &= x_4 - \omega \\
\dot{x}_4 &= \frac{K_s}{M_{us}}x_1 + \frac{B_s}{M_{us}}x_2 - \frac{K_{us}}{M_{us}}x_3 - \frac{B_s + B_{us}}{M_{us}}x_4 \\
&\quad - \frac{1}{M_{us}}u + \frac{B_{us}}{M_{us}}\omega \\
z &= x_1 \\
y &= x (\triangleq [x_1 \ x_2 \ x_3 \ x_4]^T)
\end{aligned} \tag{1}$$

where x_1 is the suspension travel, x_2 is the body vertical velocity, x_3 is the tire deflection, x_4 is the tire vertical velocity, z is the suspension travel, ω is the road surface velocity and u is the force applied by the controller. The physical parameters K, B, and M represent the suspension coefficient, the damper coefficient, and the body mass of the quarter car model, respectively. The subscripts s, and us are the abbreviations for sprung and unsprung, respectively. The generalized plant for the full state feedback control problem(LQR, $\mathscr{H}_2$, $\mathscr{H}_\infty$) is defined as follows,

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|c:c} A & B_w & B_u \\ \hline C_z & D_{zw} & D_{zu} \\ \hdashline I_{4x4} & 0_{4x1} & 0_{4x1} \end{array}\right] \begin{bmatrix} x \\ w \\ u \end{bmatrix} \tag{2}$$

whereas the generalized plant is updated for static output feedback(sky-hook controller) as follows,

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|c:c} A & B_w & B_u \\ \hline C_z & D_{zw} & D_{zu} \\ \hdashline [0 \ 1 \ 0 \ 0] & 0 & 0 \end{array}\right] \begin{bmatrix} x \\ w \\ u \end{bmatrix} \tag{3}$$

The corresponding open loop transfer functions are defined as follows,

$$\begin{aligned}
P_{zw}(s) &= C_z(sI - A)^{-1}B_w + D_{zw} \\
P_{zu}(s) &= C_z(sI - A)^{-1}B_u + D_{zu} \\
P_{yw}(s) &= C_y(sI - A)^{-1}B_w \\
P_{yu}(s) &= C_y(sI - A)^{-1}B_u
\end{aligned}$$

and the closed loop transfer function between the exogenous input and the exogenous output is defined as

$$T_{zw}(s) = P_{zw} + P_{zu}K(I - P_{yu}K)^{-1}P_{yw}$$

and the input output transfer function is given as,

$$T_{yu}(s) = K(I - P_{yu}K)^{-1}$$

3. CONTROLLER DESIGN

3.1 Linear Quadratic Regulator

The Linear Quadratic Regulator(LQR) is designed via solving the following Riccati equation,

$$PA + A^TP + Q - PB_uR^{-1}B_u^TP = 0 \quad P \succ 0$$

where $P = P^T \in \mathbb{R}^n$ is to be solved and the controller is obtained with $K = -R^{-1}B_u^TP$. The LQR design problem minimizes the following objective,

$$J = \int_0^\infty (x^TQx + u^TRu)dt \quad Q \succeq 0 \quad R \succ 0$$

where the controller is $u = Kx$ and hence the LQR design can be interpreted as a special $\mathscr{H}_2$ optimal control problem as stated in Boyd et al. (1994).

3.2 $\mathscr{H}_2$ Optimal state feedback controller

The $\mathscr{H}_2$ optimal state feedback controller design problem as an LMI problem as stated in Boyd et al. (1994) is given below,

$$\begin{gathered}
\min \rho \\
trace(Z) < \rho \\
AP + (AP)^T + B_uW + (B_uW)^T + B_wB_w^T \prec 0 \\
\begin{bmatrix} -Z & C_zP + D_{zu}W \\ (C_zP + D_{zu}W)^T & -P \end{bmatrix} \prec 0
\end{gathered}$$

where the controller is calculated as $K = WP^{-1}$ and the closed loop transfer function satisfies $||T_{zw}||_2 < \sqrt{\rho}$. In order to design an applicable $\mathscr{H}_2$ optimal state feedback controller, the given design problem is converted into the following using $\mathbb{D}$ stability as shown in Duan and Yu (2013),

$$\begin{gathered}
\min \rho \\
trace(Z) < \rho \\
AP + (AP)^T + B_uW + (B_uW)^T + B_wB_w^T \prec 0 \\
\begin{bmatrix} -Z & C_zP + D_{zu}W \\ (C_zP + D_{zu}W)^T & -P \end{bmatrix} \prec 0 \\
L \otimes P + M \otimes (AP + B_uW) + M^T \otimes (AP + B_uW)^T \prec 0
\end{gathered}$$

where L and M define the LMI region for the $\mathbb{D}$ stability and $\otimes$ represents the Kroenecker product.

3.3 $\mathscr{H}_\infty$ Optimal state feedback controller

The $\mathscr{H}_\infty$ optimal state feedback controller design problem as an LMI problem with $\mathbb{D}$ stability as represented in Boyd et al. (1994) is given below,

$$\begin{gathered}
\min \gamma \\
P \succ 0 \\
\begin{bmatrix} (AP + B_uW)^T + (AP + B_uW) & * & * \\ B_w^T & -\gamma I & * \\ (C_zP + D_{zu}W) & D_{zw} & -\gamma I \end{bmatrix} \prec 0 \\
L \otimes P + M \otimes (AP + B_uW) + M^T \otimes (AP + B_uW)^T \prec 0
\end{gathered}$$

where the controller is calculated as $K = WP^{-1}$ and the closed loop system satisfies $||T_{zw}||_2 < \gamma$.

3.4 Sky-hook controller

The sky-hook controller for the active suspension system is defined as the feedback of the body vertical velocity and is expressed as follows,

$$u = K_{sky}\, x_2 \quad K_{sky} \in \mathbb{R} \tag{4}$$

The sky-hook controller can be viewed as a static output feedback controller, such that $u = K_{sky} y$, where $y = x_2$. Therefore the first step is to check for the existence of a stablizing sky-hook controller. Using the generalized plant given in Eq 3, the characteristic polynomial for the transfer function $T_{yu}(s)$ is defined as follows,

$$\begin{aligned} p_c(s, K_{sky}) &= |sI - A - B_u K_{sky} C_y| \\ p_c(s, K_{sky}) &= |sI - A - B_u K_{sky} \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}| \end{aligned} \tag{5}$$

It is possible to use several methods for checking the stability of the polynomial given in Eq 5, for instance, Root locus, Routh table and Generalized Nyquist theorem as stated in Söylemez et al. (2003), can be used for this purpose. We will use a direct substitution method for numerical interval calculation. For the numerical calculation of the stability interval of the sky-hook controller, the substitution $s = jw$ into the characteristic polynomial $p_c(s)$ is given in Eq 6.

$$p_c(jw, K_{sky}) = |jwI - A - B_u K_{sky} \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}| \tag{6}$$

Hence, the stability interval can acquired from Eq 7.

$$\begin{aligned} Re\left(|jwI - A - B_u K_{sky} \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}|\right) = 0 \\ Im\left(|jwI - A - B_u K_{sky} \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}|\right) = 0 \end{aligned} \tag{7}$$

Since the controller is scalar, if an interval is acquired being $K_{sky} \in [k^-, k^+]$, it is possible to solve the following problem via gridding and choosing a norm.

$$\begin{gathered} \min ||T_{zw}|| \\ T_{zw} = P_{zw} + P_{zu} K_{sky} (I - P_{yu} K_{sky})^{-1} P_{yw} \\ K_{sky} \in [k^-, k^+] \end{gathered} \tag{8}$$

For the solution of the problem Eq 8 with norm ∞ the matlab function `hinfstruct` is also available Apkarian and Noll (2006). As a result of this paper, it has been observed that problems defined in Eq 9 and Eq 10 are suitable to design sky-hook controllers.

An $\mathscr{H}_2$ optimal sky-hook controller can be designed via solving the following BMI problem,

$$\begin{gathered} \min \rho \\ trace(Z) < \rho \\ < AP + B_u\, K_{sky} C_y\, P >_S + B_w B_w^T \prec 0 \\ \begin{bmatrix} -Z & * \\ (C_z P + D_{zu}\, K_{sky} C_y P)^T & -P \end{bmatrix} \prec 0 \\ L \otimes P + M \otimes (A + B K_{sky} C_y) P + M^T \otimes P \cdot \\ (A + B K_{sky} C_y)^T \prec 0 \end{gathered} \tag{9}$$

where $< X >_S \triangleq X + X^T$ is used for compact notation. The sky-hook controller design problem is not convex, due to the term $K_{sky}\, P$, but the problem can be solved using BMI solvers. With a similar approach the $\mathscr{H}_\infty$ optimal state feedback controller design problem is defined as follows,

$$\begin{gathered} \min \gamma \\ P \succ 0 \\ \begin{bmatrix} < AP + B_u K_{sky} C_y P >_S & * & * \\ B_w^T & -\gamma I & * \\ (C_z P + D_{zu} C_y K_{sky} P) & D_{zw} & -\gamma I \end{bmatrix} \prec 0 \\ L \otimes P + M \otimes (A + B K_{sky} C_y) P + M^T \otimes P \cdot \\ (A + B K_{sky} C_y)^T \prec 0 \end{gathered} \tag{10}$$

where the closed loop transfer function satisfies $||T_{zw}||_\infty < \gamma$. Here, both BMI problems are solved with $\mathbb{D}$ stability, which is important in designing practically applicable controllers and it is important to get a feasible BMI solution, since otherwise the BMI problems may give inconsistent results or a guess for the initial point is needed.

4. SIMULATION RESULTS

In this section all BMI problems are solved via PENBMI by Kočvara and Stingl (2003), on a computer with 2.8 GHz Intel core i7-7820HQ processor and 16 GB 2133Mhz ram. As can be seen from Table 1, two models named Model M1 and Model M2 are used for simulations.

Table 1. Model parameters

Symbol	Quanser Model M1	Jalili and Esmailzadeh (2001) Model M2
M_s	2.45 kg	972.2 kg
M_{us}	1 kg	113.6 kg
K_s	900 N/m	42719.6 N/m
K_{us}	1250 N/m	101105 N/m
B_s	7.5 $Nsec/m$	1095 $Nsec/m$
B_{us}	5 $Nsec/m$	14.6 $Nsec/m$

The LQR controller for Model M1 is given in Eq 11 Apkarian and Abdossalami (2013), where the closed loop norm is determined as $||Tzw||_2 = 0.2369$.

$$K = [24.6621 \ 47.6059 \ 38.7426 \ 4.5090] \tag{11}$$

A circle with a radius 60 at $s = -5$ is chosen as the $\mathbb{D}$ region for the BMI problems, so that the region includes all the closed loop poles of the LQR design and is chosen so that the control signal is limited for practical applicability. The matrices for the $\mathbb{D}$ region are given in Eq 12.

$$L = \begin{bmatrix} -60 & -5 \\ -5 & -60 \end{bmatrix} \quad M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{12}$$

Plugging in the model parameters for Model M1 into Eq 5 gives the characteristic equation given in Eq 13.

$$\begin{aligned} p_c(s) = &s^4 + (15.56 - 0.408 K_{sky}) s^3 + (2533 - 2.04 K_{sky}) s^2 \\ &+ (5663.0 - 510.2 K_{sky}) s + 459184 \end{aligned} \tag{13}$$

The stability interval is obtained as $-\infty < K_{sky} < 6.02$. It took approximately 0.7 seconds to acquire the stability interval. The solution of the gridding problem Eq 8 for the 2-norm and the ∞-norm results in Fig 1.

The solution of Eq 9 for Model M1 is given in Eq 14 and the solution of Eq 10 is given in Eq 15. Using Model M1, it takes approximately 1.5 seconds to solve the BMI problems.

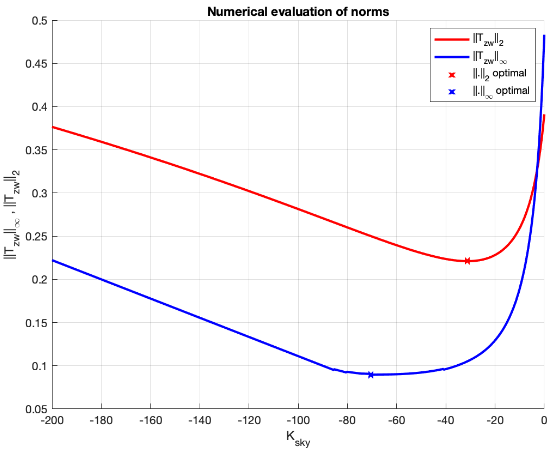

Fig. 1. $||T_{zw}||_\infty$ and $||T_{zw}||_2$ norms against the sky-hook parameter-Model M1

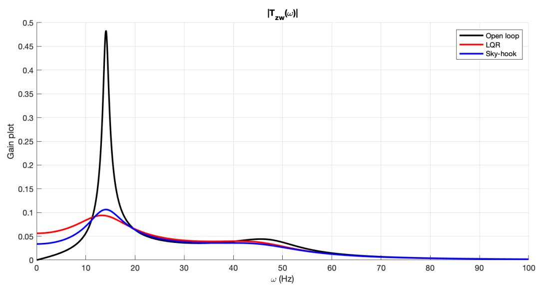

Fig. 2. Bode: A comparison of bode gain plots of Model M1 for the BMI solutions given in Eq 14 or Eq 15 agains LQR

$$P = \begin{bmatrix} 0.0496 & * & * & * \\ -0.6452 & 16.6870 & * & * \\ 0.0070 & 0.1411 & 0.0314 & * \\ -0.6366 & 6.2110 & -0.5008 & 26.5571 \end{bmatrix}$$

$$\begin{aligned} Z = 0.0496 \qquad & \gamma = 0.2228 \\ K_{sky} = -30.4754 \ & ||Tzw||_2 = 0.2211 \end{aligned} \tag{14}$$

$$P = \begin{bmatrix} 11.13 & * & * & * \\ -149.8 & 2679.0 & * & * \\ -2.05 & 45.35 & 1.911 & * \\ -36.29 & 144.2 & -2.711 & 1495.0 \end{bmatrix}$$

$$\begin{aligned} \rho = 733.5 \qquad & \gamma = 27.08 \\ K_{sky} = -30.4754 \ & ||Tzw||_\infty = 0.091 \end{aligned} \tag{15}$$

From Fig 1 it can be seen that, the ∞-norm can be further minimized, but the BMI solutions coincide at the $\mathscr{H}_2$-optimal solution, due to the prescribed $\mathbb{D}$ region.

A comparison of the Bode gain plot of the closed loop transfer function $|T_{jw}|$ is shown in Figure 2, comparing open-loop, LQR 11 and sky-hook obtained from the BMI problem Eq 15 or Eq 14.

Using the parameters of Model M2, the stability interval for the sky-hook controller is obtained as $-\infty < K_{sky} < 763.6$. It takes approximately 0.8 seconds to solve for the stability interval. Gridding the sky-hook controller results in Figure 3. The solution of Eq 9 for Model M2 is given in Eq 16 and the solution of Eq 10 is given in Eq 17. Using Model M2, It takes approximately 1.5 seconds to solve the BMI problems.

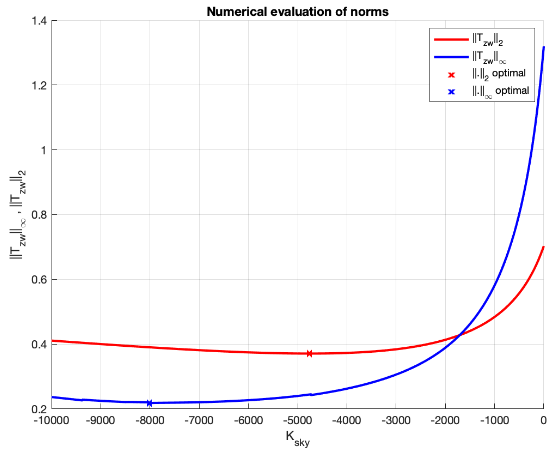

Fig. 3. $||T_{zw}||_\infty$ and $||T_{zw}||_2$ norms against the sky-hook parameter-Model M2

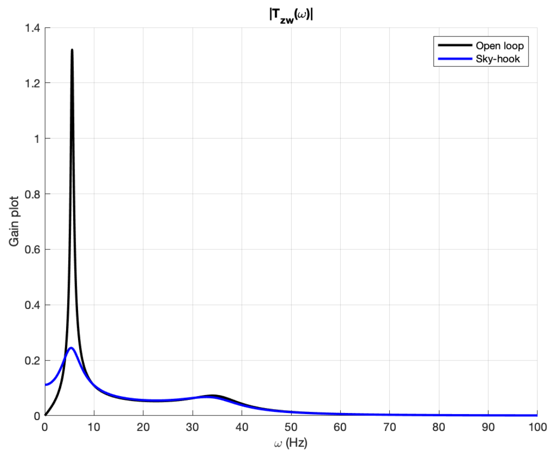

Fig. 4. Bode: A comparison of bode gain plots of Model M2 for the BMI solutions given in Eq 16 or Eq 17

$$P = \begin{bmatrix} 0.1376 & * & * & * \\ -0.5578 & 4.2430 & * & * \\ 0.0004 & 0.0578 & 0.0414 & * \\ -0.5578 & 0.9295 & -0.5000 & 28.9902 \end{bmatrix}$$

$$\begin{aligned} Z = 0.1376 \qquad & \gamma = 0.37091 \\ K_{sky} = -4760.91 \ & ||Tzw||_2 = 0.3709 \end{aligned} \tag{16}$$

$$P = \begin{bmatrix} 0.1376 & * & * & * \\ -0.5578 & 4.243 & * & * \\ 0.0004141 & 0.0578 & 0.04144 & * \\ -0.5578 & 0.9295 & -0.5 & 28.99 \end{bmatrix}$$

$$\begin{aligned} \rho = 0.1376 \qquad & \gamma = 0.3709 \\ K_{sky} = -4760.91 \ & ||Tzw||_\infty = 0.2445 \end{aligned} \tag{17}$$

A comparison of the Bode gain plot of the closed loop transfer function $|T_{jw}|$ is shown in Figure 4.

5. EXPERIMENTAL RESULTS

The experiments are carried out on the QUANSER active suspension set as described in Apkarian and Abdossalami (2013), which is referred as Model M1 in the previous section. Three types of road profiles are used during the experiments namely, a square bump to analyze low frequency performance, a half rectified sine wave at the critical frequency and a random road profile to analyze the high frequency performance of the control method. The square bump road profile is defined in Eq 18 and the sine wave road profile is defined in Eq 19.

$$z_r(t) = Mu(t - t_0) - Mu(-t + \tau + t_0) \tag{18}$$

The expression $u(t)$ is the unit step function and the critical frequency f_{crit} is chosen from Fig 2 as $f_{crit} = 14Hz$, where the amplitude is chosen as $M = 0.002$.

$$z_r(t) = M|sin(2\pi f_{crit} t)| \tag{19}$$

The random road profile is chosen as a grade A random road as described by Tyan et al. (2009), where the road profile generation is defined in Eq 20 with $\omega(t)$ white noise input.

$$\dot{z}_r(t) = -0.635 z_r(t) + \omega(t) \tag{20}$$

The results for the square bump road profile are given in Table 2. It can be seen that, the sky-hook controller uses smaller control signal, the maximum amplitude for the suspension travel is smaller than the LQR controllers maximum amplitude, and that the energy of the suspension travel has similar values for both control methods.

Table 2. Performance indication values for the square bump.

	Open loop	LQR	Sky-hook
$\|\|z\|\|_\infty / \|\|z_r\|\|_\infty$	0.9956	1.238	1.068
$\|\|z\|\|_2 / \|\|z_r\|\|_2$	0.3152	0.2396	0.2645
$\|\|u\|\|_\infty / \|\|z_r\|\|_\infty$	0	478.7	338.0
$\|\|u\|\|_2 / \|\|z_r\|\|_2$	0	89.77	60.37

Using the values from Table 3, it can be concluded that the sky-hook controller uses smaller control signal and performs almost as good as the LQR controller.

Table 3. Performance indication values for the half sine wave.

	Open loop	LQR	Sky-hook
$\|\|z\|\|_\infty / \|\|z_r\|\|_\infty$	0.7023	0.6729	0.933
$\|\|z\|\|_2 / \|\|z_r\|\|_2$	0.3931	0.6266	0.8192
$\|\|u\|\|_\infty / \|\|z_r\|\|_\infty$	0	697.7	496.9
$\|\|u\|\|_2 / \|\|z_r\|\|_2$	0	604.7	339.9

From the performance indication values given in Table 4, it is obvious that the sky-hook controller uses smaller control signal and has similar performance compared with the LQR controller, where a slight improvement is observed for the peak value of the suspension travel.

The square bump used during the experiments is depicted in Figure 5.

A comparison of the suspension travel for the open loop, closed loop with the LQR controller and the closed loop with the sky-hook controller is shown in Figure 6. The control signals for the the same cases are given in Figure 7.

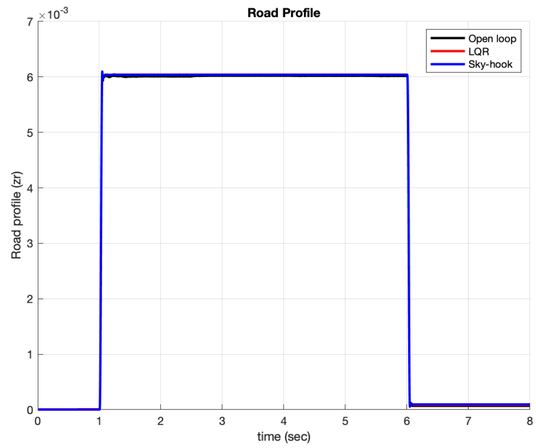

Fig. 5. Road profile

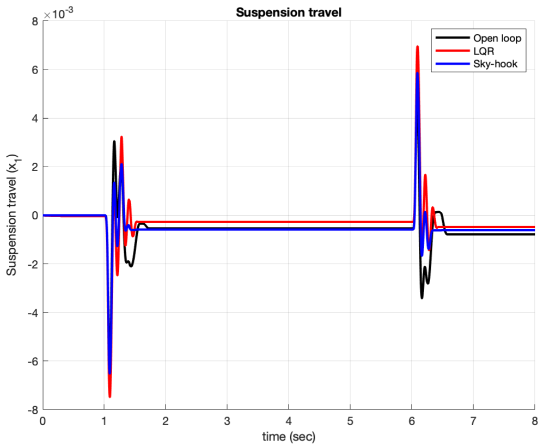

Fig. 6. Comparison: Suspension travel

6. CONCLUSION

In this paper, the sky-hook control method, which is a well known practical method for vibration isolation in suspension systems, is posed as a BMI problem in the framework of $\mathscr{H}_2$ and $\mathscr{H}_\infty$ optimal control design. For this purpose, methods for finding the stability interval are proposed first. Then the problem is given as a BMI problem. Since BMI problems are not convex, the design of the sky-hook controller using BMIs is converted into a $\mathbb{D}$ stability problem based on the LQR controller, which is widely used for suspension systems. The proposed BMI problem with $\mathbb{D}$ stability is used to design sky-hook controllers for two different suspension systems. It has been shown that the resulting sky-hook controllers, which use

Table 4. Performance indication values for the random road profile.

	Open loop	LQR	Sky-hook
$\|\|z\|\|_\infty / \|\|z_r\|\|_\infty$	0.6516	0.9447	0.8062
$\|\|z\|\|_2 / \|\|z_r\|\|_2$	0.6950	0.6156	0.6177
$\|\|u\|\|_\infty / \|\|z_r\|\|_\infty$	0	1128.0	430.7
$\|\|u\|\|_2 / \|\|z_r\|\|_2$	0	640.6	338.6

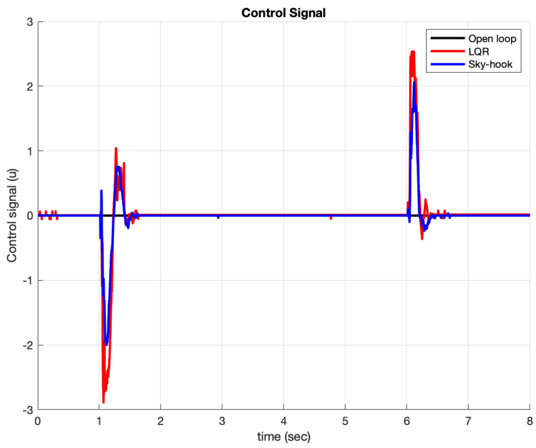

Fig. 7. Comparison: Control Signal

only one sensor instead of 4 sensors by the with the LQR controller performs almost as good as the LQR controller and hence, is preferable over the LQR controller in terms of performance and cost for serial manufacture.

It has also been shown that, using $\mathbb{D}$ regions can speed up the solving time of BMIs used for the design of the sky-hook controller. The time required to solve the BMIs for the sky-hook controller design problem, takes almost the same amount of time to calculate a stability interval for the sky-hook controller. Therefore, the BMI method is faster than calculating a stability interval for the sky-hook controller and gridding the interval for a suitable controller.

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