# Nonlinear Model Predictive Control applied to Concentrated Solar Power Plants \*

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**Abstract:** This papers presents a nonlinear model predictive controller (NMPC) for temperature control in solar collector fields. The proposed NMPC uses feedback linearization for handling the systems nonlinearities in a mixed integer quadratic programming (MIQP) formulation that makes the constraints convex in the new coordinates, making the controller solution an optimal one. Several simulations are shown with real data and a validated model of solar field to illustrate the advantages of the proposed strategy over other general-purpose NMPC methods, showing great improvement in constraint satisfaction.

*Keywords:* Nonlinear Model Predictive Control; Concentrated Solar Power Plant; Renewable Energy; Mixed Integer Quadratic Programming.

## 1. INTRODUCTION

Renewable energies have gathered great momentum in the last years due to the need for increasing demand and reducing the environmental impact caused by nonrenewable energy sources like oil or coal (Camacho et al., 2012). The power grids along the world are expanding and heliothermic power plants appear like a good and sustainable alternative, as only part of the energy provided by the sun is enough to supply the current planet demands (Camacho et al., 2012).

In Concentrated Solar Power (CSP) Plants, a collector system is used to concentrate the solar irradiance into an absorber where a heat transfer fluid (HTF) passes through acquiring energy to feed a Rankine cycle that generates electricity. In this case, the temperature has to be precisely controlled to improve the energy conversion efficiency and provide an optimized operation in the power unit (Roca et al., 2008a), and the same can be said to other applications as heating of furnaces (Beschi et al., 2012) and air-conditioning as in (Zambrano et al., 2008).

Although solar energy looks promising, it is not always available due to its intermittency caused by cloudy or rainy days, turning it difficult to meet the power demand and challenging the energy management systems. Currently, the research community is focused on improving the energy storage unit efficiency and operation to account for this problem (Camacho et al., 2012), along with the development of better control techniques to make solar energy more efficient and reliable. This paper proposes a solution for the control problem of a CSP field, where the solar irradiance acts as a disturbance since the energy source depends on the weather and can not be manipulated by the control system. This particularity of CSP plants makes the long term energy planning harder than traditional power units. On top of that, these solar fields present nonlinear dynamics and dead-time behavior, thus, nonlinear advanced control technique could benefit the CSP operation.

From the plant complexity and the solar intermittency a good choice for the control framework would be the Model Predictive Control (MPC), that take advantage of using the disturbances predictions to generate an optimal planning for the HTF flow. Several works in the literature have proposed MPC controllers for different solar plants: in (Martins Lima et al., 2015) a linear Filtered Dynamic Matrix Control (FDMC) was proposed for the AQUASOL desalination solar plant; in (Gálvez-Carrillo et al., 2009) a nonlinear general-purpose MPC approach known as Nonlinear EPSAC (NEPSAC) was proposed for the control of the ACCUREX solar plant; (Santos et al., 2011) proposed a nonlinear robust predictive controller using a Dead-Time Compensator (DTC) with Generalized Predictive Controller (GPC) and feedback linearization for handling the system dynamics, applying it in the AQUASOL plant (Roca et al., 2008a); (de Andrade et al., 2013) uses a general-purpose nonlinear MPC method named Practical Nonlinear MPC (PNMPC) (Plucenio et al., 2007) for the ACCUREX plant; finally, (Elias et al., 2018) formulates an PNMPC using Mixed Logical Dynamical (MLD) (Bemporad and Morari, 1999) in order to reduce the energy dissipated through the field in cloudy situations by deactivating parts of it.

This work proposes a hybrid nonlinear MPC that uses feedback linearization to handle nonlinearities and can

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activate/deactive parts of the solar field to avoid energy dissipation as side effect of passing clouds. The main difference of the proposed approach lies in the prediction model and the way that the feedback linearization is handled together with the constraints, arriving to the optimal solution helped by mixed integer programming.

This work gets even more important as technology for solar irradiance forecasting evolves (Alzahrani et al., 2017), such that the controller can use long term knowledge about the solar energy onto the field for better planning. In this scenario, general-purpose nonlinear MPC techniques, such as PNMPC and NEPSAC, might not be sufficient since their prediction is not exactly the nonlinear one, leading to prediction errors in the MPC such that its latest predictions may worsen the controller's performance. With a controller that can accurately optimize its actions according to the knowledge of the future disturbances and with the help of a good model, the irradiance forecasting advances may be able to enhance the plant efficiency and energy generation.

To show the enhancing capabilities, the authors show here the results, under real disturbance data and model parameters. The nonlinear models used in this work are well known in the literature, being validated in (Roca et al., 2008b). The results shown for the proposed method are compared with other general-purpose nonlinear MPC to display the quality of the proposed controller. The simulations are held in different scenarios, in which the MPC knows the future of the solar irradiance and can make the best decisions.

The paper is organized as follows: section 2 describes the solar field proposed for study and its mathematical model. In section 3 the proposed constraints for the controller are shown. Section 4 describes the proposed MPC formulation. Results under simulation scenarios are shown in section 5 for different strategies. Finally, section 6 concludes the paper.

## 2. PLANT DESCRIPTION

The proposed plant of study is described in this section along with a description of its nonlinearities. The main focus of the present work is the exploitation of the model to formulate a mixed integer quadratic programming problem. Although there are more complex models, the model used here bases itself in solid arguments and thermodynamics principles, and was shown to perform well in the literature as can be seen in (Roca et al., 2008b) (Gálvez-Carrillo et al., 2009).

The main idea of the proposed solar plant is to have the HTF passing across the field and acquiring internal energy, coming from a cold storage tank and going to another storage tank, which can then be used for the main stage of the plant. Fig. (1) shows an schematic of the proposed configuration.

Equation (1) presents the dynamics of the solar field. This model is known as lumped parameters model (Elias et al., 2018). The derivation of this model can be seen in (Roca et al., 2008b). Table (1) presents the values of the parameters of the model, and are the same as the ones used in (Dietrich et al., 2016). In (1),  $T_{out}$  represents the output



Fig. 1. Solar collector field schematic

temperature of the HTF, I is the direct solar irradiance onto the collectors,  $T_a$  is the ambient temperature at the fields place,  $T_{in}$  is the temperature of the HTF as it enters the field,  $\dot{m}$  is the mass flow of HTF and  $\overline{T}$  stands for the average temperature in the field, given by the mean of the HTF input and output temperatures.

symbol	description	value	
ρ	HTF density	$975 \ kg/m^3$	
$c_p$	HTF specific heat	$4190 \ J/kgK$	
A	collector straight section	$1.7453e^{-4} m^2$	
β	model irradiance	$0.1024 \ m$	
Н	thermal losses	4 J/sK	
$L_{eq}$	equivalent collector lenght	5.67 m	
$n_{eq}$	process parameter	588	
$d_c$	transport delay	$50 \ s$	
Table 1. Solar plant parameters (Dietrich et al.,			

Table 1. Solar plant parameters (Dietrich et al. 2016)

This plant uses water as its heat transfer fluid. This makes it necessary that we guarantee that there is no phase change in the working fluid, in order to make the proposed model valid.

$$\rho c_p A \dot{T}_{out}(t) = \beta I(t) - \frac{H}{L_{eq}} (\overline{T}(t) - T_a(t))$$

$$- \frac{c_p}{n_{eq} L_{eq}} (T_{out}(t) - T_{in}(t)) \dot{m}(t - d_c)$$
(1)

A state space model of equation (1) can be computed considering  $x = T_{out}$  be the state which in this case is the process output y = x,  $u = \dot{m}$  is the control input and  $w = [I T_a T_{in}]^T$  are the disturbances. Moreover, taking  $A = \frac{-H}{2L_{eq}\rho c_p A}$ ,  $B_{u_x} = \frac{-1}{n_{eq}L_{eq}\rho A}$ ,  $B_{u_w} = \frac{1}{n_{eq}L_{eq}\rho A}$ ,  $B_w = [\beta_I \frac{H}{L_{eq}\rho c_p A} - \frac{H}{2L_{eq}\rho c_p A}]$  and C = 1, the final formulation can be observed in equation (2), where it can be seen that the system is almost linear, taking out the bilinearity given between the control input and state and disturbances.

$$\dot{x} = f(x, u, w) = Ax + (B_{u_x}x - B_{u_w}w)u + B_ww$$
  

$$y = h(x) = Cx$$
(2)

A discrete representation of the system can be obtained using the Euler discretization, as presented in equation (3), where k is a timestamp and  $T_s$  is the discretization sample time. Since the sampling times used in this will work be large ones, with the least value being of three minutes, while the transport delay of the plant is fifty seconds, the dead time of the solar field model is neglected without losing too much performance, while greatly simplyfing the controller's design.

$$\dot{x} \simeq \frac{x[k+1] - x[k]}{T_s} = f(x[k], u[k], w[k])$$

$$x[k+1] = x[k] + T_s f(x[k], u[k], w[k]) =$$

$$x[k+1] = (1 + T_s A)x[k] + T_s (B_{u_x} x[k] - B_{u_w} w[k])u[k]$$

$$+ T_s B_w w[k]$$
(3)

## 3. CONSTRAINTS

Although the model is one of the most important things for a MPC controller, its ability to handle multiple constraints is what makes it such a powerful tool. In this section, the constraints imposed by the solar field or designed to increase its performance are proposed.

The first constraint that the controller should take into account is the control input allowed intervals, since these are performed by a pump flow. In order to maintain an hydronamic equilibrium, making the model in equation (1) more valid, and avoid the pump cavitation (Dietrich et al., 2016), the control action calculated should be in the intervals shown in equation (4), unless the field is shutdown, when in this case the value should be zero.

$$\underline{u} = 1.2 \ kg/s \le u \le 4.8 \ kg/s = \overline{u} \tag{4}$$

Other two important constraints are presented in equations (5) and (6). The first one is used to avoid stress in the absorber tubes material (Roca et al., 2009). As for the second one, its purpose is to limit the water to not get to a ninety degree celsius, making it more likely to be in the fluid state instead of vapor (Roca et al., 2009).

$$5 \ ^{\circ}C \le x - w(3) \le 25 \ ^{\circ}C$$
 (5)

$$x \le 90 \ ^{\mathrm{o}}C \tag{6}$$

The last constraint to be presented in this section is related to the energy generation. The difference in internal energy obtained by the fluid after passing through the field is given by equation (7). Looking at the equation and knowing that the mass flow and the specific heat are greater or equal to zero, the only possibility for the HTF losing energy after passing through the solar field is when the fluid's input temperature is greater than the output temperature. In this case, the plant is dissipating the fluid's energy. Since this is not desirable, the controller, in an ideal case, should be able to shutdown the supply of HTF to the field in the event of this scenario. This can be handled with the support of integer variables, as shown in equation (8). It follows the same idea as in (Elias et al., 2018) but done in a more simplified way.

$$dU = \dot{m}_i c_p (x - w(3)) dt \tag{7}$$

The proposed constraint (8) work as follows. Assume that M is big enough, like if it tends to infinity for the following derivation. The variable z represents the state of the solar field, where z = 1 when the pump is turned on and z = 0otherwise, assuming only two possible values. Following from these equations, it can be verified that when z = 1, the equations yields to equation (4), only if the difference between output and input temperatures is greater than zero, because then  $M(1-1) = 0 \le x - w(3) \le M$ . When z = 0, the only control input allowed is zero, meaning that the pump is turned off and that the difference of output and input temperature is less or equal to zero, assuming, as said, a M that tends to infinity. So, with a large enough M, the proposed equations with the hypoteshis that  $z \in \{0, 1\}$  makes for a constraint that does not allow energy dissipation.

$$\frac{uz}{M(z-1)} \le u \le \overline{u}z$$

$$M(z-1) \le x - w(3) \le Mz$$

$$z \in \{0,1\}$$
(8)

#### 4. NONLINEAR CONTROLLER DESIGN

The main contribution of this paper resides on a MPC formulation that takes into account the exact prediction model as the one presented in equation (3), making the controller's solution much more accurate when compared to the true optimal control solution. This is achieved by a sort of change of variable, in a similar way to the common nonlinear control technique known as feedback linearization. Although change of variables are a common technique for other closed-loop control formulations, it usually yields to problems when used together with a MPC due to the model transformation, making the constraints non-convex in the new coordinates.

The method proposed here is made such that the constraints, in the new representation, stay convex, leading to a optimal solution. To start the derivation, we use the prediction model in equation (3), merely changing the bilinearity in it by an optimization variable v, which will be called virtual input. The new equation is shown in equation (9). In the equation,  $B_v = B_{u_x}$  and v = (x - w(3))u.

$$x[k+1] = (1+T_s)Ax[k] + T_sB_vv[k] + T_sB_ww[k]$$
(9)

With the change of equation there could be two choices to handle it. Either write all constraints and the objective function in terms of the virtual input v, or use the original ones but make a new constraint that correlates v and the control input u, which would be the bond v = (x - w(3))u. Since this is a non-convex constraint an easier option would be to use the first idea, which is how it'll be handled.

Looking at the previous constraints, the only one that needs to be changed is equation (4), since its the only one that explicitly have the control input u on it. Afterwards a new objective function could be chosen, taking in account only x and v. The control input bounds can be rewritten as presented in equation (10), assuming  $(x - w(3)) \ge 0$ .

$$\underline{u} \le u \le \overline{u}$$
  

$$\underline{u}(x - w(3)) \le u(x - w(3)) \le \overline{u}(x - w(3))$$
  

$$u(x - w(3)) \le v \le \overline{u}(x - w(3))$$
(10)

The last equation has a problem, created from the fact that the output temperature was assumed greater or equal to the input one. From that, and assuming now that the output temperature is lower than the input temperature, the set of possible values for v becomes unfeasible. For example, if x - w(3) = -1, the constraints in equation (10) which lead to  $-1.2 \le v \le -4.8$ , which is impossible to happen. To handle this a solution with binary variables is proposed again such that, when the field starts loosing energy, the value of v should be zero, leading to a zero control input. Equation (11) shows how to acomplish this, again using M as a big number.

$$\underline{u}(x - w(3)) \le v \le \overline{u}(x - w(3)) + M(1 - z)$$
  
-  $Mz \le v \le Mz$   
 $z \in \{0, 1\}$  (11)

With that, the optimization problem will take into account the initial constraints proposed in the last section, along with using the nonlinear model for prediction. The reason for this can be seen when analyzing each of the possible scenarios for z, which again represents if the field is turned on or not. When z = 1, constraint (11) yields to  $\underline{u}(x - w(3)) \le v \le \overline{u}(x - w(3))$  and  $-M \le v \le M$ , but since M is a big value, only the first constraint ends up mattering. Analyzing it for z = 0 leads to  $\underline{u}(x - w(3)) \le v \le \overline{u}(x - w(3)) + M \simeq M$  and  $0 \le v \le 0$ , since  $\underline{u}(x - w(3)) \le 0$  in this case because the solar field is turned off (z = 0), the constraints now lead to v = 0 as desired.

Other point that has to be taken into account before using the constraints proposed is the feasibility. Constrains such as equation (4), which are written over the control input optimization variable, always allow the optimization problem to find a solution. On the other hand, the ones related to system's outputs may not be achievable. As an example, in a cloudy day, the field may not be able to generate any energy, which will make the output temperature lower than the input one. This would lead the optimization step of the controller not being able to find a solution that maintains the temperature inside constraint (5), leading to an unfeasible situation in the optimization problem. In order to fix this, these constraints use an additional slack variable, as presented in equations (12) and (13). These slack variables, written as  $s_j$  in the equations, are decision variables, put in there to account for overshoot and make the optimization feasible. Since these are wanted, ideally, to be always zero, they'll receive a big weight when placed inside the objective function in order to make them as low as possible.

$$5 \ ^{\circ}C \le x - w(3) + s_1 \le 25 \ ^{\circ}C$$
 (12)

$$x + s_2 \le 90 \ ^{\mathrm{o}}C \tag{13}$$

Next step is to propose the objective function, which is presented in equation (14). The costs chosen for the function are the slack variables, weighted over  $Q_1$  and  $Q_2$ ,



## Fig. 2. Solar Irradiances

and the increments of v represented as  $\Delta v$  weighted with R. The last term is -v, weighted with  $\rho$ , which is directly proportional to the internal energy gained after passing through the field, so that the controller tries to maximize the energy production.

$$J = \sum_{i=0}^{N-1} ||s_1[i]||_{Q_1}^2 + ||s_2[i]||_{Q_2}^2 + ||\Delta v[i]||_R^2 - \rho v[k+i]$$
(14)

Thus, the optmization problem to be solved by the MPC is:

$$\begin{array}{ll} \underset{s_1,s_2,v,x}{\text{minimize}} & J \\ \text{subject to:} & (9), \ (11), \ (12) \ \text{and} \ (13) \end{array}$$
(15)

# 5. RESULTS

The PNMPC followed the same philosophy of last sections related to the objective function and constraints when designed. The results of the application of both controllers are shown in Fig. (4) for the HTF output temperature and in Fig. (5) for the control actions. In the figures, solid and dashed lines are respectively used for the proposed method, called NMPC, and the PNMPC. The dotted lines represent the limits at each time sample for the constraint (5). The initial output temperature is set to 20 °C such that the solar field should be deactivated.

Looking at Fig. (4), it can be seen a significative improvement with the proposed technique. Firstly, the pure nonlinear formulation does not violate the constraints as much as the PNMPC. This is due to its prediction model which takes into account the system in its nonlinear form. As an example, around 9 hours of simulation, the NMPC starts lowering the HTF mass flow to the field, yielding



Fig. 3. HTF Input and Ambient Temperature



Fig. 4. Solar Field Output Temperature

to an increase of output temperature. This action pays off later, when the irradiance suddenly drops due to clouds, so that the time during which the HTF output temperature stays inside the dotted boundary and inside constraint (5) is maximized, while in constrast the PNMPC does not take this into account, leading to more violation. From Fig. (4), both techniques acomplish constraint (6). Keep in mind that the violations are to be expected since this a solar plant and the disturbance, solar irradiance, is what drives the output temperature, such that in cloudy days there might be no way to acomplish the soft constraints (6).

Aside from their performance, both controllers achieve what is desirable in terms of energy maximization. As the output temperature is close to the lower bound of (5), the temperature is as close as possible to the ambient temperature, leading to less heat losses to the ambient, which is the only one possible in the model (1). So



Fig. 5. Control Action

the choice of making the output temperature as low as possible is within the design of the controllers, as long as it makes the constraints achiveble. But one important point to be taken is that, due to PNMPC having a bad prediction model for seeing further into the future, it ends up making more bad control choices and violating the lower bound more often, leading to more energy generation. This behaviour shows that analyzing only energy produced is not enough to compare performance, since the NMPC generates less energy by doing a better job at fullfiling all constraints.

Lastly, looking at the control inputs in Fig. (5), it can be seen that both controllers achieve the deactivation when the output temperature is lower than the input one. This can be seen at the beggining of the simulation since the initial output temperature is 20 °C. Even though both controllers respect the constraints, the proposed controller makes it more smoothly, while the PNMPC has to change between its minimum and maximum control bounds more constantly to account for its poor predictions.

Table (2) shows more details about the performance of each controller. The energy and violation are shown for each irradiance scenario as in Fig. (2). Analyzing the performance, the violation of the constraint (5), represented as minutes of violation in the table, shows vastly improvements with the proposed technique, especially since there are times, like when the sun is clouded, that there's no way to maintain the system inside the constraints, thus making for an uncontrollable scenario. The energy gained by the HTF for the PNMPC is greater than the NMPC, which could be seen as a bad thing, but, as discussed before, the PNMPC having a better result in this metric is actually expected due to its poor performance when compared to the NMPC. Also, looking at the differences, the gain in

Indexes	NMPC	PNMPC	difference
scenario 1			
constraint violation	$32 \min$	$43 \min$	25.6~%
energy generated	$73.0 \ \mathrm{MJ}$	73.5 MJ	0.7~%
scenario 2			
constraint violation	$38 \min$	$50 \min$	24.0~%
energy generated	$75.7 \mathrm{~MJ}$	76.4 MJ	0.9~%
scenario 3			
constraint violation	$36 \min$	$53 \min$	32.1 %
energy generated	$75.5 \mathrm{~MJ}$	76.6 MJ	1.4 %
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Table 2. Controllers performances

constraint violation far outperform the increase in energy generation, making the plant more safe and long lasting.

## 6. CONCLUSIONS

This work proposed an alternative way to control simple solar fields making the use of its nonlinear model for better predictions when used with the MPC framework. It also developed a simple set of constraints for handling an optimal deactivation for the solar field, such that the plant waste less energy. The results were compared with a general-purpose nonlinear controller in order to demonstrate the capabilities of the method described here, showing promissor results for long term energy generation planning in these types of plants when a well forecasting about the future disturbances can be made.

As a future work the authors intend to generalize the formulation here presented, such that it can be used in other solar field configurations, and also try to increase its robustness by adding some sort of filtering to the predictions, as is done in the DTC-MPC method.

#### REFERENCES

- Alzahrani, A., Shamsi, P., Dagli, C., and Ferdowsi, M. (2017). Solar irradiance forecasting using deep neural networks. *Procedia Computer Science*, 114, 304 – 313.
- Bemporad, A. and Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints. Automatica, 35(3), 407 - 427.
- Beschi, M., Visioli, A., Berenguel, M., and Yebra, L. (2012). Constrained temperature control of a solar furnace. *IEEE Transactions on Control Systems Tech*nology, 20, 1263–1274.
- Camacho, E., Berenguel, M., Rubio, F., and Martinez Plaza, D. (2012). Control of Solar Energy Systems.
- de Andrade, G., Pagano, D., Álvarez, J., and Berenguel, M. (2013). A practical nmpc with robustness of stability applied to distributed solar power plants. *Solar Energy*, 92, 106–122.
- Dietrich, J.D.V., Normey-Rico, J.E., Berenguel, M., and Roca, L. (2016). Temperature control of large solar fields using pnmpc approach - practical nonlinear model predictive control (in portuguese).
- Elias, T., Mendes, P., and Normey-Rico, J. (2018). Mixed logical dynamical nonlinear model predictive controller for large-scale solar fields. Asian Journal of Control, 1–10.
- Gálvez-Carrillo, M., Keyser, R., and Ionescu, C. (2009). Nonlinear predictive control with dead-time compensator: Application to a solar power plant. *Solar Energy*, 83, 743–752.

- Martins Lima, D., Normey-Rico, J., and Santos, T. (2015). Temperature control in a solar collector field using filtered dynamic matrix control. *ISA transactions*, 62.
- Plucenio, A., Pagano, D., Bruciapaglia, A., and Normey-Rico, J. (2007). A practical approach to predictive control for nonlinear processes. *IFAC Proceedings Volumes*, 40(12), 210 – 215. 7th IFAC Symposium on Nonlinear Control Systems.
- Roca, L., Berenguel, M., Yebra, L., and Alarcon, D. (2008a). Preliminary modeling and control studies in aquasol project. *Desalination*, 222, 466–473.
- Roca, L., Berenguel, M., Yebra, L., and Alarcón-Padilla, D.C. (2008b). Solar field control for desalination plants. *Solar Energy*, 82(9), 772 – 786.
- Roca, L., Guzmán, J., Normey-Rico, J., Berenguel, M., and Yebra, L. (2009). Robust constrained predictive feedback linearization controller in a solar desalination plant collector field. *Control Engineering Practice*, 17, 1076–1088.
- Santos, T., Roca, L., Guzman, J.L., Normey-Rico, J.E., and Berenguel, M. (2011). Practical mpc with robust dead-time compensation applied to a solar desalination plant.
- Zambrano, D., Bordons, C., and Garcia-Gabin, W. (2008). Model development and validation of a solar cooling plant. *International Journal of Refrigeration*, 31, 315– 327.