Power Management in Serial Hybrid Electric Vehicle:
A singular perturbation approach *

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Abstract: In this paper, a control based on singular perturbation approach is designed to solve the problem of power management for a serial hybrid electric vehicle. In this case, the power supply system consists of an Internal Combustion Engine (ICE) mechanically bound to a Permanent Magnet Synchronous Machine (PMSM) through a flywheel. Both sources are torque-controlled and connected to the traction motor by simple (without battery) DC link. The contribution of this paper is to apply the singular perturbation approach to design a control, which takes into account the different dynamic of the considered power topology. The proposed strategy is based on two-closed loops cascade control (fast and slow), which allows to impose the constraints of slower (faster respectively) ICE torque dynamics (PMSM current dynamics respectively) and to protect the power devices.

Keywords: Serial hybrid electric vehicle, Permanent magnet synchronous machine, Internal combustion engine, Singular perturbation approach.

1. INTRODUCTION

This paper deals with the power management issue of a serial hybrid electric vehicle. In Fig. 1, the system is form by three parts. The first part is the power generator consisting of an Internal Combustion Engine (ICE) connected trough a flywheel to a reversible Permanent Magnet Synchronous Machine (PMSM) and it's inverter. The second part is the capacitor, which if connected to a battery was initially used to filter PWM harmonics of the inverter and thus protect the battery from it. Here without the battery, this capacitor is used as a small energy storage to ensure the energy transfer on the DC link between the power generator and the propulsion motor. And the last part is the propulsion motor, i.e. the traction motor and considers in the following to be an electrical load.

In this case, two critical objectives can be draw: first the DC link voltage and second the rotational speed of the ICE-PMSM. Both of them have to be kept at a certain level to ensure the power production and transmission to the propulsion motor.

In Paladini et al. (2007) a multi objective genetic algorithm optimization has been applied for the supervisory powertrain control of a hybrid electric vehicle using charge sustaining mode. However, this approach is based on a priori knowledge of the power load, thus real-time control is not straightforward implementable.

Rizzoni, Giorgio and Onori, Simona (2015) resume 15 years of development on energy management of hybrid electric vehicle. Different methods are developed: Rule-Based, Equivalent Consumption Minimization, Dynamic Programming, Optimal Method and Pontryagin's Minimun Principle. Moreover, implementations with steady state models are provided.

In this paper, the transient state is also considered around a substantially constant ICE-PMSM speed. Consequently, the ICE torque dynamic model is approximated by a first-order linear model with constant parameter. This approximation is made on this prelimary work and depending on future implementation results of the proposed control, a more sophisticate model may be considered (Tashiro and Maruyama (2017) and Hrovat and Sun (1997)).

Based on the considered power topology (ICE, flywheel, PMSM, Capacitor), the main objective of this paper is to propose a power management solution, which satisfies the power requirements of the considered serial hybrid electric vehicle. To achieve the objective, a control based on singular perturbation approach is proposed.

This paper is organized as follows. In Section 2, the idea of the singular perturbation approach is recalled and the model of the power system is given. Based on this model, Section 3 proposes the control design. Section 4 reports the simulation results. Some final conclusions and future works are given in Section 5.
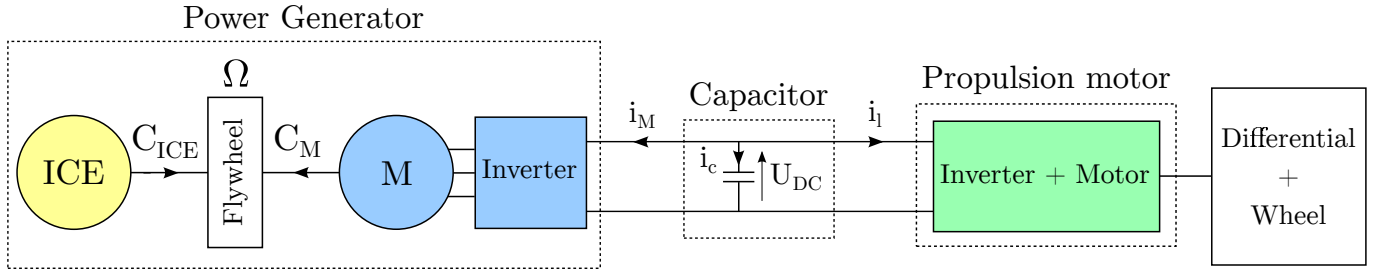
Fig. 1. Serial hybrid electric vehicle

2. SINGULAR PERTURBATION APPROACH AND MODELLING

The control scheme challenge is to drive the rotationnal speed under the main constraint of slow variation of ICE torque C_{ICE} and the secondary constraint of small capacitor with low variation of his voltage U_{dc}. To achieve this goal, a singular perturbation approach Kokotovic et al. (1986); Khalil (1996) is used. In this section, first a brief recall on such approach is introduced, then the ICE-PMSM modelling is presented. The control design of the ICE-PMSM system is developed in Section 3.

2.1 Brief recall of singular perturbation approach

Let us consider the following nonlinear system:

$$\dot{x} = f(x, z, \varepsilon) \qquad (1)$$
$$\varepsilon \dot{z} = g(x, z, \varepsilon) \qquad (2)$$

with $x \in R^m$, $z \in R^n$, ε a small positive parameter and f, g two analytical vector fields of appropriate dimensions. Roughly speaking, x can be seen as the slow state and z as the fast variable. Nevertheless, this statement must be clarified and some assumptions and theoretical developments must be added. First of all, it is usual to decompose the system (1)-(2) into two decoupled time scale dynamics. For this purpose, it is important to be able to compute the so-called slow manifold $z = \phi(x, \varepsilon)$.

It is the function ϕ that must verify the following equation:

$$\varepsilon \dot{\phi}(x, \varepsilon) = g(x, \phi(x, \varepsilon), \varepsilon) \qquad (3)$$

where

$$\phi(x, \varepsilon) = \sum_{i=0}^{\infty} \alpha_i(x) \frac{\varepsilon^i}{i!} \qquad (4)$$

is computed iteratively Vasil'eva (1963). For example the so-called frozen solution verifies

$$0 = g(x, \alpha_0(x), 0).$$

The following assumption is generally requested for the existence of α_0.

Assumption 1. The Jacobian $\{\frac{\partial g(x,z,0)}{\partial z}\}$ is regular in the considered slow and fast state spaces $x \in D_x \subset R^M$ and $z \in D_z \subset R^n$, respectively.

This assumption is directly linked to the implicit function theorem and in the nonlinear case more than one solution is possible (this particular case is outside the scope of this short presentation). Now, it is important to know if the system (1)-(2) converges on a slow manifold that is given by the well-known Tikhonov's theorem reformulated in Hunter (2004). But before recalling the theorem, it is necessary to analyze the fast dynamics on the boundary-layer. For this, a new state variable $\eta = z - \phi$ is introduced and η converges rapidly to zero if the system behaviour converges on the slow manifold. This manifold is the z behaviour recovered when the fast transient time is ended ("outside the boundary-layer").

The η dynamics are equal to:

$$\dot{\eta} = \frac{1}{\varepsilon} g(x, \phi(x, \varepsilon) + \eta, \varepsilon) - \frac{\partial \phi(x, \varepsilon)}{\partial t}. \qquad (5)$$

Setting $\varsigma = \frac{t}{\varepsilon}$, (5) may be rewritten as follows:

$$\frac{\partial \eta}{\partial \varsigma} = g(x, \phi(x, \varepsilon) + \eta, \varepsilon) - \varepsilon \frac{\partial \phi(x, \varepsilon)}{\partial \varsigma}. \qquad (6)$$

Assumption 2. The system (6) is at least locally in η and uniformly in x exponentially stable.

Hereafter, Tikhonov's Theorem without considerations of time domain, existence and uniqueness of the attractivity domain with respect to η solution (for example local Lipschitz condition around zero is ensured by Assumption 1) is introduced.

Theorem 1. Assume that Assumptions 1-2 hold. There exist $\varepsilon > 0$ sufficiently small, such that the dynamics (1)-(2) tend to the slow dynamic (7)

$$\dot{x} = f(x, \phi(x, \varepsilon), \varepsilon) \text{ and } \dot{\eta} = 0. \qquad (7)$$

In many applications, $\phi(x, \varepsilon)$ in (7) is approximated by it frozen solution α_0 (given in (4)):

$$\dot{x} = f(x, \alpha_0(x), 0) \text{ and } \dot{\eta} = 0. \qquad (8)$$

Remark 1. This paper often limits itself to the frozen solution of the slow manifold, i.e. $\phi \simeq \alpha_0(x)$. Nevertheless, for example, when the slow dynamics has a behavior too close to fast dynamics it is necessary to do a better approximation of ϕ.

2.2 Model on two time scales

In the sequel, the flywheel dynamic is represented by:

$$\dot{\Omega} = \frac{C_{ICE} + \alpha i_q}{J} \qquad (9)$$

where Ω, C_{ICE}, i_q, J and α respectively denote the rotation speed, the ICE torque, the current in Park reference, the inertia and the PMSM torque constant.

Remark 2. Here a PMSM with non-salient poles (see Glumineau and De Leon-Morales (2015)) is considered, i.e. $C_M = \alpha\, i_q$.

The torque respond of the ICE is modeled by a first-order linear model:

$$\dot{C}_{ICE} = \frac{C^*_{ICE} - C_{ICE}}{\tau} \qquad (10)$$

where τ is the time constant of the ICE and C^*_{ICE} a control input.

The capacitor used here have a constant capacity and negligible losses. Furthermore, the switches of the inverter are considered as ideal. From these considerations, the connection of the "ICE-PMSM" system to an active load (propulsion motor) via DC link is represented by:

$$\dot{U}_{dc} = \frac{-\alpha\, i_q \dfrac{\Omega}{U_{dc}} - i_l}{C} \qquad (11)$$

where U_{dc} and C are the DC link voltage and the DC link capacitor. Furthermore i_l refers to the load current.

To sum up, the system under consideration is a 3th order nonlinear state space model:

$$\begin{aligned}
\dot{x}_1 &= \frac{x_2 + \alpha\, i_q}{J} \\
\dot{x}_2 &= \frac{C^*_{ICE} - x_2}{\tau} \\
\dot{x}_3 &= \frac{-\alpha\, i_q \dfrac{x_1}{x_3} - i_l}{C}
\end{aligned} \qquad (12)$$

where $x = [x_1\, x_2\, x_3]^T$ is the state space and $x_1 = \Omega$, $x_2 = C_{ICE}$ and $x_3 = U_{dc}$.

The control input is defined as $u = [i_q\, C^*_{ICE}]^T$.

Remark 3. The model (12) contains two time scales, the fast one given by x_3 dynamic and the slow one which are represented by the dynamics of x_1 and x_2.

3. CONTROL DESIGN

3.1 Control diagram

The block diagram scheme of the proposed controls based on singular perturbation approach is depicted in Fig. 2. It is composed of two subsystems : inner loop (fast control) based on one fast actuators (PMSM) to drive the DC bus voltage; outer loop block (slow control) based on slow actuator (ICE) to control the rotational speed of the power generator.

3.2 Fast control design

The fast control is a PI voltage control (13) that assigns a value to i_q in order to control the DC bus voltage $x_3 = U_{dc}$ to it's reference U^*_{ref}:

$$i_q = \frac{x_3}{\alpha\, x_1}(K_P\,(x_3 - x_3^*) + K_I\, x_4) \qquad (13)$$

where $x_4 = \int (x_3 - x_3^*) dt$.

The control input (13) applied to system (12) leads to the closed loop nonlinear system:

$$\begin{aligned}
\dot{x}_1 &= \frac{x_2 + \dfrac{x_3}{x_1}(K_P\,(x_3 - x_3^*) + K_I\, x_4)}{J} \\
\dot{x}_2 &= \frac{C^*_{ICE} - x_2}{\tau} \\
\dot{x}_3 &= \frac{-K_P\,(x_3 - x_3^*) - K_I\, x_4 - i_l}{C} \\
\dot{x}_4 &= x_3 - x_3^*
\end{aligned} \qquad (14)$$

Let us introduce the following change of variables:

$$\begin{aligned}
\varepsilon\, \xi_1 &= x_3 - x_3^* \\
\varepsilon^2\, \xi_2 &= x_4
\end{aligned} \qquad (15)$$

Then system (14) can be rewritten as follows:

$$\begin{aligned}
\dot{x}_1 &= \frac{x_2 + \dfrac{x_3}{x_1}(K'_P\, \xi_1 + K'_I\, \xi_2)}{J} \\
\dot{x}_2 &= \frac{C^*_{ICE} - x_2}{\tau} \\
\varepsilon\dot{\xi}_1 &= \frac{-K'_P\xi_1 - K'_I\xi_2 - i_l}{C} \\
\varepsilon\dot{\xi}_2 &= \xi_1
\end{aligned} \qquad (16)$$

where the tuning parameters of the fast control (13) are given by the following equations

$$K_P = \frac{K'_P}{\varepsilon} \quad \text{and} \quad K_I = \frac{K'_I}{\varepsilon^2}. \qquad (17)$$

The tuning choice of K'_P and K'_I is made such that the polynomial $s^2 + \frac{K'_P}{C}s + \frac{K'_I}{C}$ is Hurwitz. According to the recall of section (2.1) and more precisely equations (1) and (2), the dynamics of (16) look as a slow-fast singular perturbed form:

$$\dot{X} = f(X, \xi, \varepsilon) \qquad (18)$$

$$\varepsilon\dot{\xi} = g(X, \xi, \varepsilon) \qquad (19)$$

where $X = [x_1\, x_2]^T$, $\xi = [\xi_1\, \xi_2]^T$ and ε a small positive parameter.

Now the frozen solution of the fast dynamic equation (16) is computed as follows:

$$\Phi_0 = [0, -\frac{i_l}{K'_I}]^T. \qquad (20)$$

Proposition 1. For all $x_1 > \Omega_{min} > 0$, there exists a sufficiently small parameter $\varepsilon \geq 0$ such that the dynamics (18)-(19) can be approximated by it slow dynamics described by the following system:

$$\dot{X} = f(X, \Phi_0, 0) = \begin{pmatrix} \dfrac{x_2 - i_l\dfrac{x_3}{x_1}}{J} \\[2mm] \dfrac{C^*_{ICE} - x_2}{\tau} \end{pmatrix} \text{ and } \dot{\eta} = \dot{\xi} - \dot{\Phi}_0 = 0 \qquad (21)$$

Remark 4. In practice, due to the ICE non reversibility and the necessity to produce the power, the hypothesis on the rotational speed (i.e. $x_1 > \Omega_{min} > 0$) is verified.

Proof. The Jacobian

$$\frac{\partial g(X, \xi, 0)}{\partial z} = \begin{bmatrix} -K'_P & -K'_I \\ 1 & 0 \end{bmatrix} \qquad (22)$$

is regular ($K'_I \neq 0$), which makes the Assumption 1 verified.
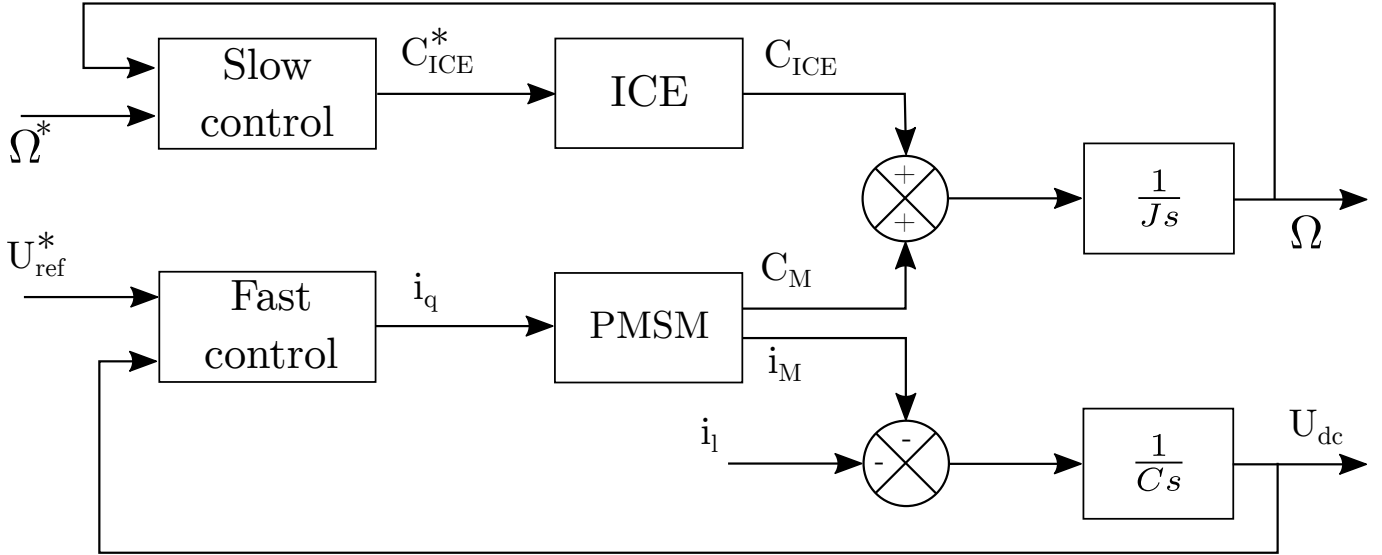
Fig. 2. Singular perturbation structure

Setting $\eta := \xi - \Phi_0$. Then the fast dynamic becomes:

$$\begin{aligned}
\dot{\eta}_1 &= \frac{-K'_P \eta_1 - K'_I \eta_2}{\varepsilon C} + O(\varepsilon^0) \\
\dot{\eta}_2 &= \frac{\eta_1}{\varepsilon} + O(\varepsilon^0)
\end{aligned} \qquad (23)$$

The system (23), with respect to the choice of gains K'_P and K'_I is stable, which makes the Asumption 2 verified for all $x_1 > \Omega_{min}$. Consequently for $\varepsilon \ll \Omega_{min}$ assumptions of Tikhonov's theorem (see Theorem 1) are verified and the proof follows.

Remark 5. Hereafter, the slow control (slow closed loop control) will be designed by using the model (21) such that its dynamic is slower than the dynamic (23).

3.3 Slow control design

The desired equilibrium point is the following one $x_1^* = \Omega^*$, where Ω^* is the desired rotational speed. The desired load current i_l represents the driving cycle of the hybrid electric vehicle.

The control C^*_{ICE} is designed as:

$$C^*_{ICE} = J\tau(\ddot{x}_1^* + \lambda_0(\dot{x}_1^* - \dot{x}_1) + \lambda_1(x_1^* - x_1)) + x_2 \quad (24)$$

where λ_0 and λ_1 are tuning parameters.

From the control (24) and the reduced model (21), the slow dynamics become:

$$\dot{X} = \begin{pmatrix} \dfrac{1}{J}(x_2 - i_l \dfrac{x_3}{x_1}) \\ J(\ddot{x}_1^* + \lambda_0(\dot{x}_1^* - \dot{x}_1) + \lambda_1(x_1^* - x_1)) \end{pmatrix} \qquad (25)$$

The tuning choice of λ_0 and λ_1 is made such that the polynomial function $s^2 + \lambda_0 s + \lambda_1$ is Hurwitz.

By taking into account that its dynamic must be very slow, C^*_{ICE} is considered as a slow input and has only a slow effect on the convergence of the dynamic (25).

Note that the error $\tilde{x}_1 = x_1^* - x_1$ satisfies:

$$\ddot{\tilde{x}}_1 = -\lambda_0 \dot{\tilde{x}}_1 - \lambda_1 \tilde{x}_1 + \frac{1}{J}\frac{\partial}{\partial t}(i_l \frac{x_3}{x_1}) \qquad (26)$$

In our considered case, the flywheel J is designed sufficiently large with respect to the power demand i_l and its variation $\dot{i}_l$, consequently for λ_0 and λ_1 sufficiently large, $\frac{i_l}{J}$ and $\frac{\dot{i}_l}{J}$ can be considered close to zero almost everywhere.

4. SIMULATION RESULTS

4.1 Settings

Considering the given parameters in the table 1, the performance of the proposed control (13)-(24) is tested with the benchmark proposed by Fig. 3. In the benchmark, the rotational speed starts above the idle speed at 1200RPM and quickly increases to 2500RPM in order to avoid the ICE torque limitation. For the same reason, the reference of the DC bus voltage is set equal to 400V. The load current varies between 0A and 75A. This power cycle is representative of a vehicle power demand, where the load is raising and lowering between 0 and 30kW.

Table 1. System parameters

Symbol	Description	Value	Unit
τ	ICE dynamic	0.125	s
α	PMSM torque constant	0.49	
J	Inertia	40	$g.m^2$
C	Capacitor	1	mF

4.2 Control parameter's tuning

The parameters K'_P and K'_I of the fast control (13) (fast loop) are tuned such that the second order polynomial $s^2 + \frac{K'_P}{C}s + \frac{K'_I}{C}$ is Hurwitz and the DC link voltage is kept close to 400V with $\pm 10V$ of margin. Then according to (17), K_P and K_I are calculated by fixing ϵ sufficiently small to achieve two times-scales (see table 2).

The parameters λ_0 and λ_1 of the slow control (24) (slow loop) are fixed (see table 2) such that:

- the convergence is slower than the fast loop (19)
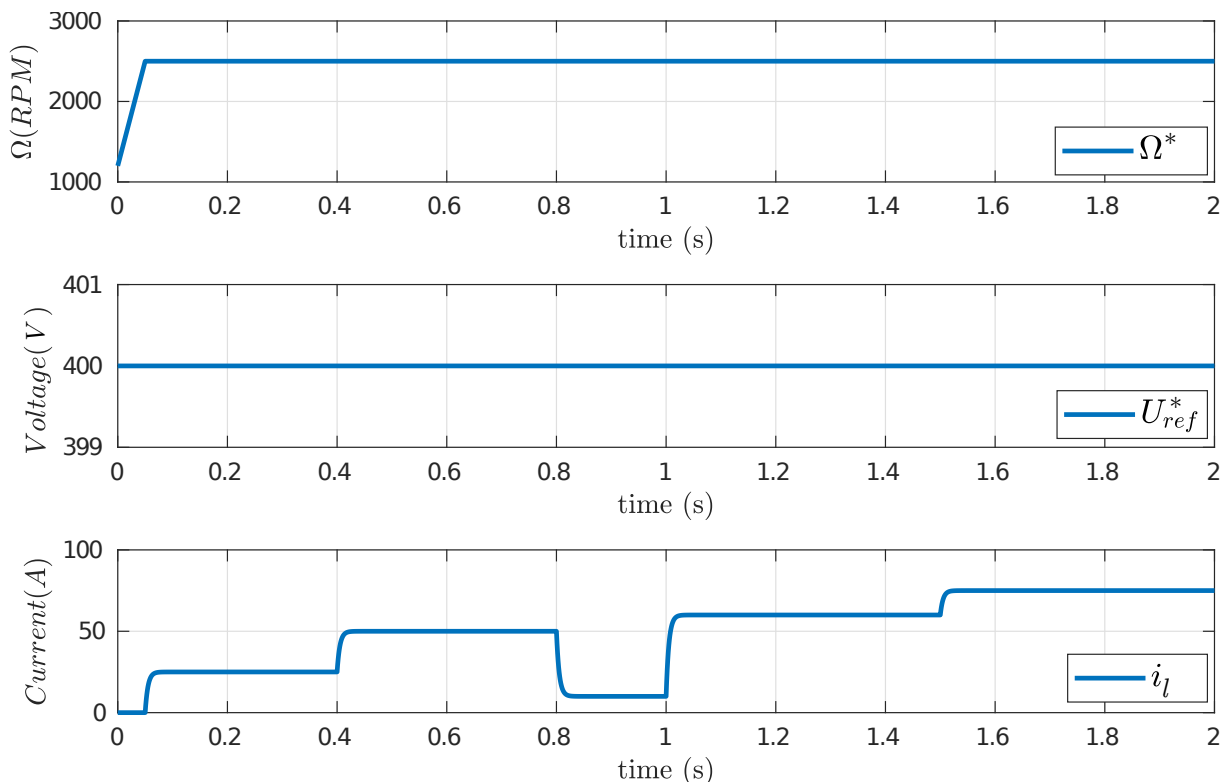- the polynomial function $s^2 + \lambda_0 s + \lambda_1$ is Hurwitz

Fig. 3. Benchmark of the proposed power management – Top left: Rotational speed reference – Middle: DC link voltage reference – Bottom : Load current.
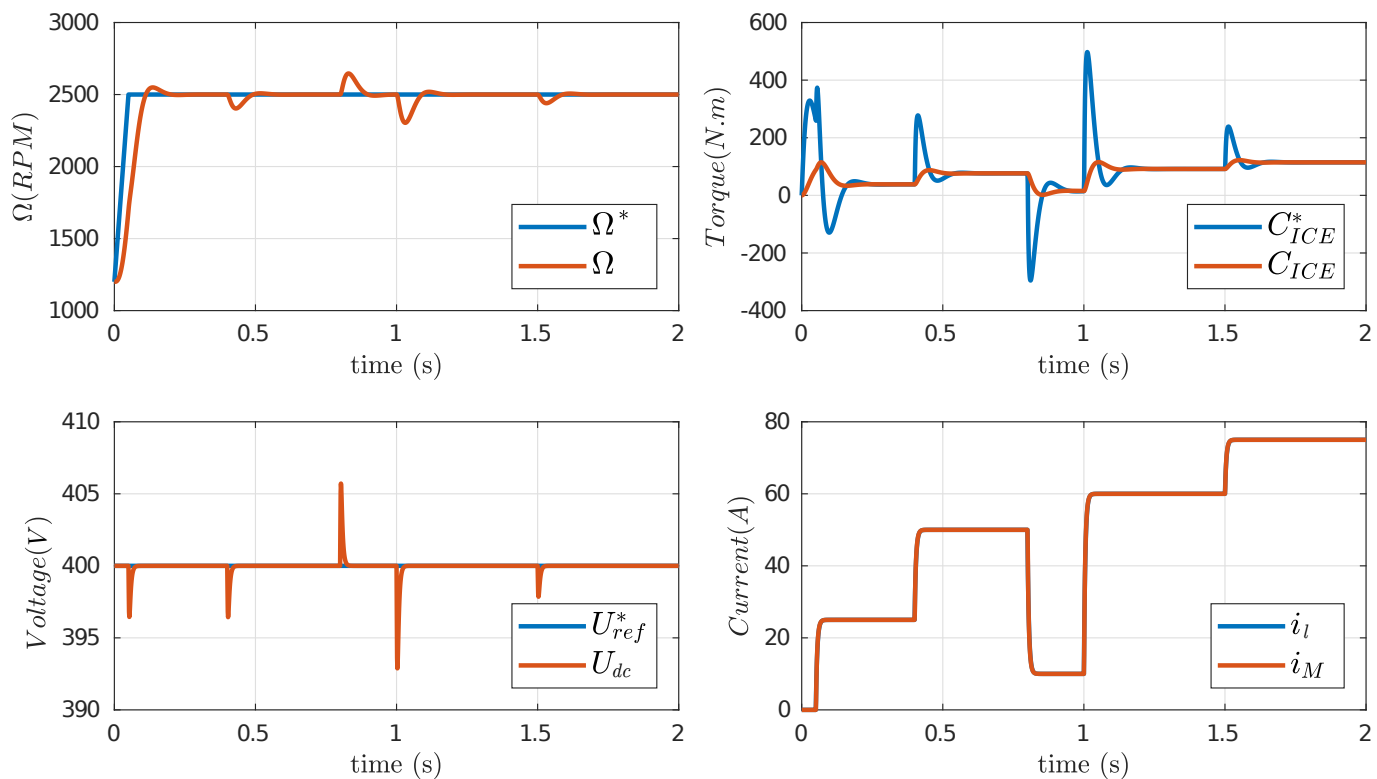

Fig. 4. Simulation results with the singular perturbation control and the current load i_l varies between 0A and 75A. – Top left: Rotational speed – Top right: ICE Torque – Bottom left: DC link voltage – Bottom right : Load current and PMSM current.

- the transient error is kept at $\pm 200 RPM$ when the power demand occur.

Table 2. Control parameters

Symbol	Value
K_P	1.84
K_I	846
ϵ	0.1
λ_0	64.4
λ_1	2100

4.3 Simulation

Fig. 4 shows the response of the system according to the reference trajectories of the benchmark (Fig. 3). It can be remarked that a very good tracking of the DC bus is ensured by the proposed fast control (13) with a smooth response of the ICE during fast power demands of the load. Moreover, the energy balance is well achieved, characterized by a rotational speed reaching the desired set point (2500 RPM) at steady state by the proposed slow control (24).

5. CONCLUSION AND FUTURE PERSPECTIVES

This paper has proposed a control based on singular perturbation approach to solve the problem of power management for a serial hybrid electric vehicle. The proposed strategy is based on two-closed loops (fast and slow), which allows to impose the constraints of slower (faster respectively) ICE torque dynamics (PMSM current dynamics respectively) and to follow the desired power management. The main goal was to develop a control strategy able to satisfy the requirement of the considered particular power topology.

Future works will focus on testing the control strategy on real conditions with a test bench and investigate a more general modeling of ICE taking account the delay response.

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