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Robust Anomaly Detection Based on a Dynamical Observer for Continuous Linear Roesser Systems

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Abstract: Monitoring of industrial systems for anomalies such as faults and cyber-attacks as unknown and extremely undesirable inputs in the presence of other inputs (like disturbances) is an important issue for ensuring the safety and the reliability of their operation. In this study, a robust anomaly detection filter is proposed for continuous linear Roesser systems using dynamic observer framework. Sufficient conditions for the existence of the observer and its sensitivity to anomaly as well as its robustness to disturbances are addressed via linear matrix inequalities (LMIs). The mentioned sensitivity and robustness are based on the H_- and H_∞ performance indices, respectively. Finally, the performance of the proposed observer is demonstrated through a numerical example.

Keywords: Fault detection and diagnosis, Anomaly Detection, 2D-Roessor Systems, Dynamic Observer

1. INTRODUCTION

Anomaly detection is one of the most challenging issues in industrial control systems. Generally, abnormality can be occurred due to faults and cyber-attacks. These two unknown inputs can affect the performance of the system and lead to the unwanted shutdowns and financial losses. Therefore, development of fast anomaly detection with maximum possible precision is crucial (Ding et al. (2018)). One of the obstacles to this goal is the existence of some unknown inputs such as disturbance and measurement noise. The effect of these inputs can be mistaken for the occurrence of the fault or cyber-attack, and a false alarm can be generated. Therefore, a diagnosis method with the ability of anomaly detection robust to noise and disturbances is in a great advantage (Wu et al. (2016)).

Observers are the most popular tool in model-based diagnosis methods and they are designed in various classes, including unknown input observers, deadbeat observers, and H_∞ observers. A robust event-triggered fault detection filter in the presence of network induced delays, data packet dropout, and disturbances is addressed by Wu et al. (2016) where the optimization problem is solved for all frequencies. Zhou et al. (2017) and Zhang et al. (2015) considered a similar problem for fault detection, where distinct finite frequency bandwidths for the faults and disturbances are assumed and it is claimed that these frequency can be determined with sufficient knowledge of the plant. Negash et al. (2016) and Luo et al. (2019) used perfect decoupling to design unknown input observers for cyber-attack detection, although their case study is different. Luo et al. (2019) investigated false data injection attacks for smart grids, while Negash et al. (2016) studied the cyber-attack detection in the communication network of a group of UAVs. Xu et al. (2018) combined both robust and perfect disturbance decoupling methods to obtain a residual for the wider class of systems, while maintaining its satisfactory fault detection performance. Silvestre et al. (2017) proposed a deadbeat observer to detect the occurred fault, with only a few past measurements for unstable linear parameter-varying systems.

Fault diagnosis has also gained attention for other dynamical systems than ordinary differential equation (ODE) systems. Two dimensional (2D) systems (Baniamerian et al, 2017) fall into this category which have more than one independent variable despite of ODE systems where the time is the only independent variable. These systems are widely used for image processing (Roesser (1975)), repetitive processes (Xu et al. (2003)), and modeling of the distributed parameter systems (Dillabough et al. (2014)). Roesser model is one of the main types of 2D systems, which is introduced by Roesser (1975).

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Fault diagnosis has also attracted attention for Roesser systems. Wang et al. (2015) proposed a deadbeat observer for fault detection of Roesser systems. They have considered the system with disturbances, and tried to cancel the effects of them on residual, completely. This approach is restrictive, because of conditions for the existence of the extinct left null space of disturbance matrix from fault matrix. Ding et al. (2015) proposed a robust H_-/H_∞ observer to maximize the effect of the faults and minimize the effect of the disturbances on the residual simultaneously, for a specified frequency bandwidth and generalized Kalman-Yakubovich-Popov lemma of discrete 2D Roesser systems was used to derive sufficient conditions for the existence of the mentioned observer. The main required assumption for their method is that the bandwidths of the faults and disturbances are known. Duan et al. (2019) proposed a similar approach for fuzzy discrete Roesser systems.

Fault detection is the only investigated problem in the above papers. It is reasonable to assume that the bandwidth of the faults and disturbances is known (although it is difficult to obtain), but this assumption does not generally hold for cyber-attacks. The set of actuators and sensors that can be affected by cyber-attacks is predictable, but the frequency bandwidth of these attacks cannot be obtained due to the unknown nature of cyber-security threats. On the other hand, design of detection observer for all frequencies can be restricting due to the low degrees of freedom in conventional static Luenberger observers. Dynamic observers can be a suitable tool to design detection filter for all frequencies with desirable robustness to the disturbances. The output injection error term is filtered in the feedback route for these observers, before using in estimation equation. Gao et al. (2016) proposed a unified framework about H_∞ dynamic observer for ODE systems, which is not applicable for Roesser model, due to matrix differential operator.

To the best of our knowledge, there is no research paper on dynamic observer design for 2D systems in the literature. In this regard, the main aim of this paper is to propose a design method of H_-/H_∞ dynamic observer for continuous 2D Roesser systems to detect anomalies in the presence of energy bounded disturbances.

The rest of the paper is organized as follows. Section 2 contains some preliminaries and problem formulation. Section 3 presents dynamic observer design, and its sufficient existence conditions. Section 4 shows the applicability of the proposed method through a numerical example. Finally, the conclusion is stated in Section 5.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider a 2D Roesser system affected by anomaly (fault/attack) and disturbance as:

$$\begin{aligned}\begin{bmatrix}\dfrac{\partial x^h(x,t)}{\partial x}\\ \dfrac{\partial x^v(x,t)}{\partial t}\end{bmatrix} =&\mathfrak{D}_{x,t}X(x,t) = \begin{bmatrix}A_1 & A_2\\ A_3 & A_4\end{bmatrix}\begin{bmatrix}x^h(x,t)\\ x^v(x,t)\end{bmatrix}\\ &+\begin{bmatrix}B_1\\ B_2\end{bmatrix}u(x,t)+\begin{bmatrix}B_{f1}\\ B_{f2}\end{bmatrix}f(x,t)\\ &+\begin{bmatrix}B_{d1}\\ B_{d2}\end{bmatrix}d(x,t),\\ y(x,t) =&[C_1\ C_2]\begin{bmatrix}x^h(x,t)\\ x^v(x,t)\end{bmatrix}+D_f f(x,t)\\ &+D_d d(x,t),\end{aligned} \tag{1}$$

where $x^v(x,t) \in \mathbb{R}^{n_h}$, $x^h(x,t) \in \mathbb{R}^{n_v}$, $y(x,t) \in \mathbb{R}^p$, $u(x,t) \in \mathbb{R}^m$, $f(x,t) \in \mathbb{R}^{n_f}$, and $d(x,t) \in \mathbb{R}^{n_d}$ denote the horizontal system states, vertical system states, output, control input, fault/attack signal, and disturbance, respectively, and $X(x,t) = [x^{v^\intercal}(x,t), x^{h^\intercal}(x,t)]^\intercal$. The matrix coefficients in (1) are $A_1 \in \mathbb{R}^{n_h\times n_h}$, $A_2 \in \mathbb{R}^{n_h\times n_v}$, $A_3 \in \mathbb{R}^{n_v\times n_h}$, $A_4 \in \mathbb{R}^{n_v\times n_v}$, $B_1 \in \mathbb{R}^{n_h\times m}$, $B_2 \in \mathbb{R}^{n_v\times m}$, $B_{f1} \in \mathbb{R}^{n_h\times n_f}$, $B_{f2} \in \mathbb{R}^{n_v\times n_f}$, $B_{d1} \in \mathbb{R}^{n_h\times n_d}$, $B_{d2} \in \mathbb{R}^{n_v\times n_d}$, $C_1 \in \mathbb{R}^{p\times n_h}$, $C_2 \in \mathbb{R}^{p\times n_v}$, $D_f \in \mathbb{R}^{p\times n_f}$, and $D_d \in \mathbb{R}^{p\times n_d}$. Moreover, the boundary conditions of (1) are assumed to be bounded:

$$\|x^h(0,t)\|_\infty < \infty, \|x^v(x,0)\|_\infty < \infty.$$

Next lemma provides the stability analysis of 2D Roesser systems.

Lemma 1. (Galkowski (2002)). The 2D Roesser system (1) with $u = f = d = 0$ is asymptotically stable, if there exist positive definite matrices P_h and P_v with appropriate dimensions, such that:

$$\begin{bmatrix}A_1^\intercal & A_3^\intercal\\ A_2^\intercal & A_4^\intercal\end{bmatrix}\begin{bmatrix}P_h & 0\\ 0 & P_v\end{bmatrix}+\begin{bmatrix}P_h & 0\\ 0 & P_v\end{bmatrix}\begin{bmatrix}A_1 & A_2\\ A_3 & A_4\end{bmatrix}<0. \tag{2}$$

For anomaly detection, the following dynamical observer is proposed:

$$\begin{aligned}\mathfrak{D}_{x,t}\hat{X}(x,t) &= A\hat{X}(x,t)+Bu(x,t)+v(x,t),\\ v(x,t) &= L_i\xi(x,t)+L_p(y(x,t)-C\hat{X}(x,t)),\\ \mathfrak{D}_{x,t}\xi(x,t) &= \Phi\xi(x,t)+\Gamma(y(x,t)-C\hat{X}(x,t)),\\ r(x,t) &= V_2\xi(x,t)+V_1(y(x,t)-C\hat{X}(x,t)),\end{aligned} \tag{3}$$

where $r(x,t)$ is the residual that reveals the effects of the fault/attack in the presence of disturbances. The main goal is to maximize the effect of fault/attack $f(x,t)$ on residual $r(x,t)$ while minimizing its sensitivity to the disturbance $d(x,t)$.

Criterion 1. The system (1) is said to have H_- performance with index β with respect to the input $f(x,t)$, if for a given $\beta > 0$ and zero boundary conditions, the following inequality for $\forall t_1, t_2 > 0$ holds:

$$\begin{aligned}&\int_0^{t_2}\int_0^{t_1} r^\intercal(\tau_1,\tau_2)r(\tau_1,\tau_2)\,\mathrm{d}\tau_1\mathrm{d}\tau_2\\ &\qquad > \beta^2\int_0^{t_2}\int_0^{t_1} f^\intercal(\tau_1,\tau_2)f(\tau_1,\tau_2)\,\mathrm{d}\tau_1\mathrm{d}\tau_2.\end{aligned} \tag{4}$$

Criterion 2. The system (1) is said to have H_∞ performance with index γ with respect to the input $d(x,t)$, if for a given $\gamma > 0$ and zero boundary conditions, the following inequality for $\forall t_1, t_2 > 0$ holds:

$$\begin{aligned}&\int_0^{t_2}\int_0^{t_1} r^\intercal(\tau_1,\tau_2)r(\tau_1,\tau_2)\,\mathrm{d}\tau_1\mathrm{d}\tau_2\\ &\qquad < \gamma^2\int_0^{t_2}\int_0^{t_1} d^\intercal(\tau_1,\tau_2)d(\tau_1,\tau_2)\,\mathrm{d}\tau_1\mathrm{d}\tau_2.\end{aligned} \tag{5}$$

It is desired is to detect anomalies in the presence of disturbances. The detectability measure is defined in *Criterion 1* while the robustness with respect to disturbances is presented in *Criterion 2*. The goal of this paper is to design a dynamical detection filter, which satisfies the H_-/H_∞ performance index and guarantees the stability of the observer error dynamics.

3. ROBUST OBSERVER DESIGN

The dynamics of estimation error as $e(x,t) = X(x,t) - \hat{X}(x,t)$, using (1) and (3) is given as:

$$\begin{aligned}\mathfrak{D}_{x,t}e(x,t) =&(A - L_pC)e(x,t) - L_i\xi(x,t)\\ &+ (B_f - L_pD_f)f(x,t) + (B_d - L_pD_d)d(x,t),\\ \mathfrak{D}_{x,t}\xi(x,t) =&\Phi\xi(x,t) + \Gamma Ce(x,t) + \Gamma D_f f(x,t)\\ &+ \Gamma D_d d(x,t),\\ r(x,t) =& V_2\xi(x,t) + V_1Ce(x,t) + V_1D_f f(x,t)\\ &+ V_1D_d d(x,t).\end{aligned} \tag{6}$$

It should be noted that the coefficient matrices of the observer have partitioned form as:

$$L_i = \begin{bmatrix} L_{i1} & L_{i2} \\ L_{i3} & L_{i4}\end{bmatrix}, L_p = \begin{bmatrix} L_{p1} \\ L_{p2}\end{bmatrix}, \Phi = \begin{bmatrix} \phi_1 & \phi_2 \\ \phi_3 & \phi_4\end{bmatrix}, \Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_2\end{bmatrix}. \tag{7}$$

Equation (6) can be rewritten in an augmented compact form as:

$$\begin{aligned}\mathfrak{D}_{x,t}Z(x,t) &= A_zZ(x,t) + B_{fz}f(x,t) + B_{dz}d(x,t),\\ r(x,t) &= C_zZ(x,t) + D_{fz}f(x,t) + D_{dz}d(x,t),\end{aligned} \tag{8}$$

where:

$$\begin{aligned}&Z = \begin{bmatrix} Z^h \\ Z^v\end{bmatrix}, Z^h = \begin{bmatrix} e^h \\ \xi^h\end{bmatrix}, Z^v = \begin{bmatrix} e^v \\ \xi^v\end{bmatrix},\\ &A_z = \begin{bmatrix} A_{z1} & A_{z2} \\ A_{z3} & A_{z4}\end{bmatrix}, A_{zk} = \begin{bmatrix} A_k - (L_pC)_k & -L_{ik} \\ (\Gamma C)_k & \Phi_k\end{bmatrix}, k = 1,\dots,4\\ &B_{fz} = \begin{bmatrix} B_{fz1} \\ B_{fz2}\end{bmatrix}, B_{fzj} = \begin{bmatrix} B_{fj} - L_{pj}D_f \\ \Gamma_jD_f\end{bmatrix}, j = 1,2\\ &C_z = [C_{z1}\ C_{z2}], C_{zj} = [V_1C_j\ 0], j = 1,2\\ &D_{fz} = V_1D_f, D_{df} = V_1D_d.\end{aligned} \tag{9}$$

The following theorem presents the main result of this paper for the design a dynamic observer (3).

Theorem 1. The 2D system (6) is stable with H_- performance of index β, and H_∞ performance of index γ, if there exist matrices $P_h^1 > 0$, $P_h^2 > 0$, $P_v^1 > 0$, $P_v^2 > 0$, Y_{p1}^1, Y_{p2}^3, Y_{i1}^1, Y_{i2}^1, Y_{i3}^3, Y_{i4}^3, Y_2^1, Y_4^2, $Y_{\phi1}^2$, $Y_{\phi2}^2$, $Y_{\phi3}^4$, $Y_{\phi4}^4$, and W such that the following LMIs hold:

$$\begin{bmatrix} \psi_{11}^1 & \psi_{11}^2 & \psi_{12}^1 & \psi_{12}^2 & \psi_{13}^1 \\ * & \psi_{11}^4 & \psi_{12}^3 & \psi_{12}^4 & \psi_{13}^2 \\ * & * & \psi_{22}^1 & \psi_{22}^2 & \psi_{23}^1 \\ * & * & * & \psi_{22}^4 & \psi_{23}^2 \\ * & * & * & * & \psi_{33}\end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} \lambda_{11}^1 & \psi_{11}^2 & \lambda_{12}^1 & \psi_{12}^2 & \lambda_{13}^1 \\ * & \psi_{11}^4 & \psi_{12}^3 & \psi_{12}^4 & \lambda_{13}^2 \\ * & * & \lambda_{22}^1 & \psi_{22}^2 & \lambda_{23}^1 \\ * & * & * & \psi_{22}^4 & \lambda_{23}^2 \\ * & * & * & * & \lambda_{33}\end{bmatrix} < 0, \tag{11}$$

where:

$$\begin{aligned}&\psi_{11}^1 = P_h^1A_1 - A_1^\intercal P_h^1 - Y_{p1}^1C_1 - C_1^\intercal(Y_{p1}^1)^\intercal - C_1^\intercal WC_1,\\ &\psi_{11}^2 = -Y_{i1}^1 + C_1^\intercal(Y_1^2)^\intercal,\\ &\psi_{11}^4 = Y_{\phi1}^2 + (Y_{\phi1}^2)^\intercal,\\ &\psi_{12}^1 = P_h^1A_2 - A_3^\intercal P_v^1 - Y_{p1}^1C_2 - C_1^\intercal(Y_{p3}^2)^\intercal - C_1^\intercal WC_2,\\ &\psi_{12}^2 = C_1^\intercal(Y_2^4)^\intercal - Y_{i2}^1,\\ &\psi_{12}^3 = Y_1^2C_2 - Y_{i3}^3,\\ &\psi_{12}^4 = Y_{\phi2}^2 + (Y_{\phi3}^4)^\intercal,\\ &\psi_{13}^1 = P_h^1B_{f1} - Y_{p1}^1D_f - C_1^\intercal WD_f,\\ &\psi_{13}^2 = Y_1^2D_f,\\ &\psi_{22}^1 = P_v^1A_4 - A_4^\intercal P_v^1 - Y_{p2}^3C_2 - C_2^\intercal(Y_{p2}^3)^\intercal - C_2^\intercal WC_2,\\ &\psi_{22}^2 = -Y_{i4}^3 + C_2^\intercal(Y_2^4)^\intercal,\\ &\psi_{22}^4 = Y_{\phi4}^4 + (Y_{\phi4}^4)^\intercal,\\ &\psi_{23}^1 = P_v^1B_{f2} - Y_{p2}^3D_f - C_2^\intercal WD_f,\\ &\psi_{23}^2 = Y_2^4D_f,\\ &\psi_{33} = \beta^2I - D_f^\intercal WD_f,\\ &\lambda_{11}^1 = P_h^1A_1 - A_1^\intercal P_h^1 - Y_{p1}^1C_1 - C_1^\intercal(Y_{p1}^1)^\intercal + C_1^\intercal WC_1,\\ &\lambda_{12}^1 = P_h^1A_2 - A_3^\intercal P_v^1 - Y_{p1}^1C_2 - C_1^\intercal(Y_{p3}^2)^\intercal + C_1^\intercal WC_2,\\ &\lambda_{13}^1 = P_h^1B_{d1} - Y_{p1}^1D_d + C_1^\intercal WD_d,\\ &\lambda_{13}^2 = Y_1^2D_d,\\ &\lambda_{22}^1 = P_v^1A_4 - A_4^\intercal P_v^1 - Y_{p2}^3C_2 - C_2^\intercal(Y_{p2}^3)^\intercal + C_2^\intercal WC_2,\\ &\lambda_{23}^1 = P_v^1B_{d2} - Y_{p2}^3D_d + C_2^\intercal WD_d,\\ &\lambda_{23}^2 = Y_2^4D_d,\\ &\lambda_{33} = D_d^\intercal WD_d - \gamma^2I,\\ &W = V_1^\intercal V_1,\end{aligned}$$

and the dynamic observer matrices in (3) can be computed as:

$$\begin{aligned}&L_{p1} = (P_h^1)^{-1}Y_{p1}^1, L_{p2} = (P_v^1)^{-1}Y_{p2}^3, L_{i1} = (P_h^1)^{-1}Y_{i1}^1,\\ &L_{i2} = (P_h^1)^{-1}Y_{i2}^1, L_{i3} = (P_v^1)^{-1}Y_{i3}^3, L_{i4} = (P_v^1)^{-1}Y_{i4}^3,\\ &\Gamma_1 = (P_h^2)^{-1}Y_1^2, \Gamma_2 = (P_v^2)^{-1}Y_2^4, \phi_1 = (P_h^2)^{-1}Y_{\phi1}^2,\\ &\phi_2 = (P_h^2)^{-1}Y_{\phi2}^2, \phi_3 = (P_v^2)^{-1}Y_{\phi3}^4, \phi_4 = (P_v^2)^{-1}Y_{\phi4}^4.\end{aligned} \tag{12}$$

Proof: LMI (10) is related to the fault/attack detection, and (11) is related to the disturbance attenuation and error dynamic stability. The proof of (10) and (11) are similar. So, the detailed proof of (11) is omitted for the sake of brevity. The condition (4) can rewritten as:

$$\begin{aligned}&\int_0^{t_2}\int_0^{t_1}\left[r^\intercal(\tau_1,\tau_2)r(\tau_1,\tau_2) - \beta^2f^\intercal(\tau_1,\tau_2)f(\tau_1,\tau_2)\right]\mathrm{d}\tau_1\mathrm{d}\tau_2\\ &\qquad\pm\int_0^{t_2}(Z^h(t_1,\tau_2))^\intercal P_hZ^h(t_1,\tau_2)\,\mathrm{d}\tau_2\\ &\qquad\pm\int_0^{t_1}(Z^v(\tau_1,t_2))^\intercal P_vZ^v(\tau_1,t_2)\,\mathrm{d}\tau_1,\end{aligned} \tag{13}$$

where $V^h(t_1,t_2) = (Z^h(t_1,t_2))^\intercal P_h Z^h(t_1,t_2)$, $V^v(t_1,t_2) = (Z^v(t_1,t_2))^\intercal P_v Z^v(t_1,t_2)$, $P_h > 0$, and $P_v > 0$. Equation (13) can be rewritten as:

$$\begin{aligned}\int_0^{t_2}\int_0^{t_1}\Big[& r^\intercal(\tau_1,\tau_2)r(\tau_1,\tau_2) - \beta^2 f^\intercal(\tau_1,\tau_2)f(\tau_1,\tau_2)\\ &- \frac{\partial V^h(\tau_1,\tau_2)}{\partial\tau_1} - \frac{\partial V^v(\tau_1,\tau_2)}{\partial\tau_2}\Big]\,\mathrm{d}\tau_1\mathrm{d}\tau_2\\ &+\int_0^{t_2}(Z^h(t_1,\tau_2))^\intercal P_h Z^h(t_1,\tau_2)\,\mathrm{d}\tau_2\\ &+\int_0^{t_1}(Z^v(\tau_1,t_2))^\intercal P_v Z^v(\tau_1,t_2)\,\mathrm{d}\tau_1 > 0,\end{aligned} \tag{14}$$

where:

$$\begin{aligned}&\frac{\partial V^h(\tau_1,\tau_2)}{\partial\tau_1} = (Z^h(\tau_1,\tau_2))^\intercal P_h\left[\frac{\partial Z^h(\tau_1,\tau_2)}{\partial\tau_1}\right]\\ &+\left[\frac{(\partial Z^h(\tau_1,\tau_2))^\intercal}{\partial\tau_1}\right]P_h Z^h(\tau_1,\tau_2),\\ &\frac{\partial V^v(\tau_1,\tau_2)}{\partial\tau_2} = (Z^v(\tau_1,\tau_2))^\intercal P_v\left[\frac{\partial Z^v(\tau_1,\tau_2)}{\partial\tau_2}\right]\\ &+\left[\frac{(\partial Z^v(\tau_1,\tau_2))^\intercal}{\partial\tau_2}\right]P_v Z^v(\tau_1,\tau_2).\end{aligned} \tag{15}$$

Regarding (8), (9), (14), and (15), the fault detectability condition (4) can be rewritten as:

$$\begin{aligned}\int_0^{t_2}\int_0^{t_1}\Bigg[&\Big(\Big[(Z^h(\tau_1,\tau_2))^\intercal\ (Z^v(\tau_1,\tau_2))^\intercal\Big]\begin{bmatrix}C_{z1}^\intercal\\ C_{z2}^\intercal\end{bmatrix}\\ &+ f^\intercal(\tau_1,\tau_2)D_{fz}^\intercal\Big)\Big([C_{z1}\ C_{z2}]\begin{bmatrix}Z^h(\tau_1,\tau_2)\\ Z^v(\tau_1,\tau_2)\end{bmatrix}\\ &+ D_{fz}f(\tau_1,\tau_2)\Big) - \beta^2 f^\intercal(\tau_1,\tau_2)f(\tau_1,\tau_2)\\ &-\Big((Z^h(\tau_1,\tau_2))^\intercal P_h\Big[A_{z1}Z^h(\tau_1,\tau_2)\\ &+A_{z2}Z^v(\tau_1,\tau_2) + B_{fz1}f(\tau_1,\tau_2)\Big]\\ &+\Big[(Z^h(\tau_1,\tau_2))^\intercal A_{z1}^\intercal + (Z^v(\tau_1,\tau_2))^\intercal A_{z2}^\intercal\\ &+ f^\intercal(\tau_1,\tau_2)B_{fz1}^\intercal\Big]P_h Z^h(\tau_1,\tau_2)\Big) -\\ &\Big((Z^v(\tau_1,\tau_2))^\intercal P_v\Big[A_{z3}Z^h(\tau_1,\tau_2) + A_{z4}Z^v(\tau_1,\tau_2)\\ &+B_{fz2}f(\tau_1,\tau_2)\Big] + \Big[(Z^h(\tau_1,\tau_2))^\intercal(A_{z3}^\intercal\\ &+(Z^v(\tau_1,\tau_2))^\intercal A_{z4}^\intercal\\ &+f^\intercal(\tau_1,\tau_2)B_{fz2}^\intercal\Big]P_v Z^v(\tau_1,\tau_2))\Big)\Bigg]\,\mathrm{d}\tau_1\mathrm{d}\tau_2 + \Delta,\end{aligned} \tag{16}$$

where:

$$\begin{aligned}\Delta &= \int_0^{t_2}(Z^h(t_1,\tau_2))^\intercal P_h Z^h(t_1,\tau_2)\,\mathrm{d}\tau_2\\ &+\int_0^{t_1}(Z^v(\tau_1,t_2))^\intercal P_v Z^v(\tau_1,t_2)\,\mathrm{d}\tau_1.\end{aligned}$$

It can be verified that the inequality (16) is satisfied if the double integrated function is positive. Therefore, the following condition can be derived:

$$\Big[(Z^h)^\intercal\ (Z^v)^\intercal\ f^\intercal\Big]\begin{bmatrix}\psi_{11} & \psi_{12} & \psi_{13}\\ * & \psi_{22} & \psi_{23}\\ * & * & \psi_{33}\end{bmatrix}\begin{bmatrix}Z^h\\ Z^v\\ f\end{bmatrix} < 0, \tag{17}$$

where:

$$\begin{aligned}&\psi_{11} = P_h A_{z1} + A_{z1}^\intercal P_h - C_{z1}^\intercal C_{z1},\\ &\psi_{12} = P_h A_{z2} + A_{z3}^\intercal P_v - C_{z1}^\intercal C_{z2},\\ &\psi_{13} = P_h B_{fz1} - C_{z1}^\intercal D_{fz},\\ &\psi_{22} = P_v A_{z4} + A_{z4}^\intercal P_v - C_{z2}^\intercal C_{z2},\\ &\psi_{23} = P_v B_{fz2} - C_{z2}^\intercal D_{fz},\\ &\psi_{33} = \beta^2 I - D_{fz}^\intercal D_{fz}.\end{aligned} \tag{18}$$

The positive definite matrices P_h and P_v are assumed to have partitioned form as:

$$P_h = \begin{bmatrix}P_h^1 & 0\\ 0 & P_h^2\end{bmatrix},\ P_v = \begin{bmatrix}P_v^1 & 0\\ 0 & P_v^2\end{bmatrix}. \tag{19}$$

Noting (17), (18) and (19), and the matrices in (12), the inequality (10) can be obtained. The inequality (11) can be obtained in a similar way. The matrices P_h and P_v are dummy variables that only need to be positive definite. So, they can be used as degree of freedom to ensure stability of system (6). According to *Lemma 1*, the system (6) without consideration of fault/attack and disturbance is asymptotically stable, if there are matrices $P_h > 0$ and $P_v > 0$ such that the following inequality holds:

$$\begin{bmatrix}A_{z1}^\intercal & A_{z3}^\intercal\\ A_{z2}^\intercal & A_{z4}^\intercal\end{bmatrix}\begin{bmatrix}P_h & 0\\ 0 & P_v\end{bmatrix} + \begin{bmatrix}P_h & 0\\ 0 & P_v\end{bmatrix}\begin{bmatrix}A_{z1} & A_{z2}\\ A_{z3} & A_{z4}\end{bmatrix} < 0. \tag{20}$$

Considering (12), (19), and (20), the error dynamic stability can be guaranteed by (11). ■

Remark 1. It is desired to guarantee the detectability of attack, and derive the related LMI, assuming the stability of (8). As mentioned before, there are some papers that solve the fault detection problem using a static observer where it is assumed that the fault and disturbance can affect the system in two known different finite frequency bandwidths. This is not the case for cyber-attacks, where its specifications can be unknown unlike faults and disturbances. Therefore, the cyber-attack effect on the residual should be maximized in all frequencies.

4. NUMERICAL SIMULATION

In this section, two numerical examples are represented to demonstrate the efficacy of the proposed observer design method. The first example is about fault detection that the matrices B_f and D_f are not zero and the second example with $B_f = 0$ is related to the sensor false data injection attack detection.

Example 1. Consider system (1) with the following matrices:

$$\begin{aligned}&A_1 = \begin{bmatrix}-1 & 1\\ 0 & -0.5\end{bmatrix}, A_2 = \begin{bmatrix}1\\ 6\end{bmatrix}, A_3 = [0\ 0], A_4 = -3\\ &B_1 = \begin{bmatrix}1\\ 0\end{bmatrix}, B_2 = B_{f2} = B_{d2} = 1, B_{f1} = \begin{bmatrix}0\\ 1\end{bmatrix}, B_{d1} = \begin{bmatrix}1\\ 0\end{bmatrix},\\ &C_1 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, C_2 = \begin{bmatrix}0\\ 1\end{bmatrix}, D_f = \begin{bmatrix}-0.9\\ 1\end{bmatrix}, D_d = \begin{bmatrix}0.5\\ 0.5\end{bmatrix}.\end{aligned} \tag{21}$$

The control input is assumed $u(x,t) = 0.1$, and fault and disturbance signals are taken as:

$$f(x,t) = \begin{cases} 0.1 & \text{if}(3 < x < 6)\&(5 < t < 8) \\ \dfrac{x}{150}\left|\sin\left(\dfrac{t-11}{2}\right)\right| & \text{if}(x > 9)\&(t > 11) \\ 0 & \text{otherwise} \end{cases},$$

$$d(x,t) = \frac{x}{2}\sin(t).$$

The observer (3) is designed using Theorem 1 with parameters $\beta = 10$ and $\gamma = 1$. Figures 1, 2, and 3 show the fault, the disturbance and the residual signal, respectively.

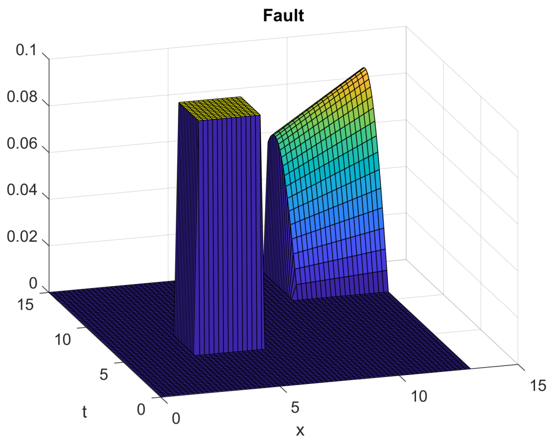

Fig. 1. The fault signal corresponding to Example 1.

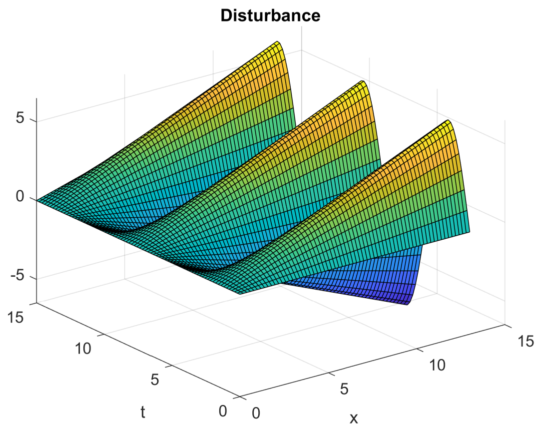

Fig. 2. The disturbance signal corresponding to Example 1.

As shown in Fig. 3, the residual signal converge to zero, before the occurrence of fault and after the injection of the fault, it suddenly takes value about 50 times of the fault magnitude. At the same time, the effect of the disturbance signal is attenuated on the residual signal.

Example 2. This example is about sensor false data injection into the 2D system (1). Consider the matrices of the system with $B_f = 0$ as:

$$A_1 = \begin{bmatrix} -5 & 1 \\ 0 & -2.5 \end{bmatrix}, A_2 = \begin{bmatrix} 0.3 \\ 1 \end{bmatrix}, A_3 = [0 \ 0.5], A_4 = -1,$$

$$C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

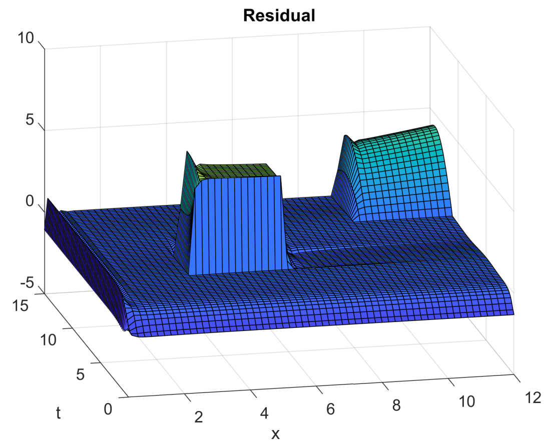

Fig. 3. The residual signal corresponding to Example 1.

The other matrices are the same as (21). The control input, γ and β are considered as in Example 1 as well. The attack and disturbance signals are considered as:

$$f(x,t) = \begin{cases} \dfrac{x}{50}\sin(t) & \text{if}(x > 3)\ \&\ (t > 5) \\ \dfrac{x}{150}\left|\sin\left(\dfrac{t-11}{2}\right)\right| & \text{if}(x > 9)\&(t > 11) \\ 0 & \text{otherwise} \end{cases},$$

$$d(x,t) = \begin{cases} 5 & \text{if}(3 < x < 6) \text{ or } (8 < x < 12) \\ 0 & \text{otherwise} \end{cases}.$$

Figures 4, 5, and 6 show the attack, the disturbance and the residual signal, respectively.

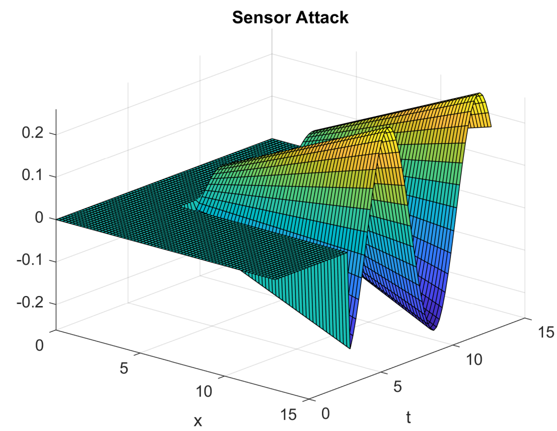

Fig. 4. The sensor false data injection attack corresponding to Example 2.

As shown in Fig. 6, the injected sensor attack can be detected using the proposed approach in the presence of large disturbance.

5. CONCLUSION

In this paper, the problem of robust fault/attack detection for continuous Roesser systems is considered. In this regard, a H_-/H_∞ dynamical observer is proposed to get the residual sensitive to the anomalies with attenuated effect of disturbances. The sufficient conditions for the stability and H_-/H_∞ performance of the proposed dynamic observer is

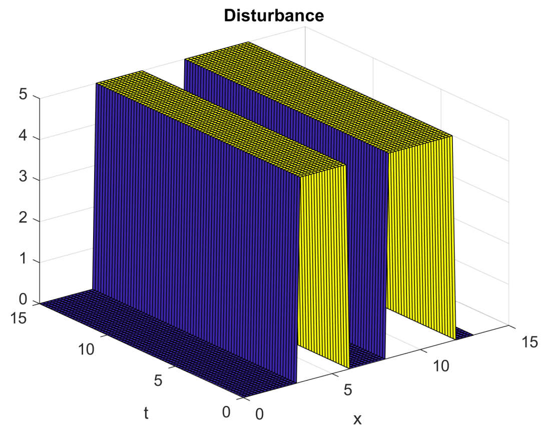

Fig. 5. The disturbance signal corresponding to Example 2.

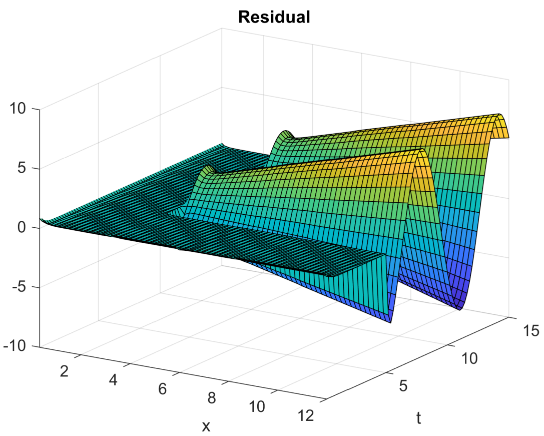

Fig. 6. The residual signal corresponding to Example 2.

derived. Finally, the performance of the proposed dynamical observer is validated through a numerical simulation. As an extension to the current research, investigation on the dynamical observer design for 2D singular systems is considered.

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