Design of a Steering Law for Control Moment Gyro Clusters based on a Set of Initial Positions

Louis Dubois * **Hélène Evain** * **Vincent Meyniel** *
Daniel Alazard ** **Mathieu Rognant** ***

* *CNES, Toulouse, France (e-mail: helene.evain@cnes.fr).*
** *ISAE SUPAERO, Toulouse, France*
*** *ONERA, Toulouse, France*

Abstract: This paper addresses the issue of controlling a spacecraft with a redundant pyramidal Control Moment Gyro (CMG) cluster. A rapid analysis of the mathematical problem to steer the attitude control system is carried out with a focus on how to use the redundancy to start the attitude maneuvers in optimal conditions. A set of initial positions that ensures no singularity is encountered during an unidirectional maneuver is defined thanks to the study of the topology of the cluster and simulations. The desired initial position is derived onboard the spacecraft from the direction of the maneuver. After reaching this position without creating torque errors thanks to the null-motion trajectories, the Moore-Penrose steering law is used. The cases of a pyramidal six-CMG cluster without CMG failures and with two failures are studied. Finally, simulations show the characteristics of the designed steering laws.

Keywords: Satellite Attitude Control, Control Moment Gyro, Redundancy, Steering Law, Singularity.

1. INTRODUCTION

Control Moment Gyros (CMGs) are actuators used to control the orientation of spacecraft in space. They are based on the same physical principle as reaction wheels, which is the transfer of angular momentum from the actuators to the spacecraft. Instead of changing the magnitude of each actuator angular momentum (as for reaction wheels), its direction is changed in CMGs. CMGs provide larger torques than reaction wheels for the same mass and power consumption, and since they do not rely on fuel, they are often integrated in massive spacecraft like the International Space Station and in agile Earth-observation satellites. Nevertheless, this technology is expensive and complex to steer due to the singularities. The case of an agile small satellite requiring large reorientation maneuvers possibly along any axis is studied here. Vectors are in bold lower cases and matrices in bold upper cases.

A single-gimbal CMG is composed of a flywheel that spins at a constant rate, providing an angular momentum along a unit vector $\mathbf{x}_i$: $\mathbf{h}_i = h_f \mathbf{x}_i$. Torques are then created by rotating the angular momentum vector of an angle σ_i called gimbal angle, along an axis $\mathbf{z}_i$ fixed in the satellite frame. The created torque is then along $\mathbf{y}_i$ so that $(\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i)$ is a direct frame, and equals $\boldsymbol{\tau}_i = h_f \dot{\sigma}_i \mathbf{y}_i$. A drawing of the principle is given in Fig. 1. Let's note $\mathbf{X}(\boldsymbol{\sigma})$ and $\mathbf{Y}(\boldsymbol{\sigma})$ the rectangular matrices where each column is respectively the axis $\mathbf{x}_i$ or $\mathbf{y}_i$ of the CMG number i in the cluster, expressed in the cluster frame. $\boldsymbol{\sigma}$ is the vector gathering the gimbal angles of the CMGs. We define an initial position of the cluster as a vector $\boldsymbol{\sigma}_0$ such that the angular momentum of the cluster (sum of the CMG
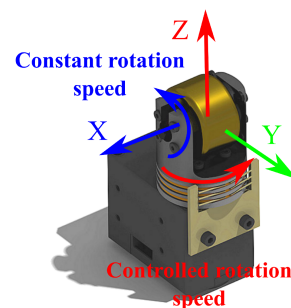

Fig. 1. CMG physical principal

angular momentum) is null. It is assumed the cluster starts at a null angular momentum at the beginning of the maneuvers. The torque created by the cluster can be expressed as (1) assuming h_f is equal for all CMGs.

$$\boldsymbol{\tau} = h_f \mathbf{Y}(\boldsymbol{\sigma})\dot{\boldsymbol{\sigma}} \tag{1}$$

where $\dot{\boldsymbol{\sigma}}$ gathers the gimbal velocities of the CMGs. To control the attitude of a satellite, a required velocity $\dot{\boldsymbol{\sigma}}$ is given to the CMGs by the steering law so that the desired torque $\boldsymbol{\tau}$ calculated by the attitude controller can be achieved. The cluster is said to be in a singularity when (1) cannot be pseudo-inverted. This happens when $\mathbf{Y}$ is not full-rank, meaning that torques cannot be created along an axis and that the satellite is not fully controllable. Singularities have been deeply studied by Bedrossian et al. (1991a), Kurokawa (1998) and Wie (2004). The total amount of angular momentum that a cluster of CMGs can provide to the satellite is limited and is called the angular momentum envelope or the external singularities.

The singularities located inside this envelope are called internal and are to be avoided or passed.

Steering the CMG cluster comes down to inverting equation (1) under constraints like avoiding the singularities and the gimbal velocity saturations for instance. One constraint we add is for the steering law to be calculated in real-time onboard satellites. Many steering laws have been proposed in the literature over the years and a summary is proposed by Kurokawa (2007). The simplest one is the Moore-Penrose where a pseudo-inversion of $\mathbf{Y}(\boldsymbol{\sigma})$ is carried out. However, in singular positions, the gimbal velocity tends to infinity and the actuators saturate. Therefore, adding a term in the pseudo-inverse as proposed by Bedrossian et al. (1991b) in the Singular Robust Inverse method makes the inversion possible, but errors are created and the system cannot begin in singular initial positions. The Perturbed Singular Robust Inverse as described by Oh and Vadali (1989) and Wie (2003) consists in adding perturbation terms in the matrices of pseudo-inversion when needed, making it possible to begin in initial positions but also creating torque errors. In the preferred directions from Kurokawa (1997), the steering law limits the possible gimbal angles to remain in angular momentum areas without singularities. The maneuver capacities are decreased but steering inside these domains is easy. Other real-time steering laws include the Extended Kalman Filter (EKF) based laws where constraints can be added and prioritized while the problem remains invertible (see Evain et al. (2016)).The EKF-based method is theoretically able to start in any gimbal position and avoid singularities, but tuning the parameters is complex. Another method of particular interest for this paper is the Preferred Gimbal Angles from Vadali et al. (1990). It consists in choosing an optimal initial position for the forthcoming maneuver by back-integration of the maneuver. Then the usual pseudo-inversion can be used. This method has also been investigated by Kurokawa (1998), according to whom it presents advantages despite two major issues : initial gimbal angle determination and final return of the angular momentum of the cluster to zero. One difference in the work we propose compared to Vadali et al. (1990) is the method to select the initial positions by studying the topology of the cluster and then simulating to find the minimal sets while minimizing the distance of the initial positions. In addition, the initial gimbal reorientation uses null motion and the singular robust inverse in case of singularities in the paper of Vadali et al. (1990) while we propose to also use the topology of the cluster to steer exactly and globally to the desired gimbal angles.

In this paper, we assume that most of the maneuvers are known beforehand. The steering law has been developed relying on the knowledge of the topology of the cluster. As can be expected and will be shown in the following sections, only a small set of initial positions is needed when six CMGs are active in order to be able to reach the angular momentum envelope in any direction. Once at a desired initial position, the Moore-Penrose steering law can be used as in the Preferred Gimbal Angles method.

This paper begins by describing the topology of the six-CMG cluster needed to design the steering law, with and without actuator failures. Then the steering law is described, in two parts: definition of the sets of initial
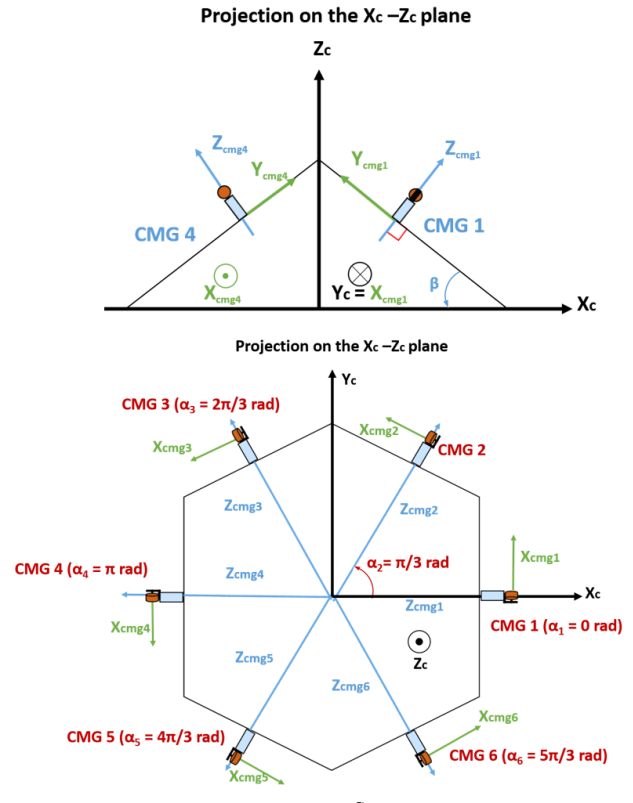

Fig. 2. Cluster configuration chosen

positions to reach the angular momentum envelope and the null-motion trajectories to follow; and secondly the choice of the initial positions among the set depending on the torques required and the actual steering law. Finally, simulations show a possible implementation of the developed steering law.

2. SIX-CMG CLUSTER CONFIGURATION AND SOME ELEMENTS OF TOPOLOGY

2.1 Choice of the cluster configuration

Taking into account the singularity issues, a cluster of at least four single-gimbal CMGs is necessary for a three-axis attitude control. As explained by Kurokawa (1998) and Evain et al. (2019), a pyramidal six-CMG cluster has the advantages not only of redundancy and failure robustness (compared to roof-type clusters for instance) but also of being much easier to steer than pyramidal four-CMG clusters. Indeed, there are initial configurations that ensure singularities are only located near the angular momentum envelope if unidirectional maneuvers are considered. Thus this configuration is studied and shown in Fig 2 in the cluster frame noted $(\mathbf{x}_c, \mathbf{y}_c, \mathbf{z}_c)$.

Two cases are then studied, the nominal six-CMG cluster and this cluster with two CMG failures, symmetrically located. One CMG failure is not studied here for brevity.

2.2 Elements of topology for the six-CMG cluster

Due to the redundancy, all the positions $\boldsymbol{\sigma}$ such that the satellite angular momentum of the cluster equals a
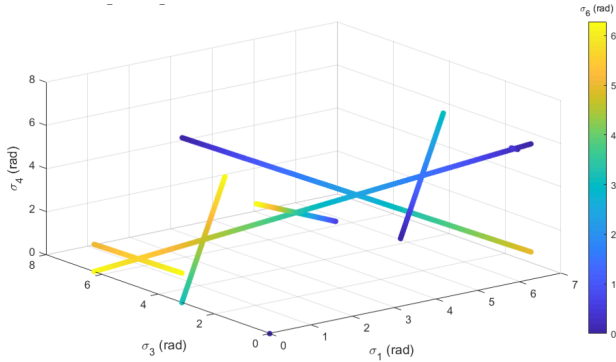
Fig. 3. Null-motion manifold of the six-CMG cluster with two CMG failures

particular vector are called null-motion manifolds. Indeed, when moving in such a manifold, the angular momentum remains constant so no motion is created. The manifold we are interested in corresponds to a null angular momentum of the cluster, thus is the solution to (2). To simplify, this manifold will be noted M_0.

$$\mathbf{h} = h_f \sum_1^6 \mathbf{x}_i(\sigma_i) = \mathbf{0} \qquad (2)$$

There are six unknowns for three equations. By choosing three σ_i values, we finally obtain a polynomial equation of degree 8 with no obvious solution. Therefore, the solutions are derived numerically. There is a wide variety of possible initial positions to test for our steering law, but since only a small set is necessary, we will focus on specific structures that are particularly interesting for steering the cluster. We concentrate on finding parts of the manifold in which it is possible to steer the cluster without creating a torque error. This happens when all positions taken while steering from a position $\boldsymbol{\sigma}_{01}$ in M_0 to $\boldsymbol{\sigma}_{02}$ belong to M_0 as well. The following paths have been determined as belonging to M_0:

- $\boldsymbol{\sigma} = (\alpha, 0, -\alpha, \alpha, 0, -\alpha)$
- $\boldsymbol{\sigma} = (\alpha, -\alpha, 0, \alpha, -\alpha, 0)$

where α can be any angular value. In particular, the zero position belongs to both paths and is therefore an intersection point.

2.3 Topology for the case of two symmetrical CMG failures

We study the case of two CMG failures (two CMGs keeping a null angular momentum) symmetrically located so that the cluster keeps two axes of symmetry. This failure case is interesting since its M_0 structure is similar to some of the six-CMG cluster: M_0 is composed of straight paths. Indeed, in the other cases, the M_0 does not contain straight paths as in the previous section, so it is harder to steer from one initial position to another one without creating torque errors. The steering law still applies but with this disadvantage. We choose to have CMG number 2 and 5 out. The same equation as (2) can be derived in our case with only one degree of freedom in σ_i. M_0 is then represented in Fig. 3. The various paths can be deduced :

(1) $(\sigma_1, \sigma_3, \sigma_4, \sigma_6) = (\alpha, -\alpha, \alpha, -\alpha)$
(2) $(\sigma_1, \sigma_3, \sigma_4, \sigma_6) = (\alpha, -\alpha, -\alpha, \alpha)$
(3) $(\sigma_1, \sigma_3, \sigma_4, \sigma_6) = (\alpha, \alpha, \alpha, \alpha)$

(4) $(\sigma_1, \sigma_3, \sigma_4, \sigma_6) = (\alpha, \alpha, -\alpha, -\alpha)$

The first path is called the main path since all the others cross it. The intersection points have also been determined. Thus, if steering from a path to another is needed, it can be carried out without torque errors by following the main path and through the intersection points.

3. STEERING LAW DEVELOPPED

3.1 Description the steering law

The design of the steering law is in two steps. The first part deals with the calculations that can be carried out on ground since they need to be done only once. The set of initial positions is found and associated with the possible angular momentum directions where no singularity is encountered before the angular momentum saturation. The distance to singularities in the angular momentum domain is assessed to check the robustness to realization errors.

The second part is implemented onboard the spacecraft: given the forthcoming maneuver or the required torque at the beginning of the maneuver (depending on the available data), an initial position is chosen. The cluster is then steered following the paths defined in the previous section to the desired configuration, and finally the Moore-Penrose steering law is calculated.

3.2 Selection of the set of the initial positions on ground

The objective is to select the smallest set possible and preferably on the paths defined before. To do so, the following procedure has been carried out :

(1) Mesh the paths in M_0
(2) From each point of the mesh, simulate maneuvers in every direction of the meshed angular momentum domain with a Moore-Penrose steering law and in open-loop. Stop each maneuver whenever the external angular momentum envelope is reached or when a singularity is too close. For this last criterion, the determinant of $\mathbf{Y}(\boldsymbol{\sigma})\mathbf{Y}(\boldsymbol{\sigma})^T$ is evaluated and the threshold has been set at 0.5, when the lack of efficiency may cause actuator saturations. In a singular position, the determinant would be null.
(3) For each initial position, a mapping of the location of the internal singularities encountered (inside the angular momentum envelope) is obtained. Polygons representing the borders of the singular regions are generated.
(4) Search among the generated mappings the smallest set enabling to carry out maneuvers in any direction without encountering internal singularities, and if possible this set should be located on the smallest number of paths.

It was noted that in a single path, some areas remain unreachable as shown in Fig. 4. Therefore initial positions on at least two paths are needed to cover the entire angular momentum domain. In Fig. 4, the mapping of the angular momentum domain is given in latitude and longitude in the CMG cluster frame. For the two-failure case, five initial positions on three different paths ensure a
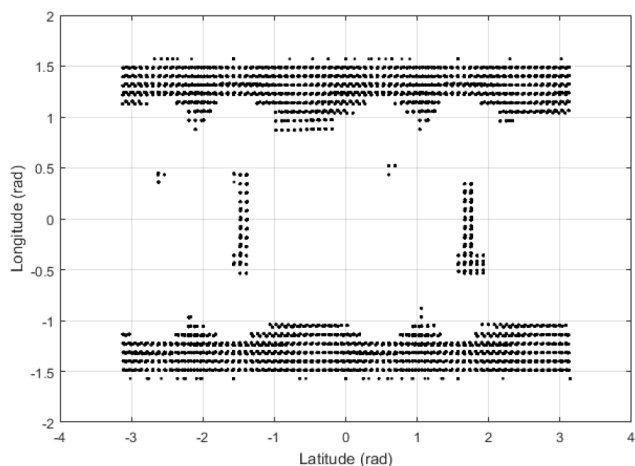
Fig. 4. Unreachable angular momentum envelope in the given directions for the first path for the two-failure case
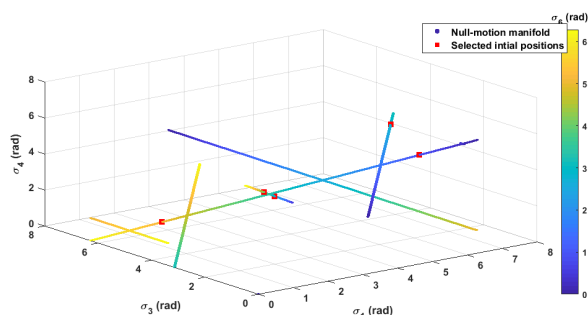

Fig. 5. Positions of the set of selected initial positions for the two-failure cases

complete mapping of the angular momentum domain with no internal singularity. Their positions are shown on Fig. 5.

For the six-CMGs case, it appeared that with three initial positions in total, located on the two paths, 99.9% of the angular momentum domain is reachable. Instead of introducing another path and initial positions, it was decided to keep the three positions and check in the small remaining domain the behavior of steering law which has proven satisfactory as shown in section 5.

4. STEERING LAW IMPLEMENTED IN THE SATELLITE

Once the sets and mappings are defined from on-ground calculations, the following procedure is implemented in the spacecraft as a steering law.

4.1 Selection of the initial position in the defined set

First the direction of the following maneuver is retrieved or at least the first torque command. From it, the various maps are checked to find whether the direction belongs to the polygons of singular regions. The initial positions that can be used to achieve the maneuver are deduced. If more than one position fits, then the most robust one is selected. The robustness is evaluated by the distance of the direction of torque to the nearest encountered singularity in the
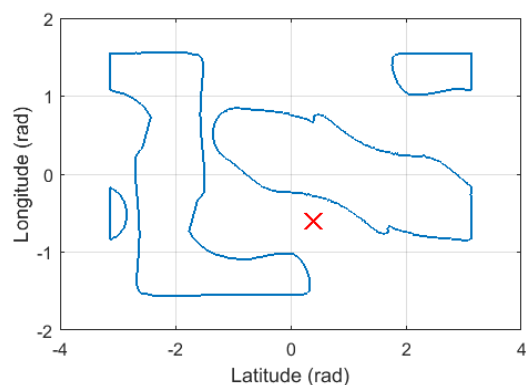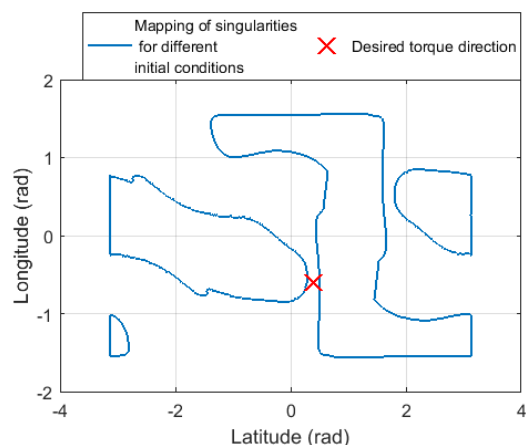

Fig. 6. Mappings for two initial positions

maps. An example of the maps of two initial positions for the two-failure case are given in Fig. 6 with a desired direction represented by the red cross. In this algorithm, the second initial position will be selected because it has more margins to singularities.

4.2 Steering to the desired initial position

Then, the cluster is steered to the desired initial position in the null-motion manifold, from an initial position that belongs to the main path in the two-failure case, and to one of the two paths for the six-CMGs case. The steering law ensures that the paths are followed to prevent torque errors with a Proportional-Derivative controller. If the desired initial position is located on a different path from the initial one, the cluster is steered to an intersection point.

For other CMG failure cases that do not present the same topology characteristics, steering to the initial position without creating torque error require more complex and curved trajectories.

4.3 Moore-Penrose steering law

Finally, when the cluster is in the desired initial position, the required attitude can be reached by using the Moore-Penrose pseudo-inverse since the singularities have been avoided beforehand.

In the next section, simulations are carried out to show the behavior of the steering law.

5. SIMULATIONS

5.1 Presentation of the control loop

The control loop implemented is composed of an attitude controller like shown in Fig 7. A Proportional-Derivative controller with a phase-lead transfer function to improve the phase margins was selected. This controller was tuned thanks to an H-infinity synthesis to ensure a time response of the system, robustness margins and low excitation of the actuators at high frequency as shown in Evain et al. (2017). The inputs of the system are the required attitude and angular velocity of the spacecraft. To improve the performance, a feedforward term (3) has been added, with a structure based on previous work (see Genin and Viaud (2018)).

$$\boldsymbol{\tau}_{ff} = K_{ff} I_{sat} \dot{\boldsymbol{\omega}}_d (t + \Delta T) \qquad (3)$$

with K_{ff} a gain to tune, I_{sat} the minimal moment of inertia of the satellite, $\dot{\boldsymbol{\omega}}_d$ the required angular acceleration and ΔT the feedforward delay to tune. A MATLAB/Simulink simulator has been developed, including the steering law and the attitude controller as presented in Fig 7.

5.2 Example of a maneuver with two CMG failures

Fig 8 shows simulation results for a simple maneuver in the two-failure case, and Fig 9 shows the corresponding behavior in the cluster's angular momentum space. As one can see, the threshold set on the determinant of $\mathbf{Y}(\boldsymbol{\sigma})\mathbf{Y}(\boldsymbol{\sigma})^T$ is respected almost all along the motion maneuver and never falls to zero, which means that no singularity has been encountered.

The systems behavior is then composed of four stages :

(1) The cluster is in its nominal position, which belongs to the null-motion manifold.
(2) The attitude controller sets a torque value to be created by the CMGs. The steering law finds the appropriate initial position with the most robustness and sets it as a target. This position belongs to another path, so the system moves along the null-motion path towards the corresponding intersection point. This position is singular, so the value of the determinant of $\mathbf{Y}(\boldsymbol{\sigma})\mathbf{Y}(\boldsymbol{\sigma})^T$ falls to zero. It corresponds to the brown arrow on Fig 9.
(3) The system switches path once on the intersection point, and reaches the selected initial position. One can see that the determinant of $\mathbf{Y}(\boldsymbol{\sigma})\mathbf{Y}(\boldsymbol{\sigma})^T$ reaches values above the threshold. See black arrow on Fig 9.
(4) From the selected position, the system uses the Moore-Penrose steering law to create torques. As it is now far from the singularities, this maneuver is efficient and can catch up from the starting delay. See green arrow on Fig 9.

5.3 Test of the singular direction for the six-CMG cluster

For the six-CMGs case, a check of the steering laws behavior in the 0.1% of the angular momentum domain uncovered with the three initial positions was necessary (see section 3.2). Simulation results for a maneuver requiring angular momentum in this remaining direction are presented on Fig 10 from one of the initial position.

As one can see, a flat response in the angular momentum norm and a drop of the value of the determinant of $\mathbf{Y}(\boldsymbol{\sigma})\mathbf{Y}(\boldsymbol{\sigma})^T$ shows that the cluster approaches a first singularity at 1.2 s but the determinant value is just below 0.5. A second drop of the value of the determinant at 1.7 s shows it slowly approaches the external angular momentum envelope. Finally at 3 s, a second flat response at the approximate same value of the angular momentum and another drop of the determinant shows that the system passes next to the same singularity while returning to the null-motion manifold. Though, the determinant never falls beneath 0.5, which means that the system did not hit this singularity. In addition, the gimbal velocities did not saturate as one can see on the plot.

This can be explained thanks to the threshold value set to 0.5 for the creation of the mappings. This value is indeed high enough to ensure a safety margin.

6. CONCLUSION

The complexity of passing the singularities have been put in the initial positions calculations. The main advantage of this method is its easy implementation onboard, however some delay can arise if the cluster is configurated at the desired initial position only when the maneuver starts. This method is not robust to maneuvers other that those tested when defining the initial positions, here for instance to maneuvers other than along one direction. Also, it does not ensure that the initial gimbal position is reached at the end of the maneuver. Nevertheless for missions with predefined maneuvers, which is a common feature, this paper provides a simple strategy to guarantee the angular momentum domain available even with CMG failures with quantified robustness to singularities.

Comparisons in particular with the EKF-based method are scheduled to be carried out experimentally. The experiment will take place in microgravity during a parabolic flight campaign at Novespace. The verification of the real-time features of the steering law will therefore also be demonstrated.

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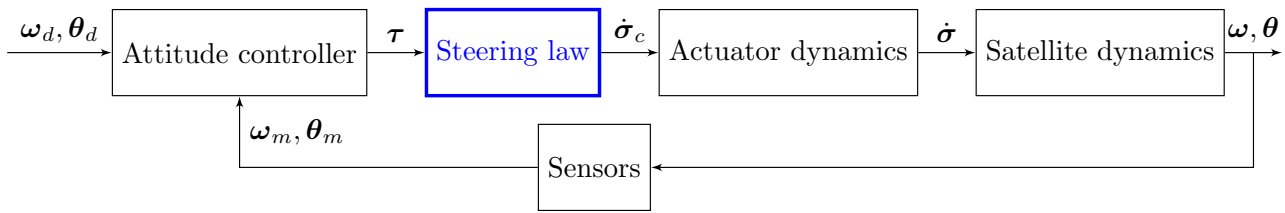
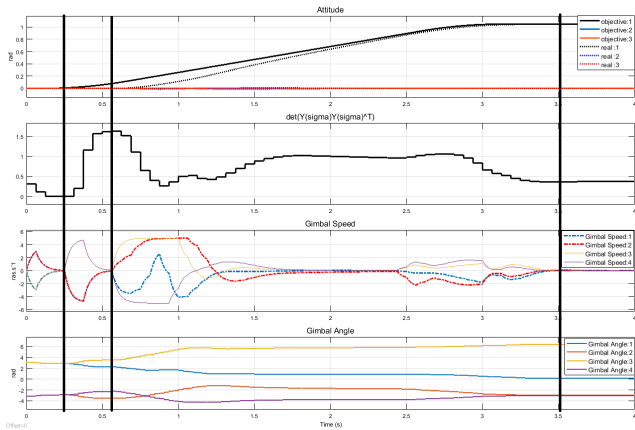Fig. 7. Block diagram of the satellite attitude control tested.

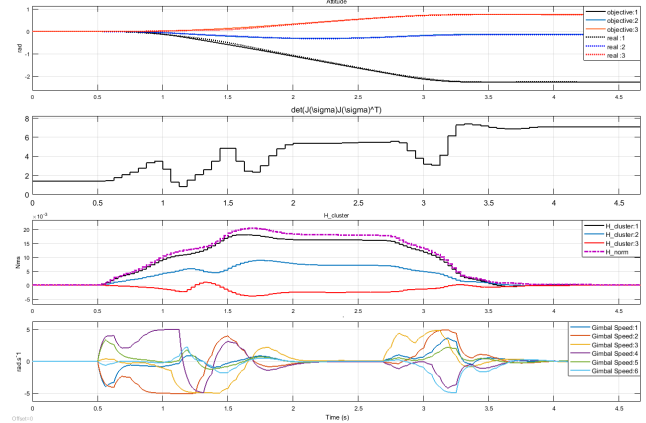Fig. 8. Simulation results for the two-failure case
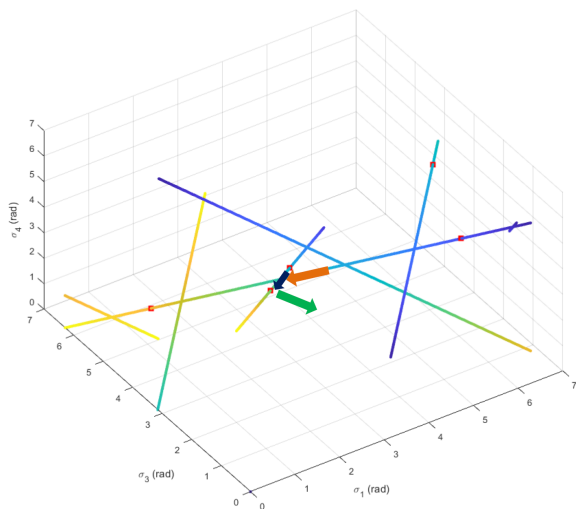

Fig. 10. Simulation results for the six-CMG case

Fig. 9. Steering procedure in M_0

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