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Abstract: The interconnection of complex devices in network structures has been a challenging topic in the system identification research domain. This study presents the model identification of autonomous vehicles in platoon formation, which can be cast as a dynamic network. The paper presents the comparison between two network structures: (i) a vehicle-based network, which considers the interconnection between the vehicles based only on the velocity measurements, and (ii) a sensor-based network that considers the available sensor, i.e. the velocity and the relative distance measurements. The comparison is based on the difference between the identified transfer functions and the true ones, and the analysis of the identified air resistance coefficient variances. In addition, the paper presents the identifiability requirements for both network topologies. Simulation results show that for the same data set the variance of the identified parameters can be almost five times smaller if the system is represented as a sensor-based network, but some conditions to guarantee the identifiability of this network structure must be fulfilled.

Keywords: Identification, dynamic network model, identifiability, network topologies, interconnected systems, autonomous vehicles

1. INTRODUCTION

In this paper, we study the parameter identification of autonomous vehicles in platoon formation, which can be represented as distributed control system. The concept of vehicle platooning is to form a convoy of vehicles driving close behind each other to increase the freeways traffic throughput and also reduce fuel consumption for the follower vehicles, transportation costs and greenhouse gas emissions (Liang et al., 2016). The study of platooning of autonomous vehicles has gain more attention as the traffic flow is increasing as the world economy grows, especially in freeways, due to the expansion of the freight transport (Liang et al., 2016). The road freight is responsible for the transportation of 49.0 % in Europe Union (EU Commission, 2018), 45.9 % in the USA (Wilmsmeier and Spengler, 2015) and 61.1 % in Brazil (CNT, 2019) of all goods. To achieve higher fuel efficiency, with economies up to 20% of fuel consumption (Liang et al., 2016), small spacing between two vehicles is required, which increases the risk of accident. One solution for this issue is the formation of autonomous vehicle platoons, which may maintain a desired spacing policy. Many studies can be found in the literature where controllers are designed with this objective (Dai et al., 2018; Liang et al., 2016; Sedran, 2016). However, most of these studies consider that the dynamic behavior of all vehicles in the platoon is the same, i.e. the same mathematical model is considered. Note that the vehicles masses or brake performances can vary, due to the load and unload of goods or depreciation of the breaking systems, which may cause accidents in autonomous vehicle if these changes are not considered when the platoon enter in a upper- or downhill road. Considering the fuel consumption, one of the main concerns is the air resistance forces in the vehicle aerodynamic, which changes considering the distances from the leader, weight, velocity and type of the vehicle (Alam et al., 2015; Sandberg, 2001). This parameter is extremely important to estimate the fuel consumption and to design reliable controllers.

The interconnection of a platoon can be represented as a dynamic network model, where one vehicle influences the dynamic behavior of the other, as a distributed control system. It is well know that complex network structure cannot be operated, designed, and maintained without the help of models (Dankers et al., 2016). The dynamic network modeling is defined as an interconnection of...
transfer functions or modules where the interconnecting signals (terminals) are considered as nodes/vertices in the network, and proper transfer function are considered as links/edges (Goncalves and Warnick, 2008; Van den Hof et al., 2013; Gevers et al., 2018). In this paper, it is assumed that the interconnection structure (topology) of the network is known, and our goal is to identify the transfer functions in the network as in Van den Hof et al. (2013).

In this context, this study focuses on how to identify the transfer function of each vehicle in a platoon based on data from autonomous vehicle formation. Also, a comparison between different network model representations of the dynamic system and its identified parameter variances are considered: (i) a vehicle-based network structure considering the velocity measurements and (ii) a sensor-based network structure considering velocity and the relative distance measurements.

2. MATHEMATICAL MODELING AND PROBLEM STATEMENT

This study considers a platooning formation of heavy-duty vehicles (HDVs). Figure 1 illustrates the platoon system architecture for an N vehicle platoon. The lead vehicle, with index i = 1, is to the left and the last vehicle is to the right. The control architecture for vehicle speed control is shown in front of each vehicle. The information flow in the system is given by the arrows (Alam et al., 2015).

Each HDV has two layers of controllers and a connection to the wireless network to send relevant information between the vehicles. The bottom layer is the Cruise control (CC), where the controller acts in the speed of the vehicle considering the speed set by the driver. In the layer above the advanced CC (ACC) uses a radar sensor to control the desired spacing to the preceding vehicles (Liang et al., 2016). Based on this structure, a suboptimal decentralized controller can be designed to maintain the vehicle formation, as presented in (Alam et al., 2015). In this paper we consider that when the gain of the controllers for each HDV are known, which is normally the case, it is possible to identify the transfer function of each HDV. Therefore, the model of each HDV and the dynamic network models can be represented as follows.

2.1 HDVs Mathematical Modeling

The dynamic equation of center of mass of the HDVs can be described by

\[ m_v \ddot{v} = k_e T_e - k_b F_{brake} - k_d v^2 - k_f \cos \alpha - k_g \sin \alpha, \]

where \( v \) is the vehicle velocity, \( \dot{v} \) is its derivative (acceleration), \( m_v \) denotes the accelerated mass and \( T_e \in R \) denotes the net engine torque. \( k_e, \ k_b, \ k_d, \ k_f, \) and \( k_g \) denote the characteristic vehicle and environmental coefficients for the brake, air resistance, road friction, and gravitation respectively. \( \alpha \) is the terrain slope and \( F_{brake} \) is the action of the brakes. Further explanation about the parameters can be found in Alam et al. (2015).

The nonlinear model (1) can be linearized with respect to cruise velocity \( v_o \), an engine torque \( T_{e,o} \) which maintains the velocity, a fixed time gap between the vehicles \( \tau_{s,o} \), and a constant slope \( \alpha_o \).

The linearized equation of the HDV can be represented as

\[
\begin{aligned}
\dot{v}_i &= - \frac{2k_d v_i}{m_{t,i}} v_i + k_e T_{e,i}, \quad \text{(leader)} \\
\dot{v}_i &= - \frac{2k_d v_i}{m_{t,i}} v_i + \frac{k_d v_i^2}{m_{t,i}} d_{(i-1)i} + k_e T_{e,i}, \quad \text{(followers)},
\end{aligned}
\]

where \( \tilde{k}_{d,i} > 0 \) is the coefficient related to the air resistance and vehicle velocity and \( k_{d,i} > 0 \) is the coefficient or the air resistance related to the distance between two vehicles. Typical values for \( k_d \) ranges from 0.5 to 1.1 (Sandberg, 2001), and \( \tilde{k}_{d,i} = k_i(1 - \Phi(d)/100) \), where \( \Phi = 41.29 - 0.414d \), \( d = v_o \) (Alam et al., 2015).

Assumption 1. The mass \( m_{t,i} \) and the cruise velocity \( v_o \) are known or can be measured.

Assumption 2. The coefficients \( k_{e,i} \) that transforms the engine torque \( (T_{e,i}) \) into linear force is known.

The platoon dynamics can be represented in a compact form for \( N \) vehicles

\[
\begin{aligned}
\dot{v}_1 &= \Theta_1 v_1 + k_{e,1} T_{e,1}, \quad \text{(leader)}, \\
\dot{v}_i &= \Theta_i v_i + \delta_i d_{(i-1)i} + k_{e,i} T_{e,i}, \quad \text{(followers)},
\end{aligned}
\]

where \( i = 2, ..., N \) is the number of HDVs in formation,

\[
\Theta_1 = - \frac{2k_d v_1}{m_{t,1}} \Theta_i = - \frac{2k_d v_i}{m_{t,i}} \quad \text{and} \quad \delta_i = - \frac{k_d v_i^2}{m_{t,i}} \cdot (2)
\]

Considering the HDVs velocity as \( (v_1, v_2, ..., v_N) \) and relative distance between the HDVs \( (d_{12}, d_{23}, ..., d_{(N-1)N}) \), to maintain the formation one can design a state-feedback controller with the following control law (Alam et al., 2011)

\[
T_{e,1} = - K_{i}^3 v_1 + r_{v,1}.
\]

\[
T_{e,i} = - K_{i}^1 v_{(i-1)} - K_{i}^3 d_{(i-1)i} - K_{i}^3 v_i + r_{v,i}, \quad \text{(4)}
\]

where \( T_{e,i} \) is the input torque for the \( i \)th vehicle, the \( K_{i}^{1,2,3} \) are the feedback gains and \( r_{v,i} \) are the preset reference velocity.

The implementation of this control law gives the following closed-loop representation

\[
\begin{aligned}
\dot{v}_1 &= (\Theta_1 - k_{e,1} K_{i}^3) v_1 + r_{v,1}, \quad \text{(leader)} \\
\dot{v}_i &= (\Theta_i + k_{e,i} K_{i}^3) v_i + (\delta_i + k_{e,i} K_{i}^3) d_{(i-1)i} - k_{e,i} K_{i}^1 v_{(i-1)} + r_{v,i}. \quad \text{(followers)}
\end{aligned}
\]

2.2 Problem Statement

The objective of this study is to identify the transfer function model of each HDV and also the parameters \( k_d \) and \( k_f \), which are linked to the air resistance coefficient. For that, measurements from the vehicle velocity \( (v_i) \) and relative distance \( (d_{(i-1)i}) \) are gathered with sample time
To proceed the identification procedure we first obtain the discrete transfer function using the Euler Backward approximation for the velocity equations

\[ V_i(q^{-1}) = -\frac{K_i k_e, t_s}{(K_i k_e, t_s - \Theta_i t_s + 1 - q^{-1})} V_{i-(1)}(q^{-1}) + \frac{K_i k_e, t_s}{(K_i k_e, t_s - \Theta_i t_s + 1 - q^{-1})} D_{i-(1)}(q^{-1}), \]

and for the relative distance

\[ D_{i-(1)}(q^{-1}) = (V_i(q^{-1}) - V_{i-(1)}(q^{-1}) t_s) / (q^{-1} - 1), \]

where \( q^{-1} \) is the backward shift operator, i.e. \( q^{-1} u(t) = u(t-1) \). Based on the discrete transfer function the problem is cast in the dynamic network model representation.

### 2.3 Dynamic network model representation

In order to represent the studied problem in the framework of the identification of dynamical networks (Dankers et al., 2016; Gevers et al., 2018), we consider two network structures: (i) where each \( L \) node is the velocity of \( N \) HDVs and (ii) where each \( L \) node corresponds to a particular sensor, i.e. velocity and relative distance, where the node signals can be denoted as \( \omega_1(t), \ldots, \omega_L(t) \). These node signals are related to each other and to external excitation signal \( r_j \) and with noise signals \( e_j \) by the following network equation, which we call network model and in which the matrix \( G^0 \) will be called the network matrix:

\[
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_L
\end{bmatrix} = \begin{bmatrix}
G_{12} & \cdots & G_{1L} \\
G_{21} & \cdots & G_{2L} \\
\vdots & \ddots & \vdots \\
G_{L1} & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_L
\end{bmatrix}
+ K^0(q) \begin{bmatrix}
\gamma_1 \\
\gamma_2 \\
\vdots \\
\gamma_L
\end{bmatrix}
+ H^0(q) \begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_L
\end{bmatrix},
\]

or

\[ \omega(t) = G^0(q) \omega(t) + K^0(q) r(t) + H^0(q) e(t), \]

with the following properties (Gevers et al., 2018):

- \( G_{ij} \) are proper but not necessarily strictly proper transfer functions. Some of them may be zero, indicating that there is no link from \( \omega_j(t) \) to \( \omega_i(t) \);
- there is a delay in every loop going from one \( \omega_p(t) \) to itself;
- the network is well-posed so that \( I - G^0 \) is proper and stable;
- all node signals \( \omega_p(t), p = 1, \ldots, L \) are measurable.

\( K^0 \) reflects how the external excitation signals affect the node signals.

- \( e \in \mathcal{R}^L \) is an unmeasured disturbance, referred to as the process noise with positive definite covariance matrix \( \Sigma \), \( H(q) \) is a \( L \times L \) stable rational matrix.

- the external excitation signals \( r_j \) are assumed to be uncorrelated with all noise signals \( e_j, j = 1, \ldots, L \).

### 3. Dynamic Network Identification

Among different identification methods for closed-loop system, we selected one of the most general method called Instrumental Variable (IV) (Söderström and Stoica, 1983).

#### 3.1 The Instrumental Variable Method

In the case of network identification, the interconnection of the modules could result in

\[ \dot{\omega}_j(t) |_{t = 1, \theta} = \phi(t)^T \theta_o + \nu_j(t). \]

Here \( \theta_o \) is called the true parameter vector, and assumes that \( \nu_j(t) \) is a stationary stochastic process that is independent of the input signal (Söderström and Stoica, 1983). In this case the estimation is consistent if (and essentially only if)

\[ E [\phi(t) \phi^T(t)] \text{ is nonsingular}, \quad (9) \]

\[ E [\phi(t) \nu(t)] = 0, \quad (10) \]

where \( E \) denoted the expectation operator. In the studied case, \( \nu(t) \) is correlated noise which violates the second condition, as the disturbance \( \nu(t) \) is correlated with the delayed output variables present in \( \phi(t) \). One way to overcome this issue is to use the Instrumental variables (IV), which is a generalization of the LS estimate and is expressed as

\[ \hat{\theta}^{IV}_M = \left[ \frac{1}{M} \sum_{t=1}^M z(t) \phi^T(t) \right]^{-1} \left[ \frac{1}{M} \sum_{t=1}^M z(t) \nu(t) \right], \]

where \( z(t) \) is a vector of instrumental variables. There are several techniques to select the IV, for example instrumental variable in four steps, IV recursive, etc (for more details (Söderström and Stoica, 1983)). In this work the choice of the instrumental is the input signal of the network, the variable \( r_1 \) (the velocity set point for the platoon leader) where

\[ z(t) = [ r_1(t) \ r_1(t-1) \ r_1(t-2) \ \cdots \ r_1(t-M) ]. \]

Before proceeding to the parameter identification step, an important definition has to be stated, according to (Gevers et al., 2013):

**Definition 1. (Identifiability at \( \theta_1 \)**) Consider a model at a given parameter value \( \theta_1 \). The model is locally identifiable at \( \theta_1 \) if there exists a \( \delta > 0 \) and a data set \( z(\cdot) = \{ u(\cdot), x_0 \} \) such that, for all \( \theta \in \| \theta - \theta_1 \| \leq \delta \), the outputs of the model with these two different parameter values \( \theta \) and \( \theta_1 \), both driven by the same data set are identical (i.e. \( \omega(t, \theta) = \omega(t, \theta_1) \forall t > 0 \)) only if \( \theta = \theta_1 \). The model is globally identifiable at \( \theta_1 \) if the same holds for all \( \delta > 0 \).

The model is structurally identifiable if it is identifiable at all \( \theta \).

### 4. HDV Dynamic Network Representation and Parameter Identification

In this section, we present the identification procedure involving a platoon of three HDVs. The identification procedure is evaluated comparing the true value parameters to the identified ones and its variances. In the simulation procedure, similar as proposed by Alam et al. (2015), first it is considered that the HDV platoon is moving with constant speed of 70 km/h \( (v_o = 70 \text{ km/h}) \), meaning that the distance between the vehicles are constant as previously set \( (\tau_{r,s} = 1 \text{s}) \). The disturbance in the network is added by the leader that is forced to accelerate through a step input from 70 km/h to 80 km/h and after 60 second resume the cruise speed to 70km/h. This results in a step excitation in the platoon, as shown in Figure 2. The radar sensor, which measures the relative distance between the two HDVs, are considered to have white noise with variance 0.1. It is important to highlight that different from the work proposed by Dai et al. (2018); Liang et al. (2016); Sedran (2016), each vehicle has differently dynamic behavior,
meaning that $m_{t,1} \neq m_{t,2} \neq m_{t,3}$, $k_{d,1} \neq k_{d,2} \neq k_{d,3}$ and $k_{d,1} \neq k_{d,3} \neq k_{d,3}$.

The simulation parameters are (Vehicle 1) $k_{d,1} = 0.6$, $k_{e,1} = 0.148 \times 10^{-3}$, $K_{d}^1 = 0.98 \times 10^3$, $m_{t,1} = 40000$ kg; (Vehicle 2) $k_{e,2} = 0.148 \times 10^{-3}$, $K_{d}^2 = -6.56 \times 10^3$, $K_{d}^2 = -500.35 \times 10^3$, $K_{d}^3 = 590.03 \times 10^3$, $m_{t,2} = 30000$ kg; (Vehicle 3) $k_{e,3} = 0.148 \times 10^{-3}$, $K_{d}^1 = -6.8 \times 10^3$, $K_{d}^2 = -590.35 \times 10^3$, $K_{d}^3 = 700.03 \times 10^3$, $m_{t,3} = 50000$ kg and $v_o = 19.44$ m/s (70 km/h).

4.1 Vehicle-Based Dynamic Network Representation for the Platoon Formation of HDVs

Assuming that the $i^{th}$ vehicle controls the headway distance by using only information from the immediate preceding vehicle, the discrete transfer function from the lead vehicle’s velocity $v_1(q^{-1})$ to the tail-end vehicle’s velocity $V_N(q^{-1})$ can be expressed as

$$V_2(q^{-1}) = G_1^v(q^{-1})G_2^v(q^{-1})\cdots G_{N-1}^v(q^{-1})V_1(q^{-1}),$$

where

$$V_2(q^{-1}) = G_i^v(q^{-1})V_{i-1}(q^{-1}), \quad i = 2, \ldots, N,$$

the superscript $(v)$ means the vehicle-based dynamic network structure.

Figure 3 represents the block diagram of equation (14), which is represented as a branch network.

4.2 Sensor-Based Dynamic Network Representation for the Platoon Formation of HDVs

The network matrix $G^0$ from equation (7) represents the interconnection of the HDVs. Taking the closed-loop discrete equations of the platoon formation, equations (5) and (6), and the measurements from velocity sensor and relative distance sensor, the network matrix can be expressed as (13).

The identifiability of the dynamic network must be analyzed in order to obtain the correct identification of the network parameters. Considering that the only excitation signal is given by the leader of the platoon ($re_1(t) \neq 0$) and the parameters of the HDV no. 2 are desired, we must guarantee independent excitations to their in-neighbours, here we have $v_1(t), v_2(t)$ and $d_{12}(t)$ are uncorrelated signals, otherwise the parameter subset is non-identifiable. In figure 4 we can see that the signal $v_1(t)$ is correlated with $v_2(t)$ and $d_{12}(t)$ when there is no noise in the measurements. This means that the network is not identifiable unless there is uncorrelated noise in the measurements of velocity and relative distance, i.e. $re_2(t) \neq re_3(t) \neq 0$. For the HDV no. 3 we have a similar result, where we must guarantee the signals $v_2(t), v_3(t)$ and $d_{23}(t)$ are uncorrelated, to guarantee the network identifiability. Normally, in practice all measurements have noise and that is an advantage for the dynamic network identification.
The parameters are identified using the IV method (11), (25) and (26). Based on that we define

\[
\hat{\theta}_{(j-2)}^s = \left[ a_{0,i}^s, a_{1,i}^s, b_{0,i}^s \right],
\]

\[
\begin{align*}
\hat{\phi}_{(j-2)}^s &= \left[ -v^s_1[k] - v^s_{i-1}[k-1] v_{i-1}(j-1) \right], \\
\hat{\phi}_{(j-1)}^s &= \left[ \Theta_i^s, e_{i-1}^s, d_{i-1}^s \right], \\
\hat{\phi}_{(j-1)}^s &= \left[ -v^s_i[k] - v^s_{i-1}[k-1] d_{i-1}(j-1) \right].
\end{align*}
\]

and consider the instrumental variable as (12).

The identification procedure uses the same data set presented in the previous section. As the system is represented as a sensor-based dynamic network, which considers the measurements of \(v_1(t), v_2(t)\) and \(d_{12}(t)\), where \(d_{12}(t)\) (the radar sensor of HDV no. 2) has noisy measurements for the identification of HDV no. 2. Similar regressor is considered for the third vehicle, but in this case it uses the measurements of \(v_2(t), v_3(t)\) and \(d_{23}(t)\), where \(d_{23}(t)\) (the radar sensor of HDV no. 3) has noisy measurements for the identification of HDV no. 3. To obtain the desired parameters \((k_{d,i,j}^s and \(k_{d,i,j}^s\)), first we compute \(\Theta_i^s\) and \(\delta_i^s\) using equations (29) and (28), respectively, and after equations presented in (2). The true values, the estimated mean values and its variance are presented in Table 1.

**REFERENCES**


Table 1. Air resistance coefficient identification results \( n = 1000 \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Estimated Values (Vehicle-Based Structure)</th>
<th>Estimated Values (Sensor-Based Structure)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{1,2} )</td>
<td>1.100 ( \times 10^4 )</td>
<td>1.097 ( \times 10^4 )</td>
<td>2.979 ( \times 10^{-6} )</td>
</tr>
<tr>
<td>( \hat{k}_{1,2} )</td>
<td>8.228 ( \times 10^{-1} )</td>
<td>2.729 ( \times 10^{-3} )</td>
<td>6.97 ( \times 10^{-7} )</td>
</tr>
<tr>
<td>( k_{1,3} )</td>
<td>8.000 ( \times 10^{-1} )</td>
<td>2.983 ( \times 10^{-2} )</td>
<td>3.673 ( \times 10^{-9} )</td>
</tr>
<tr>
<td>( \hat{k}_{1,3} )</td>
<td>5.340 ( \times 10^{-1} )</td>
<td>6.704 ( \times 10^{-3} )</td>
<td>8.209 ( \times 10^{-10} )</td>
</tr>
</tbody>
</table>

Fig. 5. Bode Diagram from the difference between identified and true transfer function. The first row is linked to the HDV no. 2 and the second row HDV no. 3. The first column are the vehicle-based identification results and the other two columns are the sensor-based identification results.


