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Bayesian Optimization Approach to Input Shaper Design for Flexible Beam Vibration Suppression ⋆

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Abstract: This paper tackles the problem of suppressing vibrations of a flexible beam mounted on a mobile robot for inspection purposes. The adopted approach is an input shaper design along with Bayesian optimization. The latter methodology is employed to find out the optimal shaping parameter, taking into account non-ideal behaviors as controller hysteresis and time delays. Experimental results bolster the performance of the proposed approach.

Keywords: flexible beam, vibration suppression, Bayesian optimization, input shaping, feedforward control, adaptive control

1. INTRODUCTION

The European Organization for Nuclear Research (CERN) has several accelerators and laboratory environments where robotic survey is ideal because of its large area and possible radiation hazard. Mobile robots and robotic trains are the preferred tools for visual inspection of these environments since they can cover long distances relatively fast. A significant challenge in the navigation of mobile robots is the avoidance of obstacles in tunnels. In accelerator tunnels, robots must be able to pass under the beam, while being able to inspect the top of the machinery. In order to satisfy these conflicting requirements, a camera positioning system had been designed. It can raise a pan-tilt camera to 1.7 m height in order to inspect the top of the accelerator. A retractable flexible beam was used as the camera stand to make the system fast and lightweight. The beam's height can be changed with a motor, and the camera orientation can be modified with a 2-axis pan-tilt module on top. It goes without saying that such a setup may be used for other inspection applications in field robotics.

Nowadays, most of the structural elements of robotic arms and positioning systems are made from rigid steel links. Rigid links have high stiffness, low elasticity, and high mass. Rigid links need high torque actuators to reach high speeds. The reduced mass of flexible beam positioning systems allows faster movements with less actuator power. However, controlling the precise position of the beam tip is challenging because of the vibrations of the beam. The vibration is most dominant on the top of the beam, which can make visual inspection very uncomfortable.

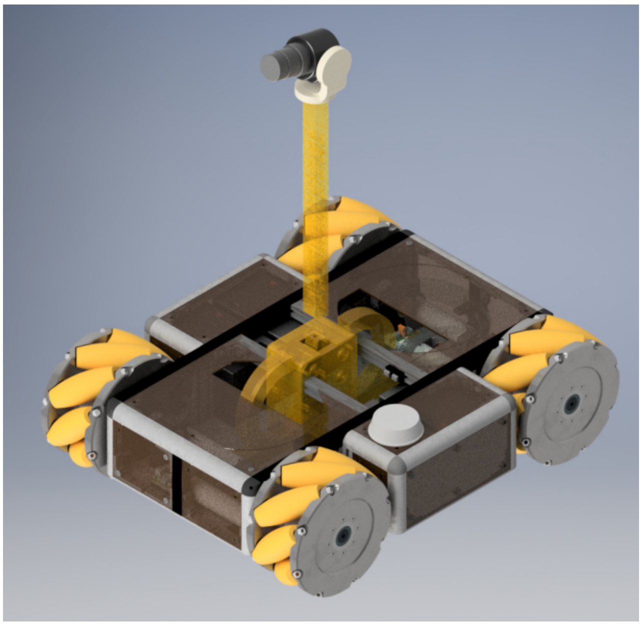

Fig. 1. CAD model of the mobile robot with the flexible beam.

With reference to Fig. 1, the mobile robot and the flexible beam create a flexible inverted pendulum with a tip mass on a cart. The robot has four omnidirectional wheels, and thus it is holonomic. The flexible beam has only one actuator to change the height of the camera. This

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makes the system underactuated since no controller can directly influence the horizontal motion of the beam. The motion control of the robot is accomplished through four independent motors, one for each wheel. The high mass of the robot base limits the acceleration of the system.

We decided to utilize input shaping, a feed-forward control method, to compensate for the vibrations of the beam. In our current work, we propose a novel approach where input shaper design is formulated as an optimization problem. Based on the system response, our algorithm attempts to find out the optimal shaping parameters. This approach automatically takes into account the non-ideal behaviour of controllers, such as hysteresis and time delays. By the usage of Bayesian optimization, we can surpass the performance of traditional input shapers with as few as 20 trials.

The outline of the paper is as follows. The next section provides state of the art and the contribution of the work. The input shaping technique is revised in Section 3. In Section 4, we give a brief introduction to Bayesian optimization. The experimental setup and results are discussed in Section 5. Section 6 gives the conclusion and proposes further research directions.

2. STATE OF THE ART

The control of flexible structures dates back more than 50 years. There have been several efforts made to model the vibrations of beams. These methods can be grouped into lumped parameter models and partial differential equation models. Patil and Gandhi (2014) modeled the flexible beam with tip mass through the Euler-Lagrange method. Park and Youm (2001) introduced a dynamic model of moving elastic beam based on the Euler-Bernoulli beam model. Xu and Yu (2004) described elastic inverted pendulums based on a Hamiltonian principle as a coupling of both ordinary and partial differential equations. Trivedi et al. (2011) modelled an elastic beam as a connection of rigid segments. Dwivedy and Eberhard (2006) gave a comprehensive review of different approaches for the modelling of flexible manipulators.

Based on the system models, many researchers have proposed feedback control methods for stabilizing flexible inverted pendulums. Tang and Ren (2009) designed a control system based on a proportional controller and a low pass filter for a flexible inverted pendulum. Banavar and Dey (2010) introduced a closed-loop controller for flexible inverted pendulum on a cart based on the energy-Casimir method. Yu et al. (2012) investigated the control of a flexible pendulum, with both linear-quadratic regulator (LQR) and fuzzy control. Kim et al. (2003) designed a flexible beam based X-Y positioning system with robust feedback and a preshaping controller. Gorade and Kurode (2015) and Peng et al. (2018) presented a sliding-mode control approach for beam stabilization. Gandhi et al. (2016) implemented a non-linear feedback controller based on energy shaping. Dadios et al. (2006) has synthesised a controller based on a simulated system model with genetic algorithms. Singla (2011) developed a vibration suppression method for cart flexible pendulum based on zero vibration derivative (ZVD) input shaper.

Input shaping has been widely used for the vibration suppression of flexible structures, since the seminal paper of Singer and Seering (1990). The big appeal of this feed-forward method lies in its simplicity and ability to eliminate vibrations of flexible structures. However, input shaping is very sensitive to system identification errors. Several methods has been proposed to alleviate this sensitivity, such as ZVD, extra insensitive (EI) (Singhose et al. (1994)), and specified insensitivity (SI) (Singhose et al. (1996)) shapers. Adaptive input shaping methods are meant to identify system parameters on the fly. This has the advantages to accommodate changes in system structure, for example, in the case of cranes, when the mass of the container changes. Pereira et al. (2012) presented a method for algebraic identification of input shapers. Cole and Wongratanaphisan (2011) developed a least-squares optimization method for direct input shaper design. Stergiopoulos et al. (2009) implemented an adaptive input shaping scheme on a crane. Ramli et al. (2018) proposed a neural network-based adaptive input shaping method.

The primary real-world application of input shaping is in cranes and flexible robotic arms. There have been relatively few attempts to use them with mobile robots. Freese et al. (2007) developed a mine-detecting robot, where the arm is compensated for flexible vibrations. Hamaguchi et al. (2005) used input shaping to reduce the sloshing of fluids in a tank located on top of a mobile robot.

2.1 Contribution

Our contribution can be found in the complexity of the robotic systems. Since the robot has four directly actuated wheels, with no suspension, a high amount of vibration is introduced by the unevenness of the surface and the non-ideal control of the movement. This high noise makes traditional input shaping methods, where only the model of the beam is considered, resulting in a suboptimal solution. The usage of Bayesian optimization ends in a much simpler adaptive shaping method. The method also allows for the direct comparison of input shaping parameters, without the need to perform further trials. It is also able to reach this performance in as few as 20 trials.

3. INPUT SHAPING

Input-shaping is a feed-forward control method, which was developed for the vibration suppression of flexible beams. Input shaping changes the control signal by applying a finite impulse response (FIR) filter to it. The filter, the so-called input shaper, suppresses the flexible poles of the beam.

In the simplest case, the beam can be modelled with a second-order harmonic oscillator. In this case, the position of the beam y_o can be described by the following equation

$$y_o(t) = \left[\frac{A_0 \omega}{\sqrt{1-\xi^2}} e^{-\xi\omega(t-t_0)} \right] \sin\left(\omega\sqrt{1-\xi^2}\,(t-t_0) \right), \tag{1}$$

where A_0 denotes the amplitude of the vibration, ξ is the damping factor, and ω natural frequency of the beam, t the current time point, and t_0 the time of the impulse. The system response to a set of impulses can be calculated

trough superposition. For the sake of simplicity, we denote $\omega_d = \omega\sqrt{1-\xi^2}$ as the damped natural frequency.
By applying a second impulse, precisely with a delay of half a wave length, the two impulses cancel each other out. In Fig. 2, it can be observed that after half a wavelength the vibration of the beam is completely suppressed.

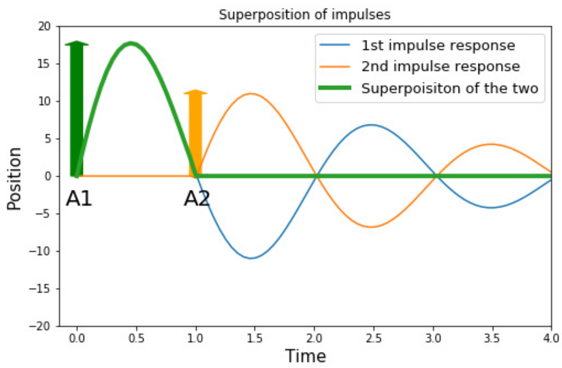

Fig. 2. Combined effect of two impulses.

In case of several impulses, the residual vibration of the sequence after the last impulse becomes

$$y_\Sigma(t) = \sum_{i=1}^{n}\left[\frac{A_i\omega}{\sqrt{1-\xi^2}}e^{-\xi\omega(t-t_0)}\right]\sin\left(\omega_d\left(t-t_0\right)\right). \quad (2)$$

To ensure that the flexible beam reaches its desired position, the amplitude of the impulses must be equal to the desired control amplitude yielding

$$\sum_{i=0}^{n} A_i = 1. \quad (3)$$

Since the free vibration of the beam is periodical, there is an infinite number of solutions to this problem. The control impulses can be applied at any period. However, we can choose the time optimal solution to ensure the fast response of the system. This adds an additional constraint, we are thus looking for the parameters $\{A_i, t_i\}$ subject to $\min(t_i)$. The simplest solution to this problem is the Zero Vibration (ZV) shaper. It contains two impulses. The four parameters of the shaper are the time location of the impulses, t_i, and the impulse size A_i . However, the constraints simplifies the problem to two parameters: the factor K determines the proportion of the amplitudes of the impulse , while t_1 is the location of the second impulse. To analytically calculate K and t_1, the beam natural frequency and the damping factor are needed

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+K} & \frac{K}{1+K} \\ 0 & \frac{\pi}{\omega\sqrt{1-\xi^2}} \end{bmatrix}, \quad (4)$$

$$K = e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)}. \quad (5)$$

If we consider a perfect controller, the residual vibration only depends on the identification precision of the beam's natural frequency and the damping factor. There have been several works that proposed input shapers that are robust to the identification of these parameters. By using a longer set of impulses we can reduce the amount of residual vibration even if the true parameters are slightly different to the modelled ones. Each degree of robustness is achieved by adding 0.5 vibration period to the control duration.

4. BAYESIAN OPTIMIZATION

The advantage of using optimization methods to fit real-world systems lies in their robustness. Compared to purely analytic methods, it allows for a finer approximation of the system behaviour. Analytical methods always contain assumptions, which simplify the dynamics of the system. The more complex the system, the higher the chance that these simplifications will result in a suboptimal controller design. The two significant points of failure of these methods can be that the model is not a good representation of system behaviour or, in the case of non-linear systems, the optimization algorithm might diverge.

To compare the different input shapers, we decided to place a gyroscope on top of the flexible beam. Based on the gyroscope data, the loss function is calculated as the root mean square error (RMSE) compared to the stationary beam. In our case, modelling the behaviour of the mobile robot and the flexible beam would result in an overly complicated representation of the system. To avoid this problem, we can consider our robot as a black box. Moreover, since the wheels and the surface where the robot is moving is not perfectly smooth, there is a vibration noise introduced to the measurements.

Most machine learning methods employ gradient-based optimization, such as deep learning or logistic regression. The error gradient shows the direction to update the model to reduce the loss. This results in a relatively quick and robust way of finding the local optimum of parameter values. This method allowed the proliferation of deep learning, where it is used to fit models with millions of parameters. There are two main ways of calculating the error gradients numerically, namely automatic differentiation and finite difference method. In our case, it is impossible to perform automatic differentiation since we do not have a model of the system. Finite difference methods are likewise not applicable, because the function evaluations are noisy.

The most straightforward method to use for black-box optimization is a random search. The algorithm evaluates random parameter values and picks the best one as a solution. Random search is simple to implement; however, it suffers from the curse of dimensionality. It is very inefficient since it does not use any information regarding the topology of the loss surface. Although there are extensions for noisy problems, these require a much higher number of evaluations. Similarly to the random search, grid search does not use any information from the previous function evaluations. It evaluates parameters in a grid. This results in a very high number of evaluations to fit the whole parameter surface. In robotic systems, it is very impractical, because frequently the corner points of parameter space contain dangerous or irrelevant states. The significant advantage of the grid search is that it yields a good representation of the loss surface. On the other hand, Bergstra and Bengio (2012) empirically showed that grid search is inferior to random search.

Bayesian optimization is a global optimization method. It can find out the optimal parameters for black-box functions without the use of gradients. It is mainly used for finding the optimum in problems where function evaluations are costly, such as oil drilling and hyper-parameter tuning of machine learning algorithms. It is ideal for use in robotics, where obtaining data samples is very expensive compared to computer vision. Calandra et al. (2016) used it to find optimal gait parameters for a two-legged robot. Cully et al. (2015) created routines for robots to rapidly adapt control strategies to injuries. Berkenkamp et al. (2016) created a parameter tuning algorithm that takes into account the safety of the robot.

Bayesian optimization is an iterative algorithm. It works by fitting a model to the loss surface of the optimization problem. Based on this, it determines which parameters to evaluate next. After a new evaluation, the model is fit again, and the procedure is repeated. The most popular model is a Gaussian process, but there are examples of using decision trees in the literature. Gaussian processes model functions in discrete intervals. Instead of fitting a parametric model, such as a line or a sine wave, the function value is determined by key points and a kernel function, which shows how the function value changes between the key points. The model returns not just the most likely value for a given point in parameter space, but also the uncertainty of the prediction. This uncertainty is capable of modelling the noise of the black-box function. It can also show the epistemic uncertainty of the model. In Fig. 3, a Gaussian Process model of $x\sin(x)$ can be seen. It showcases the ability of GPs to precisely predict the values of a function from a small number of points. It can be observed that the uncertainty is zero in points the algorithm has probed, and gradually increases in regions where there are few key points.

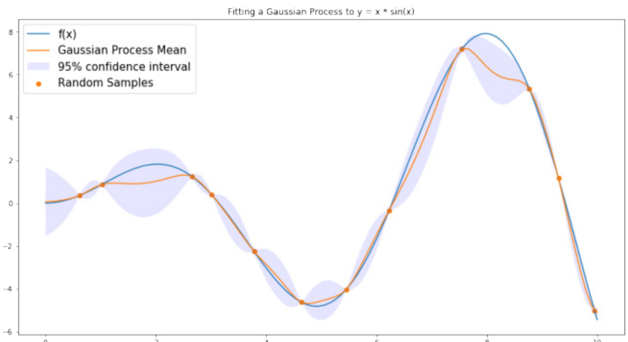

Fig. 3. Gaussian Process model of $x\sin(x)$.

The acquisition function determines which set of parameters evaluate at the next optimization step. The loss surface is mapped into the acquisition surface. The most promising point is then chosen with a local optimization algorithm, such as the limited-memory Broyden–Fletcher–Goldfarb–Shanno (L-BFGS) one. Acquisition functions are a trade-off between exploration and exploitation. By emphasizing exploitation, the algorithm converges to regions that have a small predicted loss. In this case, the algorithm will quickly converge, but it might only find local optima of the parameters. If the acquisition function prefers exploration, it will try to find not just the minimum of the function but, at the same time, probe regions where uncertainty is higher. This will result in a more significant number of evaluations and a slower convergence, and thus a higher probability of finding the global optimum.

5. EXPERIMENTS

5.1 Experimental setup

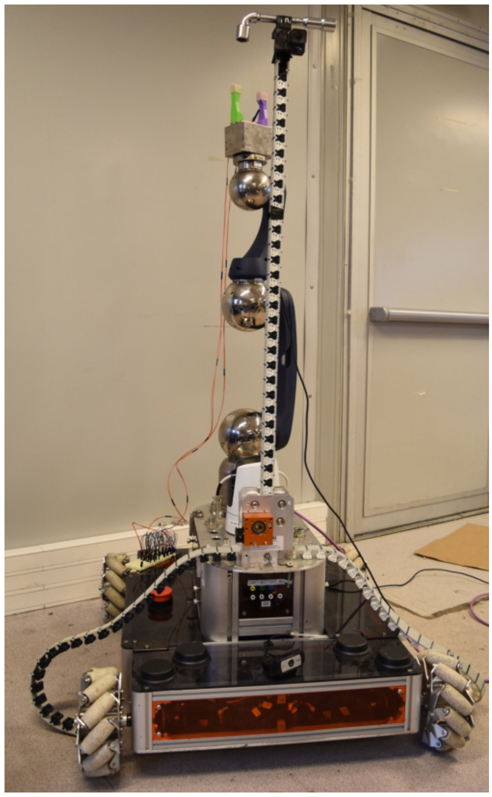

Fig. 4. Experimental mobile robot.

The robotic system, which can be seen on Fig. 4, consists of the mobile robot base and a flexible beam camera system. The mobile robot base has a height of 20 cm, and it is equipped with omnidirectional wheels whose diameter is 20.32 cm (8 in). The gear reduction between the motor and the wheels is made with a timing belt. The wheels have no suspensions. The structure does not damp the vibrations from the omnidirectional wheels and the uneven surface. For safety reasons, four lead-acid battery packs are used as energy storage the energy storage. Their weight can stabilize the robot base, which makes it possible to add a robotic arm on top. The weight of the robot is around 30 kg, limiting the acceleration of the robot.

Maxon EPOS2 controllers control the wheel motors. The main controller runs on Ubuntu Linux. It sends movement commands at a frequency of 150 Hz to the wheel actuators. We have tuned the values of the controller parameters to take into account the weight of the mobile robot. Even though the motors receive the commands in a 2 ms window, their movements are not perfectly synchronised. By inspecting the motor currents and velocities, we could see a mismatch between them. The mismatch could be attributed to the unevenness of the ground surface, and the non-ideal behaviour of the actuators.

The flexible beam is a Serapid Rigibelt. It is a purely mechanical actuator, manufactured from non-magnetic materials. This allows it to work in high magnetic fields, such as magnetic resonance imaging machines. The belt is a push chain consisting of two form-fitting zipper chains. The complex structure of the chain makes dynamic modelling of the beam arduous. The models available in the literature are concerned mostly with homogeneous-isotropic beams, and they do not depict its movement faithfully. The beam vibration is dominant in the horizontal direction. In Fig. 5, the flexible beam and the pushing mechanism are depicted. There was a noticeable backlash both in the gearbox connecting to the actuator and also in the connection of the housing and the beam. This backlash allows high amplitude vibrations to pass through the beam.

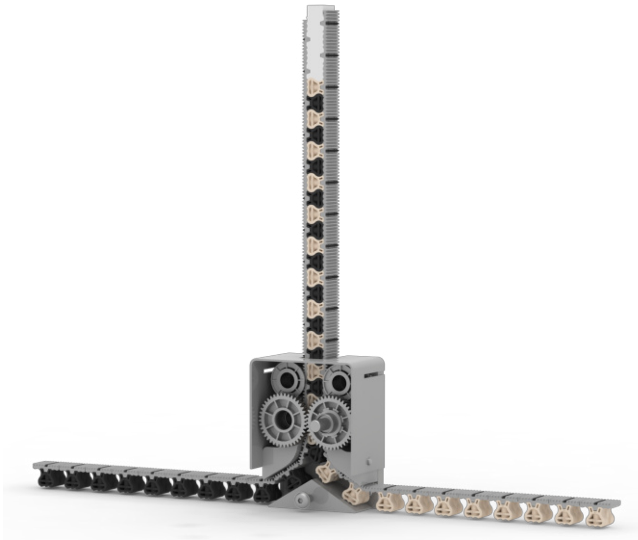

Fig. 5. Serapid Rigibelt, flexible beam.

5.2 Vibration measurement

To measure the vibrations, a VMU931 IMU is placed on top of the beam. The IMU has a built-in accelerometer, a gyroscope, and a magnetometer. In our applications, we utilized only the gyroscope, since it had lower noise than the other components. When the beam is moving freely, its movement can be modelled by a harmonic oscillator.

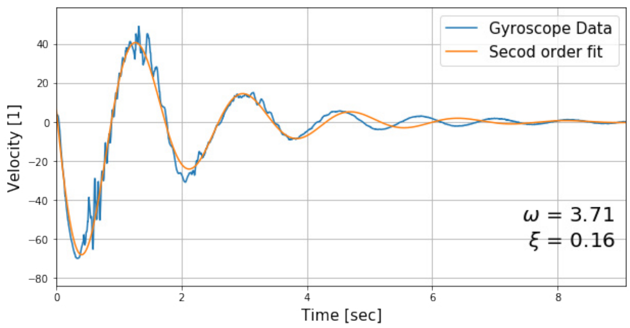

Fig. 6. Free vibration of the beam.

However, during the movement, the gyroscope measurements are noisy because the structure does not damp the vibrations. Without filtering of the signal, Bayesian optimization algorithms were unable to find an optimal parameter set because of the high variance of the measurements. To reduce the variance, we apply a low-pass Butterworth filter on the signal.

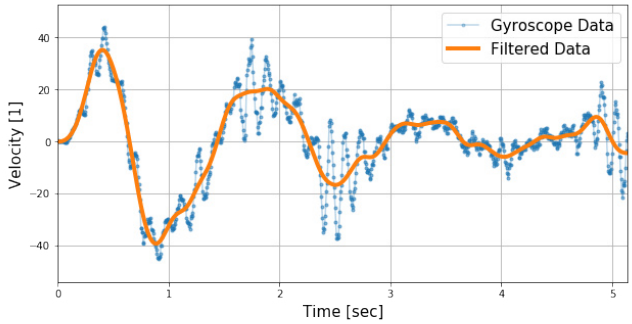

Fig. 7. Comparison between raw and filtered data.

The time period of vibration of the beam is around 1 s. To evaluate the different feed-forward filters, we have accelerated the robot from standing position to constant velocity in a second. Afterward, the robot is moved with a constant velocity for another 3 s, this gives us time to observe the settling of the beam oscillation. To make sure that we get the ideal parameters for moving the robot in both directions, a single function evaluation consisted of one forward and one backward movement.

5.3 Optimization experiments

After careful consideration, we have decided to use fmfn / Bayesian Optimization Library (Nogueira, 2014). The library contains all the basic acquisition functions and regressors, along with logging and plotting features. For the optimization loss function, we have used the root mean squared error (RMSE). We considered that, in an ideal case, the top of the beam would stay stationary. In this case, the error is the deviation of the measured speed from the stationary condition.

For the Gaussian process kernel, we employed the Matern covariance from Minasny and McBratney (2005). In our experience, it resulted in a more robust convergence than the Gaussian radial-basis function. For each dimension, we set the length-scales separately, and we updated it with maximum likelihood estimation after subsequent trials.

The most crucial part of applying Bayesian Optimization is choosing an acquisition function and tuning the parameters to achieve the right balance between exploration and exploitation. In our experiments, the most often failure came from the acquisition function, quickly converging to a local optimum. We have performed experiments with expected improvement, probability of improvement, and upper confidence bound acquisition functions. Upper confidence bound with a $\kappa = 1.0$ exploration-exploitation parameter resulted in a relatively fast and stable convergence to optima in different dimensional models. Experiments were carried out on four different input shapers. The simplest shaper was a ZV one with two free parameters, namely the amplitude (A_1) and the time shift (t_1). After an initial pulse, the algorithm searched for the second impulse that could eliminate the vibration. The algorithm converged to optimal parameter values in approximately twenty trials. The loss surface predicted by the Gaussian process model can be seen in Fig. 8. It can be seen that the algorithm quickly converged to points around the global minimum. The surface shows the loss values depending on the t_1 and A_1 parameters of the ZV shaper. The red dots show the function evaluation points. As can be seen, the loss surface is convex, and most of the evaluations are in the neighborhood of the global maximum.

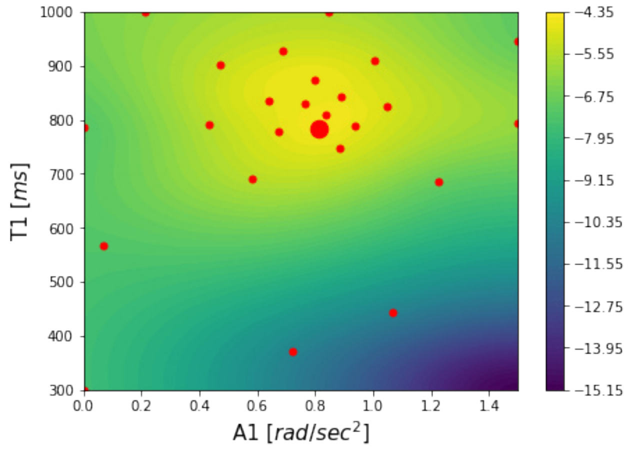

Fig. 8. Loss surface of a two impulse shaper after 25 steps.

We have evaluated the usage of two, three, and four impulse input shapers. The algorithm had directly searched for the A_i and t_i values. This resulted in respectively three, five, and seven parameters, since $t_0 = 0$. The increase in the number of parameters resulted in the increase of trials needed for convergence to 35-70 trials. Albeit the increased complexity, these filters did not result in an improvement of the beam oscillation. They have achieved a comparable loss value to the ZV shaper while needing 2-3 times more time for convergence. This can be attributed to the fact that a flexible beam has only one flexible pole, and a simple ZV shaper with its two free parameters were sufficient to reduce the vibrations.

To verify the precision of the input shapers, we have compared the vibration of the unshaped movement, that is the ZV shaper, based on the free-vibration of the beam, and the shaper resulting from Bayesian optimization. The ZV shaper derived from the beam had parameters $K = 0.59$ and $t_1 = 858$ ms . The shaper obtained through Bayesian optimization had corresponding parameters of $K = 0.56$ and $t_1 = 810$ ms . In Figure 9, the results of ten test rounds can be seen between the three options. The unshaped movement had a loss of 8.53. The analytical shaper achieved a loss of 7.17. Finally, the Bayesian shaper had a loss of 6.6. It is important to note that, even in the ideal case, the RMSE loss would not converge to 0. The input shaper needs at least half a period to eliminate the vibrations, and this means that a small movement of the flexible beam in the opposite direction of the base movement would be unavoidable.

Input shaping is most effective for reducing the high amplitude vibrations. To get rid of the high frequency vibrations, image stabilization algorithms might be used in tandem. This way both high and low frequency motion of the video feed could be avoided.

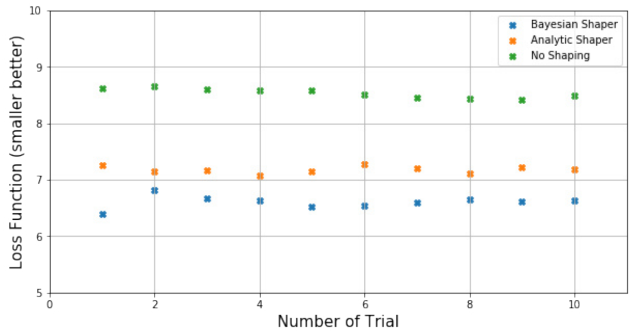

Fig. 9. Comparison of residual vibrations.

6. CONCLUSION

In this paper, we proposed a method for designing input shapers for vibration reduction in complex underactuated robots. The Bayesian optimization method converges to a solution quickly and robustly. It was simple to implement even on systems where dynamic modeling is not feasible. This allows its application not just in structured laboratory environments, but highly sophisticated on-the-field robots. We would like to emphasize the ease with which this method can be applied to different systems. Since the optimization process can be regarded as a black box, it needs no time consuming dynamic modelling.

Mobile robots give a platform which can be easily extended with additional sensors and robotic arms depending on the needs of the specific intervention. By changing the robot, the parameters of the input shapers should also be modified. In future works, a system based on Bayesian optimization could be developed, which could find the optimal input shaper parameters in a lower amount of trials. By considering the previous Gaussian Process as a prior distribution, the robot could be capable of seamlessly accommodating structural changes even during operation, similarly to how transfer learning functions in deep neural networks.

In our current work, we considered a robot with velocity control. By designing the input shaper at a lower level of actuator control, such as current/torque control, there might be a possibility to achieve a lower amount of residual vibrations. This would increase the complexity of the optimization problem, and it would require the usage of a safety-aware optimization algorithm. The feasibility of these approaches could be a further direction for future investigation.

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