Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

The Constrained Total Least Squares Solution for Virtual Reference Feedback Tuning ⋆

Cristiane Silva Garcia * **Alexandre Sanfelici Bazanella** *

* *Department of Automation and Energy, Federal University of Rio Grande do Sul, Porto Alegre, Brazil (e-mail: cristiane.garcia, bazanella@ufrgs.br)*

Abstract: Virtual Reference Feedback Tuning (VRFT) is a direct data-driven control design method employed to tune a controller's parameters aiming to achieve a prescribed closed-loop performance. Its primary formulation leads to a biased estimate in the presence of noise, so an instrumental variable (IV) alternative has been proposed and this alternative has been favoured whenever the noise level is significant. Even though VRFT thus formulated has been very successful, the bias reduction through the IV approach comes at the cost of an important increase in the variance of the parameters' estimate. In this work we propose a different solution for the parameters estimation in VRFT which reduces bias without increasing the variance — the Constrained Total Least Squares (CTLS). The effectiveness of the proposed solution is illustrated by three case studies, showing that the mean square error of the parameters' estimate is smaller when compared to previously proposed solutions and, most importantly, that the closed-loop performance is significantly better.

Keywords: Data-based control, model following control, virtual reference feedback tuning, errors in variables identification, constrained total least squares.

1. INTRODUCTION

The problem of adjusting the parameters of a feedback controller is usually approached either by using model-driven methods, data-driven (DD) methods, or a mixture of those. In the model-driven control framework the model of the process to be controlled must be known and this information is employed directly to calculate the controller's parameters. On the other hand, within the direct DD control framework the controller's parameters are estimated directly from a batch of data collected from the process, without identifying the process model (Bazanella et al., 2011). Besides reducing drastically the human effort spent in the design, it has been shown that DD design can also provide much better closed-loop performance than model-driven design when adjusting the parameters of low-order controllers (Campestrini et al., 2016a).

Different DD methods have been proposed in the literature along the past two decades. The noniterative or *one-shot* DD methods are those in which the data are collected just once, either in open-loop or in closed-loop with a previously set controller. A few well-known examples of noniterative DD methods are: the Correlation-based Tuning (CbT) (Karimi et al., 2007); the Optimal Controller Identification (OCI) (Campestrini et al., 2016a); and the Virtual Reference Feedback Tuning (VRFT) (Campi et al., 2002).

⋆ This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. This work was financed in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

Among these DD methods, VRFT has received considerable attention in the literature and has proven to be very effective in applications. The VRFT method uses the data to generate the (virtual) input and output signals of the ideal controller and then identifies the best matching controller considering the available controller structure. Therefore, the original problem of shaping the closed-loop response is recast as a controller identification problem, and its solution is found, in the most common case of linearly parametrized controllers, by solving a least squares (LS) problem, as described in Campi et al. (2002). In the presence of noise, that approach results in a biased estimate and to reduce the bias the same work proposed an instrumental variable (IV), which comes with the drawback of increasing the variance of the estimate. The same work also deals with the so-called mismatched case, i.e. when the ideal controller can not be represented using the available controller structure.

Later, the VRFT method was also extended and improved to deal with more general cases: the case when the process has non-minimum phase zeros (Bazanella et al., 2011); the multivariable case considering a diagonal parametrization of the reference model matrix (Nakamoto, 2004), or a reference model matrix fully parametrized (Campestrini et al., 2016b); and the case where the solution is obtained recursively, allowing it to be implemented in hardware with processing restrictions (Garcia and Bazanella, 2017), to cite a few. However, all the above modifications use the same solution proposed in Campi et al. (2002) to deal with the noisy data, hence giving rise to the same increase in the variance of the estimate. This is clearly presented in van Heusden et al. (2010), where the authors evaluate the effect of such noisy data in the estimate of the controller's parameters using different DD methods.

On the other hand, in the literature dealing with LS methods, the original VRFT solution is known as Ordinary Least Squares (OLS), and it is only one among others: the Total Least Squares (TLS) (Golub and Van Loan, 1980); the Constrained Total Least Squares (CTLS) (Abatzoglou and Mendel, 1987); and the Data Least Squares (DLS) (DeGroat and Dowling, 1993). The TLS problem is closely related to the Errors-in-Variables (Söderström, 2018) in the system identification literature, and the CTLS is a particular case of TLS that, as will be shown in this paper, fits nicely in the VRFT problem. With that in mind, we propose to apply the CTLS solution to the VRFT problem to reduce the estimation bias *in lieu* of the IV solution. In order to evaluate the proposed solution three simulation examples are presented, and the results are compared against the primary LS and the IV solutions of the VRFT.

The remaining of this paper is organized as follows: Section 2 shows some statements; in Section 3 the VRFT method is presented; the CTLS and original solutions are presented in Section 4; in Section 5 three simulation case studies illustrate the application of the proposed solution; finally, Section 6 shows some conclusions and future work.

2. PRELIMINARIES

Consider a linear discrete-time monovariable process

$$y(t) = G(q)u(t) + H(q)v(t), \quad (1)$$

where q is the forward-shift operator, $G(q)$ is the process transfer function, $H(q)$ is the noise model, $u(t)$ is the input signal, $y(t)$ is the output signal, and $v(t)$ is a zero-mean white noise with variance σ^2.

When the data is collected during an open-loop experiment, the input $u(t)$ is an exogenous signal, while the output $y(t)$ is affected by the noise. In this case the output is given directly by (1).

On the other hand, for data gathered during a closed-loop experiment exciting the reference, the process' input is

$$u(t) = C_0(q)\left(r(t) - y(t)\right), \quad (2)$$

where $C_0(q)$ is the original controller, that is, the controller operating in the process while performing the experiment, and $r(t)$ is the reference signal. In this case, replacing (2) in (1) gives

$$y(t) = T_0(q)r(t) + S_0(q)H(q)v(t), \quad (3)$$

where $T_0(q) = \left[1 + G(q)C_0(q)\right]^{-1} G(q)C_0(q)$ is the closed-loop response obtained with the original controller $C_0(q)$, and $S_0(q) = \left[1 + G(q)C_0(q)\right]^{-1}$ is the original sensitivity transfer function. In a similar way, (2) may be rewritten, using (1), as

$$u(t) = S_0(q)C_0(q)r(t) - S_0(q)C_0(q)H(q)v(t). \quad (4)$$

3. THE VRFT METHOD

The VRFT is a noniterative data-driven method, that is, the controller's parameters can be estimated using data collected from a single experiment as described in Campi et al. (2002). This way, aside from the richness of the input data no special experiment is required.

This method is usually employed to adjust the parameters of a linear parametrized controller, that is,

$$C(q, \rho) = \rho^{\mathrm{T}}\bar{C}(q),$$

where $C(q, \rho)$ is the controller's transfer function, $\bar{C}(q)$ is a vector representing the controller structure and ρ is the parameters vector. The controller's class is defined as

$$\mathcal{C} = \{C(q, \rho) \mid \rho \in \Omega \subseteq \mathbb{R}^m\}$$

where Ω is the subset of all allowed parameters vectors, and m is the number of parameters.

The method's goal is to solve the following problem

$$\min_{\rho} J_y(\rho) = \bar{\mathrm{E}}\left[y(t, \rho) - y_d(t)\right]^2,$$

where $J_y(\rho)$ is the reference cost criterion to be minimized, $\bar{\mathrm{E}}\left[x(t)\right] = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} \mathrm{E}\left[x(t)\right]$, while $y(t, \rho)$ is the closed-loop process output signal obtained with the controller $C(q, \rho)$, and $y_d(t)$ is the desired output signal, i.e. the reference model's response. This cost function may be rewritten as

$$J_y(\rho) = \bar{\mathrm{E}}\left[\left(T(q, \rho) - M(q)\right) r(t)\right], \quad (5)$$

where $r(t)$ is the reference signal, $T(q, \rho)$ is the transfer function of the closed-loop with the controller $C(q, \rho)$, and $M(q)$ is the reference model, which corresponds to the desired closed-loop transfer function. Moreover, from (5), the ideal controller could be calculated as

$$C_d(q) = \left[G(q) - M(q)G(q)\right]^{-1} M(q), \quad (6)$$

if $G(q)$ were available. It is assumed that $C_d(q)$ is in the controller's class $\mathcal{C}$, which it is equivalent to say that

$$\exists\ \rho_d \in \Omega \mid C(q, \rho_d) = C_d(q),$$

where ρ_d is the ideal parameters vector.

The VRFT method starts with an open- or closed-loop experiment, where an input signal $u(t)$ excites the process and is collected along with the output signal $y(t)$. After the experiment, the *virtual* part takes place. In possession of the collected output $y(t)$ and pretending that the ideal controller $C(q, \rho_d)$ is already in the loop, one can calculate the virtual reference $\bar{r}(t)$ that would produce the same output:

$$\bar{r}(t) = M^{-1}(q)y(t) \quad (7)$$

The virtual error $\bar{e}(t)$, which is the ideal controller's input, is obtained from

$$\bar{e}(t) = \bar{r}(t) - y(t). \quad (8)$$

Because the controller is linearly parametrized, it is possible to define a regressor vector $\varphi(t)$ as

$$\varphi(t) = \bar{C}(q)\bar{e}(t). \quad (9)$$

Furthermore, by replacing (8) and then (7) in (9), the regressor vector is given by

$$\varphi(t) = \bar{C}(q)R(q)y(t), \quad (10)$$

where $R(q) = M^{-1}(q) - 1$.

Using the above information, the VRFT method transforms the problem of minimizing the cost function $J_y(\rho)$ presented in (5) into an LS identification of the controller $C(q, \rho)$, which consists in minimizing the following cost:

$$J^{VR}(\rho) = \bar{\mathrm{E}}\left[u(t) - \rho^{\mathrm{T}}\varphi(t)\right]^2$$

Assuming that the system is not affected by noise and that there is an ideal controller $C(q, \rho_d)$ such that $J_y(\rho) = 0$, that is $T(q,\rho_d) = M(q)$, the minimum of $J_y(\rho)$ is proven to coincide with the minimum of $J^{VR}(\rho)$. In the sequence, we present the CTLS solution, and the original (OLS) and IV solutions.

4. SOLUTIONS FOR THE VRFT LS PROBLEM

Consider first the noise-free case, then writing the ideal controller's input-output relationship in matrix form gives

$$\Phi_0 \rho_d = u_0, \tag{11}$$

where the controller's noise-free output vector is given by $u_0 = [u_0(1)\, u_0(2)\, \dots\, u_0(N)]^{\mathrm{T}} \in \mathbb{R}^N$ and $\Phi_0 \in \mathbb{R}^{N\times m}$ is the regressor matrix defined as

$$\Phi_0 = [\varphi_0(1)\ \ \varphi_0(2)\ \ \dots\ \ \varphi_0(N)]^{\mathrm{T}}.$$

Here, N is the number of samples. $\varphi_0(t)$ is obtained replacing (1) in (10) and setting $v(t) \equiv 0$ as follows:

$$\varphi_0(t) = \bar{C}(q)R(q)G(q)u(t).$$

However, this is not a realistic case, since the signals collected from actual processes are always affected by noise. In practice, the calculated regressor matrix Φ and the measured output vector u are given by

$$\Phi = \Phi_0 + \Delta_\Phi \tag{12}$$

$$u = u_0 + \delta_u, \tag{13}$$

where

$$\Phi = [\varphi(1)\ \ \varphi(2)\ \ \dots\ \ \varphi(N)]^{\mathrm{T}} \tag{14}$$

$$u = [u(1)\ \ u(2)\ \ \dots\ \ u(N)]^{\mathrm{T}}, \tag{15}$$

and $\Delta_\Phi \in \mathbb{R}^{N\times m}$ and $\delta_u \in \mathbb{R}^N$ represent the noise contributions in Φ and u, respectively. That allows us to use Φ_0 and u_0 from (12) and (13) to rewrite (11) as

$$(\Phi - \Delta_\Phi)\,\rho = u - \delta_u. \tag{16}$$

Different classes of least squares problems have been proposed in the literature depending on how the noise affects the regressor matrix Φ and the output vector u. As mentioned before, in the original VRFT approach the controller's parameters are estimated through the solution of the OLS problem, considering a linearly parametrized controller. Such solution fits best the situation where only u is affected by noise, which is not the VRFT's case. Therefore, the estimate is biased in the presence of noise, which may be counteracted by employing an IV with the drawback of increasing the variance of the estimate.

Observe that other LS solutions could be applied to the VRFT problem. The TLS approach, for example, considers that the input and output are affected by noise. When the noises affecting Φ and u are white and uncorrelated the TLS have a closed solution. Unfortunately, that is not the VRFT's case either, rendering that solution unfit for that case. On the other hand, the CTLS approach considers a different constraint: that the same noise source affects Φ and u. In practice, the CTLS solution is the one that better suits the VRFT's problem, as will be shown. Finally, the DLS approach also presents a closed solution, but it deals with the case where only Φ is affected by noise. Therefore, this approach fits better the VRFT's problem when data from an open-loop experiment is used. However, the DLS does not fully exploits the problem structure in the optimization, leading to suboptimal results. Because of that, this solution is not applied in this work. The following subsections present the CTLS solution applied to the VRFT problem and the original OLS and IV approaches as presented in the literature.

4.1 VRFT with CTLS

The CTLS problem was developed in Abatzoglou and Mendel (1987) from the TLS problem, with the constraint that Φ and u are affected by the same noise source (this will be clarified in a moment). The optimization problem is formulated from (16) as

$$\min_{v,\rho} \|[\Delta_\Phi\ \ \delta_u]\|_{\mathrm{F}}^2 \quad \text{s. t. } (\Phi - \Delta_\Phi)\,\rho = u - \delta_u, \ \text{ and}$$

$$[\Delta_\Phi\ \ \delta_u] = [P_1 v\ \ P_2 v\ \ \dots\ \ P_{m+1} v]$$

where $\|\cdot\|_{\mathrm{F}}$ corresponds to the Frobenius norm, that is, $\|X\|_{\mathrm{F}}^2 = \Sigma_{i,j}|x_{ij}|^2$, whereas, $v \in \mathbb{R}^N$ is a vector with the noise samples:

$$v = [v(1)\ \ v(2)\ \ \dots\ \ v(N)]^{\mathrm{T}}$$

Also, $P_i \in \mathbb{R}^{N\times N}$ with $i = 1,\dots,m+1$ are Toeplitz matrices representing filters. These filters describe how the single noise source v affects every column of the regressor matrix and the output vector. Thereby, the CTLS solution minimizes the Frobenius norm of the noise contributions. This problem was simplified in Abatzoglou and Mendel (1987) in order to remove the decision variable v, resulting in the following theorem. The proof is in the reference.

Theorem 1. Let $[\Delta_\Phi\ \ \delta_u] = [P_1 v\ \ P_2 v\ \ \dots\ \ P_{m+1} v]$. The CTLS solution is given by

$$\min_\rho \begin{bmatrix}\rho\\-1\end{bmatrix}^{\mathrm{T}} [\Phi\ \ u]^{\mathrm{T}} \left(\Gamma_\rho K^{-1} \Gamma_\rho^{\mathrm{T}}\right)^{-1} [\Phi\ \ u] \begin{bmatrix}\rho\\-1\end{bmatrix}$$

where $\Gamma_\rho = \sum_{i=1}^{m}(P_i\rho_i) - P_{m+1}$ and $K = \sum_{i=1}^{m+1} P_i^{\mathrm{T}} P_i$.

In order to apply the CTLS solution successfully, one needs to determine the filters' impulse responses and to generate the matrices P_i with $i = 1,\dots,m+1$ that will be used in the optimization. Those filters will be different depending whether the data are collected from an open- or closed-loop experiment. Therefore, the main contributions of the present work are: to formulate the VRFT problem as a CTLS problem, and to determine the filters for the open- and closed-loop case. These filters are presented below.

Filters for the closed-loop experiment: In this case, both signals $u(t)$ and $y(t)$ are affected by noise. The input $u(t)$ is given in (4) and the output $y(t)$ is presented in (3). Replacing (3) in (10) gives

$$\varphi(t) = \underbrace{\bar{C}(q)R(q)T_0(q)r(t)}_{\varphi_0(t)} + \underbrace{\bar{C}(q)R(q)S_0(q)H(q)v(t)}_{\Delta_\Phi(t)},$$

where the regressor vector $\varphi(t)$ was split in two terms: one purely from the reference signal, $\varphi_0(t)$; and another from the noise contribution alone, $\Delta_\Phi(t)$. This equation may be rewritten in vector form as (12). Therefore, the matrices P_i, $i = 1,\dots,m$ are generated from the filters

$$F_i(q) = -\bar{C}_i(q)R(q)S_0(q)H(q). \tag{17}$$

The first m filters come from the above equation.

The last filter $F_{m+1}(q)$ is obtained observing that (4) may also be split in two terms:

$$u(t) = \underbrace{C_0(q)S_0(q)r(t)}_{u_0(t)} - \underbrace{S_0(q)C_0(q)H(q)v(t)}_{\delta_u(t)}.$$

It is straightforward to see that the filter that will be used to generate the matrix P_{m+1} is given by

$$F_{m+1} = S_0(q)C_0(q)H(q). \tag{18}$$

Filters for the open-loop experiment: In this case only the output signal $y(t)$ is affected by noise. The regressor vector may be rewritten, replacing (1) in (10), as follows

$$\varphi(t) = \underbrace{\bar{C}(q)R(q)G(q)u(t)}_{\varphi_0(t)} + \underbrace{\bar{C}(q)R(q)H(q)v(t)}_{\Delta_\Phi(t)}.$$

The filters that generate the matrices P_i with $i = 1, \ldots, m$ are defined as

$$F_i(q) = -\bar{C}_i(q)R(q)H(q). \tag{19}$$

Because the data is from an open-loop experiment, the output vector u is noise-free. Therefore the corresponding filter is given by

$$F_{m+1}(q) = 0,$$

and the matrix P_{m+1}, in turn, is all zeros.

Remark 1. From (17) and (19), it is clear that the noise affecting the regressor matrix is never white, which makes the TLS solution unsuitable to the VRFT's problem.

Remark 2. From the above, unlike in the classic system identification problem, the OLS solution will always result in a biased estimate even if the noise affecting the output were white. Nevertheless, the OLS solution will be presented next for completion.

4.2 OLS — The original VRFT solution

The OLS problem considers that only the output vector u is affected by noise. This is equivalent to considering Δ_Φ equals zero in (16) and the optimization problem is formulated as

$$\begin{aligned} \min_{\delta_u,\rho} \quad & \|\delta_u\|_2^2 \\ \text{s. t.} \quad & \Phi_0\rho = u - \delta_u. \end{aligned} \tag{20}$$

Thereby, the objective is to find a parameters vector ρ and a noise vector δ_u that perturbs as little as possible (20).

However, as stated before, in the VRFT problem the matrix Φ is always affected by noise while the output u will be affected whenever the data are collect from a closed-loop experiment. Nevertheless, the algebraic solution to the OLS problem is well known and is given by

$$\hat{\rho} = \left(\Phi^{\mathrm{T}}\Phi\right)^{-1}\Phi^{\mathrm{T}}u.$$

This solution leads to a biased estimate in the presence of noise. In order to reduce the bias of the estimate an IV is used. Among the IV options, we choose to use the one that requires a second experiment exciting the process with the same input signal to collect more data, because this particular IV guarantees the consistence of the estimate (Bazanella et al., 2011). The idea is that the noise affecting the outputs of the experiments will be uncorrelated and the estimate will be unbiased. An alternative approach is to perform a single experiment employing a periodic input signal and then split the data simulating two experiments. Suppose the data from a single experiment is split in two, generating the signals $u_1(t)$, $u_2(t)$, $y_1(t)$, and $y_2(t)$. Then, the parameters are estimated by

$$\hat{\rho} = \left(\Phi_1^{\mathrm{T}}\Phi_2\right)^{-1}\Phi_1^{\mathrm{T}}u_2, \tag{21}$$

where Φ_1 and Φ_2 must be generated from $y_1(t)$ and $y_2(t)$, respectively, using (10) and (14), while u_2 is the vector containing the samples of $u_2(t)$, as in (15). Note that another possible estimate is given by exchanging the subscripts 1 and 2 in (21). Also note that in case the experiment is performed in open-loop, the signals $u_1(t)$ and $u_2(t)$ will be identical. The IV approach reduces the bias of the estimate with the drawback of increasing its variance. This effect will be seen in the next section presenting the results obtained with the proposed and original solutions for the VRFT method.

5. SIMULATION RESULTS

In order to evaluate the results, three simulation case studies are carried out. In the first case study, the parameters are estimated from data collected during an open-loop experiment. The second and third case studies consider data collected from a closed-loop simulation. In the three cases, 100 Monte Carlo simulations are performed varying the noise realization.

The parameters' Mean Squared Error (MSE) and an estimate of the performance criterion cost function $\hat{J}_y(\hat{\rho})$ are used as metrics to compare the obtained results. The MSE is calculated as follows

$$\mathrm{MSE} = \mathrm{E}\,|\rho - \rho_d|^2 = \sum\nolimits_{i=1}^{m} \mathrm{bias}^2(\rho_i) + \mathrm{var}(\rho_i).$$

The cost function $\hat{J}_y(\hat{\rho})$ is estimated for each controller obtained in each Monte Carlo simulation, as

$$\hat{J}_y(\hat{\rho}) = \frac{1}{N}\sum\nolimits_{t=1}^{N} [y(t,\hat{\rho}) - y_d(t)]^2$$

with N chosen as twice the settling time of the reference model.

Furthermore, an estimate for the Signal-to-Noise Ratio (SNR) of the output signal is obtained as

$$\mathrm{SNR_{dB}} = 10\log\frac{y^{\mathrm{T}}y}{v^{\mathrm{T}}v}, \tag{22}$$

were $y = [y(1)\,y(2)\,\ldots\,y(N)]^{\mathrm{T}}$ is a vector with the output samples corrupted by noise.

All three case studies consider the same process and the same reference model. The open-loop process is given by

$$G(q) = \frac{0.095q}{(q-0.8)(q-0.92)}. \tag{23}$$

The reference model $M(q)$ was chosen as

$$M(q) = \frac{0.2}{q-0.8}. \tag{24}$$

From the chosen reference model and process one may calculate the ideal controller, by replacing (23) and (24) in (6). The obtained ideal controller is

$$C_d(q) = \underbrace{[0.5221\ \ 0.0337\ \ 1.5495]}_{\rho_d}\underbrace{\left[1\ \ \frac{q}{q-1}\ \ \frac{q-1}{q}\right]^{\mathrm{T}}}_{\bar{C}(q)}. \tag{25}$$

The objective is to tune a Proportional-Integral-Derivative (PID) controller, with the controller structure $\bar{C}(q)$ presented in the above equation, which guarantees the assumption that the ideal controller is in the controller's class.

Regarding the noise, in all simulations the output $y(t)$ is affected by additive coloured noise, generated by filtering a white noise sequence through the following noise model:

$$H(q) = \frac{q^2 + 0.8q + 0.3}{q^2 - 0.8q}.$$

The controller's parameters are estimated for each Monte Carlo simulation using the CTLS approach as presented in Subsection 4.1; and original (OLS) and IV approaches as presented in Subsection 4.2. The noise model $H(q)$ was employed only to generate the data and left out of the CTLS filters, because this information is usually unavailable in real-life. Moreover, when calculating the closed-loop filters for the CTLS approach the approximation $S_0(q) \approx S_d(q) = (1 - M(q))$ is employed. Note that this approximation is valid if the requested reference model is not far from the original behaviour. The minimization in the CTLS optimization problem is solved using a quasi-Newton method with the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm to estimate the Hessian matrix. That method is already implemented in the Matlab function `fminunc` and requires an initialization point, since the problem is solved iteratively. In the three case studies the same initial value $\rho_0 = [0.35\ \ 0.02\ \ 0.5]^{\mathrm{T}}$ was used for the CTLS optimization problem.

5.1 Open-loop experiment (case I)

In the first simulation two periods of a square wave with levels equal 60 and 61 were applied as input signal $u(t)$. Each period has 200 samples, totalizing 400 samples. The input signal $u(t)$ and one realization of the output signal $y(t)$ are presented in Fig. 1, after subtracting the operating point. The SNR is approximately 34 dB, calculated as presented in (22).

Fig. 2 presents the histograms of the cost functions obtained using each method. Observe that the values obtained by VRFT with IV (splitting the data) are smaller and more widely distributed when compared with the ones obtained with the OLS approach. That is because the IV approach is known to yield an estimate with pronounced variance. The CTLS solution, on the other hand, presents its values much more concentrated towards the left, evidencing the small bias and variance of the estimate.

The values calculated for the parameters' MSE are presented in the respective column of Table 1 and agree with the histograms in Fig. 2. The smallest value is obtained with the CTLS approach, while the largest value is obtained with the OLS solution.

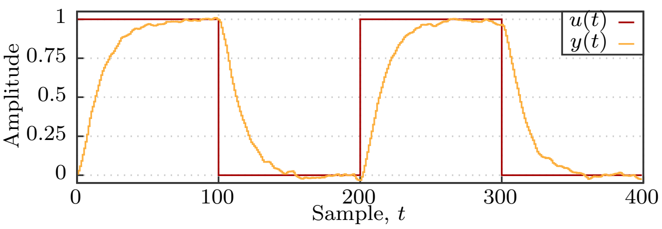

Fig. 1. Input $u(t)$ and output $y(t)$ data of the open-loop experiment.

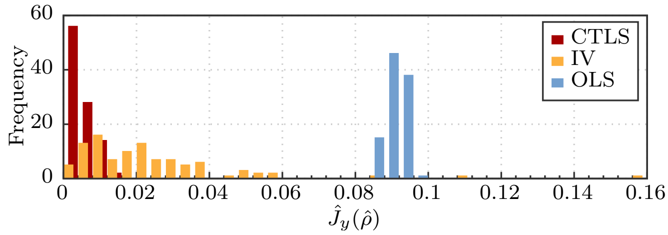

Fig. 2. Histograms of $\hat{J}_y(\hat{\rho})$ for the open-loop experiment.

Table 1. Calculated parameters' MSE values

Method	MSE Case I	Case II	Case III
OLS	1.8830	1.3305	2.0609
IV	0.9170	0.42449	—
CTLS	0.0128	0.02918	0.1940

5.2 Closed-loop experiment (case II)

In this second case, the process' input and output data, $u(t)$ and $y(t)$, are collected during a closed-loop experiment. The same process transfer function $G(q)$, reference model $M(q)$, and ideal controller $C_d(q)$, presented previously in (23) – (25), are considered. The controller initially operating in closed-loop is given by

$$C_0(q) = [0.2088\ 0.0135\ 0.6198] \left[1\ \ \frac{q}{q-1}\ \ \frac{q-1}{q}\right]^{\mathrm{T}}.$$

The same square wave probing signal used before was employed again, this time to excite the closed-loop reference input $r(t)$. One realization of the process' input $u(t)$ and output $y(t)$ is presented in Fig. 3, after subtracting the operating point. The SNR was approximately 26 dB, calculated as presented in (22).

The histograms of the cost function calculated for each controller and each method are presented in Fig. 4. A comparison of the results indicates that the larger values for the cost function were obtained with the original

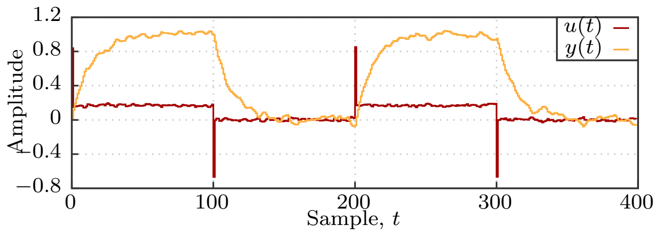

Fig. 3. One realization of the input $u(t)$ and output $y(t)$ for the closed-loop experiment.

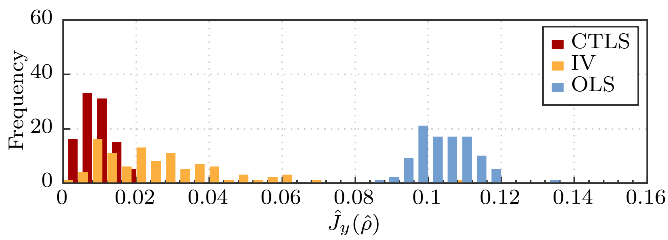

Fig. 4. Histograms of $\hat{J}_y(\hat{\rho})$ for the closed-loop experiment.

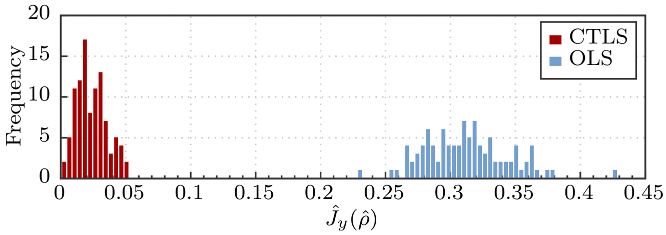

Fig. 5. Histograms of $\hat{J}_y(\hat{\rho})$, closed-loop, SNR $\approx$ 18 dB.

solution. On the other hand, the smallest values are obtained with the CTLS approach. Fig. 4 also reflects the small bias and large variance of the estimate obtained by the IV approach. Again, these results agree with the values obtained for the parameters' MSE, presented in the respective column of Table 1.

5.3 Low SNR configuration (case III)

Finally, the last case study supposes a larger noise contribution. The same input data was applied to the reference $r(t)$, the only difference is that the noise contribution was increased to reduce the SNR to approximately 18 dB. The CTLS approach presented again the smallest values for the estimated reference cost function, $\hat{J}_y(\hat{\rho})$, as shown in the histograms of Fig. 5. This figure also shows that the controller's parameters estimated with the OLS approach yielded closed-loop behaviours even further from the desired one. The IV approach presented a similar outcome and, on top of that, approximately 20% of the controllers resulted in unstable closed-loop behaviour. However, this undesired results are omitted for the sake of brevity. Instead, the MSE values calculated with the parameters estimated with the OLS and CTLS approaches are presented in Table 1.

It is also worth to mention, that the input and output signals have not been pre-filtered. It is known that the results using a filter (for example, the filter for the mismatched case or a low-pass filter) could give better outcomes. However, such filters would improve the results with all the approaches. The main objective here is to present purely a new and effective way to estimate the controller's parameters using the virtual data generate by the VRFT method.

Besides, the present work does not validates the closed-loop stability during the optimization phase. The interested reader is advised that this subject was already addressed in Van Heusden et al. (2011) for the VRFT method. That work uses the small-gain theorem to determine sufficient conditions, that are added as restrictions in the VRFT minimization problem, to guarantee closed-loop stability. Although those restrictions could also be added to the CTLS optimization problem it would exceed the scope of the present work.

6. CONCLUSIONS

The original VRFT solution is biased in the presence of noise. To reduce the bias of the estimate an IV solution is usual, which increases significantly the variance of the estimate. With that in mind, the goal of this paper was to present a different solution for the bias problem in the VRFT method, known as CTLS. As could be seen through the simulation results, this solution presents smaller values for bias and variance of the estimate and significant improvement in closed-loop performance, when compared with those obtained with the original solutions.

As future work, we intend to apply the proposed solution to the multivariable VRFT method, and also compare the results obtained with the CTLS solution against those obtained with other DD methods, as the OCI, for example. Besides, we intent to apply the CTLS solution to the Virtual Disturbance Feedback Tuning (VDFT) method, which consists of a similar approach of the VRFT's aiming at disturbance rejection instead of reference tracking.

REFERENCES

Abatzoglou, T. and Mendel, J. (1987). Constrained total least squares. In *ICASSP'87. IEEE International Conference on Acoustics, Speech, and Signal Processing*, volume 12, 1485–1488. IEEE.

Bazanella, A.S., Campestrini, L., and Eckhard, D. (2011). *Data-driven controller design: the H2 approach*. Springer Science & Business Media, Netherlands.

Campestrini, L., Eckhard, D., Bazanella, A.S., and Gevers, M. (2016a). Data-driven model reference control design by prediction error identification. *Journal of the Franklin Institute*, 354(6), 2628–2647.

Campestrini, L., Eckhard, D., Chía, L.A., and Boeira, E. (2016b). Unbiased MIMO VRFT with application to process control. *Journal of Process Control*, 39, 35–49.

Campi, M.C., Lecchini, A., and Savaresi, S.M. (2002). Virtual reference feedback tuning: a direct method for the design of feedback controllers. *Automatica*, 38(8), 1337–1346.

DeGroat, R.D. and Dowling, E.M. (1993). The data least squares problem and channel equalization. *IEEE Transactions on Signal Processing*, 41(1), 407.

Garcia, C.S. and Bazanella, A.S (2017). Recursive and traditional VRFT method implemented in a mobile application. In *XIII Simpósio Brasileiro de Automação Inteligente*, 1281–1286. SBA.

Golub, G.H. and Van Loan, C.F. (1980). An analysis of the total least squares problem. *SIAM journal on numerical analysis*, 17(6), 883–893.

Karimi, A., Van Heusden, K., and Bonvin, D. (2007). Non-iterative data-driven controller tuning using the correlation approach. In *2007 European Control Conference (ECC)*, 5189–5195. IEEE.

Nakamoto, M. (2004). An application of the virtual reference feedback tuning for an MIMO process. In *SICE 2004 Annual Conference*, volume 3, 2208–2213. IEEE.

Söderström, T. (2018). *Errors-in-Variables Methods in System Identification*. Springer, Switzerland.

Van Heusden, K., Karimi, A., and Bonvin, D. (2011). Data-driven model reference control with asymptotically guaranteed stability. *International Journal of Adaptive Control and Signal Processing*, 25(4), 331–351.

van Heusden, K., Karimi, A., and Söderström, T. (2010). On identification methods for direct data-driven controller tuning. *International Journal of Adaptive Control and Signal Processing*, 25(5), 448–465.