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State Estimation in 2D Hydrological Models using Lagrangian Sensors and Low Resolution Elevation Maps

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Abstract: In this research work, the framework to estimate 2D spatio-temporal variation in hydrodynamic variables such as water velocity (m/s) and water level (m) in complex, large scale open channels has been investigated using Lagrangian sensors. The Lagrangian sensors are passive floating platforms, which report its GPS position along with the flow of water. The 2D Saint-Venant model is simulated using HEC-RAS simulation software, the geometrical details for HEC-RAS simulations are obtained using Digital Elevation Map (DEM) of Ravi river, Pakistan. For the system model, the non-linear 2D Saint-Venant model is augmented with a Lagrangian sensor motion model. For state estimation, the GPS position of the Lagrangian sensor along with the upstream water level is assimilated in the augmented model using an Ensemble Kalman Filter (EnKF) with suitable filtering parameters in MATLAB. The hydrodynamic variables and trajectory of the Lagrangian sensor are estimated with low error.

Keywords: Data Assimilation, Ensemble Kalman Filter, 2D Saint-Venant Model, Lagrangian Sensors, Digital Elevation Maps (DEMs)

1. INTRODUCTION

The large scale and complex water bodies are a major source of water to a country, these sources are being used for irrigation, industry, etc. Due to a large number of users of these sources, the monitoring of water channels is a necessary and difficult task. The rivers in Pakistan are a major source of irrigation, which consists of 57,000 Km and these rivers are not well constructed and not well monitored. The geometry and channel parameters of the rivers are unknown and vary a lot. Furthermore, the rivers have different types of vegetation and irregular river beds which changes the friction/roughness coefficient along the channel. The geometry of large scale water bodies can be obtained using Digital Elevation Maps (DEMs). The DEMs are the height maps of a surface along with its coordinates. These maps can be obtained by different satellites or space exploration companies like NASA. Although, DEMs for rivers in Pakistan are not of high resolution as compared to other countries, which makes challenging situations to compute hydraulic profiles of the rivers. This work addresses this issue by estimating unknown states (water level and water velocity) by solving 2D hydrological models by using DEMs in HEC-RAS .

One of the major issues in monitoring large-scale water bodies is the availability of sensor data, as just the static sensors are not enough for accurate measurements at a high resolution. With the recent advancements in the field of sensors, the sensors can measure at high temporal and spatial resolution. One of these sensors is the Lagrangian sensor (Ahmad et al., 2018), which can float passively in the water body with the flow of water. These sensors report their position using a GPS. Furthermore, the Lagrangian sensor can carry other sensors as well, like pH, DO, turbidity, etc. On the other hand, the static sensors have enabled the measurements at the high temporal resolution but at specific locations (Ahmad and Muhammad, 2014). These sensors have enabled a revolution in the field of water management, which provides policies for equity, water rights and transparency within complex river basins such as Indus river basin (Wescoat et al., 2018).

In this study, our focus is on the behavior of hydrodynamic variables of large scale water bodies such as rivers (Molls and Molls, 1998). An ensemble Kalman filter has been used for data assimilation. For the mathematical model, the 2-

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dimensional Saint-Venant [1] model is used. The Lagrangian sensor is used for measurement data and the motion model for these sensors is also discussed. To take advantage of Lagrangian measurements, the Lagrangian motion model is augmented with the 2D Saint Venant Model. For the measurement model, the vector of the three outputs is considered, which consists of water elevation of upstream cell and Lagrangian position in x and y axis. The 2D Saint-Venant model is solved using HEC-RAS and state estimation is performed in MATLAB.

For data assimilation, many techniques are available in the literature. For state estimation in the 1D Saint-Venant model, the Kalman filter is being used with the Lagrangian sensor in (Affan et al., 2019). The Ensemble Kalman filter (EnKF) has the advantage over the Kalman filter as EnKF does not require the linearization of the system which requires the approximation of second-order derivatives in the 2D Saint-Venant model due to which the non-linear nature of the model is lost (Gillijns et al., 2006) & (Anderson and Moore, 1979). Also, the EnKF does not assume that the system noise or measurement noise is Gaussian as it also handles the model transitions in between the cells, which is being taken care by State Dependant Interacting Multiple models (SD-IMM) in the past. (Blackman and Popoli, 1999)

Our work differs from reported work in the following ways. First, we have used Digital Elevation Maps (DEMs) for the geometry of the water channel, which is an essential part of the system model and improves the estimates of hydrodynamic variables. Secondly, we have used only upstream boundary condition of the channel, this makes the estimation more simple because its common to have static sensors at upstream. Thirdly, we have used only one Lagrangian sensor for Lagrangian data. which shows the ability of the proposed framework to estimate with minimal measurement data.

The rest of the article is organized as follows. Section II describes the Lagrangian sensors. In Section III, the mathematical system model is discussed. In Section IV, the simulation scenario and simulation results are discussed from HEC-RAS simulation software. In Section V, the conclusion of this research work is described.

2. LAGRANGIAN SENSORS

The Lagrangian sensors can float passively with water velocity along the channel. These sensors are also popularly known as drifters or floats. The Lagrangian sensors can be equipped with multiple modalities of sensing such as temperature, pH, salinity, turbidity, and other physio-chemical parameters. An important component of Lagrangian sensors is GPS to provide positions at each step. Sensors for our work are inspired by the drifters developed by our group and reported in (Ahmad et al., 2018), where we have deployed such sensors to observe water quality data in canals and rivers. A photograph of our sensor deployed in a canal in Pakistan is shown in Fig. 1. The GPS measurements can be used to estimate the velocity

Fig. 1. The Lagrangian sensor equipped with GPS floating in a canal.

of the drifter and thereby of a steady channel. However, due to the inherent spatial variation in fluid flow due to variation in channel geometry and the occasional deviation of the sensor from forwarding movements, a simple method such as averaging does not yield good results in unsteady flows, when the channels are unstructured or when the measurements may be intermittent.

3. SYSTEM MODEL

The 2D Saint-Venant model is simulated in HEC-RAS 5.0 (Brunner, 2001), (Szymkiewicz, 2010), The model used in HEC-RAS is given as follows:

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} + q = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial h}{\partial x} + v_t\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - c_u, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial h}{\partial y} + v_t\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - c_v, \tag{3}$$

where $c_u = c_f u + fv$ and $c_v = c_f v + fu$, u and v are the velocities in x and y direction respectively, the h is the water level, c_f is the bottom friction coefficient, f is the Coriolis coefficient and g is the gravitational acceleration. The c_f is defined as follows:

$$c_f = \frac{n^2 g|v|}{R^{4/3}}, \tag{4}$$

where R is the hydraulic radius, $|v|$ is the magnitude of the velocity vector and n is the manning coefficient. The discretized version of 2D Saint-Venant is used in HEC-RAS with a sampling time of $60sec$. To incorporate the Lagrangian measurements into the model for state estimation, the Lagrangian sensor motion model is also required, such a model describes the motion of a Lagrangian sensor in 2D (Kuznetsov et al., 2003), which is written as follows:

$$X_{k+1} = X_k + u_k \Delta t, \tag{5}$$

$$Y_{k+1} = Y_k + v_k \Delta t, \tag{6}$$

where $X(m)$ and $Y(m)$ are the sensor position in x and y direction at time step k, Δt is measurement sampling time. The motion model is augmented with the 2D Saint-Venant state-space model. The state vector ϕ consist of

[1] The Saint-Venant equations are a set of hyperbolic partial differential equations that describe the flow in a fluid. These equations hold the law of conservation of mass and momentum

water level h, water velocities (u, v) and the Lagrangian sensor position X and Y, written as follows:

$$\phi = \left[h_k^1 \; u_k^1 \; v_k^1 \; \cdots \; h_k^n \; u_k^n \; v_k^n \; X_k \; Y_k\right]^T, \tag{7}$$

where the superscript is the number of cell and subscript is the time step. The path followed by the sensor is calculated by importing the water velocity profile from HEC-RAS to MATLAB. The output matrix is of order 3xJ, where J is the length of the state vector. The measurement model is defined in the following equations:

$$y_k^1 = h_k^1, \tag{8}$$
$$y_k^{N-1} = X_k, \tag{9}$$
$$y_k^N = Y_k, \tag{10}$$

where h_k^1 is upstream water level, X_k is Lagrangian position in x-axis and Y_k is Lagrangian position in y-axis. The EnKF is used for state estimation in MATLAB. The M ensembles of the state vector are generated and process noise is added into these ensembles to introduce the uncertainty (Tossavainen et al., 2008), (Burgers et al., 1998). Let β be the ensemble, The framework for state estimation using EnKF is discussed below.

$$\bar{\beta}_k^m = F(\beta_{k-1}^m) + w_{k-1}^m, \tag{11}$$
$$\eta_k = \frac{1}{M}\sum_{m=1}^{M} \bar{\beta}_k^m, \tag{12}$$
$$E_k = \left[\bar{\beta}_k^m - \eta_k \; \cdots \; \bar{\beta}_k^M - \eta_k,\right] \tag{13}$$
$$\Upsilon_k = \frac{1}{M-1} E_k E_k^T, \tag{14}$$
$$K_k = \Upsilon_k H_k^T [H_k \Upsilon_k H_k^T + R_k]^{-1}, \tag{15}$$
$$\beta_k^m = \bar{\beta}_k^m + K_k \left[y_k - H_k \beta_k^m + \varepsilon_k^m,\right] \tag{16}$$

where $\bar{\beta}_k^m$ is the estimated m_{th} ensemble at time step k, η_k is the conditional expectation of ensembles,Υ_k is the co-variance, w is the zero mean process noise, F is the process model which is augmented model in this research study, K_k is the Kalman gain and β_k^m is the updated m_{th} ensemble at time step k.

The performance of this data assimilation framework is analyzed by calculating the residual of estimated states and actual data obtained by simulations. The residual is calculated by eq. 17.

$$\epsilon(k) = \sqrt{\frac{\sum_{n=1}^{N} \|x_{actual}(k) - x_{estimated}(k))\|^2}{\sum_{n=1}^{N} \|x_{actual}(k)\|^2}} \tag{17}$$

where N is the total number of cells in the grid.

4. SIMULATION RESULTS

To solve the 2-D Saint-Venant model, a Digital Elevation Map (DEM) of Ravi River, Pakistan, is imported into HEC-RAS. The DEM resolution is 1-Arc (approximately 30 meters). The rectangular grid of 30.5 x 30.5 m each cell is formed. The manning's value of the selected region is selected as default, which is 0.06. The 2D grid is shown in Fig. 2. For the hydraulic profile of the selected region, the upstream and downstream boundary conditions are defined in HEC-RAS. At the upstream boundary, the stage hydro-graph [2] is provided as input to the system as shown in Fig. 3. The water level at upstream is increased by

[2] water level profile

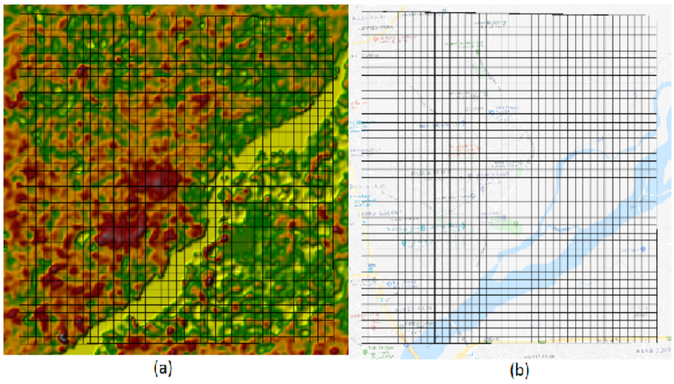

Fig. 2. Digital Elevation Map (DEM) of Ravi River, Pakistan. The grid of each cell 30.5 x 30.5 m is created on a region of section length of 1.1 Km.

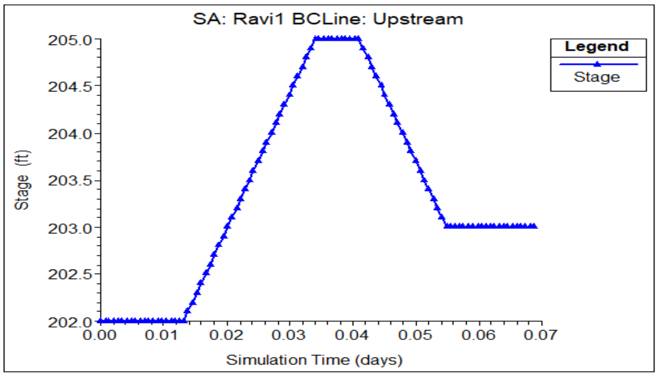

Fig. 3. The graph of the water level input at the upstream as boundary condition.

1m, which shows the phenomenon of the gate opening, the fall in water level describes the closure of the gate. The temporal variation in the system is introduced to depict the unsteady nature of water bodies. The simulation results generated using HEC-RAS are shown in Fig. 4. The Fig. 4 shows the water level and water velocity profile in Ravi River, Pakistan at time step 1 minute and 40 minutes, it shows the flow of water through water body of complex geometry. As the input at upstream is changed, the water level and water velocity in each cell show variation as shown in Fig. 5. In Fig. 5, the temporal variation at upstream, downstream and mid of the river can be seen. As the gate at upstream is opened and closed, the variation of water level and water velocity can be seen.

For data assimilation, the measurement data consists of the water level of one upstream cell and Lagrangian sensor position reported by GPS. In simulations, the Lagrangian sensor trajectory is calculated using Eqs. 5 and 6 in MATLAB by water velocity profile of the river. The Fig. 6 and 7 show the trajectory of Lagrangian sensor. The Lagrangian sensor covers the river section in 100 minutes from upstream to the downstream end.

The Lagrangian sensor data and upstream water level data are assimilated in the model using EnKF as described in section III. The estimated water level at different time steps is shown in Fig. 8 and 9, these results show the actual and estimated water level of the whole river section. The EnKF is unable to estimate the water level at the start but

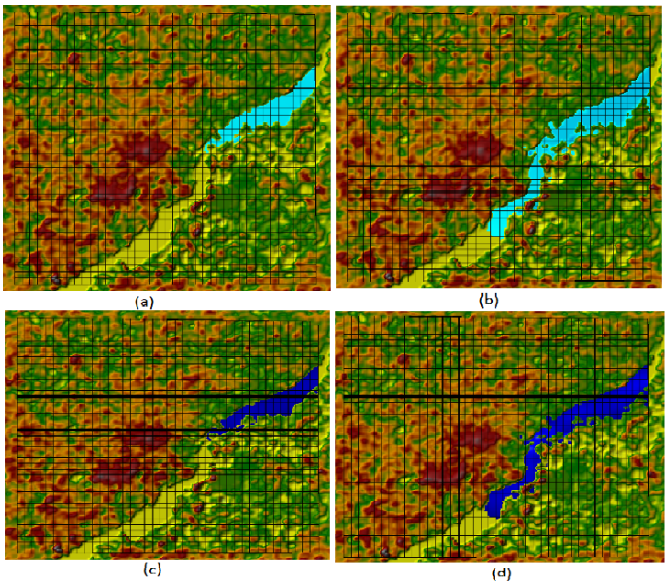

Fig. 4. (a) Water level profile at 1 minute of simulation (b) Water level profile at 40 minute of simulation (c) Water velocity profile at 1 minute of simulation (d) Water velocity profile at 40 minute of simulation

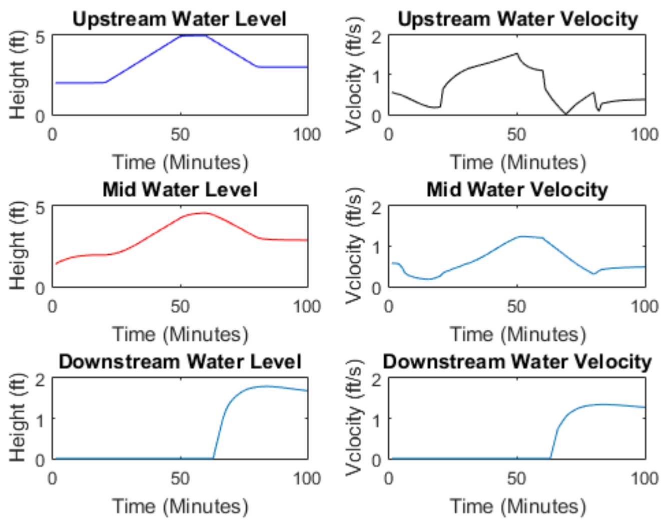

Fig. 5. Time series results of water level and water velocity of particular cells from upstream, downstream and mid point of the Ravi River.

as time increases, the EnKF can estimate the water level with high accuracy. The Fig. 10 & 11 shows the estimated y-component of water velocity profiles along with actual water velocity profile of the Ravi River. The Fig. 12 & 13 shows the estimated x-component of water velocity profiles along with actual water velocity profile of the Ravi river. The estimated Lagrangian sensor trajectory is shown in Fig. 14. The trajectory is estimated with low error. Fig. 15 shows the performance of this state estimation framework. It is clear from Fig. 15 that the framework performed with high accuracy and error reduced with the passage of time.

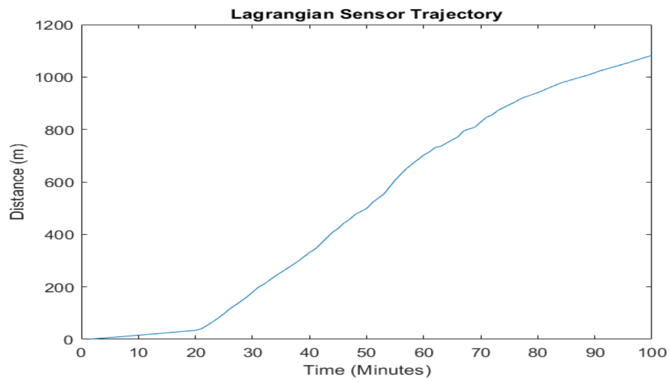

Fig. 6. Lagrangian sensor trajectory calculated by velocity profile using MATLAB.

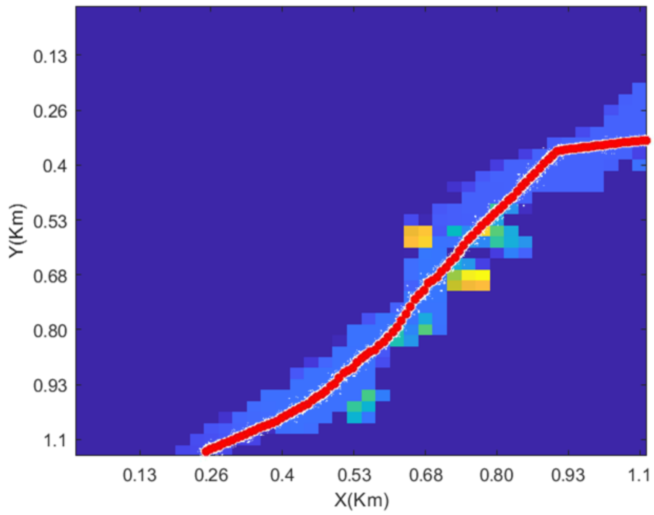

Fig. 7. Lagrangian sensor trajectory mapped on Ravi River.

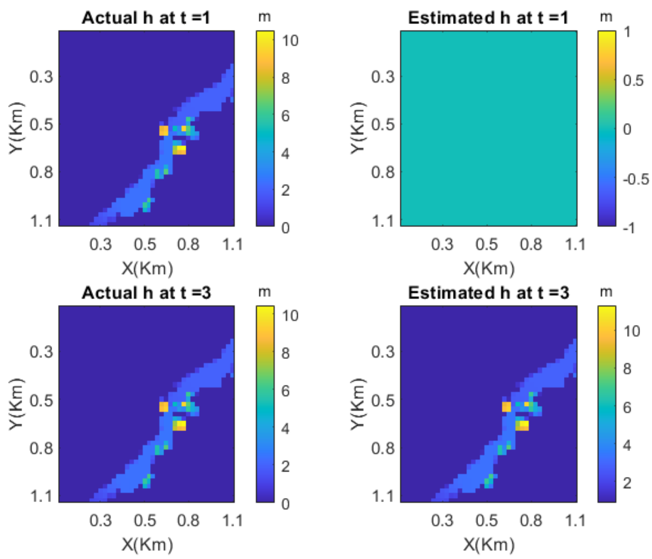

Fig. 8. Estimated and actual water level(m) at 1 minute and 3 minute.

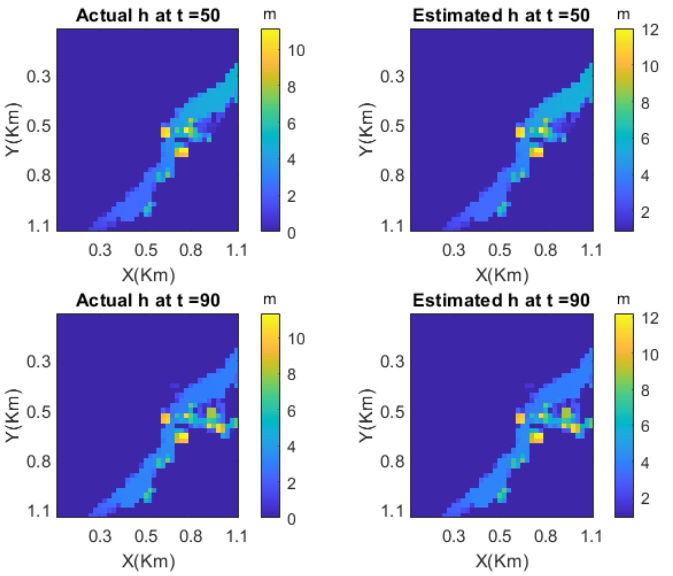

Fig. 9. Estimated and actual water level at 50 minute and 90 minute.

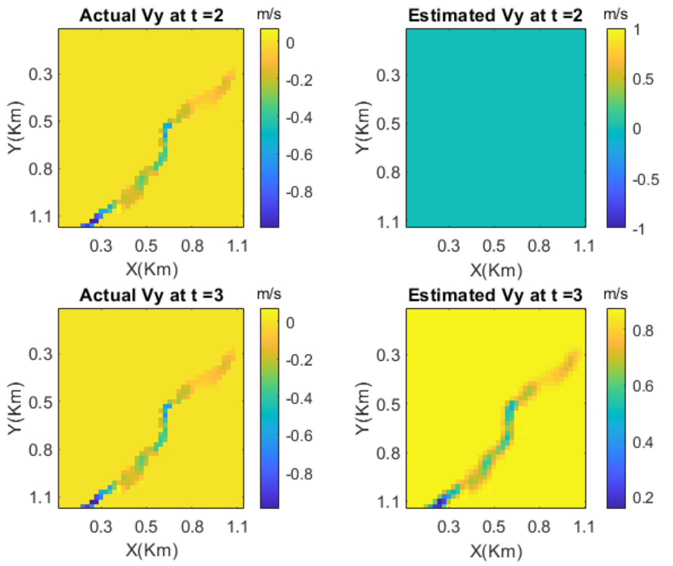

Fig. 10. Estimated and actual y-component of water velocity at 2 minute and 3 minute.

5. CONCLUSION

In this research work, an estimation framework of hydrodynamic variables in 2D such as water level and water velocities of complex water channels is proposed. This framework utilizes EnKF with Lagrangian sensor data. The Lagrangian sensors are the good passive floating source for cost-effective sensing in water bodies that provide only locations along the channel. For complex water channels, the geometry of the channel is unknown and difficult to describe. So Digital Elevation Maps (DEM) are used in HEC-RAS, which solves 2D Saint-Venant equations. The trajectory of the Lagrangian sensor is generated in MATLAB simulations and used for estimation. For this study, a 1.1Km section of the Ravi River, Pakistan is considered as a case study. The estimation framework can be used for irrigation, contamination tracking, urban flood mapping.

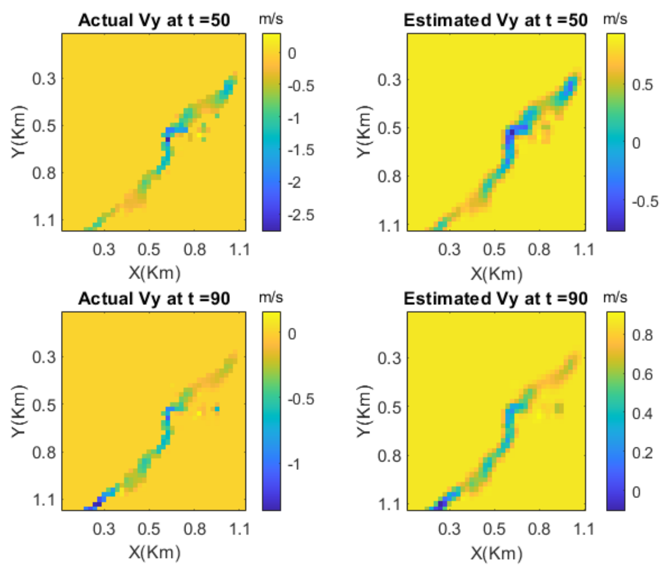

Fig. 11. Estimated and actual y-component of water velocity at 50 minute and 90 minute.

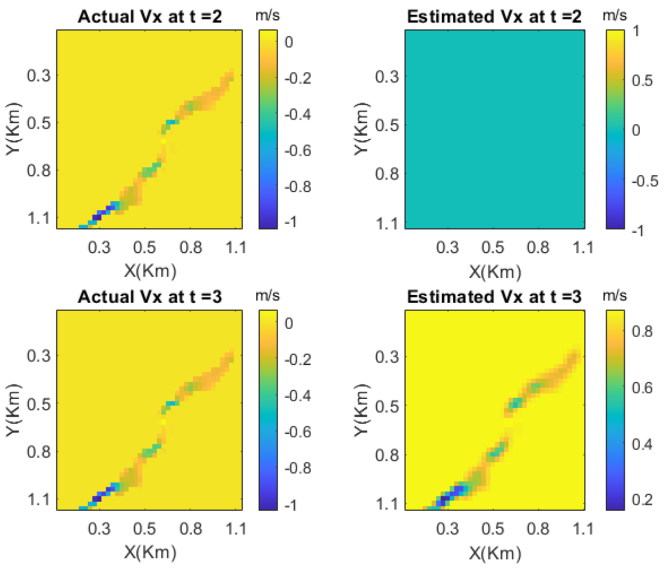

Fig. 12. Estimated and actual x-component of water velocity at 2 minute and 3 minute.

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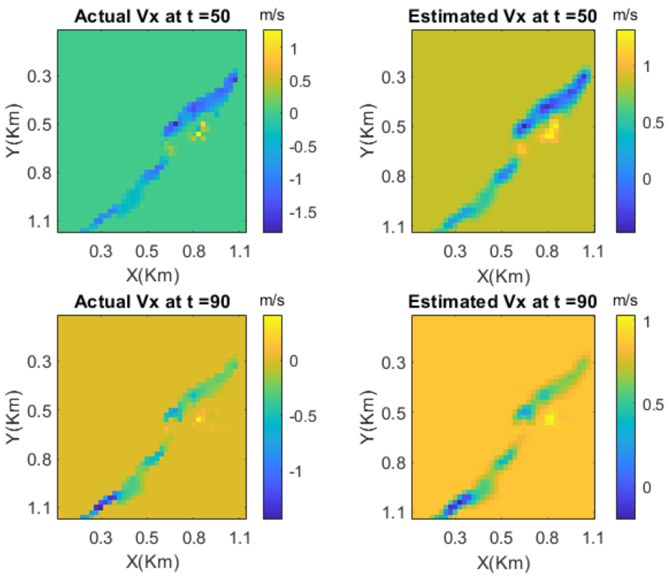

Fig. 13. Estimated and actual x-component of water velocity at 50 minute and 90 minute.

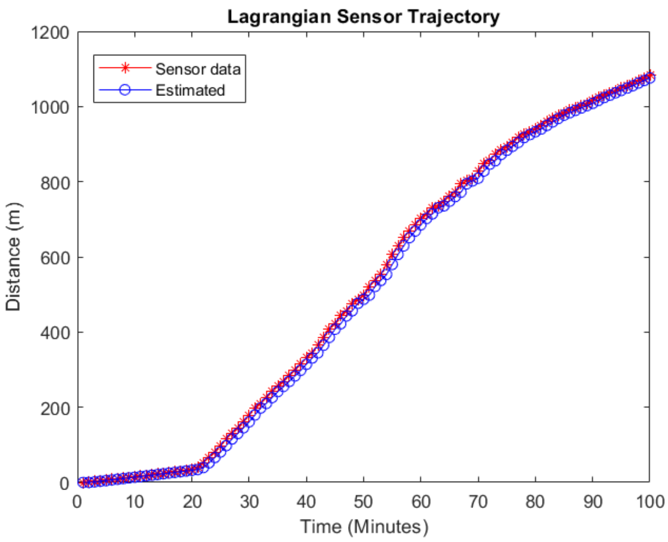

Fig. 14. Estimated and actual Lagrangian sensor trajectory

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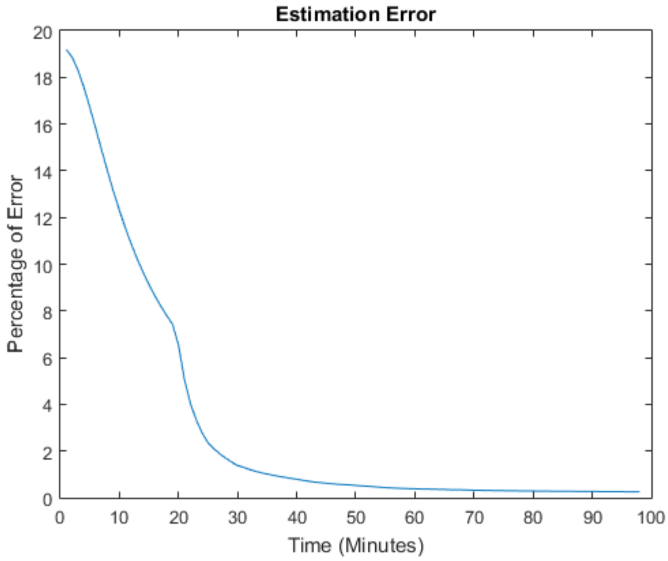

Fig. 15. Error analysis of state estimation algorithm for the 2D hydrological model using EnKF.

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