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Analysis and synthesis conditions for T-S fuzzy continuous-time systems with partially matched premises ⋆

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Abstract: A recognized challenge in the Takagi-Sugeno (T-S) fuzzy controller design concerns the use of different membership functions (MF) for controller and system. Most of the conditions available in the literature require that the controller's MF matche the system's one. Therefore, the implementation of such controllers may lead to unsafe operational conditions whenever such a match is lost. The main contribution of this paper is to provide new convex formulations for both stability analysis and controller design for T-S fuzzy systems under unmatched MF. We assume the same number of premises, yielding conditions called partially matched premises. To reduce conservatism, we use the Lyapunov approach, and we write the MF of the controller from the MF of the system. Two examples serve to compare our approach with others found in the literature. The achieved results suggest that our method outperforms the others.

Keywords: T-S fuzzy continuous-time systems, partially matched premises, Lyapunov function, LMIs

1. INTRODUCTION

A useful way to model and control nonlinear systems is through the Takagi-Sugeno (T-S) fuzzy approach, a technique used in several works and process applications (Lee and Joo, 2014; Lee et al., 2011; Li et al., 2015; Sung et al., 2012; Zhang et al., 2011). An advantage of such an approach is the use of local models, blended by membership function (MF) of the system. A control challenge, in this case, concerns the MF used to model the system and to compute the controller. Most of the works in the literature take the approach of perfect matched premises, i.e., they assume the same MF in both the system model and the controller. See for instance (Guerra et al., 2009; Lee and Joo, 2014; Tanaka and Wang, 2001; Taniguchi et al., 2001). In practice, other approaches may be more interesting such as the partially matched and the imperfect matched premises (Lam, 2018). In the partially matched premises (PMP) case, although the number of premises is the same for both the system model and the fuzzy controller, they can be different and thus the MF. In the more general case, imperfect matched premises (IMP), the number of premises are different for system model and fuzzy controller (and thus the respective MF). Conditions for interval type-2 fuzzy systems, which were recently published (Li et al., 2016a,b,c), are examples of PMP. An advantage of such an approach is to handle the case of uncertain MF. Lam and Leung (2005) investigated the PMP case and proposed analysis conditions for the T-S fuzzy system. Extensions of these investigations were performed by (Lam and Leung, 2006), by including a synthesis method, and by Ariño and Sala (2008) to handle uncertain MF. The results proposed in (Lam and Narimani, 2009) advances the previous conditions achieving less conservative results. A condition that synthesis switched controller was proposed by (de Oliveira et al., 2018). This result can be applied when we have PMP case. Lam (2011) assumes a staircase MF for the fuzzy controller and provide a stability analysis condition, using a methodology inspired by (Lam and Narimani, 2009). Only a few works address the IMP case, for instance, (Lam and Narimani, 2010) and (Chadli and Karimi, 2013). For a complete review of this theme, see (Lam, 2018).

In this paper, we focus on the PMP case and propose new convex conditions for both stability analysis of fuzzy T-S continuous-time systems and fuzzy controller design. We assume that the MF of the fuzzy controller and those of the fuzzy system are distinct from each other, even the number of premise variables is the same. Using a Lyapunov candidate function, we develop new convex conditions formulated in terms of linear matrix inequalities (LMI). We achieve a reduction in the conservatism of the proposed conditions by rewriting the fuzzy controller's MF in terms of the system's MF. We compare our proposal with similar approaches in the literature through two examples. The results suggest that our method outperforms the others.

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In the next section, we formulate the problem concerning this paper and present the structure of the fuzzy T-S model and controller are also presented. In Section III, we give some preliminary results to support the main results presented in Section IV. Two numerical examples illustrate the efficacy of our approach and establish comparisons with similar approaches in the literature. Section VI contains the main conclusions of this paper.

2. PROBLEM STATEMENT

Consider a nonlinear continuous-time system represented by:

$$\dot{x}(t) = f(x(t), u(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the control input vector of system. The function $f(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is Lipschitz and the origin is the equilibrium point of the system, i.e., $f(\mathbf{0},\mathbf{0}) = \mathbf{0}$. We assume that the nonlinear system (1) can be represented by a T-S model as (Tanaka and Wang, 2001; Feng, 2009):

$$\begin{array}{ll}\text{IF} & \phi_1(x(t)) \text{ is } M_1^i \text{ and } \cdots \text{ and } \phi_\Psi(x(t)) \text{ is } M_\Psi^i, \\ \text{THEN} & \dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1,\ldots,p,\end{array} \tag{2}$$

where $\phi_\ell(x(t)), \ell = 1, ..., \Psi$ are scalar premise variables that are supposed to depend only on states, M_ℓ^i are fuzzy sets, and p is the number of variables. The matrices $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are known. The dynamics system are described by

$$\dot{x}(t) = \sum_{i=1}^{p} w_i(x(t)) \left\{A_i x(t) + B_i u(t)\right\}, \tag{3}$$

where

$$\sum_{i=1}^{p} w_i(x(t)) = 1, \quad w_i(x(t)) \in [0,1], \quad i = 1,\ldots,p, \text{ and}$$

$$w_i(x(t)) = \frac{\mu_{M_1^i}(\phi_1(x(t))) \times \ldots \times \mu_{M_\Psi^i}(\phi_\Psi(x(t)))}{\sum_{k=1}^{p} \mu_{M_1^k}(\phi_1(x(t))) \times \ldots \times \mu_{M_\Psi^k}(\phi_\Psi(x(t)))}$$

is the normalized MF, and $\mu_{M_\ell^i}(\phi_\ell(x(t))), j = 1,\ldots,p$, is the grade of membership corresponding to the fuzzy set M_ℓ^i. Note that, $w_i(x(t))$ is a nonlinear function of $x(t)$.

We propose in this paper the following control law:

$$\begin{array}{ll}\text{IF} & \theta_1(x(t)) \text{ is } N_1^j \text{ and } \cdots \text{ and } \theta_\Psi(x(t)) \text{ is } N_\Psi^j, \\ \text{THEN} & u(t) = -K_j x(t), \quad j = 1,\ldots,p,\end{array} \tag{4}$$

where $\theta_\ell(x(t)), \ell = 1,\ldots,\Psi$ are scalar premise variables that are supposed to depend only on states and N_i^β are fuzzy sets. The matrices $K_j \in \mathbb{R}^{n\times m}$ are the gain matrices of the control law. The inferred output of the fuzzy controller is given by

$$u(t) = -\sum_{j=1}^{p} m_j(x(t)) K_j x(t), \tag{5}$$

where

$$\sum_{j=1}^{p} m_j(x(t)) = 1, \quad m_j(x(t)) \in [0,1], \quad j = 1,\ldots,p, \text{ and}$$

$$m_j(x(t)) = \frac{\mu_{N_1^j}(\theta_1(x(t))) \times \ldots \times \mu_{N_\Psi^j}(\theta_\Psi(x(t)))}{\sum_{k=1}^{p} \mu_{N_1^k}(\theta_1(x(t))) \times \ldots \times \mu_{N_\Psi^k}(\theta_\Psi(x(t)))}$$

is the normalized membership function that is a nonlinear function of $x(t)$ and $\mu_{N_\ell^j}(\theta_\ell(x(t))), j = 1,\ldots,p$ is the grade of membership corresponding to the fuzzy set N_ℓ^j.

Remark 1. Note that the T-S fuzzy controller (4)–(5) does not share the membership function of the T-S fuzzy model (2)–(3).

By replacing (4)–(5) in (2)–(3), we obtain the following closed-loop T-S fuzzy system:

$$\dot{x}(t) = \sum_{i=1}^{p} w_i(x(t)) \Big(A_i x(t) - B_i \sum_{j=1}^{p} m_j(x(t)) K_j x(t)\Big). \tag{6}$$

The two problems treated in this paper are formulated as:

Problem 1. Given the matrices K_j, $j = 1,\ldots,p$, and the T-S fuzzy model (2)–(3) with control law (4)–(5), verify if the closed-loop T-S fuzzy system (6) is asymptotically stable.

Problem 2. Given the T-S fuzzy model (2)–(3), determine the matrices K_j, $j = 1,\ldots,p$, such that the resulting closed-loop T-S fuzzy system (6) from the control law with control law (4)–(5) is asymptotically stable.

3. PRELIMINARY RESULTS

Consider the candidate Lyapunov function, $V(x(t)) : \mathbb{R}^n \to \mathbb{R}$ given by $V(x(t)) = x^T(t)Px(t)$ fulfilling:

$$\alpha_0(\|x(t)\|) \le V(x(t)) \le \alpha_1(\|x(t)\|) \tag{7}$$

with positive-definite matrix $P = P^T \in \mathbb{R}^{n\times n}$ and $\alpha_i(\cdot)$, $i = 0, 1$, are $\mathcal{K}_\infty$ functions. Additionally, from Lyapunov's theory, it is required

$$\dot{V}(x(t)) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \le -\alpha_2(\|x(t)\|), \tag{8}$$

where $\alpha_2(\|x(t)\|)$ is also a $\mathcal{K}_\infty$ function Khalil (2002).

The following lemma, found in (Wang et al., 1996) and (Chen et al., 1993), is useful to establish our main contributions.

Lemma 1. The T-S fuzzy closed-loop system (6) is asymptotically stable if there exist a symmetric matrix P such that the following LMIs hold for $i, j = 1,\ldots,p$:

$$(A_i - B_i K_j)^T P + P(A_i - B_i K_j) < \mathbf{0}. \tag{9}$$

Proof 1. By multiplying (9) successively by w_i and m_j, summing them up, pre- and postmultiplying the resulting inequality by $x^T(t)$ and $x(t)$, respectively, and using (6), we obtain $\dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) < 0$. Thus, we have that (8) $\dot{V}(x(t)) \le -\alpha_2\|x(t)\|^2$ is ensured with a small enough $\alpha_2 > 0$ and $\lambda_{\min}(P)\|x(t)\|^2 \le V(x(t)) \le \lambda_{\max}(P)\|x(t)\|^2$. Therefore, $V(x(t))$ is a Lyapunov function and the closed-loop T-S fuzzy system (6) is asymptotically stable.

Although the MF of the T-S fuzzy model (2)–(3) differs from the one of the T-S fuzzy controller (5), they can be related as established in the next lemma.

Lemma 2. Consider the MF of the T-S fuzzy model (2)–(3) and the MF of the T-S fuzzy controller (4)–(5). There exist the scalars $g_\ell \in [0,\ 2]$, for $\ell = 1,\ 2$, and a vector $h \in \mathbb{R}^p$, such that

$$m_i = g_\ell w_i + (1 - g_\ell) h_i, \tag{10}$$

where, if $g_\ell < 1$

$$m_i - g_\ell w_i \ge 0, \tag{11}$$

or if $g_\ell > 1$

$$m_i - g_\ell w_i \leq 0, \quad (12)$$

and if $g_\ell = 1$, we have the Parallel Distributed Compensation (PDC). Therefore, h verifies $\sum_{i=1}^{p} h_i = 1$ and $h_i \geq 0$.

Proof 2. With $g_\ell \in [0,\ 2]$, for $\ell = 1,\ 2$, and h_i verifying (10) for $i = 1, \ldots, p$, we sum such equation up on $i = 1, \ldots, p$, and use the convex property on w_i and m_i to get

$$\sum_{i=1}^{p} h_i = \frac{\sum_{i=1}^{p} m_i - g_\ell \sum_{i=1}^{p} w_i}{1 - g_\ell} = 1.$$

Furthermore, taking into account (11), if $g_\ell < 1$, or (12), if $g_\ell > 1$, results in $h_i \geq 0$, for $i = 1, \ldots, p$.

Remark 2. The association of the membership functions $m(x(t))$ and $w(x(t))$ proposes in the Lemma 2 is an extension that the way proposed in (Lam and Narimani, 2009). We can see that in these works consider $g_j \in [0,\ 1]$, thus only the situation $g_j < 1$ and inequality (11) is taken into account.

We can show through a simple example that our representation contemplates the representation proposed by (Lam and Narimani, 2009). Therefore, consider that $w_1 = 0.60$ and $w_2 = 0.40$. From the representation proposed by (Lam and Narimani, 2009), we consider $g_1 = g_2 = 0.80$, which result in $m_1 \in [0.48,\ 0.68]$ and $m_2 \in [0.32,\ 0.52]$. From our proposal, we consider $g_1 = 0.80$ and $g_2 = 1.20$, which results in $m_1 \in [0.48,\ 0.72]$ and $m_2 \in [0.28,\ 0.52]$. Therefore, our proposal results in intervals for $m_1(x(t))$ and $m_2(x(t))$ about $16,67\%$ bigger than the representation proposed by (Lam and Narimani, 2009).

By using Lemma 2, we replace m_j in (6) by the right side of (10) we get, after some algebraic manipulations (see (Silva et al., 2018) and (Silva et al., 2020) for details):

$$\dot{x}(t) = \sum_{i=1}^{p} \sum_{j=i}^{p} \sigma_{ij} w_i w_j 0.5(A_i + A_j - g_\ell B_i K_j - g_\ell B_j K_i) - \sum_{q=1}^{p} \sum_{i=1}^{p} \sum_{j=i}^{p} \sigma_{ij} w_i w_j h_q 0.5(1 - g_\ell)(B_i K_q + B_j K_q), \quad (13)$$

with $\sigma_{ij} = 1$ when $i = j$ and $\sigma_{ij} = 2$ otherwise.

4. MAIN RESULTS

The main contributions of this paper are given in the following theorems, that provide a solution for the stability analysis and T-S fuzzy controller design stated in problems 1 and 2.

4.1 Stability analysis condition

Theorem 1. Consider the T-S fuzzy model (2)–(3) and given gain matrices K_i, $i = 1, \ldots, p$. Suppose that there exist symmetric definite positive matrices $P \in \mathbb{R}^{n \times n}$ and $Q_{ii} \in \mathbb{R}^{n \times n}$, $i = 1, \ldots, p$, matrices $Q_{ji} = Q_{ij}^T \in \mathbb{R}^{n \times n}$, $i = 1, \ldots, p$ and $j = i + 1, \ldots, p$, and a given scalar $g_\ell \in [0,\ 2]$ such that the following LMIs are verified for $i = 1, \ldots, p$, $j = i, \ldots, p$, $q = 1, \ldots, p$, and $\ell = 1, 2$:

$$\hat{A}_{ijq\ell}^T P + P \hat{A}_{ijq\ell} + 0.5(Q_{ij} + Q_{ji}) < \mathbf{0}, \quad (14)$$

with

$$\hat{A}_{ijq\ell} = 0.5(A_i + A_j - g_\ell B_i K_j - g_\ell B_j K_i) - 0.5(1 - g_\ell)(B_i K_q + B_j K_q). \quad (15)$$

Then, the T-S fuzzy closed-loop (6) is asymptotically stable.

Proof 3. Substituting (15) into (14), multlipy it by w_i, w_j, and h_q, and sum them up on $i = 1, \ldots, p$, $j = i, \ldots, p$, and $q = 1, \ldots, p$, to get

$$\left(\sum_{i=1}^{p} \sum_{j=i}^{p} \sigma_{ij} w_i w_j 0.5(A_i + A_j - g_\ell B_i K_j - g_\ell B_j K_i) - \sum_{q=1}^{p} \sum_{i=1}^{p} \sum_{j=i}^{p} \sigma_{ij} w_i w_j h_q 0.5(1 - g_\ell)(B_i K_q + B_j K_q) \right)^T P + P \left(\sum_{i=1}^{p} \sum_{j=i}^{p} \sigma_{ij} w_i w_j 0.5(A_i + A_j - g_\ell B_i K_j - g_\ell B_j K_i) - \sum_{q=1}^{p} \sum_{i=1}^{p} \sum_{j=i}^{p} \sigma_{ij} w_i w_j h_q 0.5(1 - g_\ell)(B_i K_q + B_j K_q) \right) + \sum_{i=1}^{p} \sum_{j=i}^{p} \sigma_{ij} w_i w_j 0.5(Q_{ij} + Q_{ji}) < \mathbf{0}, \quad (16)$$

which can be rewritten as follows

$$\left(\sum_{i=1}^{p} \sum_{j=1}^{p} w_i w_j (A_i - g_\ell B_i K_j) - \sum_{q=1}^{p} \sum_{i=1}^{p} w_i h_q (1 - g_\ell) B_i K_q \right)^T P + P \left(\sum_{i=1}^{p} \sum_{j=1}^{p} w_i w_j (A_i - g_\ell B_i K_j) - \sum_{q=1}^{p} \sum_{i=1}^{p} w_i h_q (1 - g_\ell) B_i K_q \right) + \sum_{i=1}^{p} \sum_{j=1}^{p} w_i w_j Q_{ji} < \mathbf{0}. \quad (17)$$

Using Lemma 2, we replace $(1 - g_\ell) \sum_{q=1}^{p} h_q$ by $\sum_{q=1}^{p} m_q - g_\ell \sum_{q=1}^{p} w_q$ that gives

$$\left(\sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j (A_i - g_\ell B_i K_j) - \sum_{q=1}^{p}\sum_{i=1}^{p} w_i m_q B_i K_q + g_\ell \sum_{q=1}^{p}\sum_{i=1}^{p} w_i w_q B_i K_q\right)^T P + P\left(\sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j (A_i - g_\ell B_i K_j) - \sum_{q=1}^{p}\sum_{i=1}^{p} w_i m_q B_i K_q + g_\ell \sum_{q=1}^{p}\sum_{i=1}^{p} w_i w_q B_i K_q\right) + \sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j Q_{ji} < \mathbf{0}. \quad (18)$$

Moreover, the last inequality can be rearranged as

$$\left(\sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j (A_i - B_i K_j)\right)^T P + P\left(\sum_{i=1}^{p}\sum_{j=1}^{p} w_i m_j (A_i - B_i K_j)\right) P + \sum_{i=1}^{p}\sum_{j=1}^{p} w_i w_j Q_{ji} < \mathbf{0}. \quad (19)$$

Pre- and postmultiplying (19) by $x^T(t)$ and $x(t)$, respectively, and using (6), we obtain

$$\dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + \xi(t)\mathbb{Q}\xi(t) < 0, \quad (20)$$

where $\xi(t) = [w_1 x(t) \ \cdots \ w_p x(t)]$ and

$$\mathbb{Q} = \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1p} \\ Q_{21} & Q_{22} & \cdots & Q_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{p1} & Q_{p2} & \cdots & Q_{pp} \end{bmatrix} > \mathbf{0}.$$

By considering the candidate Lyapunov function (7) and its derivative (8), inequality (20) means that

$$\dot{V}(x(t)) < -\xi(t)\mathbb{Q}\xi(t) < -\alpha_2 \|x(t)\|^2 < 0,$$

for some small enough $\alpha_2 > 0$. Additionally, $V(x(t))$ fulfills the requirements to be a Lyapunov functions (see details in the proof of Lemma 1). Therefore, if (14) is verified, then the closed-loop T-S fuzzy system (6) is asymptotically stable ensured by the Lyapunov function $V(x(t))$.

4.2 Synthesis condition

Theorem 2. Consider the T-S fuzzy model (2)–(3) and suppose that there exist symmetric definite positive matrices $W \in \mathbb{R}^{n\times n}$ and $\tilde{Q}_{ii} \in \mathbb{R}^{n\times n}$, $i = 1, \ldots, p$, matrices $\tilde{Q}_{ji} = \tilde{Q}_{ij}^T \in \mathbb{R}^{n\times n}$, $Y_i \in \mathbb{R}^{n\times m}$, $i = 1, \ldots, p$ and $j = i + 1, \ldots, p$, and a given scalar $g_\ell \in [0,\ 2]$ such that the following LMIs are verified for $i = 1, \ldots, p$, $j = i, \ldots, p$, $q = 1, \ldots, p$, and $\ell = 1, 2$:

$$H_{ijq}^T + H_{ijq} + 0.5(\tilde{Q}_{ij} + \tilde{Q}_{ji}) < 0, \quad (21)$$

with

$$H_{ijq} = 0.5((A_i + A_j)W - g_\ell B_i Y_j - g_\ell B_j Y_i) - 0.5(1 - g_\ell)(B_i Y_q + B_j Y_q). \quad (22)$$

Then, the control matrices of (4)–(5) are computed as follows

$$K_i = Y_i W^{-1} \quad (23)$$

and the resulting control system formed by the T-S fuzzy model (2)–(3) and the fuzzy control law (4)–(5) is asymptotically stable.

Proof 4. Replace in (21)–(22) Y_i by $K_i W$ and pre- and postmultiply the resulting inequality by W^{-1}. Assuming $\tilde{Q}_{ij} = WQ_{ij}W$, $i = 1, \ldots, p$ and $j = 1, \ldots, p$, and $P = W^{-1}$, we recover the LMIs (14)–(15) and the proof follows similar steps of the Proof 3.

Remark 3. Note that if we consider $g_\ell = 1$, for $\ell = 1, 2$, in the theorems 1 and 2, we recover the classical conditions of analysis and synthesis of T-S fuzzy controllers presented in (Tanaka and Wang, 2001), with the control law in parallel distributed compensation (PDC) scheme, i.e., the T-S fuzzy system and the T-S fuzzy controller share the membership function.

5. EXAMPLES

We present two examples in this section. In the first one, we compare the analysis condition, Theorem 1, with similar ones found in the literature. In the second example, we use the synthesis condition, Theorem 2, to design a T-S fuzzy controller stabilizing a nonlinear system. The examples illustrate that even the T-S fuzzy system and T-S fuzzy controller do not share the same MF, the resulting T-S fuzzy closed-loop system is stable.

5.1 Example 1

Consider a nonlinear system modeled as the following T-S fuzzy system with two fuzzy rules ($p = 2$):

$$\text{IF } x_1(t) \text{ is } M_1^i, \quad \text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \quad (24)$$

with

$$A_1 = \begin{bmatrix} 2 & -10 \\ 1 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} a & -10 \\ 1 & c \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} b \\ 0 \end{bmatrix}.$$

We use this T-S fuzzy system to perform numerical comparisons in the context of stability analysis, where we confront the proposal of Theorem 1 with a method from the literature, and in the context of stabilization by comparing the achievements of Theorem 2 with another proposal from the literature.

Stability Analysis: We chose $c = 1$ to compare our method with the numerical test performed by Lam and Leung (2005). We did a grid over the parameter space $a \times b$ and for each pair (a, b), with $a \in [1,\ 6]$ and $b \in [1,\ 5]$, we design T-S fuzzy controller gains K_i, $i = 1, 2$, computed by pole placement such that the closed-loop poles of $A_i - B_i K_i$ are at -1 and -15. Next, we try to certify the stability of the respective closed-loop system using the conditions of Theorem 1. Such a procedure was repeated for $g_1 = 0.83$ and $g_2 = 1.25$. The performance of Theorem 1 is compared with that introduced by (Lam and Leung, 2005, Lemma 1) and the achieved results are given in Figure 1. The pairs (a, b) certified as stable by (Lam and Leung, 2005, Lemma 1) are marked with $\{\circ\}$. In case of the pairs (a, b) certified as stable by Theorem 1 is marked with $\{\circ, \times\}$.

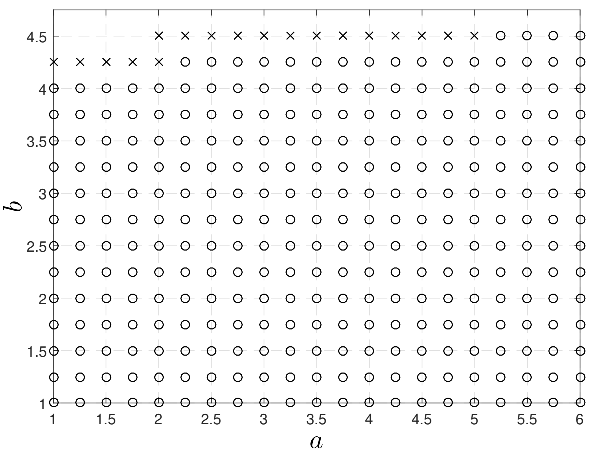

Fig. 1. Stable regions certified by (Lam and Leung, 2005, Lemma 1) ($\{\circ\}$) and by Theorem 1 with $g_1 = 0.83$ and $g_2 = 1.25$ ($\{\circ, \times\}$).

Figure 1 shows the achievements, indicating the superior performance of our proposal.

Stabilization: The objective here is given a pair (a, b), try to design a T-S fuzzy controller gain that gives a stable closed-loop system. Assuming $c = 3$, Lee (2019) used a synthesis condition based on an affine MF to design a T-S fuzzy control law. Note that although this is a PMP case, the MF of the controller has the same shape as that of the T-S fuzzy model. With $a = 3$, the parameter b is increased and for each value we applied Theorem 2 with $g_1 = g_2 = g = 0.99$. Our condition can stabilize the closed-loop system up to $b = 3.96 \times 10^{15}$, which is slight higher value than the achieved by (Lee, 2019, Theorem 1) ($b = 3.95 \times 10^{15}$). Therefore, we can say that our result is superior to the presented in (Lee, 2019) because, besides a slightly better value on the maximal b, our approach allows a completely different MF on the T-S fuzzy controller into its interval.

5.2 Example 2

Consider a T-S fuzzy model described by (2), with

$$A_1 = \begin{bmatrix} 1.74 & 0.58 \\ 0.58 & 0 \end{bmatrix}, \; A_2 = \begin{bmatrix} 1.16 & 0.58 \\ 0.58 & -0.58 \end{bmatrix}, \; B_i = \begin{bmatrix} 3.6 \\ 0.6 \end{bmatrix},$$

$i = 1, 2$, $w_1(x(t)) = \sin^2(x_{2,k})$, and $w_2(x(t)) = \cos^2(x_{2,k})$. Our interesting here is synthesis T-S fuzzy controller gains (4)–(5) that stabilize the T-S fuzzy system. We propose three different types of controller's membership functions:

$$m_1^1(x(t)) = e^{-(x_1(t)/10)^2},$$
$$m_1^2(x(t)) = \max\left(\min\left(\frac{x_1(t)+10}{10}, \frac{10-x_1(t)}{10}\right), 0\right),$$
$$m_1^3(x(t)) = w_1(x(t)),$$

with $m_2^\kappa(x(t)) = 1 - m_1^\kappa(x(t))$, $\kappa \in \{1,\ 2,\ 3\}$. The two first membership functions are different from that the T-S fuzzy system, and the last one is equal to the T-S fuzzy system. Therefore, we use the synthesis condition presented in Theorem 2, with $g_1 = 0.6$ and $g_2 = 1.4$ to get:

$$K_1 = [0.8051 \;\; 0.3925] \text{ and } K_2 = [0.7436 \;\; 0.3973]. \qquad (25)$$

With these gains we simulate the resulting T-S fuzzy closed-loop system with initial condition $x(0) = [15 \;\; -10]^T$ for each of the considered MF. The respective state response are shown in 2, where the solid lines represent the state $x_1(t)$ and dotting lines represent the state $x_2(t)$. The black lines concern $m_1^1(x(t))$, the blue ones are related to $m_1^2(x(t))$ and the red ones with $m_1^3(x(t))$. We can see that the resulting states trajectories converge to origin.

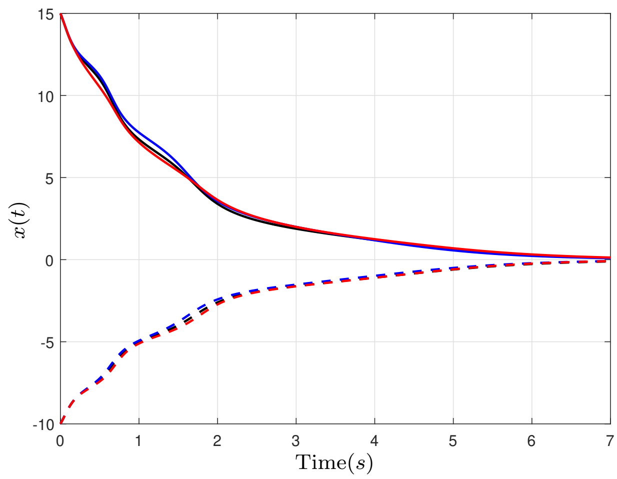

Fig. 2. State trajectories of the resulting control systems ($x_1(t)$ with solid lines and $x_2(t)$ with dashed lines), using MF m_1^1 (black), m_1^2 (blue), and m_1^3 (red).

Figure 3 shows the MF of the T-S fuzzy system, $w_1(x(t))$ (top plot, solid lines), and of the controller, $m_1^\kappa(x(t))$, for $\kappa = \{1,\ 2,\ 3\}$ (bottom plot, dotted lines with m_1^1 (black), m_1^2 (blue), and m_1^3 (red))). As expected, in all cases, the T-S fuzzy closed-loop systems remain asymptotically stable despite the PMP condition. It is interesting to note that even the MF of the controller violating the region established from the g_ℓ (see Lemma 2), the T-S fuzzy closed-loop system remains asymptotically stable. That shows that our result is conservative.

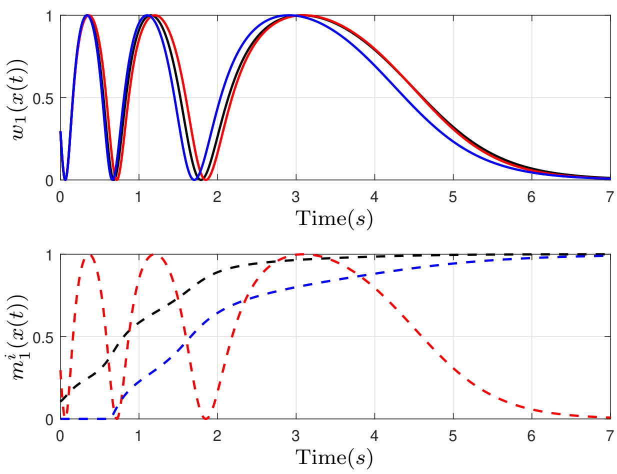

Fig. 3. Membership functions of the T-S fuzzy system (solid lines) and the controller (dashed lines).

6. CONCLUSIONS

We presented new convex conditions, formulated in terms of LMIs, for *a)* stability analysis of T-S fuzzy systems driven by T-S fuzzy controllers, and *b)* T-S fuzzy controller design, assuming partially mismatch premises (PMP) in both cases. The proposed conditions are based on a quadratic Lyapunov function. We succeed to reduce the conservatism by expressing the membership function of the controller from the membership function of the T-S fuzzy system. We presented two examples where we compare our approach with other conditions from the literature. The results suggest that our proposal outperforms the compared conditions.

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