Trajectory Planning For Autonomous Wheeled Mobile Robots With Trailer

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Abstract: This paper presents a dynamic optimization scheme for generating trajectory planning for autonomous wheeled mobile robots with tractor designed to accomplish missions in indoor environments. Such an optimization criterion problem requires a method that can yield a fast execution time and minimum traveling distance that contains geometrics, kinematics, and physical/environment constraints. The main goal is to develop optimal trajectories planning approach of an autonomous wheeled mobile robots with trailer for the execution of predefined tasks in structures environment. The developed approach can be considered as an extension of the Random Profiles Approach used for wheeled mobile robots. The results also illustrate that thanks to its time optimal trajectories planning, our scheme is well adapted to complex tasks as it can get shorter execution time for the autonomous holonomic tractor with a nonholonomic trailer.

Keywords: Trailer, Autonomous Wheeled Mobile Robot, Trajectory planning, Dynamic Optimization, Random Profiles Approach.

1. INTRODUCTION

Over the past few years, the exploitation and development of wheeled mobile robots with trailer is the subject of extensive research. A tractor-trailer is a wheeled robot with a permanent pivotal connection or semi-permanent in its structure (Li and Shao, 2016). Generally, a tractor-trailer turns with a substantially smaller turning radius than a rigid-body robot of the same length (Li et al., 2015). This class of robots is characterized by a relatively simple mechanical design, a displacement system that adapts to various types of environment and capabilities locomotion that can be used in combination or alternately as required. Thanks to their potential, mobile robots with trailers have become an indispensable tool for carrying out complex tasks, often repetitive and cumbersome, carried out in short and large volumes of work. The application fields are diverse: transportation, agriculture, military applications (mine clearance), etc.

To improve the efficiency of these wheeled mobile robots during task performance, it is necessary first to optimize the design of mechanical structure and second, to operate at optimally available capacities of the tractor-trailer. In general, these tasks can be categorized into two : (*i*) path planning and (*ii*) trajectory planning. First of all, the aim is to find the optimal paths or continuous sequences of configurations defined independently of time between two boundary situations. While, trajectory planning is a continuous sequence of time-defined configurations, which focuses on how to travel over the path in time space, taking into account kinematics and dynamics constraints. In the literature, papers treat path planning approaches are numerous relatively to those who deal with trajectory planning. These works are grouped according to: (*i*) kinematic or dynamic modeling, (*ii*) performance criteria, and (*iii*) implementation technique addressing the problem.

Various methods have been developed in this direction. Most of these methods are generally limited to taking into account geometric and/or kinematic constraints. Other methods require, in general, preliminary analytical developments that are specific to the treatment issues.

To generate the reference trajectories between the initial and final situations, we use a methodology that considers the kinematic and/or dynamics capabilities of the mobile robot wheels with trailer. Several studies have been carried out to bring solutions to the trajectory planning problem. These include methods based on the Pontryagin principle maximum (Pontriaguine et al., 1974), methods based on the phase plane (Li and Shao, 2016), the potential field methods (Koren and Borenstein, 1991), methods based on parametric optimization techniques (Von Stryk and Bulirsch, 1992) and other more specialized methods. Most of these methods are generally limited to taking into account geometric and/or kinematic constraints. Other methods require, in most cases, preliminary analytical developments that are specific to the treatment issues. As a result, the implementation can be painful, especially when taking into account the dynamic constraints due to the strong non-linearity of the problem.

The main objective of the developed work is to determine the best trajectory, according to a performance criterion. Which will bring the autonomous wheeled mobile robots with trailer (AWMRT) from an initial configuration to a final configuration ensuring the good functioning of the system, while respecting constraints, that are properly applied like geometric (task position boundary conditions, AWMRT joint constraints), kinematic (task velocities boundary conditions, robot actuator velocities and accelerations kinematic limits) and physical/environment, the AWMRT system should not collide with the obstacles in the environment and such a rigid-body mobile robot, different parts should not collide with the other. This trajectory planning issue is a complex problem of nonlinear constrained optimization. For that, we use a holonomic mobile robot, *Robotino* (Melingui et al., 2013), with a passive, non-holonomic, two-wheeled trailer. A rigid pivot joins these mobile robots.

The used approach is an extension of the random profile approach (*RPA*) for trajectory planning of tractor-trailers presented in (Hank and Haddad, 2016) for hybrid navigation of mobile robots, (Bouktir et al., 2008) for a quadrotor helicopter, (Haddad et al., 2007) and (Haddad et al., 2010) in the case of wheeled mobile robots and mobile manipulators, and initially proposed by (Chettibi and Lehtihet, 2002) in fixed base manipulators. This approach is based on transforming the original problem into a constrained parametric optimization problem in which one of the parameters is the execution time of the task. The following performances characterize this approach:

- Efficiency: It provides high-quality solutions in reasonable computational times and without prior simplification of the dynamic model,
- Simplicity of implementation: it converts the problem of trajectory planning, in searching for the optimal position of a few defined control points in bounded spaces,
- Versatility: It is operated for the planning tasks of various systems (manipulators, mobile robots, mobile manipulators and hybrid navigation).

The presented results show that AWMRT is extremely well suited to this approach. The remainder of this paper is structured as follows. Section2 concerns the kinematic model of AWMRT, and problem formulation trajectory planning is shown in section 3. Then our Proposed resolution method and adaptation are introduced in Section 4, followed by Section 5, where simulation results and experimental validation. Finally, conclusions and future work are drawn in Section 7.

2. KINEMATIC MODEL OF AWMRT

To model or operate a mobile robot, it is necessary to describe the kinematic and/or dynamic behaviors of the physical system in the form of mathematical equations. The mobile robot considered in this work consists of a holonomic mobile robot *Robotino* (Fig. **??**), connected to a non-holonomic trailer (a chassis supported by two passive wheels) with a rigid pivot joint (Fig. 1).

The set evolves in a plane environment to which a reference $\mathscr{R}_0 = (O_0, \boldsymbol{x}_0, \boldsymbol{y}_0, z_0)$ is linked. Note $\mathscr{R}_1 = (O_1, \boldsymbol{x}_1, \boldsymbol{y}_1, z_1)$, the movable frame attached to the chassis of the *Robotino*, O_1 origin, the center axis of the drive wheels. The moving reference $\mathscr{R}_2 = (O_2, \boldsymbol{x}_2, \boldsymbol{y}_2, z_2)$, attached to the trailer chassis, is located in the middle of the trailer's wheel axis. The system is defined by the generalized coordinates vector $\boldsymbol{q}$, compound variables positioning and orientation of two markers, $\mathscr{R}_1$ and $\mathscr{R}_2$, with respect to reference $\mathscr{R}_0$; and the variables characterizing the rotational angles of the wheels of the *Robotino* and the trailer. The following relation gives this vector:

$$\boldsymbol{q} = [x_1, y_1, \theta_1, \varphi_1, \varphi_2, \varphi_3, x_2, y_2, \theta_2, \varphi_4, \varphi_5] \tag{1}$$

where x_1, y_1 and θ_1: position and orientation coordinates of $\mathscr{R}_1$ with respect to $\mathscr{R}_0$; x_2, y_2 and θ_2: position and orientation coordinates of $\mathscr{R}_2$ with respect to $\mathscr{R}_0$; φ_1, φ_1 and φ_3: wheels rotation angles of the *Robotino*, φ_4 and φ_5: wheels rotation angles of trailer.

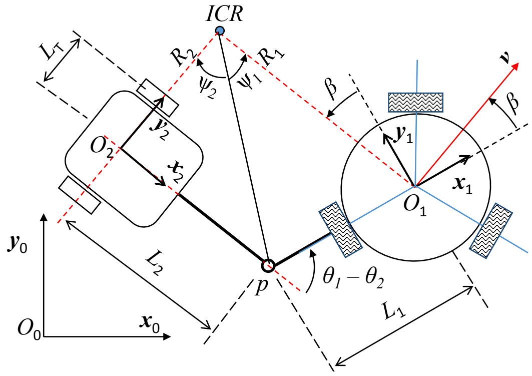

Fig. 1. AWMRT model

During the modeling, we assumed that the tractor (*Robotino*) is assimilated to a differential robot, using one of the wheels as a free-caster wheel.

Nonholonomic equations define the constraints equations for the trailer of rolling and/or pivoting without sliding wheels and holonomic constraints of the rigid connection between the two mobile robots. In the last constraints, regardless of AWMRT configurations, it is assumed that the articulation is always maintained. For this purpose, points P_1 and P_2, located in the joint center and belonging to *Robotino* and trailer, have the same position at each moment.

$$^0\boldsymbol{O}_0\boldsymbol{P}_1 = {}^0\boldsymbol{O}_0\boldsymbol{P}_2 \tag{2}$$

For either wheeled mobile robot, satisfying the rolling and/or pivoting no sideslip, can move, it is necessary to check that the perpendicular lines associated with the steering wheels (from the assumption) and fixed wheels are concurrent. For all these wheels, there must be a single zero velocity point around which the robot rotates instantaneously. Instant motion is a rotation around the intersection of all these lines; this point is named as the instantaneous center of rotation (*ICR*).

Therefore, v_x and v_y (longitudinal and lateral linear velocities) are chosen as inputs to the kinematic and/or dynamic models, and the remainders are considered as state variables. Based on the no side slip of the tractor, the steering angle is calculated by the following relationship:

$$\theta_1 = Arctang(\frac{\dot{v}_y}{\dot{v}_x}) \tag{3}$$

The orientation angle of trailer θ_2 is calculated geometrically by using the instantaneous center of rotation of the system in planar motion (Fig. 1).

$$\theta_2 = \theta_1 - \psi_1 - \psi_2 \tag{4}$$

$$\theta_2 = \theta_1 - Arctang(\frac{L_1}{R_1}) - Arctang(\frac{L_2}{R_2}) \tag{5}$$

Where R_1 and R_2 are the curvature radii of the *Robotino* and the trailer, respectively.

The kinematics of the concerned AWMRT is described by:

$$\begin{cases} \dot{\varphi}_1 = \frac{n}{r_{wT}}(-v_x \cdot \cos(\frac{\pi}{6}) + v_y \cdot \sin(\frac{\pi}{6}) + L_R \dot{\theta}_1) \\ \dot{\varphi}_2 = -v_y + \frac{n}{r_{wT}} L \dot{\theta}_1 \\ \dot{\varphi}_3 = \frac{n}{r_{wT}}(v_x \cdot \cos(\frac{\pi}{6}) + v_y \cdot \sin(\frac{\pi}{6}) + L_R \dot{\theta}_1) \\ x_2 = x_1 - L_1 \cos(\theta_1) - L_2 \cos(\theta_1) \\ y_2 = y_1 - L_1 \sin(\theta_1) - L_2 \sin(\theta_1) \\ \dot{\varphi}_4 = \frac{1}{r_{wR}}(x_2 \cos(\theta_2) + y_2 \sin(\theta_2) + L_T \dot{\theta}_2) \\ \dot{\varphi}_5 = \frac{1}{r_{wR}}(x_2 \cos(\theta_2) + y_2 \sin(\theta_2) - L_T \dot{\theta}_2) \end{cases} \tag{6}$$

where: r_{wT} and r_{wR} refer to the radii of *Robotino* and trailer wheels, respectively. L_R is the *Robotino* radius base, n is the motor reducer.

3. PROBLEM FORMULATION

Consider a AWMRT operating in a structured flat-plane environment cluttered by obstacles or free. The problem is to find the optimal trajectory $\boldsymbol{q}$(t) and the execution time T which allows to bring the tractor with its trailer from an initial configuration, $\mathbf{X}^S = [x_1^S, y_1^S, \theta_1^S, \theta_2^S]$, to a final configuration, $\mathbf{X}^G = [x_1^G, y_1^G, \theta_1^G, \theta_2^G]$, to minimize an objective function J, while respecting the constraints related to the task, namely, the robot and environment constraints.

In this work, we limit ourselves to minimize the execution duration task ($T \equiv J$). However, the same approach is also applicable to other more general forms of the cost function.

3.1 Task constraints

These constraints are defined by the limits of the conditions in positions and velocities imposed on tasks to be performed.

- Condition limits in position and orientation:
  $\boldsymbol{q}(0) = \boldsymbol{X}^S$ and $\boldsymbol{q}(T) = \boldsymbol{X}^G$
- Condition limits in velocity:
  $\dot{\boldsymbol{q}}(0) = 0$ and $\dot{\boldsymbol{q}}(T) = 0$

3.2 AWMRT constraints

These constraints are defined by the limit actuator's capacities in velocity and acceleration. They are given as follows:

- $|\dot{q}_i(t)| \leq \dot{q}^{\max}$ for $i = 4, ..6$
- $|\ddot{q}_i(t)| \leq \ddot{q}^{\max}$ for $i = 4, ..6$

Where $\dot{q}^{\max}$ and $\ddot{q}^{\max}$ are the limits actuators in velocity and acceleration.

3.3 Environment constraints

If the obstacles are present in the robot's workspace (AWMRT), the following Boolean function is defined:
$Col(\boldsymbol{q}(t)) = false \quad (0 \leq t \leq T)$

4. RESOLUTION METHOD AND ADAPTATION

4.1 Background

We start with a summary of the *RPA* for the trajectory planning of wheeled mobile robots, as shown in (Haddad et al., 2010). The next section will describe an adaptation to the case of wheeled mobile robots with a trailer. First, *RPA* considers the vector $\boldsymbol{q(t)}$ and the travel time, T, as the main unknowns of the problem. The vector $\tau(t)$ of generalized efforts is easily deduced via the inverse dynamic model of the robot. Second, *RPA* uses the concept of the trajectory profile. With the normalization of the time scale, any given trajectory $\boldsymbol{q(t)}$ may always be expressed in terms of its travel time T and its time-evolution profile $\boldsymbol{Q}$:

$$\boldsymbol{q(t)} = \boldsymbol{Q}(\xi(t)) \equiv \boldsymbol{Q}(\xi) o \xi(t) \text{ with } \xi(t) = \frac{t}{T} \tag{7}$$

With this concept, the overall problem may be reduced to finding only the $Q(\xi)^{best}$ profile of the optimal trajectory.

To facilitate the treatment of some constraints related to the wheeled mobile robot, the trajectory profile $\boldsymbol{Q}(\xi)$ is decomposed into a path and a motion on this path.

$$\boldsymbol{Q}(\xi) = P(\lambda(\xi)) \equiv P(\lambda) o \lambda(\xi) \text{ with } \lambda \in [0,\ 1] \tag{8}$$

The first one, $P(\lambda)$ with $\lambda \in [0,\ 1]$, is a time-independent vectorial function of the same size as $\boldsymbol{Q}$. It describes the geometric path of the robot in the generalized coordinate space when λ varies continuously from 0 (start) to 1 (end). The second function, $\lambda(\xi)$, is a monotonically increasing scalar function representing the motion profile that defines how path $P(\lambda)$ will be followed in the normalized time. Third, *RPA* transforms the problem to find the best trajectory class into a parametric optimization problem. Each candidate path profile (path $P(\lambda)$ and motion profile $\lambda(\xi)$) is defined by a finite set free control points linked together by parametric functions (B-spline for the path function and Cubic spline for the motion function). As a result, the entire trajectory planning problem is converted into the research of the optimal position of a few control points (Haddad et al., 2010).

Finally, in *RPA*, systematic treatment of the problem constraints is performed. These last ones are distributed in two classes. Constraints that depend on the travel time of the task (such as those kinodynamics), are resolved by transforming, via a clipping process, into permissible limits on this duration for a given trajectory profile. Other constraints, which depend only on the trajectory profile, are treated either by inclusion while constructing the profiles (e.g., task constraints, non-holonomic constraints) or by a rejection process (e.g., obstacle avoidance). This is to reduce the proportion of rejections during random selection. As a result, *RPA* is able to take benefit of the advantages offered by stochastic optimization techniques in terms of simplifying implementation, versatility, and efficiency.

4.2 Adaptation scheme

In this section, we propose to include the trailer constraints in the *RPA* framework (Haddad et al., 2010).

The trajectory planning problem of AWMRT is characterized, compared to mobile robots only (Haddad et al., 2007) and (Haddad et al., 2008), not only by non-holonomic constraints of the trailer and the holonomic constraints of the tractor-trailer connection but also by the limits of the conditions imposed on the trailer orientation with respect to the tractor. In order to adapt the *RPA* approach to this type of system, these constraints will have to be taken into account.

The holonomic constraints of the AWMRT connections are treated by a systematic process (inclusion) similar to that used to take into account the non-holonomic constraints of a wheeled mobile robot alone (Haddad et al., 2007).

It consists in randomly producing the position $(x_1(\lambda), y_1(\lambda))$ and velocity $(\dot{x}_1(\lambda), \dot{y}_1(\lambda)))$ of tractor to deduce the trailer position $(x_2(\lambda), y_2(\lambda))$ and the orientations $(\theta_1(\lambda), \theta_2(\lambda))$ of AWMRT, from the developed kinematic model, so that the conditions of no slipping between the wheels and the floor, and the trailer cannot move sideways, and the holonomic constraints of the AWMRT connection, are verified along the path.

The trajectory planning of wheeled mobile robots with trailer consists of looking not only for the optimal $\boldsymbol{q}(t)^{best}$ trajectory and T^{best} execution travel time but also for the most desirable initial and final configurations. As the AWMRT can reach these situations in various manners, the initial and final configurations require boundary conditions on the orientation of the trailer with respect to a Galilean reference frame. These conditions, to the limits imposed, are taken into account by a penalty in the cost function.

5. SIMULATION RESULTS AND EXPERIMENTAL VALIDATION

5.1 Numerical Results

This section gives the main numerical results related to the problem of minimizing the travel time of an AWMRT. The kinematic parameters and actuator limits, in velocity and acceleration, of this AWMRT are grouped in Table 1.

Table 1. Geometric and kinematics parameters of AWMRT

$L_T = 0.20$	$r_{w_T} = 0.062$	$L_1 = 0.50$
$L_R = 0.25$	$r_{w_R} = 0.050$	$L_2 = 0.45$
$\dot{q}_4^{max} = 180rd/s$	$\dot{q}_5^{max} = 180rd/s$	$\dot{q}_6^{max} = 180rd/s$
$\ddot{q}_4^{max} = 30rd/s^2$	$\ddot{q}_5^{max} = 40rd/s^2$	$\ddot{q}_6^{max} = 30rd/s^2$

The workspace is a $(25.0\text{x}25.0\text{m}^2)$ flat floor with obstacles (Fig. 2). We require the AWMRT to move from $\boldsymbol{X}^S = [4,\ 7,\ 45°,\ 45°]$ to $\boldsymbol{X}^G = [14,\ 7.7,\ 135°,\ 135°]$.

The minimum-time solution found via the modified *RPA* is shown in Fig. 3 with score $T = 125.45s$ and a runtime of 125s on a Centrino Duo laptop 1.6 GHz. The below figures show the

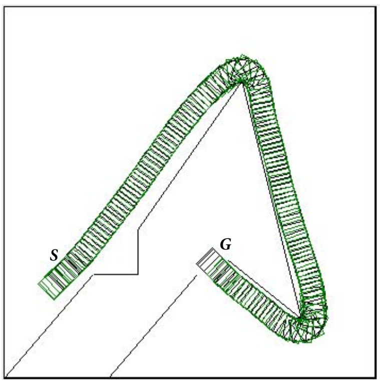

Fig. 2. Solution path obtained of AWMRT; (*Robotino* (black) and Trailer (green))

orientation angles (Fig. 2) of the AWMRT, angular velocities, and accelerations of the *Robotino* wheels. It is obvious to note that the task constraints (the initial and final orientations of AWMRT and the boundary velocities), and the constraints related to the kinematic capacities (velocities and accelerations) of the wheels *Robotino* are respected.

The obtained simulation results ensure the reliability and robustness of the proposed approach, as well as being very satisfactory in terms of quality, computation time, respecting constraints imposed, and, most importantly is the saturation constraints during the entire optimal trajectory.

These results give us the motivation to initiate an experimental validation, in which we physically test the performance of our modified approach on AWMRT.

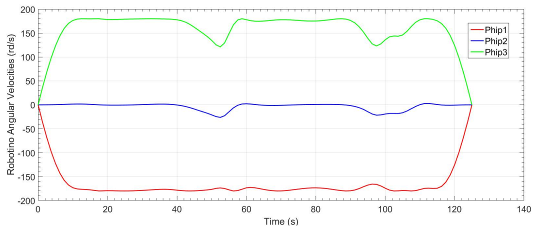

Fig. 3. Wheels angular velocities of *Robotino*

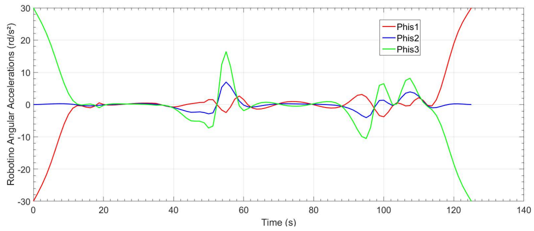

Fig. 4. Wheels angular accelerations of *Robotino*

5.2 Experimental validation

In this section, we want to track a planned trajectory by an omnidirectional mobile robot of the *Robotino* type with a passive trailer linked together by a rigid pivot joint.

The minimum-time trajectory problem is considered under constraints on position, velocity, and acceleration. Boundary velocities are null. The problem is to be dealt with a three-wheel omnidirectional-drive AWMRT. These wheels are all identical in size, two are independently driven, whereas the other is assimilated as a free-caster wheel.

Trajectory tracking aims to follow a reference trajectory, which has been planned between the initial and final configurations in a structural environment. A desirable choice, of the method for monitoring a reference trajectory, is dictated by specific requirements and constraints that must be respected during the execution phase:

- The execution time of a reference trajectory must be near the travel time of this trajectory.
- The method must be sufficiently robust in relation to certain errors (e. g. localization errors).
- The method must be flexible and adaptable enough to take into account the avoidance of obstacles in the case of an error in the modeling of mobile robots or obstacles.

To satisfy these requirements, the trajectory tracking method that we have chosen for this work is based on the principle of tracking a virtual robot (Egerstedt et al., 2001) in motion on the reference trajectory. In addition to satisfying the requirements mentioned above, this approach is characterized by its simplicity of implementation.

The criterion to be met is to keep the tracking errors in linear and angular velocities, between the real and the virtual AWMRT, within predefined limits. In our case, this criterion can be satisfied mainly by, on the one hand, taking into account the kinematic limits in velocity and acceleration of the robot during planning. On the other hand, the *Robotino* is equipped with an algorithm to control the angular velocities of the wheels. This principle enables the real AWMRT to correctly track the virtual AWMRT, which ensures certain robustness during tracking (considering that the trailer is equipped with passive wheels).

For the implementation, we used the classic PID controller (Blažič, 2011), which only needs an implicit description of the AWMRT. The controller generates the necessary commands to minimize errors in the linear longitudinal velocity (e_{v_x}), in the linear lateral velocity (e_{v_y}) and the angular velocity (e_w) between real and virtual AWMRT. The following relationships give the expressions of these errors:

$$\begin{cases} e_{v_x} = v_{x_v} - v_{x_E} \\ e_{v_y} = v_{y_v} - v_{y_E} \\ e_w = w_v - w_E \end{cases} \tag{9}$$

Where, v_{x_v}, v_{y_v} and w_v are the linear longitudinal and lateral velocities, and the angular velocity of the virtual AWMRT (reference trajectory); v_{x_E}, v_{y_E} and w_E are those of the real AWMRT (experimental trajectory) provided by the localization systems (Fig. 5).

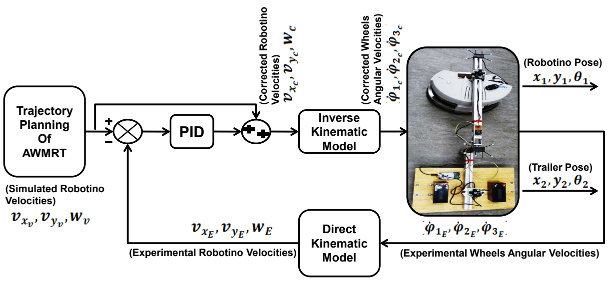

Fig. 5. Closed Lopp control scheme

Localization systems The localization of a mobile system can be defined as the determination of its configuration in relation to a reference frame.

Two localization types are developed in this work, the relative localization by odometry and the absolute localization by Marvelmind sensors. We used the odometer localization that disposes of the *Robotino*, which makes it possible to determine the position and orientation of a wheeled mobile robot with respect to a reference point related to its initial configuration. Marvelmind's internal navigation system provides precise positioning characteristics ($\pm 2\,cm$) for autonomous robots, vehicles, and helicopters in 3D environments. We placed the mobile beacons at the mass centers of both platforms.

To find out the orientation and the angular velocity of the trailer, we used a gyro sensor placed in the center of the trailer's mass and an encoder mounted on the point of connection between the *Robotino* and the trailer. The data from the sensors is acquired using a device based on *Arduino*. Fig. 6 shows the recording setup.

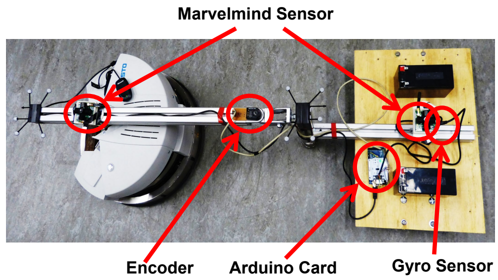

Fig. 6. Data acquisition assembly

Trajectory tracking experiments are done in the experimental indoor area. This room has floor dimensions as (9.0x6.5m^2). We used as obstacles the (1.6x0.7m^2) tables and (0.31x0.23m^2) boxes.

The workspace consists of a (9.0x6.5m^2) flat floor with obstacles (Fig. 7). Boundary conditions in positions and orientations are defined as follows: $\boldsymbol{X}^S = [2,\ 1.75,\ 0°,\ 0°]$ and $\boldsymbol{X}^G = [2,\ 5.125,\ 180°,\ 180°]$. We aimed to solve the minimum-time problem under kinematics constraints for the *Robotino* (in terms ofvelocity: $\dot{\varphi}_i^{\max} = 320\,rd/s$, acceleration: $\ddot{\varphi}_i^{\max} = 30\,rd/s^2$, $i = 1..3$ and obstacle avoidance). The Fig. **??** shows the final configuration of the AWMRT, with travelling a distance of $18\,m$.

The reference trajectory obtained by the modified *RPA* approach is illustrated in Fig. 7. The travel time is 32.11s.

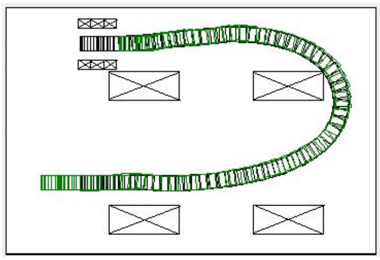

Fig. 7. Simulated trajectory of the AWMRT in problem 1; (*Robotino* (black) and Trailer (green))

The execution results of trajectory tracking module are illustrated in the figures (Fig. 8 to Fig. 10).

There is a satisfactory correlation between the simulated and the experimental trajectories.

The curves of AWMRT's rotational velocities of the *Robotino* wheels and the orientation angle in the connection point between the simulated and the experimental trajectories are closely matched. Besides, there is a saturation of the selected rotational velocities.

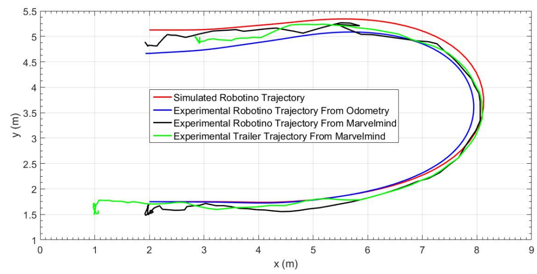

Fig. 8. Simulated and experimental paths

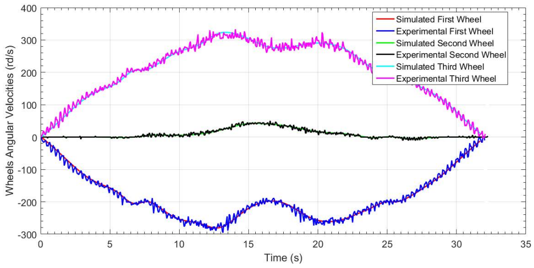

Fig. 9. Wheels angular velocities of *Robotino*

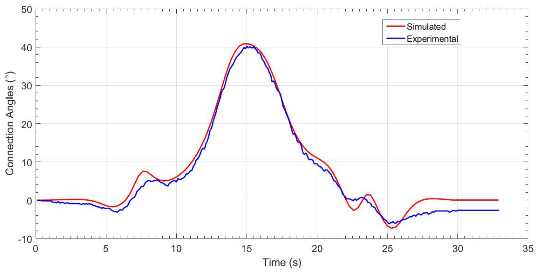

Fig. 10. Connection angle of the AWMRT

The variances found remain acceptable given the dimensions of the AWMRT.

In this section, we focused on presenting the obtained results for the test and validation of the modified *RPA* approach. The problem simulation and the experimental test were presented.

The objectives of these problems differ from one example to another. These problems were presented to evaluate the performance and efficiency of the proposed approach, in various tests (simulation and experimental validation) and levels of complexity. The proposed approach is suitable for emergency tasks, which require rapid reaction in known environments. Its efficiency depends strongly on the calculation of reference trajectories, which are optimal in terms of execution time.

6. CONCLUSIONS AND FUTURE WORK

This study follows the work on the Random Profile Approach (*RPA*) for wheeled mobile robot operation. It focuses on the problem of planning autonomous mobile robots in known environments with trailer. The aim is to generate optimal solutions for an AWMRT, to perform predefined tasks. These tasks require short decision and execution times.

We presented in detail the kinematic modeling of a holonomic mobile robot, *Robotino* type, with a passive, non-holonomic, two-wheeled trailer. A rigid pivot connects these mobile robots. We have also outlined the *RPA* principle. Then, we proposed an extension of this approach to allow the planning of optimal trajectories of AWMRT.

To test the efficiency of the proposed approach, we used it to solve various trajectory planning problems for AWMRT. To do this, we have established a numerical simulation and experimental trajectory planning tests operating in structured environments. The obtained results are very satisfactory, whether in terms of behavior, quality, execution time, or travel time. For this purpose, the efficiency of the proposed approach, in terms of execution time, increases significantly, on the one hand, with the size and complexity of the environment and, on the other hand, with the kinematic and dynamic performance of the AWMRT.

The proposed approach will be used in our future work under the following conditions:(*i*) consider the trailer orientation in relation to the tractor.;(*ii*) test the efficiency of modified *RPA* on other types of AWMRT (holonomic or non-holonomic); (*iii*) take into account dynamic constraints by developing associated models; (*iv*) increase the number of towed trailers.

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