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Opinion Dynamics of Social Networks with Stubborn Agents via Group Gossiping with Random Participants

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Abstract: Several recent models of opinion dynamics utilize gossip-based methods as an alternative to deterministic classical models. This approach is meant to be a more realistic representation of real-world communications by using random pairwise interactions. Our previous work extended the process of gossip-based models by enabling agents to communicate with a random subset of their neighbors. In this paper, we apply this idea to networks with stubborn agents. While the opinions in this model tend to oscillate, its expected dynamics is convergent, and the expected opinions and time-averaged opinions coincide over time.

Keywords: Opinion dynamics, social networks, multi-agent systems, randomized methods

1. INTRODUCTION

The challenge of understanding how opinions spread over social networks is gaining significance as humans become more interconnected through advancements in technology. In the field of control, the behavior of social actors is modeled via multi-agent systems that exhibit certain social phenomena based on their mathematical properties (Proskurnikov and Tempo, 2017). Models of opinion dynamics, in particular, have been effectively used for explaining patterns in opinion formation such as clustering, consensus, clustering, and fragmentation (Sîrbu et al., 2016).

Some of the earlier works related to opinion dynamics are the theory of social power by French (1956), the consensus model by DeGroot (1974), and the social influence network theory by Friedkin and Johnsen (1999). These models have laid the foundation for succeeding researches on opinion dynamics; however, they are deterministic and update all opinions simultaneously. Real-world social networks, on the other hand, are unpredictable and communications happen asynchronously.

More recent models involve time-varying dynamics, such as bounded confidence and gossip-based models (Proskurnikov and Tempo, 2018). Bounded confidence models, including the works of Hegselmann and Krause (2002) and Deffuant et al. (2001), have interactions that vary over time, but they only involve individuals with similar opinions.

Models based on gossip algorithms (Boyd et al., 2006) use randomized processes and have been incorporated to the classical models (Proskurnikov and Tempo, 2018). Examples of such models have been discussed in Frasca et al. (2013), Nguyen et al. (2017), and Aguilar and Fujisaki (2019). Gossip-based dynamics represents a much closer approximation of real-world communications by allowing different participants and asynchronous updates. The gossip algorithm described by Boyd et al. (2006), however, is restricted to pairwise communication. While this is effective in engineered network systems, this only represents a small portion of the ways humans communicate with one another.

The work of Aguilar and Fujisaki (2019) proposed the method of random group gossiping and applied it to the DeGroot model. This model, which belongs to a class of time-varying models for reaching consensus (Bullo, 2018; Fagnani and Frasca, 2017), allows interactions with groups composed of randomly chosen agents. We see this as an intuitive representation of how humans communicate on a daily basis, including the times when we use devices like smart phones and internet applications such as online social networks. In this paper, we continue our previous work by incorporating agents who are attached to their prior views, which is a feature of the Friedkin-Johnsen model. The opinions in this model tend to continuously fluctuate, so we analyze its expected dynamics. We then show simulation results to validate our findings.

This paper is organized as follows. Our notation and preliminaries are given in Section 2. Section 3 gives an overview of the DeGroot and Friedkin-Johnsen models. In Section 4, we describe the implementation of group gossiping with random participants on both models. Since the random group gossiping version of the Friendkin-Johnsen model is the main contribution of this paper, analysis of this model is included in this section as well. Simulation results are provided in Section 5. We state our conclusion in Section 6.

2. NOTATION AND PRELIMINARIES

We denote a directed graph as $G = (V, E)$, where $V = \{1, \ldots, n\}$ is the set of nodes and $E \subseteq V \times V$ is the set of edges. The neighbors of agent i is given by $N_i = \{j \mid (i, j) \in E\}$. A path is a sequence of edges that connects a distinct set of nodes. A node i is globally reachable if all the other nodes has a path to it. A directed graph is strongly connected if there is a path between every pair of nodes. It is weakly connected if making its edges bidirectional results to a strongly connected graph. A set of subgraphs are strongly connected components if each subgraph is strongly connected and no two subgraphs form a larger strongly connected subgraph. A strongly connected subgraph is periodic if the length of its cycles is divisible by $c \in \mathbb{Z}^+$, with $c > 1$. Otherwise, it is aperiodic.

We use $e_i \in \mathbb{R}^n$ to denote a standard basis vector, where the ith element is 1 while the rest are zeroes. The vector of ones is given by $\mathbf{1}$. The Hadamard product of two matrices P and W is denoted by $P \circ W$. Matrix W is Schur stable if and only if all of its eigenvalues are inside the unit circle.

3. DEGROOT AND FRIEDKIN-JOHNSEN MODELS

We revisit the original DeGroot and Friedkin-Johnsen models, which are some of the most influential works in opinion dynamics. Both are examples of time-invariant models with synchronous updates. In the succeeding section, we describe their time-varying versions based on random group gossiping.

In this paper, a social network is represented by a directed graph $G = (V, E)$, where each $i \in V$ corresponds to an individual or an agent and $(i, j) \in E$ if and only if agent i can interact with agent j. We assume that G is at least weakly connected, otherwise each connected component is treated as an independent social network. The opinions of the agents at time $k \in \mathbb{Z}^+$ are stored in the vector $x(k) \in \mathbb{R}^n$, and $x(0)$ contain the initial opinions. To avoid trivial cases, we assume $|V| > 1$ and $x(0) \neq \alpha\mathbf{1}$, for any $\alpha \in \mathbb{R}$.

3.1 DeGroot Model

The DeGroot model demonstrates how a group of agents can achieve consensus by iteratively combining the opinions of their neighbors. In this model, all opinions are updated simultaneously based on static interpersonal influences, resulting to a deterministic process. When consensus is reached, the model establishes the contribution of each agent on the consensus value.

Given n agents, let $W \in \mathbb{R}^{n\times n}$ be a nonnegative weight matrix, where $w_{ij} > 0$ if and only if $(i, j) \in E$. For each agent i, $w_{i1}, \dots, w_{in}$ reflects how much i gives importance to the opinions of the members of the social network, including its own opinion. The weights are normalized such that $\sum_j w_{ij} = 1$, which makes W a row-stochastic matrix.

Starting with the initial opinions $[x_1(0), \dots, x_n(0)]^T$, the succeeding opinion of each agent i at each time $k > 0$ is computed as

$$x_i(k+1) = \textstyle\sum_{j\in N_i} w_{ij} x_j(k)\,.$$

Hence, $x_i(k + 1)$ is a convex combination of the opinions of the neighbors of i. In matrix form, the model can be compactly written as

$$x(k + 1) = Wx(k). \tag{1}$$

In general, (1) corresponds to the averaging dynamics of different types of networks and multi-agent systems (Bullo, 2018; Fagnani and Frasca, 2017).

A known property of the DeGroot model is that its convergence is not dependent on the initial opinions, but rather on the topology of the social network. The opinions at time k is given by

$$x(k) = W^k x(0) \tag{2}$$

which implies that the model (1) is convergent if and only if $\lim_{k\to\infty} W^k$ exists.

Lemma 1. The $\lim_{k\to\infty} W^k$ exists if and only if all strongly connected components in G with no outgoing edges are aperiodic. If there is only one strongly connected component with no outgoing edges and all the other strongly components has a path to it, then $\lim_{k\to\infty} W^k = \mathbf{1}\pi^T$, where π is the normalized dominant left eigenvector of W.

The second part of Lemma 1 provides the consensus condition for the DeGroot model. Applying Lemma 1 to (2), we get

$$x^* = \lim_{k\to\infty} x(k) = \mathbf{1}\pi^T x(0)$$

where $\pi^T x(0)$ is the consensus value.

3.2 Friedkin-Johnsen Model

The Friedkin-Johnsen model expands the work of DeGroot by taking into consideration agents who are attached to their preexisting opinions. These stubborn agents have varying degree of openness to outside opinions based on their prejudice. As indicated in Lemma 1, convergence to different opinions in the DeGroot model is restricted to a specific network topology. The presence of stubborn agents enables disagreements to occur in the Friedkin-Johnsen even under the same condition that leads to consensus in the DeGroot model.

Interpersonal influence in the Friedkin-Johnsen model is also represented by the W matrix from the DeGroot model. The susceptibility of agents to external influence is denoted by the matrix $\Lambda = I - \text{diag}(W)$. Thus, $\lambda_i \in [0,1]$ where $\lambda_i < 1$ means agent i is stubborn and has attachment to its initial opinion, while $\lambda_i = 1$ means i is completely open to the opinions of others. Prejudices or preconceived opinions are stored in the vector $u = x(0)$. The model can then be expressed as

$$x(k + 1) = \Lambda W x(k) + (I - \Lambda)u. \tag{3}$$

Note that $\Lambda = I$ reduces (3) to the DeGroot model (1).

Based on equation (3), the opinions at time k is given by

$$x(k) = (\Lambda W)^k u + \textstyle\sum_{q=0}^{k-1} (\Lambda W)^q (I - \Lambda)u\,.$$

When all agents are either stubborn or has a path to a stubborn agent in G, ΛW becomes Schur stable. In this scenario, the Friedkin-Johnsen model (3) converges to the limit

$$\lim_{k\to\infty} x(k) = (I - \Lambda W)^{-1}(I - \Lambda)u.$$

As such, disagreements can occur in model (5) even in aperiodic strongly connected graphs, which is not permissible in the model (2).

4. GROUP GOSSIPING WITH RANDOM PARTICIPANTS

In distributed network systems, gossiping refers to a communication protocol involving random pairwise interactions. This concept has also been applied to social networks for modeling opinion dynamics. However, in these models, giving of opinions remain restricted between two agents.

We expand the idea of gossiping to permit communication with a group of random agents. We apply this approach to both the DeGroot model and the Friedkin-Johnsen model.

4.1 Consensus via Random Group Gossiping

Random group gossiping has been originally proposed for the DeGroot model in our previous work (Aguilar and Fujisaki, 2019). We summarize the model here as a prelude to the main contribution of this paper.

At each iteration of the model, agent i is selected with uniform probability. Then the chosen agent communicates with a random subset of its neighbors $S_i(k) \subseteq N_i$, where membership is determined by the Bernoulli random variable $\phi_{ij}(k)$ such that

$$\phi_{ij}(k) = \begin{cases} 1, & j \in S_i(k) \\ 0, & j \notin S_i(k) \end{cases}.$$

The matrix $P \in \mathbb{R}^{n\times n}$ defines the probability i receives the opinion of j at time k, that is

$$\mathbb{P}[\phi_{ij}(k) = 1] = p_{ij}.$$

Since we only consider the cases when $(i,j) \in E$ only if i can contact j , then we assume $p_{ij} > 0$ for all i and j. The opinion of agent i is then updated as

$$x_i(k+1) = \left(1 - \sum_{j\in S_i(k)} w_{ij}\right)x_i(k) + \sum_{j\in S_i(k)} w_{ij} x_j(k) \quad (4)$$

while the opinions of the other agents remain unchanged.

Let $A(k)$ be a random matrix based on the selected agent i and $S_i(k)$, given by

$$A(k) = I - \sum_{j\in S_i(k)} w_{ij}\, e_i e_i^T + \sum_{j\in S_i(k)} w_{ij}\, e_i e_j^T.$$

Thus, the model can be written as

$$x(k+1) = A(k)x(k) \quad (5)$$

which is a time-varying version of (1).

Depending on the structure of the social network, the model (5) can achieve probabilistic consensus, as well as consensus in expectation.

4.2 Random Group Gossiping with Stubborn Agents

We now describe the implementation of the random group gossiping scheme on the Friedkin-Johnsen model. This model follows the same communication process of (4), then takes into account the stubbornness of agents.

Let $M \in \mathbb{R}^{n\times n}$ be a matrix of interpersonal influence, similar to the matrix W of the DeGroot model. Susceptibility to external influence and prejudices are also given by the matrix Λ and the vector u, respectively, from the Friedkin-Johnsen model. For this case, $\Lambda = I - \mathrm{diag}(M)$. At each time $k > 0$, agent i is chosen with uniform probability and its opinion is updated as

$$x_i(k+1) = \lambda_i\left(\left(1 - \sum_{j\in S_i(k)} m_{ij}\right)x_i(k) + \sum_{j\in S_i(k)} m_{ij}x_j(k)\right) + (1-\lambda_i)u_i \quad (6)$$

where each $j \in S_i(k)$ is also determined by $\phi_{ij}(k)$. The other opinions retain their value.

The model can be expressed as

$$x(k+1) = A(k)x(k) + B(k)u \quad (7)$$

where

$$A(k) = \left(I - e_i e_i^T(I-\Lambda)\right)\left(I - \left(\sum_{j\in S_i(k)} m_{ij}\right)e_i e_i^T + \sum_{j\in S_i(k)} m_{ij} e_i e_j^T\right)$$

and

$$B(k) = e_i e_i^T(1-\Lambda).$$

Hence, (7) is a time-varying version of (3) that is dependent on the participating i and $S_i(k)$ at time k. The work of Frasca et al. (2013) is also a gossip-based version of the Friedkin-Johnsen model, but they only implement pairwise gossiping.

While its possible for the model (7) to reach consensus based on the values in M and P, it does not usually converge to stable opinions. Thus, the analysis on this paper is focused on the expected behavior of the model (7).

Lemma 2. Let $\bar{A} = \mathbb{E}[A(k)]$ and $\bar{B} = \mathbb{E}[B(k)]$. The expected dynamics of the model (7) is

$$\mathbb{E}[x(k+1)] = \bar{A}\mathbb{E}[x(k)] + \bar{B}u \quad (8)$$

where

$$\bar{A} = I - \frac{1}{n}\Big(I - \Lambda - \Lambda\left(P\circ M - diag\left((P\circ M)\mathbf{1}\right)\right)\Big)$$

$$\bar{B} = \frac{1}{n}(I-\Lambda). \quad (9)$$

Proof. Considering that the selection of $j \in S_i(k)$ is an independent event, then

$$\bar{A} = \frac{1}{n}\sum_i\left(\left(I - e_i e_i^T(I-\Lambda)\right)\left(I - \left(\sum_{j\in N_i} p_{ij}m_{ij}\right)e_i e_i^T + \sum_{j\in N_i} p_{ij} e_i e_j^T\right)\right)$$

$$= \frac{1}{n}\sum_i \big(I - \big(\sum_{j\in N_i} p_{ij} m_{ij}\big) e_i e_i^T + \sum_{j\in N_i} p_{ij} e_i e_j^T - e_i e_i^T (I-\Lambda) + e_i e_i^T (I-\Lambda)\big(\sum_{j\in N_i} p_{ij} m_{ij}\big) e_i e_i^T - e_i e_i^T (I-\Lambda) \sum_{j\in N_i} p_{ij} e_i e_j^T\big)$$

$$= \frac{1}{n}\big(nI - diag\big((P\circ M)\mathbf{1}\big) + P\circ M - (I-\Lambda) + (I-\Lambda) diag\big((P\circ M)\mathbf{1}\big) - (I-\Lambda)(P\circ M)\big)$$

$$= I - \frac{1}{n}\Big(I - \Lambda - \Lambda\Big(P\circ M - diag\big((P\circ M)\mathbf{1}\big)\Big)\Big)$$

and

$$\bar{B} = \frac{1}{n}\sum_i e_i e_i^T (I-\Lambda) = \frac{1}{n}(I-\Lambda). \qquad \blacksquare$$

Note that Λ, $\bar{A}$ and $\bar{B}$ can be rearranged as

$$\Lambda = \begin{bmatrix} \Lambda^{11} & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{A} = \begin{bmatrix} \bar{A}^{11} & \bar{A}^{12} \\ 0 & \bar{A}^{22} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} \frac{(I-\Lambda^{11})}{n} & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

where the Λ^{11} and $[\bar{A}^{11} \quad \bar{A}^{12}]$ correspond to all agents i that has $\lambda_i < 1$ or has a path to j in G such that $\lambda_j < 1$, and $\bar{A}^{22}$ corresponds to all the remaining agents.

Theorem 3. Let

$$x_* = \lim_{k\to\infty} \mathbb{E}[x(k)].$$

The expected dynamics of (7) converges to

$$x_* = \begin{bmatrix} \frac{(I-\bar{A}^{11})^{-1}(I-\Lambda^{11})}{n} & (I-\bar{A}^{11})^{-1}\bar{A}^{12}\bar{A}_*^{22} \\ 0 & \bar{A}_*^{22} \end{bmatrix} u \tag{11}$$

where

$$\bar{A}_*^{22} = \lim_{k\to\infty} (\bar{A}^{22})^k = \mathbf{1}(\pi_{\bar{A}^{22}})^T$$

and $\pi_{\bar{A}^{22}}$ is the normalized dominant left eigenvector of $\bar{A}^{22}$.

Proof. The following proof is similar to the argument used by Parsegov et al. (2017). From (8), the expected opinions at time k is

$$\mathbb{E}[x(k)] = \bar{A}^k u + \sum_{q=0}^{k-1} \bar{A}^q \bar{B}\, u. \tag{12}$$

Applying (12) on (10) results to

$$\mathbb{E}[x(k)] = \begin{bmatrix} (\bar{A}^{11})^k & \sum_{q=0}^{k-1} (\bar{A}^{11})^{k-q-1} \bar{A}^{12} (\bar{A}^{22})^k \\ 0 & (\bar{A}^{22})^k \end{bmatrix} u + \begin{bmatrix} \frac{1}{n}\sum_{q=0}^{k-1} \bar{A}^q (I-\Lambda^{11}) & 0 \\ 0 & 0 \end{bmatrix} u. \tag{13}$$

Note that $\sum_{j\in N_i} \bar{A}_{ij} = 1 - \frac{1-\lambda_i}{n}$. This implies that in $\bar{A}^{11}$, every node in the corresponding graph has a path to a node i such that $\sum_{j\in N_i} \bar{A}_{ij} < 1$. Thus, $\bar{A}^{11}$ is Schur stable and $\lim_{k\to\infty} (\bar{A}^{11})^k = 0$. $\bar{A}^{22}$ is stochastic with a positive diagonal, so Lemma 1 can be applied to it, resulting to $\lim_{k\to\infty} (\bar{A}^{22})^k = \mathbf{1}(\pi_{\bar{A}^{22}})^T$. Using the previous statements on (13), we get

$$x_* = \begin{bmatrix} 0 & (1-\bar{A}^{11})^{-1}\bar{A}^{12}\bar{A}_*^{22} \\ 0 & \bar{A}_*^{22} \end{bmatrix} u + \begin{bmatrix} (1-\bar{A}^{11})^{-1}(I-\Lambda^{11})/n & 0 \\ 0 & 0 \end{bmatrix} u$$

from which (11) can be obtained. $\blacksquare$

Corollary 4. If $\lambda_i < 1$ for all agents, then $\bar{A}$ is Schur stable and the limit is x_*

$$x_* = \Big(I - \Lambda - \Lambda\Big(P\circ M - diag\big((P\circ M)\mathbf{1}\big)\Big)\Big)^{-1} (I-\Lambda)u \tag{14}$$

Proof. Applying the condition in Corollary 4 to (10) implies that

$$\Lambda = \Lambda^{11} \quad \bar{A} = \bar{A}^{11} \quad \bar{B} = \frac{(I-\Lambda^{11})}{n}$$

which converts the limit (11) to

$$x_* = \left(\frac{(I-\bar{A})^{-1}(I-\Lambda)}{n}\right) u \tag{15}$$

by reapplying Theorem 3. The limit (14) can be obtained by applying (9) on (15). $\blacksquare$

It can easily be seen that when $p_{ij} = 1$ for all i and j, and $W = P\circ M$, the limit (14) is the same as the limit of the classical Friedkin-Johnsen model.

Theorem 5. Let

$$\bar{x}(k) = \frac{1}{k+1}\sum_{q=0}^{k} x(q).$$

Then, the dynamics (7) is mean-square ergodic, such that

$$\lim_{k\to\infty} x(k) = \|\bar{x}(k) - x_*\|^2.$$

Proof. Frasca et al. (2013) provided their analysis in order to prove the mean-square ergodicity of their proposed model. This method was generalized in the work of Ravazzi et al. (2015). For completeness, we include here the proof that uses their method.

From (6)

$$\min_j x_j(0) \le x_i(k) \le \max_j x_j(0). \tag{16}$$

Let $e(k) = x(k) - x_*$. Then,

$$\bar{x}(k) - x_* = \frac{1}{k+1}\sum_{q=0}^{k} x(q) - x_* = \frac{1}{k+1}\sum_{q=0}^{k} e(q). \tag{17}$$

The expected squared Euclidean norm of (17) is

$$\mathbb{E}\|\bar{x}(k) - x_*\|^2 = \mathbb{E}\left\|\frac{1}{k+1}\sum_{q=0}^{k} e(q)\right\|^2. \tag{18}$$

Note that

$$\left(\sum_{q=0}^{k} e(q)\right)^2 = \sum_{q=0}^{k} e(q)^T e(q) + 2\sum_{q=0}^{k-1}\sum_{r=1}^{k-q} e(q)^T e(q+r). \quad (19)$$

By combining (18) and (19), we get

$$\mathbb{E}\|\bar{x}(k) - x_*\|^2 = \frac{1}{(k+1)^2}\mathbb{E}\left[\sum_{q=0}^{k} e(q)^T e(q) + 2\sum_{q=0}^{k-1}\sum_{r=1}^{k-q} e(q)^T e(q+r)\right]^2. \quad (20)$$

Based on (16), there is a constant upper bound for $(x(k) - x_*)^T(x(k) - x_*)$ for all k. Let the upper bound be η. Then, from (20)

$$\mathbb{E}\left[\sum_{q=0}^{k} e(q)^T e(q)\right] \le \sum_{q=0}^{k} \eta \le \eta(k+1). \quad (21)$$

Also, from (20)

$$\begin{aligned}\mathbb{E}[e(q)^T e(q+r)] &= \mathbb{E}\big[\mathbb{E}[e(q)^T e(q+r)|x(q)]\big] \\ &= \mathbb{E}\big[e(q)^T \mathbb{E}[e(q+r)|x(q)]\big] \\ &= \mathbb{E}\big[e(q)^T \mathbb{E}[x(q+r) - x_*|x(q)]\big] \\ &= \mathbb{E}[e(q)^T(\mathbb{E}[x(q+r)|x(q)] - x_*)]. \quad (22)\end{aligned}$$

By recursively applying (8) on $\mathbb{E}[x(q+r)|x(q)]$ until $x(q)$ is reached, we have

$$\mathbb{E}[x(q+r)|x(q)] = \bar{A}^r x(q) + \sum_{s=0}^{r-1} \bar{A}^s \bar{B}\, u. \quad (23)$$

Using the same principle on x_* yields

$$x_* = \bar{A}^r x_* + \sum_{s=0}^{r-1} \bar{A}^s \bar{B}\, u. \quad (24)$$

Then applying (23) and (24) on (22) produces

$$\begin{aligned}\mathbb{E}[e(q)^T e(q+r)] &= \mathbb{E}[e(q)^T(\mathbb{E}\bar{A}^r x(q) - \bar{A}^r x_*)] \\ &= \mathbb{E}[e(q)^T \bar{A}^r (x(q) - x_*)] \\ &= \mathbb{E}[e(q)^T \bar{A}^r e(q)] \\ &\le \mathbb{E}[e(q)^T \rho^r e(q)] \\ &\le \eta\rho^r \quad (25)\end{aligned}$$

where ρ is a constant such that $v^T \bar{A} v \le v^T \rho v$ for any vector v.

Applying (25) on (20) yields

$$\begin{aligned}\mathbb{E}\|\bar{x}(k) - x_*\|^2 &\le \frac{1}{(k+1)^2}\left(\eta(k+1) + 2\sum_{q=0}^{k-1}\sum_{r=1}^{k-q} \eta\rho^r\right) \\ &\le \frac{\eta}{(k+1)^2}\left(k+1+2\sum_{q=0}^{k-1}\sum_{r=1}^{k-q}\rho^r\right) \\ &\le \frac{\eta}{k+1}\left(1+2\sum_{r=1}^{k}\rho^r\right) \\ &\le \frac{\eta}{k+1}\left(1+\frac{2}{1-\rho}\right).\end{aligned}$$

This completes the proof. ∎

5. SIMULATION RESULTS

In this section, we consider one of the social networks under the Twitter ego dataset of the Stanford Network Analysis Project (McAuley and Leskovec, 2012). This particular network contains 50 nodes that are weakly connected via 278 directed edges. Since the edges are based on followed profiles, we added self-loops to all the nodes to take into account the weights of personal opinions. The dataset does not include information about interpersonal influences and frequency of interactions. We assigned random weights and probabilities to the edges, which correspond the values in the W and P matrices, respectively. We also randomly generated the initial opinions such that each $x_i(0) \in [0,1]$.

Figure 1 shows the result of the DeGroot model version with random group gossiping. The structure of the network allowed the model to reach probabilistic consensus.

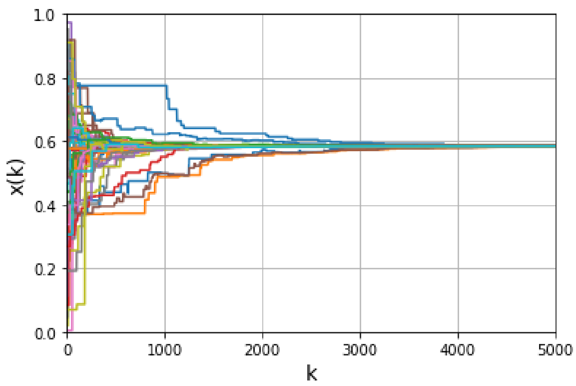
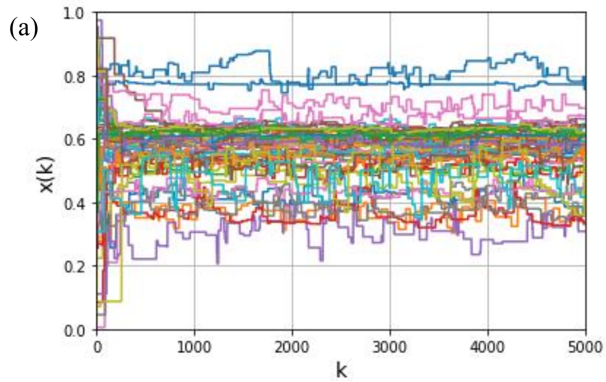


Fig. 1. Consensus via random group gossiping.

Figure 2 demonstrates the application of random group gossiping on the Friedkin-Johnsen model. For this numerical example, M has the same values as W. The openness of agents to external influences are defined by $\Lambda = I - \mathrm{diag}(M)$, while prejudices are given by $u = x(0)$. Figure 2 (a) shows the behavior of random group gossiping with stubborn agents. Even if the same weights and probabilities are used in this numerical example, the model did not converge to stable opinions. However, Figure 2 (b) shows that the average of the opinions over time approaches the expected value of the opinions in Figure 2 (c) as time k increases. Figures 2 (b) and (c) exhibit the ergodic property of the model that is stated in Theorem 5.



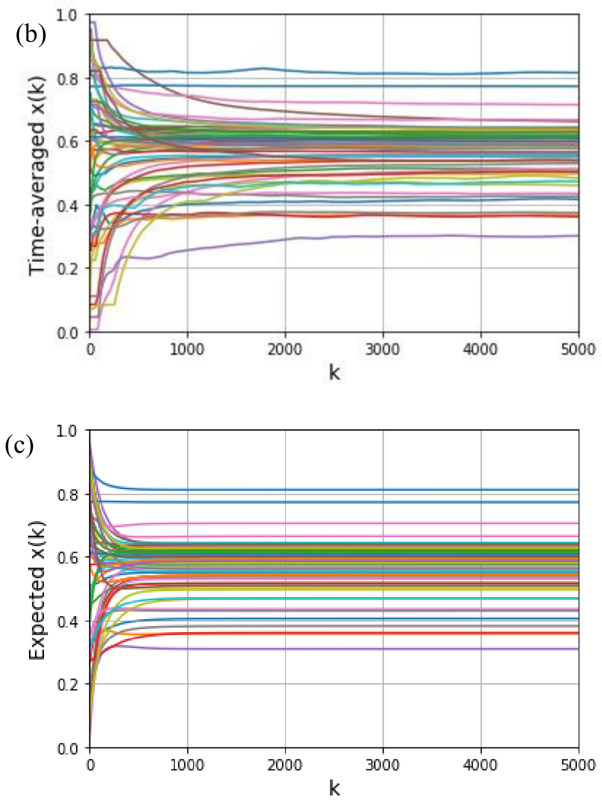


Fig. 2. (a) Random group gossiping with stubborn agents, (b) time-average of the model, and (c) the expected dynamics of the model.

6. CONCLUSION

In this study, we applied group gossiping with random participants for modeling the opinion formation process in social networks with stubborn agents. Our proposed model is a time-varying version of the Friedkin-Johnsen model, where the opinion update at each iteration is determined by a random agent and a random subset of the agent's neighbors. This approach expands the dynamics of pairwise gossiping in order to represent more real-world communication scenarios and capture the irregularity of human interactions.

While our proposed model is not guaranteed to converge to stable opinions, we have shown that its expected dynamics is convergent regardless of the network topology. Additionally, its time-averaged opinions and expected opinions approach the same values as the number of iterations increases. The behavior of our model was demonstrated through a numerical example that uses a real-world social network.

REFERENCES

Aguilar, E. and Fujisaki, Y. (2019). Gossip-based model for opinion dynamics with probabilistic group interactions. *51st ISCIE International Symposium on Stochastic Systems Theory and Its Applications*.

Boyd, S., Ghosh, A., Prabhakar, B., and Shah, D. (2006). Randomized gossip algorithms. *IEEE Transactions on Information Theory*, 52(6), 2508–2530.

Bullo, F. (2018). *Lectures on Network Systems*. CreateSpace. Available: http://motion.me.ucsb.edulbook-lns.

Deffuant, G., Neau, D., Amblard, F., and Weisbuch, G. (2001). Mixing beliefs among interacting agents. *Advances in Complex Systems 3*, 87–98.

DeGroot, M.H. (1974). Reaching a consensus. *Journal of the American Statistical Association*, 69(345), 118-121.

Fagnani, F. and Frasca, P. (2017). *Introduction to Averaging Dynamics Over Networks*. New York, NY, USA: Springer-Verlag.

Frasca, P., Ravazzi, C., Tempo, R., and Ishii, H. (2013). Gossips and prejudices: Ergodic randomized dynamics in social networks. *Proc. IFAC NecSys 2013 Workshop*, 212-219.

French, J., Jr. (1956). A formal theory of social power. *Physchol. Rev. 63*, 181–194.

Friedkin, N.E. and Johnsen, E.C. (1999). Social influence networks and opinion change. *Advances in Group Processes*, 16, 1-29.

Hegselmann, R. and, Krause, U. (2002). Opinion dynamics and bounded confidence models, analysis, and simulation. *Journal of Artificial Societies and Social Simulation*, 5(3), 1-33.

McAuley, J. and Leskovec, J. (2012). Learning to Discover Social Circles in Ego Networks. *Advances in Neural Information Processing Systems*, 539–547.

Nguyen, L.T.H., Wada, T., Masubuchi, I., Asai, T., and Fujisaki, Y. (2017). Opinion formation over signed gossip networks. *SICE Journal of Control, Measurement, and System Integration*, 10(3), 266-273.

Parsegov, S., Proskurnikov, A., Tempo, R., and Friedkin, N. (2017). Novel multidimensional models of opinion dynamics in social networks. *IEEE Trans. Autom. Control*, 62(5), 2270–2285.

Proskurnikov, A.V. and Tempo, R. (2017). A tutorial on modeling and analysis of dynamic social networks. Part I. *Annual Reviews in Control*, 43, 65-79.

Proskurnikov, A.V. and Tempo, R. (2018). A tutorial on modeling and analysis of dynamic social networks. Part II. *Annual Reviews in Control*, 45, 166-190.

Ravazzi, C., Frasca, P., Tempo, R., and Ishii, H. (2015). Ergodic randomized algorithms and dynamics over networks. *IEEE Trans. Control of Network Syst.*, 2(1), 78–87.

Sîrbu, A., Loreto, V., Servedio, V.D., and Tria, F. (2017). Opinion dynamics: models, extensions and external effects. *Participatory sensing, opinions and collective awareness*, 363–401.