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Enforcing Opacity in Modular Systems

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Abstract: In discrete-event systems, the opacity of a secret ensures that some behaviors or states cannot be inferred with certainty from partial observation of the system. Enforcing opacity in a discrete-event system, encoded by a finite labelled transition system (LTS), is a way to avoid information leakage. Checking opacity is decidable but costly (EXPTIME in the worst cases). This paper addresses opacity for modular systems in which every module, represented by an LTS, has to protect its own secret (a set of secret states $\mathcal{S}$) w.r.t. a local attacker. Once the system is composed, we assume a coalition between the attackers that share their local view (called the global attacker). Assuming the global attacker can observe all interactions between modules, we provide a reduced-complexity opacity verification technique and an algorithm for constructing local controllers that enforces opacity for each secret separately.

Keywords: opacity, supervisory control, labelled transition systems, modular systems.

1. INTRODUCTION

Online services are an increasingly important part of the global economy. Since we depend on them for financial transactions, confidential communications, and administrative processes, we need to enforce security and privacy. In addition to protecting stored data, it is also important to guarantee secrecy of the tasks performed online. Indeed, launching an attack at an appropriate moment may dramatically increase the chances of success of the attack. For instance, a phishing attack may be more effective when a user is about to complete a transaction. Hence, during a transaction, a system should remain as obscure as possible to actors that are not involved in the exchange.

Security properties of a system can be evaluated using formal methods. In particular, we are interested in the property of opacity. Opacity was first introduced in the context of modelling security protocols (Mazaré, 2005). The notion was later studied with respect to transition systems, in which a predicate is considered opaque if an attacker cannot deduce the predicate's truth with any certainty (Bryans et al., 2008). Opacity unifies two security properties: anonymity and secrecy. It can be used to guarantee that an attacker is either unable to distinguish that an action was performed by some agent i, or unable to decide which secret decision was taken by agent i. Opacity indicates whether an attacker can infer a "secret" about the system behavior, assuming the attacker knows the system's structure and has partial observation of the system's actions. If the attacker's estimate of the system's behavior never reveals the secret, the system is opaque. There are several variants of opacity: for instance, *initial-state opacity* ensures that an attacker can never distinguish when the system starts at a secret initial state, and *current-state opacity* ensures that an attacker can never distinguish when the system is currently in a secret state. See (Jacob et al., 2016) for a review of these variants.

This paper considers opacity in a modular architecture. Modular systems are typically modeled using parallel composition of transition systems, which can then be used for diagnosis and control applications, e.g., (de Queiroz and Cury, 2000; Contant et al., 2006). The monolithic transition system obtained by assembling modules may be very large. In the general case, opacity verification for modular systems is EXPSPACE-complete (Masopust and Yin, 2019). Saboori and Hadjicostis (2010) found conditions that improve the complexity of initial-state opacity verification, and Tong and Lan (2019) extended that to current-state opacity verification. We can further improve performance by avoiding the computation of the monolithic system; we want to verify and enforce properties only on relevant sets of components. In this paper, we verify current-state opacity in systems where each module has an individual secret, and we use an iterative approach to improve performance in many cases. When a modular system is not opaque, we want to enforce opacity using supervisory control, i.e., restrict behaviors of the system to ensure its opacity. The work of Badouel et al. (2007) and Dubreil et al. (2008, 2010) apply supervisory control theory (Ramadge and Wonham, 1982) to enforce opacity. We use these techniques to construct local controllers that enforce opacity of each module's secret in composed systems. That is, we find modular controllers that collectively ensure the monolithic system does not leak individual secrets.

This paper is organized as follows. First, we introduce our notation and background on discrete event systems, opacity, and supervisory control. Then, we present a reduced-complexity algorithm for verifying opacity in modular systems, assuming the attacker can observe the interface between modules. With this algorithm, we show that we do not always need to construct the monolithic system to verify opacity. Finally, we present an algorithm for syn-

thesizing local controllers that enforces opacity in modular systems.

2. PRELIMINARIES

2.1 Languages and Labelled Transition Systems

Let Σ be an alphabet of events. Then, Σ^* denotes the set of all finite words composed of elements in Σ. We can define a language as $\mathcal{L} \subseteq \Sigma^*$. A *word* is a sequence of events $t \in \Sigma^*$, which has length $|t|$. The *concatenation* of $t_1 = \sigma_1 \cdots \sigma_{|t_1|}$ and $t_2 = \sigma'_1 \cdots \sigma'_{|t_2|}$ is defined as $t_1.t_2 = \sigma_1 \cdots \sigma_{|t_1|}\sigma'_1 \cdots \sigma'_{|t_2|}$. The *prefix-closure* of a language $\mathcal{L}$ is defined as $\overline{\mathcal{L}} = \{t \in \Sigma^* \mid (\exists t' \in \Sigma^*)\ t.t' \in \mathcal{L}\}$. Given an alphabet $\Sigma' \subseteq \Sigma$, we define the *projection operator* on finite words $P_{\Sigma'} : \Sigma^* \to \Sigma'^*$, which removes from a word of Σ^* all the events that do not belong to Σ'. Formally, $P_{\Sigma'}$ is recursively defined as follows: $P_{\Sigma'}(\varepsilon) = \varepsilon$ and for $u \in \Sigma^*, \sigma \in \Sigma$, $P_{\Sigma'}(u.\sigma) = P_{\Sigma'}(u).\sigma$ if $\sigma \in \Sigma'$, or $P_{\Sigma'}(u)$, otherwise. Let $\mathcal{L} \subseteq \Sigma^*$ be a language. The definition of projection for words extends to languages: $P_{\Sigma'}(\mathcal{L}) = \{P_{\Sigma'}(s) \mid s \in \mathcal{L}\}$. Conversely, let $\mathcal{L} \subseteq \Sigma'^*$. The *inverse projection* of $\mathcal{L}$ is $P^{-1}_{\Sigma'}(\mathcal{L}) = \{s \in \Sigma^* \mid P_{\Sigma'}(s) \in \mathcal{L}\}$.

Our model is a deterministic *labelled transition system (LTS)*. An LTS G is defined as a four-tuple:

$$G = (\mathcal{Q}, \Sigma, \delta, q_0),$$

where $\mathcal{Q}$ is a finite set of *states*, Σ is the alphabet of events, $\delta : \mathcal{Q} \times \Sigma \to \mathcal{Q}$ is a *partial transition map*, and $q_0 \in \mathcal{Q}$ is the *initial state*.

The transition map easily extends to words: for a state $q \in \mathcal{Q}$, a word $t \in \Sigma^*$, where $|t| \geq 1$, and event $\sigma \in \Sigma$, we can recursively define $\delta(q, t.\sigma)$ as $\delta(q, t.\sigma) = \delta(\delta(q, t), \sigma)$. We will write $\delta(q, \sigma)!$ (resp., $\delta(q, t)!$) when action σ is *allowed* from state q, i.e., $\exists q' \in \mathcal{Q}, \delta(q, \sigma) = q'$ (resp., when sequence t is allowed from q).

Let $\mathcal{L}(G)$ denote the language of the LTS G; that is, $\mathcal{L}(G) = \{t \in \Sigma^* \mid \delta(q_0, t)!\}$. By definition, this language is prefix-closed: $\mathcal{L}(G) = \overline{\mathcal{L}(G)}$. We denote the language of G w.r.t. a set of states $\mathcal{X} \subseteq \mathcal{Q}$ as $\mathcal{L}_{\mathcal{X}}(G)$, where $(\forall t \in \Sigma^*)\ \delta(q_0, t) \in \mathcal{X} \Leftrightarrow t \in \mathcal{L}_{\mathcal{X}}(G)$.

The notion of a projection extends to an LTS. To show the partial observation of a system G, we construct the *determinization* of G. In this structure, only observable events Σ_o are used as transitions. Each state in the structure represents the set of states in which the original system could be after observing a word in Σ_o^*. More formally, we define the *unobservable reach* of a state $q \in \mathcal{Q}$, w.r.t. an alphabet $\Sigma_o \subseteq \Sigma$, as $\mathrm{UR}_{\Sigma_o}(q) = \{q' \in \mathcal{Q} \mid (\exists t \in (\Sigma \setminus \Sigma_o)^*)\ \delta(q, t) = q'\}$. Then, we define the *determinization* of G w.r.t. Σ_o as $Det_{\Sigma_o}(G) = (\mathcal{X}, \Sigma_o, \delta_{\Sigma_o}, x_0)$, where $\mathcal{X} = 2^{\mathcal{Q}}$, $x_o = \mathrm{UR}_{\Sigma_o}(q_0)$, and, for $x \in \mathcal{X}$ and $\sigma \in \Sigma_o$, $\delta_{\Sigma_o}(x, \sigma) = \bigcup_{q \in x} \mathrm{UR}_{\Sigma_o}(\delta(q, \sigma))$. Thus, the set of observable words is:

$$P_{\Sigma_o}(\mathcal{L}(G)) = \mathcal{L}(Det_{\Sigma_o}(G)). \tag{1}$$

In this paper, we work with systems designed by composing n modules, described by transition systems $G_1, G_2, \ldots, G_n$, that synchronize on common events. We can build a single monolithic system by composing all modules. We define the *parallel composition* operation for both languages and LTSs, which are used interchangeably in proofs. Given two languages $\mathcal{L}_1 \subseteq \Sigma_1^*$ and $\mathcal{L}_2 \subseteq \Sigma_2^*$, we define their parallel composition as $\mathcal{L}_1||\mathcal{L}_2 = P^{-1}_{\Sigma_2}(\mathcal{L}_1) \cap P^{-1}_{\Sigma_1}(\mathcal{L}_2)$. From this, we make two observations:

Remark 1. For languages $\mathcal{L}_1 \in \Sigma_1^*$, $\mathcal{L}_2 \in \Sigma_2^*$, $t \in \mathcal{L}_1||\mathcal{L}_2 \Leftrightarrow P_{\Sigma_1}(t) \in \mathcal{L}_1 \wedge P_{\Sigma_2}(t) \in \mathcal{L}_2$. As a consequence, parallel composition is commutative ($\mathcal{L}_1||\mathcal{L}_2 = \mathcal{L}_2||\mathcal{L}_1$) and associative ($(\mathcal{L}_1||\mathcal{L}_2)||\mathcal{L}_3 = \mathcal{L}_1||(\mathcal{L}_2||\mathcal{L}_3)$).

Remark 2. Given $t \in \mathcal{L}_1 \subseteq \Sigma_1^*$ and $\mathcal{L}_2 \subseteq \Sigma_2^*$, then $\{t\}||\mathcal{L}_2 \subseteq \mathcal{L}_1||\mathcal{L}_2$.

We define the parallel composition of two LTSs, $G_i = (\mathcal{Q}_i, \Sigma_i, \delta_i, q_{0,i})$ for $i \in \{1, 2\}$, by $G_1||G_2 = (\mathcal{Q}_1 \times \mathcal{Q}_2, \Sigma_1 \cup \Sigma_2, \delta_1 \times \delta_2, (q_{0,1}, q_{0,2}))$, where $\delta_1 \times \delta_2$ is defined as follows:

$$\delta_1 \times \delta_2((q_1, q_2), \sigma) = \begin{cases} (\delta_1(q_1, \sigma), \delta_2(q_2, \sigma)), & \text{if } \delta_1(q_1, \sigma)! \wedge \delta_2(q_2, \sigma)!; \\ (\delta_1(q_1, \sigma), q_2), & \text{if } \delta_1(q_1, \sigma)! \wedge \sigma \notin \Sigma_2; \\ (q_1, \delta_2(q_2, \sigma)), & \text{if } \sigma \notin \Sigma_1 \wedge \delta_2(q_2, \sigma)!; \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

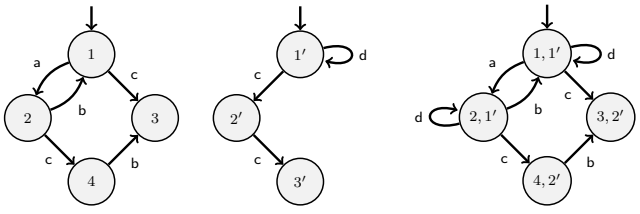
Performing the parallel composition on languages and on LTSs is equivalent:

$$\mathcal{L}(G_1||G_2) = \mathcal{L}(G_1)||\mathcal{L}(G_2). \tag{2}$$

The *interface* between two modules G_i and G_j, where $i, j \in \{1, \ldots, n\}$ and $i \neq j$, is the shared alphabet $\Sigma_{i,j} = \Sigma_i \cap \Sigma_j$. The two LTSs synchronize on these events during the parallel composition, modelling joint events between the modules. Furthermore, for modules $G_1, \ldots, G_n$, we denote the interface of shared events by:

$$\Sigma_s = \bigcup_{i,j \in \{1,\ldots,n\}, i \neq j} \Sigma_{i,j}$$

Example 3. When we compose the two LTSs in Figures 1(a) and 1(b), we get $G_1||G_2$ as pictured in Figure 1(c). Note that the parallel composition prevents event c at state $2'$ in G_2 because the two LTSs must synchronize on the shared events in $\Sigma_{1,2} = \{\mathsf{c}\}$. □

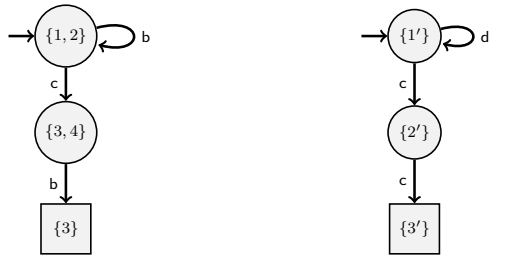


(a) G_1 (b) G_2 (c) $G_1||G_2$

Fig. 1. LTSs G_1, G_2, and $G_1||G_2$ with $\Sigma_s = \{\mathsf{c}\}$

2.2 Opacity

To model security properties in systems, we use the concept of *current-state opacity*. For a system G, we assume the presence of an attacker that has access to an alphabet $\Sigma_a \subseteq \Sigma$. Furthermore, the system has a set of secret states $\mathcal{S} \subseteq \mathcal{Q}$. The *secret language* of a system is the set of words that end in a secret state: $\mathcal{L}_{\mathcal{S}}(G) = \{t \in \mathcal{L}(G) \mid \delta(q_0, t) \in \mathcal{S}\}$. Since our transition functions are deterministic, every sequence $t \in \mathcal{L}(G)$ is either in $\mathcal{L}_{\mathcal{S}}(G)$ or in $\mathcal{L}(G) \setminus \mathcal{L}_{\mathcal{S}}(G)$, where $\setminus$ represents set subtraction. Let us recall a general



(a) $Det_{\Sigma_{a,1}}(G_1)$ (b) $Det_{\Sigma_{a,2}}(G_2)$

Fig. 2. LTSs $Det_{\Sigma_{a,1}}(G_1)$ and $Det_{\Sigma_{a,2}}(G_2)$ with $\Sigma_{a,1} = \{\mathsf{b},\mathsf{c}\}$ and $\Sigma_{a,2} = \{\mathsf{c},\mathsf{d}\}$

definition of opacity from Cassez et al. (2012), where $\mathcal{L}$ and $\mathcal{L}_\mathcal{S}$ need not be regular.

*Definition 4. For a language $\mathcal{L} \subseteq \Sigma^*$ and secret language $\mathcal{L}_\mathcal{S} \subseteq \mathcal{L}$, $\mathcal{L}_\mathcal{S}$ is **opaque** w.r.t. $\mathcal{L}$ and Σ_a iff $\forall t_1 \in \mathcal{L}_\mathcal{S}$, $\exists t_2 \in \mathcal{L} \setminus \mathcal{L}_\mathcal{S}$ where $P_{\Sigma_a}(t_1) = P_{\Sigma_a}(t_2)$.*

Applying this to an LTS G with a set of secret states $\mathcal{S} \subseteq \mathcal{Q}$, $\mathcal{S}$ is opaque w.r.t. G and Σ_a when $\mathcal{L}_\mathcal{S}(G)$ is opaque w.r.t. $\mathcal{L}(G)$ and Σ_a. Checking this condition amounts to verifying a reachability property of the determinization of G w.r.t. Σ_a. More formally:

Proposition 5. Let $Det_{\Sigma_a}(G) = (\mathcal{X}, \Sigma_a, \delta_{\Sigma_a}, x_0)$. $\mathcal{S}$ is opaque w.r.t. G and Σ_a if and only if $\forall x \in \mathcal{X}$, $\forall t \in \Sigma_a^*$ such that $x = \delta_{\Sigma_a}(\{x_0\}, t)$, $x \setminus \mathcal{S} \neq \emptyset$.

This property follows from Def. 4. It implies a simple algorithm to verify opacity of a secret $\mathcal{S}$ w.r.t. an LTS G and attacker alphabet Σ_a:

Algorithm 1 (Cassez et al., 2012). Verify opacity of $\mathcal{S}$ w.r.t. G and Σ_a.

```
1: function IsOpaque(G, Σ_a)
2:     Det_{Σ_a}(G) = (X, Σ_a, δ_{Σ_a}, x_0)
3:     If (∃t ∈ Σ_a^*, δ_{Σ_a}(x_0, t) \ S = ∅) return false
4:     else return true
```

Example 6. Using the LTSs from Figure 1, suppose that the set of secret states in G_1 (resp. G_2) is $\mathcal{S}_1 = \{3\}$ ($\mathcal{S}_2 = \{3'\}$). To verify if $\mathcal{S}_i$ is opaque, we determinize G_i w.r.t. $\Sigma_{a,i}$, as pictured in Figure 2. Since state $\{3\}$ is accessible from the initial state of $Det_{\Sigma_{a,1}}(G_1)$, and since $\{3\} \setminus \mathcal{S}_1 = \emptyset$, $\mathcal{S}_1$ is not opaque w.r.t. G_1 and $\Sigma_{a,1}$. Similarly, $\mathcal{S}_2$ is not opaque w.r.t. G_2 and $\Sigma_{a,2}$. □

Algorithm 1 runs in EXPTIME. In general, one cannot obtain better complexity, as the opacity problem is PSPACE-complete (Cassez et al., 2012). Indeed, one can select a subset X of $\mathcal{S}$ in polynomial time and then check whether $\{q_0\}$ is co-reachable from X in PSPACE. The hardness part comes from a straightforward reduction from the language inclusion problem. We will show later that in many cases, modularity can help reduce the cost of opacity verification.

2.3 Supervisory Control for Opacity

When opacity does not hold, we want to restrict the system's behavior to ensure secrets are not revealed. Using techniques from supervisory control, we can synthesize a controller to enforce opacity. We partition events in Σ into events that the controller can observe, Σ_o, and those that that it cannot observe, Σ_{uo}, such that $\Sigma = \Sigma_o \uplus \Sigma_{uo}$. We also partition Σ into events the controller can control, Σ_c, and those it cannot control, Σ_{uc}, such that $\Sigma = \Sigma_c \uplus \Sigma_{uc}$. We assume that $\Sigma_c \subseteq \Sigma_o$. The goal of a controller is to enforce a desired subset of behavior $K \subseteq \mathcal{L} \subseteq \Sigma^*$ by enabling or disabling events in Σ_c. Below, we recall two properties of a controller's language.

*Definition 7. [From Ramadge and Wonham (1982)]. A language $K \subseteq \mathcal{L}$ is **controllable** w.r.t. $\mathcal{L}$ and Σ_c iff $\overline{K}.(\Sigma \setminus \Sigma_c) \cap \mathcal{L} \subseteq \overline{K}$.*

*Definition 8. [Based on Lin and Wonham (1988)]. A language $K \subseteq \mathcal{L}$ is **observable** w.r.t. $\mathcal{L}$ and Σ_o iff $(\forall t, t' \in K)(\forall \sigma \in \Sigma)\ \pi_{\Sigma_o}(t) = \pi_{\Sigma_o}(t') \wedge t\sigma \in K \wedge t'\sigma \in \mathcal{L} \implies t'\sigma \in K$.*

Formally, we define a controller for G as a function $C : \mathcal{L}(Det_{\Sigma_o}(G)) \to \{\gamma \in 2^\Sigma \mid \Sigma_{uc} \subseteq \gamma\}$ and as an LTS $\mathcal{C} = (Q_\mathcal{C}, \Sigma_o, \delta_\mathcal{C}, q_{0,\mathcal{C}})$ such that for each $t \in \mathcal{L}(Det_{\Sigma_o}(G))$, $\delta_\mathcal{C}(q_{0,\mathcal{C}}, t.\sigma)! \iff \sigma \in C(t)$. A controller C is *fully permissive* iff $C(t) = \Sigma$ for every $t \in \Sigma_o^*$. The controlled system denoted by C/G generates the language $\mathcal{L}(C||G)$. The controller is *valid* if $\mathcal{L}(C/G)$ is both controllable and observable w.r.t. $\mathcal{L}(G)$, Σ_o, and Σ_c. Since C has the alphabet Σ_o, it is inherently observable, so it suffices to simply ensure that it is controllable.

There exists a largest sublanguage K of $\mathcal{L}(G)$ that does not violate the opacity of $\mathcal{S}$. When $\Sigma_c \subseteq \Sigma_o$, there exists a supremal controllable and observable prefix-closed sublanguage $K^\uparrow$ of $K \subseteq \mathcal{L}(G)$, which is the largest regular language included in K that can be enforced by control (Dubreil et al., 2010). If $K^\uparrow \neq \emptyset$, then there exists a *supremal controller* $\mathcal{C}$ such that $\mathcal{L}(\mathcal{C}/G) = K^\uparrow$.

To compute a regular supremal controller, there are two possible assumptions about the attacker's knowledge: either the attacker has full knowledge of both G and $\mathcal{C}$'s construction, or the attacker only has knowledge of G's construction. In the former case, Dubreil et al. (2010) proposed an algorithm to synthesize the supremal controller. This algorithm has a running time in $\mathcal{O}(|\mathcal{Q}| \times 2^{|\mathcal{Q}|})$ under two assumptions: first, $\Sigma_c \subseteq \Sigma_o$, and second, Σ_o is comparable to Σ_a (that is, $\Sigma_o \subseteq \Sigma_a$ or $\Sigma_a \subseteq \Sigma_o$). In the latter case, where an attacker only knows G, Tong et al. (2018) proposed a synthesis algorithm with running time in $\mathcal{O}(2^{2(|\mathcal{Q}| \times 2^{|\mathcal{Q}|} + |\Sigma_c|)})$, and only assumes $\Sigma_c \subseteq \Sigma_o$. In this paper, we focus on this latter case, and do not assume that the observable and controllable alphabets are comparable.

3. VERIFYING OPACITY IN MODULAR SYSTEMS

The techniques for verifying opacity in a single system can be extended to systems composed of multiple modules. One can compute the monolithic system by performing the parallel composition of all modules, and then verify the opacity of the secrets from all modules. However, the monolithic system may be large and expensive to construct. In this section, we propose optimizations that, in some cases, avoid the construction of the entire monolithic system.

3.1 Problem Formulation

We are interested in systems composed of multiple modules, $G_1, \ldots, G_n$, where the resulting system $G_1 || \cdots || G_n$

has an alphabet $\Sigma = \bigcup \Sigma_i$. In such a system, we want to make sure the secrets within each module are kept confidential. We assume that we have n attackers, $\mathcal{A}_1, \ldots, \mathcal{A}_n$, each observing one of the modules with the attacker alphabet $\Sigma_{a,i} \subseteq \Sigma_i$. We assume that if $\sigma \in \Sigma_{a,i}$ and $\sigma \in \Sigma_i$, then $\sigma \in \Sigma_{a,i}$; that is, an event that is observable in one module is observable in all modules. Each attacker is trying to identify when its module is in a secret state of $\mathcal{S}_i$. We are interested in the case where all of these attackers share knowledge; the coalition of attackers can effectively observe events in $\Sigma_a = \bigcup \Sigma_{a,i}$. For simplicity, we refer to the attackers' collective view of the system as $\mathcal{A}$. Modeling coalitions of attackers that exchange all of their observations amounts to saying that every attacker observes every module. This is clearly a conservative approach. If a system is opaque for such a set of very intrusive attackers, then it is opaque for attackers that can acquire less knowledge of possible states of components. The way an attacker $\mathcal{A}_i$ obtains information on a component G_j is unspecified. We assume the information is immediately available to all attackers, taking a conservative approach to opacity verification, regardless of communication limits among attackers.

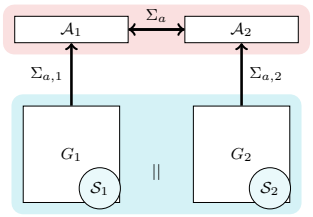


Fig. 3. Modular architecture with attackers.

We want to prevent $\mathcal{A}$ from identifying with certainty that G_i is in a secret state in $\mathcal{S}_i$. Since we are working with the attackers' collective view, we need to ensure that every secret state $(\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$ is opaque in the monolithic system $G_1 || \cdots || G_n$ w.r.t. the combined attacker alphabet, Σ_a. Since we want to ensure this for each secret $\mathcal{S}_i$, $i \in \{1, \ldots, n\}$, the modular opacity problem can be formulated as follows.

Problem 9. Given n modules $G_1, \ldots G_n$ with secrets $\mathcal{S}_1, \ldots \mathcal{S}_n$, and a global attacker view $\mathcal{A}$ with alphabet Σ_a, is $\mathcal{S} = \bigcup_{i=1}^{n} (\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$ opaque w.r.t. $G_1 || \cdots || G_n$ and Σ_a?

As shown in Section 2.2, a simple solution is to determinize the full system $G_1 || \cdots || G_n$ w.r.t. the alphabet Σ_a and check for states that are a subset of $\mathcal{S}$. If we let $m = \max_{i \in \{1,\ldots,n\}} |Q_i|$, then this algorithm is $\mathcal{O}(2^{m^n})$ because we first construct a system from n systems of size m and then determinize this result. However, it would be desirable to verify opacity without constructing this monolithic system, whenever possible.

In this paper, we assume that the modules' shared alphabet is a subset of the attacker's alphabet: $\Sigma_s \subseteq \Sigma_a$. That is, the attacker observes the interface between modules, acting as middleware in a network. This is the same assumption made in Saboori and Hadjicostis (2010) for verifying initial-state opacity. Under this assumption, we present optimizations for verifying current-state opacity.

3.2 Opacity and Composition

When $\Sigma_s \subseteq \Sigma_a$, we can avoid determinizing the monolithic system by, instead, determinizing the local modules individually. This reduces the complexity of verifying opacity, making it applicable to large modular systems. Formally, this is shown in the following theorem and corollary.

Theorem 10. [From de Queiroz and Cury (2000)]. Let $\mathcal{L}_1 \subseteq \Sigma_1^*$ and $\mathcal{L}_2 \subseteq \Sigma_2^*$ be languages and let $\Sigma_s \subseteq \Sigma' \subseteq \Sigma_1 \cup \Sigma_2$. Then, $P_{\Sigma'}(\mathcal{L}_1 || \mathcal{L}_2) = P_{\Sigma' \cap \Sigma_1}(\mathcal{L}_1) || P_{\Sigma' \cap \Sigma_2}(\mathcal{L}_2)$.

Corollary 11. Let G_1, G_2 be LTSs and let $\Sigma_s \subseteq \Sigma' \subseteq \Sigma_1 \cup \Sigma_2$. Then, $\mathcal{L}(Det_{\Sigma' \cap \Sigma_1}(G_1) || Det_{\Sigma' \cap \Sigma_2}(G_2)) = \mathcal{L}(Det_{\Sigma'}(G_1 || G_2))$.

Proof.

$$\begin{aligned}
&\mathcal{L}(Det_{\Sigma' \cap \Sigma_1}(G_1) || Det_{\Sigma' \cap \Sigma_2}(G_2)) \\
&= P_{\Sigma' \cap \Sigma_1}(\mathcal{L}(G_1)) || P_{\Sigma' \cap \Sigma_2}(\mathcal{L}(G_2)), && \text{By (1).} \\
&= P_{\Sigma'}(\mathcal{L}(G_1) || \mathcal{L}(G_2)), && \text{By Thm. 10.} \\
&= P_{\Sigma'}(\mathcal{L}(G_1 || G_2)) = \mathcal{L}(Det_{\Sigma'}(G_1 || G_2)) && \text{By (1) \& (2).} \blacksquare
\end{aligned}$$

Thus, when computing what attackers can observe in the system, we can build $\mathcal{A} = Det_{\Sigma_{a,1}}(G_1) || \cdots || Det_{\Sigma_{a,n}}(G_n)$ rather than $Det_{\Sigma_a}(G_1 || \cdots || G_n)$. In this system, the state set is $\mathcal{X} = 2^{\mathcal{Q}_1} \times \cdots \times 2^{\mathcal{Q}_n}$ while the secret state set is $\mathcal{S} = \bigcup_{i=1}^{n} (2^{\mathcal{Q}_1} \times \cdots \times 2^{\mathcal{Q}_{i-1}}) \times 2^{\mathcal{S}_i} \times (2^{\mathcal{Q}_{i+1}} \times \cdots \times 2^{\mathcal{Q}_n})$. Following from Proposition 5, we can formalize when one of these individual secrets $\mathcal{S}_i$ is opaque w.r.t. the composed system and attacker's alphabet:

Theorem 12. Let $G_i = (\mathcal{Q}_i, \Sigma_i, \delta_i, q_{0,i})$ with $\mathcal{S}_i \subseteq \mathcal{Q}_i$. Let the attacker's view of the system be $Det_{\Sigma_a \cap \Sigma_1}(G_1) || \cdots || Det_{\Sigma_a \cap \Sigma_n}(G_n) = (\mathcal{X}, \Sigma_a, \delta_{\Sigma_a}, x_0)$ and assume $\Sigma_s \subseteq \Sigma_a$. $\mathcal{S}_i$ is opaque w.r.t. the composed system $G_1 || \cdots || G_n$ and Σ_a iff $\forall x \in \mathcal{X}, \forall t \in \Sigma_a^*$ such that $x = \delta_{\Sigma_a}(x_0, t)$, $x \notin (2^{\mathcal{Q}_1} \times \cdots \times 2^{\mathcal{Q}_{i-1}}) \times 2^{\mathcal{S}_i} \times (2^{\mathcal{Q}_{i+1}} \times \cdots \times 2^{\mathcal{Q}_n})$.

Proof. Since parallel composition is commutative and associative, we can prove that the secret $\mathcal{S}_1$ of module G_1 is opaque, without loss of generality. Let $Det_{\Sigma_a}(G_1 || \cdots || G_n) = (\mathcal{X}', \Sigma_a, \delta'_{\Sigma_a}, x'_o)$. By definition, $\mathcal{S}_1$ is opaque w.r.t. $G_1 || \cdots || G_n$ and Σ_a iff $\forall x \in \mathcal{X}'$, $\forall t \in \Sigma_a^*$ such that $x = \delta'_{\Sigma_a}(\{x'_0\}, t)$, $x \setminus \mathcal{S}_1 \times \mathcal{Q}_2 \times \cdots \times \mathcal{Q}_n \neq \emptyset$—in other words, when $x \notin 2^{\mathcal{S}_1 \times \mathcal{Q}_2 \times \cdots \times \mathcal{Q}_n}$. Thus, it is opaque when $\mathcal{L}_{2^{\mathcal{S}_1 \times \mathcal{Q}_2 \times \cdots \times \mathcal{Q}_n}}(Det_{\Sigma_a}(G_1 || \cdots || G_n)) = \emptyset$. By Corollary 11, $\mathcal{L}_{2^{\mathcal{S}_1 \times 2^{\mathcal{Q}_2} \times \cdots \times 2^{\mathcal{Q}_n}}}(Det_{\Sigma_a \cap \Sigma_1}(G_1) || \cdots || Det_{\Sigma_a \cap \Sigma_2}(G_n)) = \mathcal{L}_{2^{\mathcal{S}_1 \times \mathcal{Q}_2 \times \cdots \times \mathcal{Q}_n}}(Det_{\Sigma_a}(G_1 || \cdots || G_n)) = \emptyset$. $\blacksquare$

By stating that $\mathcal{S}_i$ is opaque w.r.t. $G_1 || \cdots || G_n$ and Σ_a, we equivalently mean $\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1} \times \mathcal{S}_i \times \mathcal{Q}_{i+1} \times \mathcal{Q}_n$ is opaque w.r.t. $G_1 || \cdots || G_n$ and Σ_a.

Corollary 13. Let $G_i = (\mathcal{Q}_i, \Sigma_i, \delta_i, q_{0,i})$ with $\mathcal{S}_i \subseteq \mathcal{Q}_i$. If every $\mathcal{S}_i$ is opaque w.r.t. $G_1 || \cdots || G_n$ and Σ_a, and $\Sigma_s \subseteq \Sigma_a$, then $\mathcal{S} = \bigcup_{i=1}^{n} (\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$ is opaque w.r.t. $G_1 || \cdots || G_n$ and Σ_a.

Proof. Let the attacker's view of the system be $\mathcal{A} = (\mathcal{X}, \Sigma_a, \delta_{\mathcal{A}}, q_{0,\mathcal{A}})$. Since every $\mathcal{S}_i$ is opaque w.r.t. $G_1 || \cdots || G_n$ and Σ_a, we know that $\forall x \in \mathcal{X}, x \notin (2^{\mathcal{Q}_1} \times \cdots \times 2^{\mathcal{Q}_{i-1}}) \times 2^{\mathcal{S}_i} \times (2^{\mathcal{Q}_{i+1}} \times \cdots \times 2^{\mathcal{Q}_n})$ by Theorem 12. Thus, $\forall x \in \mathcal{X}, x \notin \bigcup_{i=1}^{n} (2^{\mathcal{Q}_1} \times \cdots \times 2^{\mathcal{Q}_{i-1}}) \times 2^{\mathcal{S}_i} \times (2^{\mathcal{Q}_{i+1}} \times \cdots \times 2^{\mathcal{Q}_n})$. Thus, $\mathcal{S}$ is opaque w.r.t. $G_1 || \cdots || G_n$ and Σ_a. $\blacksquare$

Similar to the results of Saboori and Hadjicostis (2010), this gives a compositional opacity verification algorithm with a reduced complexity: we can determinize all modules, compose them, and verify opacity of secret states.

Theorem 14. When $\Sigma_s \subseteq \Sigma_a$, opacity verification of modular systems can be performed in $\mathcal{O}(2^{mn})$.

Proof. Theorem 12 shows that opacity of a secret can be checked on the composition of determinized modules. Let $m = \max_{i\in\{1,\dots,n\}} |Q_i|$. First, each module is determinized, giving at most 2^m states per module. Then, all n modules are composed and we check for opacity. Thus, the complexity is $\mathcal{O}(2^{mn})$. ■

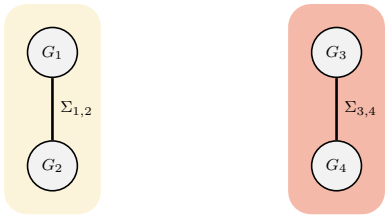
This is exponentially better than the previous algorithm, but this algorithm can still be improved.

In general, a secret $\mathcal{S}_i$ that is opaque w.r.t. G_i and Σ_a need not be opaque w.r.t. $G_1||\cdots||G_n$ and Σ_a. In other words, opacity of secrets is not preserved by composition. Note that synchronization on common events may forbid behaviors in a component, and thus reduce ambiguity as to whether a subsystem is in a secret state or not. Furthermore, synchronizing with another component may produce new observations, and hence reveal that G_i is in a state of $\mathcal{S}_i$. Nevertheless, there are clearly exceptions, such as when $\Sigma_s = \emptyset$.

Remark 15. Assume $\Sigma_s = \emptyset$. Then, $\mathcal{S}_i$ is opaque w.r.t. G_i and Σ_a iff $\mathcal{S}_i$ is opaque w.r.t. $G_1||G_2$ and Σ_a, for all $i \in \{1,2\}$.

We can use Remark 15 to split up a modular opacity verification problem into multiple subproblems to reduce the complexity. That is, we can partition the modules into sets that have no shared events with modules in other sets. Jezequel and Fabre (2012) use an *interaction graph* to illustrate this idea. The graph is defined by vertices, which are the LTS modules, and edges, where G_i has an edge to G_j iff $\Sigma_{i,j} \neq \emptyset$. We partition the LTSs into connected components, where the i-th connected component is called M_i. We can independently verify opacity for each connected component of modules; by Remark 15, opacity of a secret $\mathcal{S}_i$ holds for the whole system iff it holds for the parallel composition of the LTSs in a connected component.

Example 16. Suppose that we have four modules G_1, G_2, G_3, G_4. Furthermore, suppose $\Sigma_{1,3}, \Sigma_{1,4}, \Sigma_{2,3}, \Sigma_{2,4} = \emptyset$ and $\Sigma_{1,2}$, $\Sigma_{3,4} \neq \emptyset$. Then, we yield the interaction graph in Figure 4 with two connected components: M_1 and M_2. We can verify opacity independently within each connected component M_i. □



(a) M_1 (b) M_2

Fig. 4. Interaction graph of four modules for Example 16

We can treat each connected component as a separate modular system and perform the algorithms on them separately. Throughout the rest of this paper, we assume that $G_1, \dots, G_n$ are all part of the same connected component.

Furthermore, similar to observations from Saboori and Hadjicostis (2010) w.r.t. initial state opacity, current-state opacity holds under composition when the shared alphabet $\Sigma_s \subseteq \Sigma_a$. The following lemma and theorem echo those found in Saboori and Hadjicostis (2010), but specifically address current-state opacity.

Lemma 17. Let $\mathcal{L}_1 \subseteq \Sigma_1^*$, $\mathcal{L}_2 \subseteq \Sigma_2^*$, and $\Sigma_s \subseteq \Sigma_a$. Suppose that $t_1, t_1' \in \mathcal{L}_1$ such that $P_{\Sigma_{a,1}}(t_1) = P_{\Sigma_{a,1}}(t_1')$, and suppose that $t_2 \in \mathcal{L}_2$. Then, $\exists t \in \{t_1\}||\{t_2\}$ iff $\exists t' \in \{t_1'\}||\{t_2\}$ such that $P_{\Sigma_a}(t) = P_{\Sigma_a}(t')$.

Proof. ($\Rightarrow$) Suppose that $\exists t \in \{t_1\}||\{t_2\}$.

$$\begin{aligned} & P_{\Sigma_a}(\{t_1\}||\{t_2\}) \\ &= P_{\Sigma_{a,1}}(\{t_1\})||P_{\Sigma_{a,2}}(\{t_2\}), && \text{by Theorem 10.} \\ &= P_{\Sigma_{a,1}}(\{t_1'\})||P_{\Sigma_{a,2}}(\{t_2\}), && \text{since } P_{\Sigma_{a,1}}(t_1) = P_{\Sigma_{a,1}}(t_1'). \\ &= P_{\Sigma_a}(\{t_1'\}||\{t_2\}), && \text{by Theorem 10.} \end{aligned}$$

Since the two languages $\{t_1\}||\{t_2\}$ and $\{t_1'\}||\{t_2\}$ are equal after projecting onto Σ_a, there must exist a $t' \in \{t_1'\}||\{t_2\}$ such that $P_{\Sigma_a}(t) = P_{\Sigma_a}(t')$.

($\Leftarrow$) Now, suppose $\exists t' \in \{t_1'\}||\{t_2\}$. By the same reasoning, $\exists t \in \{t_1\}||\{t_2\}$ such that $P_{\Sigma_a}(t) = P_{\Sigma_a}(t')$. ■

Theorem 18. For $i \in \{1,2\}$, let $\mathcal{L}_i \subseteq \Sigma_i^*$ be a language and $\mathcal{L}_{\mathcal{S}_i} \subseteq \mathcal{L}_i$ be its corresponding secret language. Suppose that the shared alphabet $\Sigma_s \subseteq \Sigma_a \subseteq \Sigma$. If $\mathcal{L}_{\mathcal{S}_i}$ is opaque w.r.t. $\mathcal{L}_i$ and $\Sigma_{a,i}$, then $\mathcal{L}_{\mathcal{S}_i}$ is opaque w.r.t. $\mathcal{L}_1||\mathcal{L}_2$ and Σ_a.

Proof. Let $t \in \mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2$, $t_1 = P_{\Sigma_1}(t) \in \mathcal{L}_1$, and $t_2 = P_{\Sigma_2}(t) \in \mathcal{L}_2$. Since $\mathcal{L}_{\mathcal{S}_1}$ is opaque wrt $\mathcal{L}_1$ and $\Sigma_{a,1}$, by Definition 4, $\exists t_1' \in \mathcal{L}_1 \backslash \mathcal{L}_{\mathcal{S}_1}$ such that $P_{\Sigma_{a,1}}(t_1) = P_{\Sigma_{a,1}}(t_1')$. By Lemma 17, $\exists t' \in \{t_1'\}||\{t_2\}$ such that $P_{\Sigma_a}(t) = P_{\Sigma_a}(t')$. Thus, $t' \in \mathcal{L}_1||\mathcal{L}_2$ by Remark 2. So, $(\forall t \in \mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2)\ \exists t' \in (\mathcal{L}_1||\mathcal{L}_2) \backslash (\mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2)$ such that $P_{\Sigma_a}(t) = P_{\Sigma_a}(t')$. Thus, $\mathcal{L}_{\mathcal{S}_1}$ is opaque w.r.t. $\mathcal{L}_1||\mathcal{L}_2$ and Σ_a. This generalizes for $\mathcal{L}_{\mathcal{S}_i}$, where $1 \leq i \leq n$. ■

Corollary 19. Consider LTSs $G_1, \dots, G_n$ with secret state sets $\mathcal{S}_i$ for $1 \leq i \leq n$. Let $\Sigma_s \subseteq \Sigma_a$. If $\mathcal{S}_i$ is opaque w.r.t. G_i and $\Sigma_{a,i}$, then $\mathcal{S}_i$ is opaque w.r.t. $G_1||\cdots||G_n$ and Σ_a.

Corollary 20. If $\forall i \in \{1,\dots,n\}$, $\mathcal{S}_i$ is opaque w.r.t. $G_1||\cdots||G_n$ and Σ_a, then $\mathcal{S} = \bigcup_{i=1}^{n} = \mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1} \times \mathcal{S}_i \times \mathcal{Q}_{i+1} \times \mathcal{Q}_n$ is opaque w.r.t. $G_1||\cdots||G_n$ and Σ_a.

Because opacity is preserved by composition when $\Sigma_s \subseteq \Sigma_a$, we can introduce another optimization, reminiscent of the techniques used by Saboori and Hadjicostis (2010). Rather than building the composed system at the start of the algorithm, we can verify that $\mathcal{S}_i$ is opaque w.r.t. $G_1||\cdots||G_n$ and Σ_a by incrementally checking if it is opaque for larger and larger compositions, starting with the local module G_i. If, at any point, $\mathcal{S}_i$ is opaque w.r.t. a composition's result and Σ_a, it must be opaque w.r.t. the whole system $G_1||\cdots||G_n$ and Σ_a by Theorem 18. This is formalized in Algorithm 2.

This method allows us to verify opacity for each module's secret $\mathcal{S}_i$ much more efficiently. In the worst case, we

Algorithm 2 Verify opacity of $\mathcal{S}_i$ w.r.t. $G_1||\cdots||G_n$ and Σ_a.

input: G_i and its secret states $\mathcal{S}_i$, all LTSs $G_1, \ldots, G_n$, and the attacker alphabets $\Sigma_{a,j}$ for $j \in \{1, \ldots, n\}$. Assumes $\Sigma_s \subseteq \Sigma_a$ (pairwise shared events must be observed by the attackers).
output: true iff $\mathcal{S}_i$ is opaque wrt $G_1||\cdots||G_n$ and Σ_a.

```
if S_i is opaque w.r.t. Det_{Σ_{a,i}}(G_i) and Σ_a then
    return true                                   ▷ By Corollary 19.
D ← Det_{Σ_{a,i}}(G_i)
for all G_j ∈ {G_1, ..., G_n} \ {G_i} do
    D ← D||Det_{Σ_{a,j}}(G_j)
    if S_i is opaque w.r.t. D and Σ_a then
        return true                               ▷ By Corollary 19.
return false
```

still need to build $Det_{\Sigma_{a,1}}(G_1)||\cdots||Det_{\Sigma_{a,n}}(G_n)$ to verify opacity of each $\mathcal{S}_i$. However, for systems where all modules are already individually opaque, we only need to construct $Det_{\Sigma_{a,i}}(G_i)$ for each module, allowing us to verify opacity without building the monolithic system.

Theorem 21. Algorithm 2 is in $\Theta(2^{mn})$.

Proof. Let $m = \max_{i\in\{1,\ldots,n\}} |\mathcal{Q}_i|$, the maximum size of a module's state set. In the worst case, we multiply n structures of size 2^m and we check each of these states to verify if it is opaque. Thus, the algorithm is in $\Theta(2^{mn})$. ■

Example 22. Let us consider again the determinized LTS of Figure 2. We note that neither $\mathcal{S}_1 = \{3\}$ nor $\mathcal{S}_2 = \{3'\}$ are opaque w.r.t. their respective modules and Σ_a. Thus, we must compose $Det_{\Sigma_{a,1}}(G_1)||Det_{\Sigma_{a,2}}(G_2)$ to verify whether each $\mathcal{S}_i$ is opaque. The result is in Figure 5. From this, we can observe that $\mathcal{S}_1$ is not opaque w.r.t. $G_1||G_2$ and Σ_a, but $\mathcal{S}_2$ is because $\{3'\}$ is no longer accessible. □

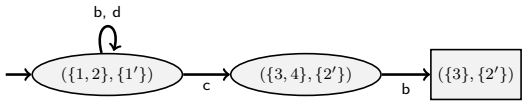


Fig. 5. $Det_{\Sigma_{a,1}}(G_1)||Det_{\Sigma_{a,2}}(G_2)$ with $\Sigma_s = \{\mathsf{c}\}$ and $\Sigma_a = \{\mathsf{b},\mathsf{c},\mathsf{d}\}$

3.3 Heuristics for Opacity Verification

We run Algorithm 2 for every secret $\mathcal{S}_i$. The worst case complexity only occurs in interconnected systems (i.e., with one connected component in the interaction graph) where $\mathcal{S}_i$ is not opaque. This is considerably better than taking the product of all modules and determinizing the result, which would be in $\mathcal{O}(2^{m^n})$. Nevertheless, it would be desirable to check for some secrets simultaneously. In particular, if we add G_j to the composition, we can check if $\mathcal{S}_j$ is opaque in the composition at the same time as checking $\mathcal{S}_i$. This means we do not need to run the algorithm separately for $\mathcal{S}_j$. This has the potential to reduce the number of times the algorithm must be run.

The runtime of Algorithm 2 depends on the selection of which LTS to compose first. Composing G_i with one LTS might immediately make $\mathcal{S}_i$ opaque w.r.t. the composed system and Σ_a, but with another, it might not. Thus, while correctness does not depend on the ordering, algorithm runtime does. Some heuristics to note include composing LTSs in the order of the most shared events or the fewest local events. Heuristics for choosing the order of LTSs in incremental verification algorithms are discussed in Brandin et al. (2004).

4. CONTROLLER SYNTHESIS

In cases where secrets are not opaque, we can use supervisory control to enforce opacity. In particular, we are interested in generating controllers locally for every module to enforce the local module's secret. Leveraging the fact that opacity holds over composition when we assume $\Sigma_s \subseteq \Sigma_a$, we present an algorithm for generating more permissive controllers than those produced by standard algorithms.

4.1 Problem Formulation

We are interested in synthesizing n controllers $C_1, \ldots, C_n$. Each controller C_i is local to a single G_i and enforces the opacity of $\mathcal{S}_i$ w.r.t. the controlled monolithic system and Σ_a. That is, while the local controller C_i knows the construction of the whole system, it is only concerned about keeping the local secret $\mathcal{S}_i$ and it can only observe and control local events in module G_i.

Globally, we have an observable alphabet $\Sigma_o \subseteq \Sigma$ and controllable alphabet $\Sigma_c \subseteq \Sigma_o$. Since each controller is local to its own module, C_i has the observable alphabet $\Sigma_{o,i} = \Sigma_o \cap \Sigma_i$ and the controllable alphabet $\Sigma_{c,i} = \Sigma_c \cap \Sigma_i \subseteq \Sigma_{o,i}$. As in Section 3.1, we let $\Sigma_{a,i} = \Sigma_a \cap \Sigma_i$.

Problem 23. Suppose we have n modules G_i with secrets $\mathcal{S}_i$ and a global attacker $\mathcal{A}$ with alphabet Σ_a. Furthermore, suppose that we want to build local controllers C_i that each have an observable alphabet $\Sigma_{o,i}$ and a controllable alphabet $\Sigma_{c,i}$ such that $\Sigma_{c,i} \subseteq \Sigma_{o,i} \subseteq \Sigma_i$, and each controller enforces the opacity of only one secret $\mathcal{S}_i$. How can we generate a controller C_i for every G_i such that $\mathcal{S} = \bigcup_{i=1}^{n} (\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$ *is opaque w.r.t. $(C_1/G_1)||\cdots||(C_n/G_n)$ and Σ_a?*

4.2 Enforcing Opacity using Control

First, we want to ensure that if we perform control to enforce the opacity of one secret, we do not expose another secret.

Theorem 24. Let $\Sigma_s \subseteq \Sigma_a$. If $\mathcal{S}_1$ is opaque w.r.t. C_1/G_1 and $\Sigma_{a,1}$, then for any valid controller C_2 for G_2, $\mathcal{S}_1$ is opaque w.r.t. $(C_1/G_1)||(C_2/G_2)$ and Σ_a.

Proof. By Theorem 18 and Corollary 19, opacity is preserved by composition when $\Sigma_s \subseteq \Sigma_a$. Thus, regardless of what we compose with C_1/G_1, $\mathcal{S}_1$ is opaque w.r.t. the composition and Σ_a. ■

This leads us to a simple, but naive, approach for synthesizing controllers to enforce opacity of $\mathcal{S}_i$ w.r.t. the system $G_1||\cdots||G_n$ and Σ_a.

Proposition 25. To enforce the opacity of $\mathcal{S}_i$ w.r.t. the system $G_1||\cdots||G_n$ and Σ_a, we can get the supremal controller for enforcing opacity in the local module. By doing this for every module, we find that $\mathcal{S}$ is opaque w.r.t. $(C_1/G_1)||\cdots||(C_n/G_n)$ and Σ_a.

Proof. We know that $\mathcal{S}_i$ is opaque w.r.t. C_i/G_i and $\Sigma_{a,i}$ by definition of the supremal controller. By Theorem 24, we know that every $\mathcal{S}_i$ is opaque w.r.t. $(C_1/G_1)||\cdots||(C_n/G_n)$ and Σ_a. By Corollary 13, we know

$$\mathcal{S} = \bigcup_{i=1}^{n} (\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$$

is opaque w.r.t. $(C_1/G_1)||\cdots||(C_n/G_n)$ and Σ_a. ■

Unfortunately, this approach can be restrictive, resulting in less permissive controllers than we would have if we considered information about other modules as well.

Example 26. Consider G_2 in Figure 2. Suppose that $\Sigma_{o,2} = \Sigma_2$ and $\Sigma_{c,2} = \{\mathsf{d}\}$ (i.e., c is uncontrollable). Since $\{3'\}$ is a secret state, we need to disable the controllable transitions leading to it. Because c is uncontrollable, we note that no valid supremal controller can exist. However, we have already shown that $\mathcal{S}_2$ is opaque w.r.t. $G_1||G_2$ and Σ_a. In the monolithic system, no control is needed to enforce opacity of $\mathcal{S}_2$. Thus, using the naive method to produce individual controllers is restrictive. □

One way to make the controllers more permissive is to use a fully permissive controller whenever $\mathcal{S}_i$ is opaque w.r.t. $G_1||\cdots||G_n$. To ensure that this works, we need to show that performing control on other modules will not affect the opacity of $\mathcal{S}_i$.

Theorem 27. Let $\mathcal{L}_1 \subseteq \Sigma_1$, $\mathcal{L}_2 \subseteq \Sigma_2$, and $\Sigma_s \subseteq \Sigma_a$. If $\mathcal{L}_{\mathcal{S}_1} \subseteq \mathcal{L}_1$ is opaque w.r.t. $\mathcal{L}_1||\mathcal{L}_2$ and Σ_a, then for any $\mathcal{L}_2' \subseteq \mathcal{L}_2$, $\mathcal{L}_{\mathcal{S}_1}$ is opaque w.r.t. $\mathcal{L}_1||\mathcal{L}_2'$.

Proof. Consider all $t \in \mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2$. Let $t_1 = P_{\Sigma_1}(t) \in \mathcal{L}_1$ and $t_2 = P_{\Sigma_2}(t) \in \mathcal{L}_2$. Since $\mathcal{L}_{\mathcal{S}_1}$ is opaque w.r.t. $\mathcal{L}_1||\mathcal{L}_2$ and $\Sigma_{a,1}$, by Definition 4, $\exists t' \in (\mathcal{L}_1||\mathcal{L}_2) \setminus (\mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2)$ such that $P_{\Sigma_a}(t) = P_{\Sigma_a}(t')$. So, we know there is a $t_1' = P_{\Sigma_1}(t') \in \mathcal{L}_1 \setminus \mathcal{L}_{\mathcal{S}_1}$. There are two cases we need to consider. In the first case, suppose $t_2 \in \mathcal{L}_2'$. Then, $t \in \mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2'$ and $t' \in (\mathcal{L}_1||\mathcal{L}_2') \setminus (\mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2')$ by Remark 1. In the second case, suppose $t_2 \notin \mathcal{L}_2'$. Then, $t \notin \mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2'$ and $t' \notin (\mathcal{L}_1||\mathcal{L}_2') \setminus (\mathcal{L}_{\mathcal{S}_1}||\mathcal{L}_2')$ by Remark 1. Thus, opacity holds: $\mathcal{L}_{\mathcal{S}_1}$ is opaque w.r.t. $\mathcal{L}_1||\mathcal{L}_2'$. ■

This means that restricting behavior of other modules will not affect the opacity of the current module's secret, allowing us to synthesize more permissive controllers. In the algorithm below, $\mathcal{C}^{\uparrow}$ represents the supremal controller generated by the algorithm from Tong et al. (2018).

Algorithm 3 Synthesizing controller C_i for module G_i.

input: G_i and its secret states $\mathcal{S}_i$, all LTSs $G_1, \ldots, G_n$, the attacker alphabets $\Sigma_{a,j}$ for $j \in \{1, \ldots, n\}$, and the observable and controllable alphabets $\Sigma_{o,i}$ and $\Sigma_{c,i}$.
output: C_i, the controller for G_i.

1: **if** $\mathcal{S}_i$ is opaque w.r.t. $G_1||\cdots||G_n$ and Σ_a **then**
2: **return** a fully permissive controller C_i
3: $G \leftarrow G_1||\cdots||G_n$
4: $\mathcal{S} \leftarrow (\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$
5: **return** $C_i \leftarrow \mathcal{C}^{\uparrow}$ enforcing opacity of $\mathcal{S}$ wrt G, Σ_a, $\Sigma_{o,i}$, and $\Sigma_{c,i}$

We compute C_i from Algorithm 3 in two steps: we first check if opacity holds, in which case a fully permissive controller suffices. Otherwise, we compute the supremal controller $\mathcal{C}^{\uparrow}$ that enforces opacity of the module's secret $\mathcal{S}_i$, w.r.t. the monolithic system $G_1||\cdots||G_n$ and attacker's alphabet Σ_a, using the algorithm from Tong et al. (2018). The resulting controller has the observable alphabet $\Sigma_{o,i}$ and controllable alphabet $\Sigma_{c,i}$: it is the controller for module G_i, only observing events local to the module. We consider the entire system to build the controller rather than a single module G_i, as it results in more permissive controllers.

By Remark 15, it suffices to only consider modules in the same connected component in the interaction graph (such as the graph in Figure 4). Enforcing opacity in the connected component will enforce it for the monolithic system. In Algorithm 3, we assume that $G_1, \ldots, G_n$ are all part of the same connected component in the interaction graph. The algorithm would still work if they were not in the same connected component, but with reduced efficiency.

Theorem 28. If we use Algorithm 3 to synthesize every C_i, then $\mathcal{S} = \bigcup_{i=1}^{n} (\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$ is opaque w.r.t. $(C_1/G_1)||\cdots||(C_n/G_n)$ and Σ_a.

Proof. By construction of the supremal controller, $\mathcal{S}$ is opaque w.r.t. $(C_i/G_i)||(G_{j_1}||\cdots||G_{j_m})$ and Σ_a. ■

Recall that we assumed that all modules are in the same connected component. By Remark 15, even if more modules in a different connected component are added, opacity still holds.

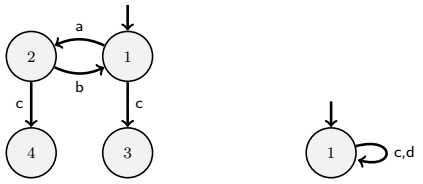
Note that the controller C_i is observable and controllable, by construction of the supremal controller. When there are other connected components, controllability still holds with the full system because controllability holds under arbitrary parallel composition.

There are two standard assumptions when synthesizing a supremal optimal controller: either the attacker knows the construction of both the system and the controller, or it only knows the system. For the first case, to compute such a controller, Dubreil et al. (2010) assume that $\Sigma_a \subseteq \Sigma_o$ or $\Sigma_o \subseteq \Sigma_a$. In general, neither of these hold. The observable alphabet for the controller is $\Sigma_{o,i}$ and the attacker's alphabet is Σ_a; that is, the attacker can observe events from other modules. Thus, in many cases, there does not yet exist an algorithm for finding a supremal controller when the attacker knows the architecture of both the system and the controller. For the second case, an algorithm from Tong et al. (2018) only requires that the controllable alphabet is a subset of the observable alphabet. Our controllers satisfy this property because $\Sigma_{c,i} \subseteq \Sigma_{o,i}$. Thus, we focus on this latter case.

Theorem 29. Algorithm 3 is in $\mathcal{O}(2^{2(m^n \times 2^{m^n} + |\Sigma_c|)})$.

Proof. Let $m = \max_{i \in \{1, \cdots, n\}} |\mathcal{Q}_i|$, the maximum size of a module's state set. Building $G_1||\cdots||G_n = (\mathcal{Q}, \Sigma, \delta, q_0)$ requires assembling (at most) n structures of size m, so $|\mathcal{Q}|$ is smaller than m^n. We then generate the supremal controller of this structure using the algorithm of Tong et al. (2018), which is $\mathcal{O}(2^{2(|\mathcal{Q}| \times 2^{|\mathcal{Q}|} + |\Sigma_c|)})$. Thus, Algorithm 3 synthesizes a controller for one module, and is in $\mathcal{O}(2^{2(m^n \times 2^{m^n} + |\Sigma_c|)})$ ■

Example 30. Consider again the example of Figure 1. Figure 5 shows $\mathcal{S}_2$ is opaque w.r.t. $G_1||G_2$ and Σ_a. Hence C_2 can be the fully permissive controller (we assume $\Sigma_{o,2} = \Sigma_2$). To enforce opacity of $\mathcal{S}_1$, we have to restrict the behavior of G_1 using the supremal controller. Assuming that $\Sigma_{o,1} = \Sigma_1$ and $\Sigma_{c,1} = \{a, b\}$, the final controller C_1 is depicted in Figure 6. Note that this is more permissive than the naive approach from Proposition 25: in this case, the naive approach does not yield a set of valid controllers. □



(a) $\mathcal{C}_1$, the controller for G_1 (b) $\mathcal{C}_2$, the controller for G_2

Fig. 6. Modular controllers produced using Algorithm 3 to enforce the opacity of $\mathcal{S}_1$ and $\mathcal{S}_1$ w.r.t. $G_1||G_2$ and Σ_a

We can now prove that synthesizing opacity-preserving controllers for each connected component of the interaction graph separately gives a set of controllers that preserve all secrets.

Theorem 31. Consider a monolithic supremal controller $\mathcal{C}$ that enforces the opacity of $\mathcal{S} = \bigcup_{i=1..n} (\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$ w.r.t. $G_1||\cdots||G_n$ and Σ_a. Then, if $\Sigma_s = \emptyset$, $\mathcal{L}((\mathcal{C}_1/G_1)||\cdots||(\mathcal{C}_n/G_n)) = \mathcal{L}(\mathcal{C}/(G_1||\cdots||G_n))$.

Proof. Let $\mathcal{S}_i' = (\mathcal{Q}_1 \times \cdots \times \mathcal{Q}_{i-1}) \times \mathcal{S}_i \times (\mathcal{Q}_{i+1} \times \cdots \times \mathcal{Q}_n)$. As $\Sigma_s = \emptyset$, every module is a separate connected component and Algo. 3 only considers the local module G_i when synthesizing C_i (recall that the Algo 3 runs only on modules in the same connected component).

Suppose $t_1.\sigma.t_2 \in \mathcal{L}_{\mathcal{S}_i}(G_i)$, with $\sigma \in \Sigma_{c,i}$ and $t_2 \in (\Sigma_i \setminus \Sigma_{c,i})^*$. Suppose there is no non-secret string $t' \in \mathcal{L}(G_i) \setminus \mathcal{L}_{\mathcal{S}_i}(G_i)$ where $P_{\Sigma_a}(t_1.\sigma.t_2) = P_{\Sigma_a}(t')$. Then, C_i must disable σ after observing $P_{\Sigma_{o,i}}(t_1)$. Now, consider all $t_1' \in \mathcal{L}(G_1||\cdots||G_n)$ where $P_{\Sigma_i}(t_1') = t_1$. Since $\Sigma_s = \emptyset$, we know that $t_1'.\sigma.t_2 \in \mathcal{L}_{\mathcal{S}_i'}(G_1||\cdots||G_n)$ and there is no corresponding non-secret string. Thus, we know that $\mathcal{C}$ must disable σ after seeing $P_{\Sigma_o}(t_1')$, as well, because $t_2 \in (\Sigma \setminus \Sigma_c)^*$. Thus, anything C_i disables is also disabled by $\mathcal{C}$: $\mathcal{L}(\mathcal{C}/(G_1||\cdots||G_n)) \subseteq \mathcal{L}((C_1/G_1)||\cdots||(C_n/G_n))$. But since $\mathcal{L}((C_1/G_1)||\cdots||(C_n/G_n))$ is controllable and observable, and because $\mathcal{S}$ is opaque w.r.t. this controlled system and Σ_a, we know $\mathcal{L}((C_1/G_1)||\cdots||(C_n/G_n)) \subseteq \mathcal{L}(\mathcal{C}/(G_1||\cdots||G_n))$; the supremal monolithic controller is maximally permissive by definition. Thus, their languages are equal. ■

5. CONCLUSIONS

Under the assumption that the attacker can observe the interface between modules, we have presented a strategy for validating and ensuring that every set of secret states $\mathcal{S}_i$ is opaque w.r.t. a composed system $G_1||\cdots||G_n$ and Σ_a by composing modules only as needed. This makes it possible to evaluate opacity in large modular systems, where traditional algorithms might not be feasible since they involve constructing the monolithic system. Furthermore, our algorithm for generating local controllers makes it possible to enforce opacity for each module separately. This is an essential technique when a controller cannot effectively control modules—for instance, if they are in different geographical locations.

In the future, we are interested in synthesizing controllers where the attacker is aware of the controllers' structure by extending techniques from Dubreil et al. (2010). We are also interested in how this could be extended to modal transition systems, another model which provides some freedom of implementation of the modules.

REFERENCES

Badouel, E., Bednarczyk, M., Borzyszkowski, A., Caillaud, B., and Darondeau, P. (2007). Concurrent secrets. *Discrete Event Dyn. Syst.*, 17(4), 425–446.

Brandin, B., Malik, R., and Malik, P. (2004). Incremental verification and synthesis of discrete-event systems guided by counter-examples. *IEEE Trans. Contr. Syst. Technol.*, 12(3), 387–401.

Bryans, J.W., Koutny, M., Mazaré, L., and Ryan, P.Y.A. (2008). Opacity generalised to transition systems. *Int. J. Inf. Secur.*, 7(6), 421–435.

Cassez, F., Dubreil, J., and Marchand, H. (2012). Synthesis of opaque systems with static and dynamic masks. *Form. Methods Syst. Des.*, 40(1), 88–115.

Contant, O., Lafortune, S., and Teneketzis, D. (2006). Diagnosability of discrete event systems with modular structure. *Discrete Event Dyn. Syst.*, 16(1), 9–37.

de Queiroz, M. and Cury, J. (2000). Modular control of composed systems. In *ACC*, 4051–4055.

Dubreil, J., Darondeau, P., and Marchand, H. (2008). Opacity enforcing control synthesis. In *WODES*, 28–35.

Dubreil, J., Darondeau, P., and Marchand, H. (2010). Supervisory control for opacity. *IEEE Trans. Automat. Contr.*, 55(5), 1089–1100.

Jacob, R., Lesage, J.J., and Faure, J.M. (2016). Overview of discrete event systems opacity. *ARC*, 41, 135–146.

Jezequel, L. and Fabre, E. (2012). Turbo planning. In *WODES*, 301–306.

Lin, F. and Wonham, W. (1988). On observability of discrete-event systems. *Info. Sci.*, 44(3), 173 – 198.

Masopust, T. and Yin, X. (2019). Complexity of detectability, opacity and a-diagnosability for modular discrete event systems. *Automatica*, 101, 290–295.

Mazaré, L. (2005). Decidability of opacity with non-atomic keys. In *FAST*, 71–84.

Ramadge, P.J. and Wonham, W.M. (1982). Supervision of discrete event processes. In *CDC*, 1228–1229.

Saboori, A. and Hadjicostis, C.N. (2010). Reduced-complexity verification for initial-state opacity in modular discrete event systems. In *WODES*, 78–83.

Tong, Y. and Lan, H. (2019). Current-state opacity verification in modular discrete event systems. In *CDC*, 7665–7670.

Tong, Y., Li, Z., Seatzu, C., and Giua, A. (2018). Current-state opacity enforcement in discrete event systems under incomparable observations. *Discrete Event Dyn. Syst.*, 28(2), 161–182.