Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Adaptive Discontinuous Control for Homogeneous Systems Approximated by Neural Networks ⋆

Mariana Ballesteros *, **Andrey Polyakov** **,***, **Denis Efimov** **,***, **Isaac Chairez** ****, **Alexander Poznyak** *

* *Department of Automatic Control, CINVESTAV-IPN Av. IPN 2508, 07360, Mexico City (e-mail: mballesteros@ctrl.cinvestav.mx)*
** *Inria, Univ, Lille, CNRS, UMR 9189 - CRIStAL, F-59000 Lille, France (e-mail: andrey.polyakov, denis.efimov@inria.fr)*
*** *Department of Control Systems and Informatics, University ITMO, 49 Av. Kronverkskiy, 197101 Saint Petesburg, Russia*
**** *Department of Bio-processes, Unidad Profesional Interdisciplinaria de Biotecnología-Instituto Politécnico Nacional, Mexico City*

Abstract: This study is devoted to the design of an adaptive discontinuous control based on differential neural networks (DNNs) for a class of uncertain homogeneous systems. The control is based on the universal approximation properties of artificial neural networks (ANNs) applied on a certain class of homogeneous nonlinear functions. The adaptation laws for the DNNs parameters are obtained with the application of the Lyapunov stability theory and the homogeneity properties of the approximated nonlinear system. The stability analysis of the closed loop system with the proposed controller is presented. The estimation error in the approximation of the uncertain homogeneous functions is considered in the stability analysis. The performance of the controller is illustrated by means of a numerical simulation of a homogeneous model.

Keywords: Homogeneous systems, Neural Networks, Adaptive Control, Discontinuous Control, System Identification.

1. INTRODUCTION

The design of automatic controllers aimed to solve the trajectory tracking problem for systems described with uncertain models and subjected to external perturbations remains as a significant area in systems theory. In the recent literature, there are numerous works related with the tracking control problem for systems with unknown elements. One remarkable approach is the application of robust techniques (Harashima et al., 1987; Orlov, 2005; Alazki and Poznyak, 2013). The results obtained with robust controllers are reliable. However, a regular robust controller has the disadvantage of being designed considering only the worst possible scenario for the uncertain system. Therefore, it is insensitive to minor system changes providing the same energy in the control signal which could not be suitable in many real plants (Doyle, 1983; Utkin et al., 2009).

On the other hand, compensating controllers, which use an approximation of the unknown dynamics have solved the trajectory tracking problems with a bounded tracking error for a wide variety of nonlinear systems. This error can be characterized by the quality of the approximation (Join et al., 2005; Castañeda et al., 2014; **?**). The approximation of system dynamics in control theory is known as non-parametric system identification. The structure of the identified systems is used to design the adaptive controllers. There are numerous algorithms for system identification (Jagannathan and Lewis, 1996; Ljung, 1999; Haber and Keviczky, 1999; Ljung, 2006) considering an extensive class of systems with uncertainties. In such cases, the design of the identifier depends on the available data entering and coming out from the system. In this work, the considered systems are known as homogeneous systems (Bernuau et al., 2014; **?**).

The homogeneous dynamic systems are represented by ordinary differential equations, whose vector fields posses a kind of symmetry under a specific nonlinear transformation. In other words, a dilation over the arguments results in a scaling over the vector fields. This characteristic allows to extend the local attributes (in a compact, i.e. the unit sphere) globally.

A homogeneous representation of nonlinear systems can be more useful than a linear approximation. For example, a linearized system may not be conclusive on the stability of the original system, meanwhile the homogeneous approximation could give a stability conclusion (Rosier, 1992). In addition, the stability of homogeneous systems has been studied and the type of stability of the system is related with the degree of homogeneity (or if some part of the approximation is not homogeneous, the stability of the equilibrium) (Bacciotti and Rosier, 2006; Bhat and Bernstein, 2005; Zimenko et al., 2017). Indeed, there exist techniques to obtain an homogeneous Lyapunov function (Hermes, 1995; Kawski, 1995; Polyakov et al., 2016) for homogeneous systems.

Homogeneous approximations of known nonlinear vector fields have been studied in Hermes (1986), Kawski (1988), among others. However, studies in the design of non-parametric iden-

⋆ This work was partially supported by the Government of Russian Federation (Grant 08-08) and the Ministry of Education and Science of Russian Federation (Project 14.Z50.31.0031).
Mariana Ballesteros is sponsored by Mexico scholarship from the National Council for Science and Technology, (Scholar reference: 550803)

tifiers or observers for homogeneous systems with uncertainties are scarce in the literature. An approach for realizing the system identification is the use of the approximation theory, which is based on the selection of a suitable function basis to estimate the uncertainty. The design of identifiers based on Artificial Neural Networks (ANN) is an example of this method (Nelles, 2013). ANN structures are known for their approximation capability of multivariate nonlinear continuous functions (Hornik et al., 1989; Park and Sandberg, 1991; Haykin, 1994). Based on this property, ANN structures have been used in control theory to approximate nonlinear dynamics of ordinary differential equations, partial differential equations and for the design of identifiers and observers. Another advantage of the ANN structures is that they can be used to approximate multi-input and multi-output systems (Lewis et al., 1998; Poznyak et al., 2001; Chairez, 2017).

This work presents the design of an adaptive controller to solve the trajectory tracking problem for uncertain homogeneous systems based on a homogeneous representation implementing differential ANNs. The **main contributions** and novelties of this work are:

- The design of an adaptive control for the tracking trajectory problem of homogeneous uncertain systems.
- The design of an adaptive discontinuous controller that uses a compensating scheme with an ANN homogeneous identifier.
- The extension of differential ANN with homogeneous functions to approximate homogeneous uncertain systems.
- A two structure controller, the first related with the compensation and the second to ensure the tracking error convergence.

This work has the following outline: Section 2 describes the dynamic system under study and provides the problem statement. Section 3 contains the justification of the ANN approximation property for the class of systems in this manuscript. In Section 4, the identification of the uncertain system based on ANN is presented. Section 5 is devoted to the adaptive control design to solve the trajectory tracking problem based on the ANN identification. Section 6 studies the performance of the suggested control scheme by means of numerical simulations. Finally, Section 7 closes the paper with some conclusions and observations for future work related with the presented approach.

Notation: The symbol $\mathbb{R}_+ = \{p \in \mathbb{R} \colon p \geq 0\}$, with $\mathbb{R}$ as the set of real numbers and $\mathbb{R}^n$ is the vector space of n real elements. For $M \in \mathbb{R}^{r\times n}$, $\text{vec}(M) = \left[m_1^\top, \dots, m_n^\top\right]^\top$. The Kronecker product is described by $\otimes$. The identity matrix of dimension n is depicted by I_n. The Euclidean norm is denoted as $\|\cdot\|$ and for the matrix space is considered the Frobenius norm $\|M\|_F = \sqrt{\text{tr}\{M^\top M\}}$, where $\text{tr}\{H\} = \sum_{i=1}^{n} H_{ii}$ is the trace of a matrix $H \in \mathbb{R}^{n\times n}$, H_{ii} is the element of the i-th column and the i-th row, $\text{diag}\{a_i\}_{i=1,\dots,m}$ is a diagonal matrix of dimension m, $a_i \in \mathbb{R}$ is the $i-th$ element of the diagonal, $[p_i]_{i=1,\dots,m}$ is a matrix with m columns $p_i \in \mathbb{R}^n$.

2. PROBLEM STATEMENT

The class of systems with uncertain model considered in this work, is described by the following ordinary differential equation:

$$\dot{z}(t) = f_0(z(t)) + \sum_{i=1}^{m} f_i(z(t))u_i,\ z(0) = z_0,\ t \in \mathbb{R}_+, \quad (1)$$

where $z \in \mathbb{R}^n$ is the states vector, $u_i \in \mathbb{R}$ are the control inputs, m is the number of inputs, $m \leq n$, $z_0 \in \mathbb{R}^n$ is the initial condition and $f_i : \mathbb{R}^n \to \mathbb{R}^n$, $i = 0, ..., m$ are unknown vector fields.

The following assumptions are considered in the design of the adaptive control.

Assumption 1. The unknown vector fields are homogeneous of degree $k_i \in \mathbb{R}$, i.e.

$$f_i(\varepsilon z) = \varepsilon^{k_i} f_i(z),\ \forall \varepsilon > 0,\ \forall z \in \mathbb{R}^n,\ i = 0, \dots, m.$$

Assumption 2. The vector fields are continuous in the unit sphere:

$$S = \{z \in \mathbb{R}^n : \|z\| = 1\}. \quad (2)$$

Assumption 3. All the states are on-line measured and bounded.

The **main goal** of the control design is to obtain a control input, such that, the trajectory tracking error

$$e = z - z^*, \quad (3)$$

is bounded, i.e.

$$\limsup_{t\to\infty} \|e(t)\| \leq \sigma(\psi^+) \leq +\infty, \quad (4)$$

where $z^* \in \mathbb{R}^n$ is the vector of the reference trajectories, ψ^+ defines the approximation quality of the unknown vector fields f_i, $i = 0, \dots, m$, and the function $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$ is a class-$\mathcal{K}$ function (See Khalil and Grizzle (2002)). This goal considers the universal approximation property of ANN structures (Hornik, 1991).

The following assumption is considered for the references trajectories.

Assumption 4. The dynamics of the reference trajectories are given by:

$$\tfrac{d}{dt} z^* = \varphi(z^*, t). \quad (5)$$

The vector field $\varphi : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is bounded and continuous.

The design of $\varphi(z^*, t)$ can be realized using diverse trajectory planning methods.

3. NEURAL NETWORK APPROXIMATION OF HOMOGENEOUS FUNCTIONS

The approximation property of ANN is based in the weighted superposition of nonlinear functions such as polynomials, radial basis functions and sigmoidal functions (Hornik et al., 1989; Cybenko, 1989; Funahashi, 1989).

The following Theorem guarantees the approximate realization of continuous mappings using bounded and monotone increasing differentiable functions.

Theorem 1. (Funahashi, 1989) Let $\phi(z)$ be a constant, bounded and monotone increasing continuous function. Let K be a compact subset (bounded closed subset) of $\mathbb{R}^n$ and $g(z_1, \dots, z_n)$ be a real valued continuous function on K. Then, for an arbitrary $\psi \in \mathbb{R}_+$, there exists and integer N and real constants c_i, θ_i and ω_{ij}, $i = 1, \dots, N$ and $j = 1, \dots, n$, such that,

$$\tilde{g}(z_1, \dots, z_n) = \sum_{i=1}^{N} c_i \phi\left(\sum_{j=1}^{n} \omega_{ij} z_j - \theta_i\right), \quad (6)$$

satisfies $\max_{z\in K}|g(z_1,\ldots,z_n)-\tilde{g}(z_1,\ldots,z_n)|<\psi$.

Notice that Theorem 1 is formulated for multi-layer ANN structures with a static structure. This property has been used for the approximation of dynamic systems.

Remark 1. In this work, the ANN structures are designed linear in the parameters (Lewis et al., 1998). In Igelnik and Pao (1995) and Lewis et al. (1998), it is stated that the parameters w_{ij}, and θ_i in (6) can be selected randomly using a uniform distribution and the universal approximation property holds by finding only the parameters c_i.

Theorem 1 states the approximation property of ANN for the estimation of continuous functions on a compact, the next Corollary extends the approximation property for the systems represented by (1). Then, the proposed ANN structure (8) provides a global approximation of the vector fields f_i.

Corollary 2. (Ballesteros et al., 2019) Consider that (1) satisfies the assumptions 1-3. Then, for any $\psi_i\in\mathbb{R}_+$ and for any Hurwitz matrix $A\in\mathbb{R}^{n\times n}$, there exist $N_i\in\mathbb{R}$ and $W_i^*\in\mathbb{R}^{n\times N_i}$, $i=0,1,..,m$ such that:

$$\|d(z,u)\|\le\psi_0\|z\|^{k_0}+\sum_{i=1}^{m}\psi_i\|z\|^{k_i}|u_i|,\quad \forall z\in\mathbb{R}^n,\ \forall u\in\mathbb{R}^m,\ \forall t\in\mathbb{R}_+, \tag{7}$$

where $d(z,u):=\dot{z}-F(z,u)$ and the approximate function $F(z,u)$ is given by:

$$F(z,u)=\|z\|^{k_0}\left[A\tfrac{z}{\|z\|}+W_0^*\phi_0\left(\tfrac{z}{\|z\|}\right)\right]+\sum_{i=1}^{m}\|z\|^{k_i}W_i^*\phi_i\left(\tfrac{z}{\|z\|}\right)u_i. \tag{8}$$

The elements of the vector functions $\phi_i:\mathbb{R}^n\to\mathbb{R}^{N_i}$, $i=0,\ldots,m$, are proposed as sigmoidal activation functions, that is:

$$(\phi_i(z))_j=\left(1+e^{-\left(\beta_{ij}^{\top}z+\alpha_{ij}\right)}\right)^{-1}, \tag{9}$$

where $\alpha_{ij}\in\mathbb{R}_+$ and $\beta_{ij}\in\mathbb{R}^n$ are properly selected parameters with $i=0,\ldots,m$ and $j=1,\ldots,N_i$.

The proof of Corollary 2 was presented in Ballesteros et al. (2019). In view of Remark 1, in the design of the identifier, the parameters α_{ij} and β_{ij}, are selected randomly with a uniform distribution. Therefore, the ANN structures are linear in the parameters and, in consequence, the identification is solved by finding the parameters W_i^*.

4. NEURAL NETWORK REPRESENTATION OF HOMOGENEOUS DYNAMIC SYSTEMS

The solution for the identification of (1) considers the representation (8). Notice that the ANN structures in (8) can be represented as follows

$$W_i^*\phi_i\left(\tfrac{z}{\|z\|}\right)=\Phi_i\left(\tfrac{z}{\|z\|}\right)w_i^*, \tag{10}$$

where $\Phi_i\left(\tfrac{z}{\|z\|}\right)=I_n\otimes\phi_i^{\top}\left(\tfrac{z}{\|z\|}\right)$ and $w_i^*=\mathrm{vec}\left(\left(W_i^*\right)^{\top}\right)$. Therefore, the identification section consists on finding the parameters w_i^* by some adaptive law, such that, the identification error $\Delta=z-\hat{z}$ converges to zero or is small enough (depending on the quality of the ANN approximation), where $\hat{z}\in\mathbb{R}^n$ represents the state vector of the following identifier:

$$\tfrac{d}{dt}\hat{z}=\|z\|^{k_0}A\tfrac{\hat{z}}{\|z\|}+\sum_{i=0}^{m}\left(\|z\|^{k_i}\Phi_i\left(\tfrac{z}{\|z\|}\right)w_iu_i+\|z\|^{k_0}\Omega_iK_i\Omega_i^{\top}\Delta\right), \tag{11}$$

where $i=0,1\ldots,m$, $u_0=1$, $A\in\mathbb{R}^{n\times n}$ is a Hurwitz matrix, and with the adaptive laws given by

$$\tfrac{d}{dt}w_i=-\|z\|^{k_0}K_i\Omega_i^{\top}\Delta, \tag{12}$$

where $K_i\in\mathbb{R}^{nN_i\times nN_i}$ are positive definite matrices and the auxiliary variables $\Omega_i:\mathbb{R}_+\to\mathbb{R}^{n\times nN_i}$ satisfy

$$\tfrac{d}{dt}\Omega_i=\|z\|^{k_0-1}A\Omega_i-\|z\|^{k_i}u_i\Phi_i\left(\tfrac{z}{\|z\|}\right). \tag{13}$$

The following Theorem sums up the result on the convergence of the identification error.

Theorem 3. (Ballesteros et al., 2019) Let assumptions 1-3 be satisfied and the control input u be designed to be a uniformly bounded function, i.e. $|u_i(t)|\le U$, $U>0$, $\forall t\in\mathbb{R}_+$. Consider that (1) can be represented in the form (8) with an estimation error given in (7) and consider the identifier (11) with the adjustment laws (12) and the auxiliary variable adjusted by (13). If $K_i\in\mathbb{R}^{nN_i\times nN_i}$, $i=0,\ldots,m$, are positive definite matrices and the control inputs u_i in (1) are such that

$$\exists z^->0,\ \exists z^+>0,\ z^-<\|z(t)\|<z^+<+\infty\quad\forall t\in\mathbb{R}_+,$$

and the following persistent exitation (PE) condition holds for all $t\in\mathbb{R}_+$ and some $\vartheta_W>0$ and $\ell_W>0$:

$$\int_t^{t+\ell_W}G^{\top}(s)G(s)ds\ge\vartheta_W I_{n\sum_{i=0}^{m}N_i}, \tag{14}$$

where the matrix $G\in\mathbb{R}^{n\times n\sum_{i=0}^{m}N_i}$ is given by

$$G=[\Omega_0,\Omega_1,\ldots,\Omega_m].$$

Then, there exist two class-$\mathcal{K}$ functions σ_1 and σ_2 such that:

$$\limsup_{t\to\infty}\|\Delta(t)\|\le\sigma_1\left(\psi^+\right), \tag{15}$$

$$\limsup_{t\to\infty}\|w_i(t)-w_i^*\|\le\sigma_2\left(\varepsilon^+\right), \tag{16}$$

where $\psi^+=\max_{i=0,\ldots,m}\{\psi_i\}$ and ψ_i are given by (7).

5. CONTROL DESIGN

The main goal of the input design for $u=[u_1,\ldots,u_m]^{\top}$ is to guarantee that the tracking error is small enough (4) and bounded. The input design considers the differential ANN identifier, therefore, the following assumption is needed.

Assumption 5. The matrix $\Xi\in\mathbb{R}^{n\times m}$, defined by

$$\Xi=\left[\Phi_i\left(\tfrac{z}{\|z\|}\right)w_i\right]_{i=1,\ldots,m},$$

is full-rank by columns (m) uniformly with respect to time.

Remark 2. Assumption 5 implies a sufficient condition to justify the controllability of the ANN identifier. This is a reasonable assumption if the closed-loop controller is developed based on the ANN approximation. The demonstration of such fact implies the introduction of a penalty function associated to the controlled Lyapunov function which led to the learning laws in (12) and (13) (Ballesteros et al., 2019). For more details on how introduce weights restrictions in the identifier structure, please see Escudero et al. (2010).

The control solution to solve the tracking control by means of the ANN identification has two main elements, a compensating element u_1 and a second structure to guarantee the tracking convergence u_2 based on the theory of discontinuous controllers.

$$u = \Pi^{-1}(t)D^{-1}(t)\,(u_1+u_2)\,, \tag{17}$$

where $\Pi(t) = \mathrm{diag}\left\{\|z\|^{k_i}\right\}_{i=1,\dots,m}$, $D(t) = H(t)\Xi(t)$. The matrix $H : \mathbb{R}_+ \to \mathbb{R}^{m\times n}$ satisfies

$$\dot{H}(t) = H(t)\Lambda, \qquad H(0) = [H_0, 0_{m,n-m}], \tag{18}$$

where $0_{m,n-m}$ is the matrix with m rows and $n-m$ columns with zero entries, $\Lambda \in \mathbb{R}^{n\times n}$, $\Lambda = \Lambda^\top$, $\Lambda > 0$ and $H_0 \in \mathbb{R}^{m\times m}$ is a diagonal positive definite matrix. The control elements are described as follows

$$\begin{aligned} u_1 = -\|z\|^{k_0}H(t)\left[\sum_{i=1}^{m}\Omega_i K_i \Omega_i^\top \Delta + \Phi_0\left(\tfrac{z}{\|z\|}\right)w_0\right. \\ \left. -\|z\|^{-k_0}\varphi(z^*,t) + A\tfrac{\hat{z}}{\|z\|}\right] - \dot{H}(t)\delta, \end{aligned} \tag{19}$$

where $\delta \in \mathbb{R}^n$, $\delta = \hat{z} - z^*$, and

$$u_2 = -\lambda_s \mathrm{Sign}(s), \tag{20}$$

with $\lambda_s > 0$. $\mathrm{Sign}(s) = [\mathrm{sign(s_i)}]_{i=1,\dots,m}^\top$, s_i is the i-th element of the manifold defined by

$$s = H\delta, \tag{21}$$

and the function

$$\mathrm{sign}(\nu) := \begin{cases} 1 & \text{if } \nu > 0 \\ [-1,1] & \text{if } \nu = 0 \\ -1 & \text{if } \nu < 0. \end{cases} \tag{22}$$

The following Theorem presents the result of the tracking error convergence with the proposed controller.

Theorem 4. Consider that assumptions 1-4 and 5 are fulfilled and consider that (1) is supplied with the input (17) using the ANN identifier structure (11). Then, the tracking error (3) is ultimately bounded by (4).

Proof. The dynamic of (21) satisfies

$$\dot{s} = \dot{H}(t)\delta + H(t)\dot{\delta}. \tag{23}$$

Taking into account the definition of δ, (5) and (11), equation (23) is such that

$$\begin{aligned} \dot{s} = \|z\|^{k_0}H(t)\left(A\tfrac{\hat{z}}{\|z\|} + \Phi_0\left(\tfrac{z}{\|z\|}\right)w_0 + \sum_{i=1}^{m}\Omega_i K_i\Omega_i^\top\Delta\right) \\ +\dot{H}(t)\delta - H(t)\varphi(z^*,t) + D(t)\Pi(t)u(t). \end{aligned} \tag{24}$$

Therefore, the dynamic (24) in closed loop with (17) is

$$\dot{s} = -\lambda_s\mathrm{Sign}(s). \tag{25}$$

Consider the following Lyapunov function candidate $V = \|s\|^2$, the time derivative is

$$\dot{V} = -2\lambda_s s^\top \mathrm{Sign}(s). \tag{26}$$

Then, the following inequality is valid

$$\dot{V} \le -2\lambda_s\sqrt{mV}. \tag{27}$$

This result confirms that $s(t) = 0$, $\forall t \ge (\lambda_s\sqrt{m})^{-1}\|s(0)\|$.

Considering the dynamic of the manifold (23) and the condition $\dot{s} = 0$, the dynamics of δ is

$$H(t)\dot{\delta} = -\dot{H}(t)\delta. \tag{28}$$

By the substitution of (18) on (28), and considering that H is row full-rank, then,

$$\dot{\delta} = -\Lambda\delta. \tag{29}$$

Therefore, the origin is an asymptotically exponentially stable equilibrium point for δ

$$\limsup_{t\to\infty}\|\delta(t)\| = 0. \tag{30}$$

Consider the following representation of (3)

$$e = z - \hat{z} + \hat{z} - z^*. \tag{31}$$

The norm of the tracking error, considering (31), satisfies the following inequality:

$$\|e\| \le \|\Delta\| + \|\delta\|, \quad \forall t \in \mathbb{R}_+. \tag{32}$$

Finally, considering Theorem 3, the bound (4) holds, with $\sigma = \sigma_1$. This completes the proof.

6. NUMERICAL RESULTS

In this section, an academic example is given to show the performance of the designed controller.

$$\begin{aligned} \dot{z_1} &= z_2^{k_0} + (z_1+z_2)^{k_1}\,, \\ \dot{z_2} &= (z_2 - z_1)^{k_0} + z_1^{k_2}u, \end{aligned} \tag{33}$$

where $k_0 = 0.2$, $k_1 = 0.1$ and $k_2 = 0.2$ and the initial condition $z(0) = [2 \quad 3]^\top$. The numerical simulations for the controller were made in Simulink Matlab® applying Dormand-Prince as integration method with a variable integration step.

The ANN was designed with a structure of three activation functions for each vector field.

$$A = \begin{bmatrix} 0 & 1 \\ -7.2 & -4.3 \end{bmatrix}. \tag{34}$$

The initial conditions of the weights are:

$$\begin{aligned} w_0(0) = [1,\ 2,\ 1,\ 1,\ 3,\ 2]^\top,\ w_1(0) = [1,\ 3,\ 1,\ 5,\ 3,\ 2]^\top, \\ w_2(0) = [1,\ 4,\ 1,\ 2,\ 1,\ 6]^\top. \end{aligned} \tag{35}$$

The initial condition of the identifier was $\hat{z}(0) = [7 \quad 5]$. The gain matrices of the identifier were $K_0 = I_2 \otimes \tilde{K}_0$, and $K_1 = K_2 = I_2 \otimes \tilde{K}_1$, with

$$\tilde{K}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 12 \end{bmatrix}, \quad \tilde{K}_1 = \begin{bmatrix} 12 & -1 \\ -1 & 5 \end{bmatrix}. \tag{36}$$

The sliding surface was developed with a gain matrix $H(t)$ having an initial condition of $H(0) = [181]$. The parameters for the control were selected as

$$\Lambda = \begin{bmatrix} 8 & -1 \\ -1 & 6 \end{bmatrix}, \text{ and } \lambda_s = 2. \tag{37}$$

These parameter were obtained using a recurrent approximation method.

Figure 1 compares the trajectories evolution of the first state of the homogeneous system, the reference and the proposed DNN identifier. Notice that either the identification and the tracking trajectory problems are solved within the first second of simulation. This figure also shows the evolution of the first state controlled with a proportional-integral-derivative PID controller with proportional gain of 20, derivative gain of 5 and integral gain of 1. Such variable does not convergence to the actual reference trajectory, as the proposed controller does.

Figure 2 presents a similar comparison between the second state of the homogeneous system, the corresponding reference and the proposed DNN identifier. In this case, the identification and the tracking trajectory problems are solved within the first second of simulation. However, larger oscillations appear at the beginning of the simulation evaluation. Notice that this figure also shows the evolution of the second state controlled with the aforementioned PID controller. Once more, the state obtained with the PID controller does not attain the reference state.

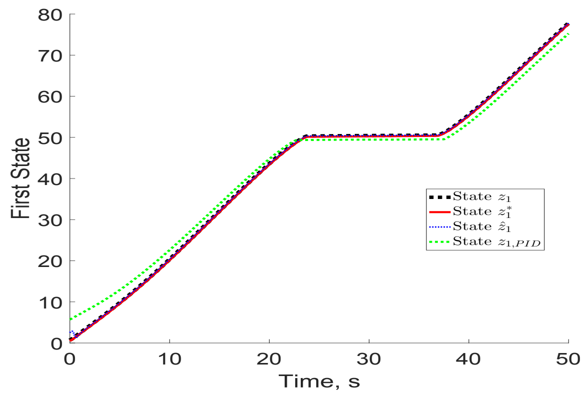

Fig. 1. Comparison between the target trajectory z_1^*, the state of the identifier $\hat{z}_1$ and the state z_1 of the system.

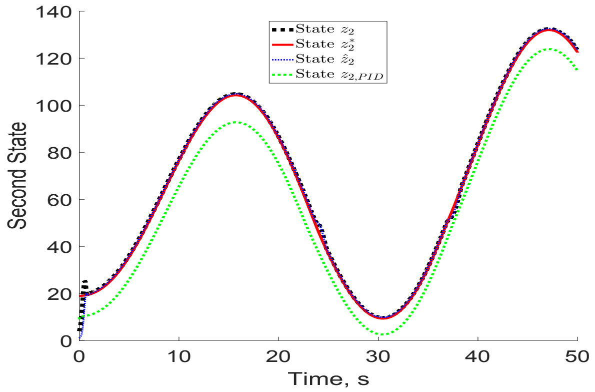

Fig. 2. Comparison between the target trajectory z_2^*, the state of the identifier $\hat{z}_2$ and the state z_2 of the system.

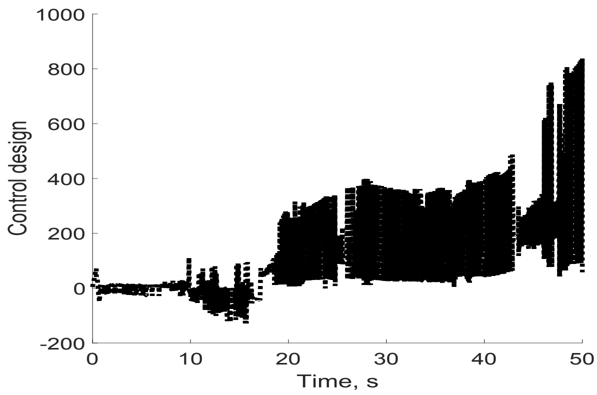

Fig. 3. Evolution of the control signal

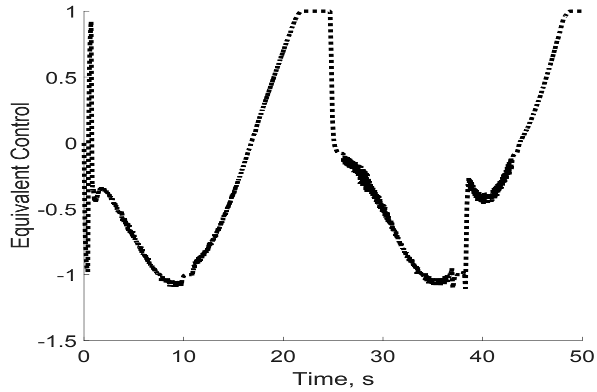

Fig. 4. Evolution of the Sliding manifold

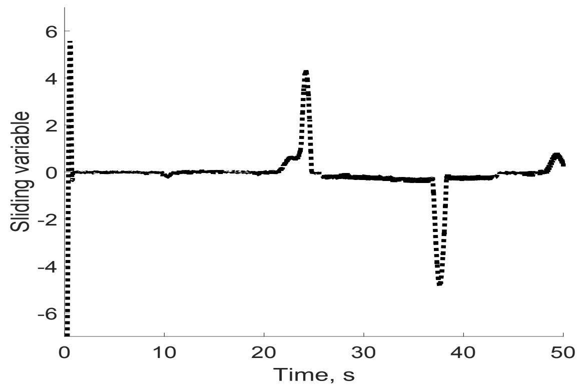

Fig. 5. Sliding variable $s(t)$ evaluated over the time evolution of the tracking error $\delta(t)$.

The comparison of PID and the proposed NN based controller demonstrates the benefits of introducing the DNN compensation. Even if the PID controller may seem too simple for the class of systems considered in this study, it offers a comparative reference to highlight the benefits of the suggested adaptive control structure.

The realization of the control action $u_1 + u_2$ appears in Figure 3. This evolution proves that tracking the states with high precision depends on increasing the control amplitude. This behavior is a consequence of the compensation based on the DNN identifier. Notice that the sliding surface implies the increment of the control amplitude. This control action exhibits the high frequency of the sliding mode realization (undesired chattering effect). This is still an opportunity area which could use an adaptive gain in the sliding control design. Figure 4 depicts the filtered version (with a first order low-pass linear filter and a time constant of 0.1 seconds) of the sliding part of the controller. The crossing of this oscillation on the zero value confirms the resolution of $\delta = 0$. The filtered sliding variable evolution (Figure 4) confirms the movement around the origin of the tracking error. This filtered information defines the resolution of the sliding mode base controller.

7. CONCLUSIONS

The design of an adaptive controller for tracking trajectory of homogeneous uncertain systems, based on differential ANN was developed in this work. The convergence of the tracking error is analyzed by means of the Lyapunov theory. The result on the stability of the tracking error proved that the bound depends on the quality of the approximation. The numerical evaluation of the suggested controller was realized over a two-states system. The suggested mixed controller using the compensation as well as the sliding resolution provides the tracking of the proposed reference trajectories. As a future extension of this work, it is plotted to extend the approximation for the case of weighted homogeneity and to use this property in the controller design to obtain a defined kind of convergence.

REFERENCES

Alazki, H. and Poznyak, A.S. (2013). A class of robust bounded controllers tracking a nonlinear discrete-time stochastic system: Attractive ellipsoid technique application. *Journal of the Franklin Institute*, 350(5), 1008–1029.

Bacciotti, A. and Rosier, L. (2006). *Liapunov Functions and Stability in Control Theory*. Springer Science & Business Media.

Ballesteros, M., Polyakov, A., Efimov, D., Chairez, I., and Poznyak, A. (2019). Differential neural network identification for homogeneous dynamical systems. In *IFAC Nonlinear and Control Systems*.

Bernuau, E., Efimov, D., Perruquetti, W., and Polyakov, A. (2014). On homogeneity and its application in sliding mode control. *Journal of the Franklin Institute*, 351(4), 1866 – 1901. doi:https://doi.org/10.1016/j.jfranklin.2014.01.007. Special Issue on 2010-2012 Advances in Variable Structure Systems and Sliding Mode Algorithms.

Bhat, S.P. and Bernstein, D.S. (2005). Geometric homogeneity with applications to finite-time stability. *Mathematics of Control, Signals and Systems*, 17(2), 101–127.

Castañeda, L.A., Luviano-Juárez, A., and Chairez, I. (2014). Robust trajectory tracking of a delta robot through adaptive active disturbance rejection control. *IEEE Transactions on Control Systems Technology*, 23(4), 1387–1398.

Chairez, I. (2017). Adaptive neural network nonparametric identifier with normalized learning laws. *IEEE transactions on neural networks and learning systems*, 28(5), 1216–1227.

Cybenko, G. (1989). Approximation by superpositions of a sigmoidal function. *Mathematics of control, signals and systems*, 2(4), 303–314.

Doyle, J.C. (1983). Synthesis of robust controllers and filters. In *The 22nd IEEE Conference on Decision and Control*, 109–114. IEEE.

Escudero, M., Chairez, I., and Garcia, A. (2010). Adaptive linearization for nonlinear systems using continuous Neural Networks. In *2010 7th International Conference on Electrical Engineering Computing Science and Automatic Control*, 116–121. IEEE.

Funahashi, K.I. (1989). On the approximate realization of continuous mappings by neural networks. *Neural Networks*, 2(3), 183 – 192. doi:https://doi.org/10.1016/0893-6080(89)90003-8.

Haber, R. and Keviczky, L. (1999). *Nonlinear System Identification Input-Output Modeling Approach*. Kluwer Academic Publishers.

Harashima, F., Xu, J.X., and Hashimoto, H. (1987). Tracking control of robot manipulators using Sliding Mode. *IEEE Transactions on Power Electronics*, (2), 169–176.

Haykin, S. (1994). *Neural networks: a comprehensive foundation*. Prentice Hall PTR.

Hermes, H. (1986). Nilpotent approximations of control systems and distributions. *SIAM journal on control and optimization*, 24(4), 731–736.

Hermes, H. (1995). Homogeneous feedback controls for homogeneous systems. *Systems & Control Letters*, 24(1), 7 – 11. doi:https://doi.org/10.1016/0167-6911(94)00035-T.

Hornik, K. (1991). Approximation capabilities of multilayer feedforward networks. *Neural networks*, 4(2), 251–257.

Hornik, K., Stinchcombe, M., and White, H. (1989). Multilayer feedforward networks are universal approximators. *Neural networks*, 2(5), 359–366.

Igelnik, B. and Pao, Y.H. (1995). Stochastic choice of basis functions in adaptive function approximation and the functional-link net. *IEEE Transactions on Neural Networks*, 6(6), 1320–1329. doi:10.1109/72.471375.

Jagannathan, S. and Lewis, F.L. (1996). Identification of nonlinear dynamical systems using multilayered neural networks. *Automatica*, 32(12), 1707–1712.

Join, C., Sira-Ramírez, H., and Fliess, M. (2005). Control of an uncertain three-tank system via on-line parameter indentification and fault detection. *IFAC Proceedings Volumes*, 38(1), 251 – 256. doi:https://doi.org/10.3182/20050703-6-CZ-1902.01844. 16th IFAC World Congress.

Kawski, M. (1988). Nilpotent Lie algebras of vector fields. *J. reine angew. Math*, 388, 1–17.

Kawski, M. (1995). Geometric homogeneity and stabilization. In *Nonlinear Control Systems Design 1995*, 147–152. Elsevier.

Khalil, H.K. and Grizzle, J.W. (2002). *Nonlinear systems*, volume 3. Prentice hall Upper Saddle River, NJ.

Lewis, F., Jagannathan, S., and Yesildirak, A. (1998). *Neural network control of robot manipulators and non-linear systems*. CRC Press.

Ljung, L. (1999). *System Identification Theory for the User*. Prentice Hall Information and System Sciences Series. Prentice Hall PTR.

Ljung, L. (2006). Some aspects on Nonlinear System Identification. *IFAC Proceedings Volumes*, 39(1), 553 – 564. doi: https://doi.org/10.3182/20060329-3-AU-2901.00085. 14th IFAC Symposium on Identification and System Parameter Estimation.

Nelles, O. (2013). *Nonlinear system identification: from classical approaches to neural networks and fuzzy models*. Springer Science & Business Media.

Orlov, Y. (2005). Finite time stability and robust control synthesis of uncertain switched systems. *SIAM Journal on Control and Optimization*, 43(4), 1253–1271.

Park, J. and Sandberg, I.W. (1991). Universal approximation using radial-basis-function networks. *Neural Computation*, 3(2), 246–257. doi:10.1162/neco.1991.3.2.246.

Polyakov, A., Efimov, D., Fridman, E., and Perruquetti, W. (2016). On homogeneous distributed parameter systems. *IEEE Transactions on Automatic Control*, 61(11), 3657–3662. doi:10.1109/TAC.2016.2525925.

Poznyak, A.S., Sanchez, E.N., and Yu, W. (2001). *Differential neural networks for robust nonlinear control: identification, state estimation and trajectory tracking*. World Scientific.

Rosier, L. (1992). Homogeneous Lyapunov Function for Homogeneous Continuous Vector Field. *Systems & Control Letters*, 19, 467–473.

Utkin, V., Guldner, J., and Shi, J. (2009). *Sliding mode control in electro-mechanical systems*. CRC press.

Zimenko, K., Efimov, D., Polyakov, A., and Perruquetti, W. (2017). A note on delay robustness for homogeneous systems with negative degree. *Automatica*, 79, 178–184.