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A two-stage algorithm for estimation of unknown parameters using nonlinear measurements

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Abstract: A suboptimal two-stage algorithm has been proposed to solve nonlinear estimation problems consist in comparison of measured and reference samples. The new algorithm consists of preliminary processing of measurements, subsampling and simplification of the errors model in nonlinear algorithm. A significant increase in computational performance determines the novelty of the presented algorithm. The effective application of the two-stage suboptimal algorithm is illustrated by an example of gravity-aided navigation.

Keywords: Bayesian methods, Particle filtering/Monte Carlo methods, Nonlinear measurements, Filtering and smoothing, parameter estimation, map-aided navigation.

1. INTRODUCTION

In signal processing it is often necessary to estimate unknown parameters based on a comparison of measured and reference sets of values, hereinafter referred to as samples. Such problems are often solved while processing sonar, radio navigation, and satellite signals, as well as in map-aided navigation (Stepanov, 1998; Bergman, 1999; B. Anonsen, 2010; Groves, 2013). A distinctive feature of this kind of problems is their nonlinear nature. Specifically, for this purpose, researchers used methods based on calculation of the cross-correlation function between the measured and reference samples and subsequent finding of its maximum (Quazi, 1981; J. Chen et al., 2006; Stepanov & Toropov, 2015). These methods are known for their comparative simplicity and clear physical meaning; on the other hand, in many cases, they provide acceptable estimation accuracy of the parameters to be determined.

At the same time, the development of applied estimation theory provides more effective estimation algorithms based on the nonlinear filtering methods (Sarkka, 2013; Candy, 2016; Stepanov & Toropov, 2016). Their application to solve specified problems became feasible due to the significant progress in the computer technology. Nevertheless, due to the nonlinear nature of the estimation problem, which has to be solved, the implementation of such algorithms may be problematic since they are subject to the "curse of dimensionality". In this connection, various suboptimal algorithms are being developed (Gao et al., 2014; Li et al., 2018). On the one hand, they provide accuracy close to the potential one, i.e., accuracy attained with the use of the optimal, in the mean-square sense, algorithm. On the other hand, they are computationally effective. The success in designing such algorithms is largely due to the possibility of considering the specific features of the nonlinear problem being solved.

Within this paper, we present a new two-stage suboptimal Bayesian algorithm to solve a problem which consists in the comparison of measured and reference samples and study the efficiency of the proposed algorithm. At the first stage, the measured sample undergoes preprocessing to reduce the measurement errors. At the second stage we derive subsample from pre-processed values and compare it with reference one to estimate required parameters. Due to preprocessing and subsampling the model of the measurement errors used at second stage can be significantly simplified in comparison with original one. As a result of simpler error model and reduced number of measurements, the nonlinear estimation algorithm can be made much simpler and more computationally tractable. The flowchart of the two-stage suboptimal algorithm is presented below.

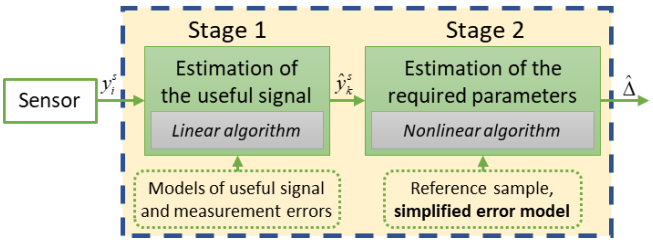

Fig. 1. The flowchart of the two-stage suboptimal algorithm; y_i^s – measurements sample; $\hat{y}_k^s$ – pre-processed subsample; $\hat{\Delta}$ – parameters estimate.

At the same time the two-stage suboptimal algorithm uses lossy processing, therefore it is necessary to develop a procedure for evaluation of its efficiency from the standpoint of the estimation accuracy of the required parameters in

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comparison with the accuracy of the optimal algorithm that use a whole set of measurements and original error model. The paper considers the design of the two-stage algorithm and evaluates of its efficiency.

The paper is structured as follows. In Section 2, the problem of comparing samples is formulated as a nonlinear filtering problem and the optimal, in the mean-square sense, algorithm for its solution is described. Section 3 discusses the proposed two-stage estimation algorithm. Section 4 gives a brief description of the procedures used to evaluate the efficiency of the algorithm. Section 5 considers an example in which the proposed algorithm is used to solve the problem of the gravity-aided navigation and analyses its efficiency.

2. NONLINEAR ESTIMATION PROBLEM STATEMENT AND THE OPTIMAL ALGORITHM OF ITS SOLUTION

To give a clear meaning of the problem to be solved, we will formulate it like map-aided navigation problem for sea vessel i.e. as aiding navigation system with position updates from a map matching procedure using a sensor and map of some geophysical field. Following (Bucy & Senne, 1971; Stepanov, 1998; Bergman, 1999), this problem is formulated in the framework of the Bayesian filtering theory and its relationship with the problem of comparing samples is discussed.

We assume that a sea vessel is equipped with a navigation system (NS), a sensor, and a field map. Scalar measurements y_i^s of the field sensor and coordinates $\mathbf{y}_i^{NS}$ from the NS being updated are generated at discrete points of time t_i with intervals of Δt. For simplicity, we assume that the map-aided problem is solved on the plane. Thus, we can write:

$$\mathbf{y}_i^{NS} = \mathbf{X}_i + \mathbf{\Delta}_i, \tag{1}$$

$$y_i^s = \phi\left(\mathbf{X}_i\right) + \varepsilon_i, \tag{2}$$

where $\mathbf{y}_i^{NS} = \left[y_1^{NS} \quad y_2^{NS} \right]_i^T$ are known NS output; ε_i are unknown values of measurement errors; $i = \overline{1 \ldots I}$; $\phi(\bullet)$ is a known nonlinear function (map) describing the field dependence on the true coordinates $\mathbf{X}_i = \left[X_1 \quad X_2 \right]_i^T$ of the vessel. Further, for simplicity, it is assumed that during the observation, NS errors $\mathbf{\Delta}_i = \left[\Delta_1 \quad \Delta_2 \right]_i^T$ are constant parameters and the map $\phi(\bullet)$ is known precisely. The above simplifications are introduced in order to discuss the essence of the proposed algorithms without focusing on details.

Having measurements (1), (2), it is required to estimate vessel position, that is, to estimate vector of unknown parameters

$$\mathbf{\Delta}_i = \mathbf{\Delta}_{i-1} = \mathbf{\Delta} \tag{3}$$

using scalar measurements

$$y_i^s = \phi\left(\mathbf{y}_i^{NS} - \mathbf{\Delta}\right) + \varepsilon_i \equiv \phi_i\left(\mathbf{\Delta}\right) + \varepsilon_i. \tag{4}$$

Note that a distinctive feature of this problem is its nonlinear nature, which is due to the nonlinear dependence of the field values on the vessel position.

It is easy to see that, actually, this problem is reduced to the comparison of the measured sample $\mathbf{Y}_i^s = \left[y_1^s, y_2^s, \ldots y_i^s \right]^T$ with its reference counterpart $\Phi_i = \left[\phi_1, \phi_2, \ldots \phi_i \right]^T$, calculated using a map along the expected trajectory. Therefore the problem under consideration is commonly solved by the methods based on maximization of the criterion, which is close in its meaning to the cross-correlation function between the compared samples (Krasovskii et al., 1979; Vaman, 2012). In this paper, following (Stepanov, 1998; Bergman, 1999), the problem is solved using nonlinear filtering, which allows us to find the optimal, in the mean-square sense, estimate (Gelb et al., 1974). With this in mind, we assume that unknown vector $\mathbf{\Delta}$ and measurement errors ε_i are random with certain statistical properties. It is known that for the optimal, in the mean-square sense, estimate of vector $\mathbf{\Delta}$ and the covariance matrix of its errors, the following relations are valid (Bucy & Senne, 1971; Stepanov, 1998):

$$\hat{\mathbf{\Delta}}\left(\mathbf{Y}_i^s\right) = \int \mathbf{\Delta}\, p\left(\mathbf{\Delta} / \mathbf{Y}_i^s\right) d\mathbf{\Delta}, \tag{5}$$

$$P_{\mathbf{\Delta}}\left(\mathbf{Y}_i^s\right) = \int \left(\mathbf{\Delta} - \hat{\mathbf{\Delta}}\left(\mathbf{Y}_i^s\right)\right)\left(\mathbf{\Delta} - \hat{\mathbf{\Delta}}\left(\mathbf{Y}_i^s\right)\right)^T p\left(\mathbf{\Delta} / \mathbf{Y}_i^s\right) d\mathbf{\Delta}, \tag{6}$$

where $p\left(\mathbf{\Delta} / \mathbf{Y}_i^s\right)$ is posterior (conditional to the set of measurements $\mathbf{Y}_i^s$), probability density function (p.d.f.) of vector $\mathbf{\Delta}$. In (5), and (6), the integrals are taken as a double integral with infinite limits. It is important to emphasize that covariance matrix (6) is a conditional one, characterizing estimation accuracy for a particular set of measurements. For the accuracy analysis, our interest is in the unconditional covariance matrix of estimation errors, defined as

$$G_{\mathbf{\Delta}} = \int\int \left(\mathbf{\Delta} - \hat{\mathbf{\Delta}}\left(\mathbf{Y}_i^s\right)\right)\left(\mathbf{\Delta} - \hat{\mathbf{\Delta}}\left(\mathbf{Y}_i^s\right)\right)^T p\left(\mathbf{\Delta}, \mathbf{Y}_i^s\right) d\mathbf{\Delta}\, d\mathbf{Y}_i^s, \tag{7}$$

where $p\left(\mathbf{\Delta}, \mathbf{Y}_i^s\right)$ is a joint p.d.f. of $\mathbf{\Delta}$ and $\mathbf{Y}_i^s$.

For the specified statistical properties of vector $\mathbf{\Delta}$ and measurement errors ε_i, the elements of matrix (7) characterize the potential estimation accuracy of $\mathbf{\Delta}$.

Specific algorithms for obtaining estimates (5) and covariance matrices (6) in the framework of the Bayesian approach are due to different approximation methods used to represent a posteriori p.d.f. $p\left(\mathbf{\Delta} / \mathbf{Y}_i^s\right)$ (Sarkka, 2013; Stepanov & Toropov, 2015; Candy, 2016). In this paper we use the p.d.f. approximation in the form

$$p\left(\mathbf{\Delta} / \mathbf{Y}_i^s\right) \approx \sum_{j=1}^{L} \mu_i^j \boldsymbol{\delta}\left(\mathbf{\Delta} - \mathbf{\Delta}^j\right), \tag{8}$$

where Δ^j, $j = \overline{1 \ldots L}$ are the grid nodes that specify possible values of $\mathbf{\Delta}$; $\boldsymbol{\delta}\left(\mathbf{\Delta} - \mathbf{\Delta}^j\right) = \delta\left(\Delta_1 - \Delta_1^j\right)\delta\left(\Delta_2 - \Delta_2^j\right)$ is a two-dimensional Dirac delta function; μ_i^j are the weights:

$$\mu_i^j = p\left(\Delta^j / \mathbf{Y}_i^s\right) \Big/ \sum_{j=1}^{L} p\left(\Delta^j / \mathbf{Y}_i^s\right). \quad (9)$$

The approximation (8) results in rather simple expressions for the calculation of estimates and covariance matrices:

$$\hat{\Delta}^{opt}\left(\mathbf{Y}_i^s\right) \approx \sum_{j=1}^{L} \mu_i^j \Delta^j, \quad (10)$$

$$P_{\Delta}^{opt}\left(\mathbf{Y}_i^s\right) \approx \sum_{j=1}^{L} \mu_i^j \left(\Delta^j \left(\Delta^j\right)^T\right) - \hat{\Delta}\left(\mathbf{Y}_i^s\right)\hat{\Delta}^T\left(\mathbf{Y}_i^s\right). \quad (11)$$

If the grid nodes are sampled stochastically, the algorithms based on approximation (8) are called Monte Carlo methods, and their various modifications are known as particle filters (Bergman, 1999; Z. Chen, 2003). In the case of a deterministic algorithm for specifying vectors Δ^j, the corresponding methods are called grid methods or point-mass method (Bucy & Senne, 1971). For the NS error model in the form (3), as shown in (K. B. Anonsen & Hallingstad, 2006), these methods show approximately the same accuracy, which depends directly on the number of nodes L.

It is easy to notice that, owing to the simplicity of relations (10), (11), the computational complexity of the algorithms for obtaining estimates and covariance matrices is mainly determined by the calculation complexity, according to (9), of the posteriori density and weights μ_i^j.

To describe errors ε_i we use *l*-dimensional Markov sequence, which can be represented using shaping filter

$$\xi_i = F\xi_{i-1} + \Gamma w_{i-1}, \quad (12)$$

so that the errors can be written as $\varepsilon_i = H\xi_i + v_i$. In these equations F, Γ and H are the known matrices of the corresponding dimensions; w_i, v_i are forcing and measuring zero-mean discrete white noises with known covariance matrices. Dimension *l* depends on the properties of the errors being described.

Introducing composite vector $x_i = \left[\Delta^T \ \xi_i^T\right]^T$ that includes two subvectors Δ and ξ_i, we can formulate the problem of filtering for this vector using measurements

$$y_i^s = \phi\left(\mathbf{y}_i^{NS} - \Delta\right) + \varepsilon_i \equiv \phi_i\left(\Delta\right) + H\xi_i + v_i. \quad (13)$$

The formula for a posteriori density $x_i = \left[\Delta^T \ \xi_i^T\right]^T$ can be written as

$$p\left(\Delta, \xi_i / \mathbf{Y}_i^s\right) = p\left(\xi_i / \Delta, \mathbf{Y}_i^s\right) p\left(\Delta / \mathbf{Y}_i^s\right). \quad (14)$$

Expression (14) makes it possible to design computationally efficient algorithms for finding optimal estimates and covariance matrices using the bank of *l*-dimensional Kalman filters (Doucet et al., 2001; Lainiotis, 1971). This is explained by the fact that density $p\left(\xi_i / \Delta, \mathbf{Y}_i^s\right)$ will be Gaussian in the case that subsector Δ is fixed, and the additional condition about the Gaussian nature of measurement errors. Its parameters can be calculated using the KFs that provide a solution to the problem of filtering subsector ξ_i using measurements in the form $\tilde{y}_i^s = y_i^s - \phi_i\left(\Delta\right) = H\xi_i + v_i$. In addition, after obtaining partial estimates generated in the KF and the corresponding covariance matrices, it is possible to derive the following recursive relations to calculate the weights (Doucet et al., 2001):

$$\mu_i^j = \eta \exp\left\{-\frac{1}{2}\frac{\left(\tilde{y}_i^s - H\hat{\xi}_{i/i-1}^j\right)^2}{\sigma_v^2 + HP_{i/i-1}^{\xi^j}H^T}\right\}\mu_{i-1}^j, \ \mu_0^j = p\left(\Delta\right), \quad (15)$$

where $\hat{\xi}_{i/i-1}^j$ is the prediction of subvector ξ_i at the i-th point of time generated in the j-th KF; $P_{i/i-1}^{\xi^j}$ is the corresponding covariance matrix; σ_v^2 is the variance of the discrete white-noise sensor error; μ_0^j are prior values of the weights calculated in accordance with the p.d.f. $p\left(\Delta\right)$; η is normalizing factor.

In general, the described algorithm is rather simple in its structure, but the total amount of computations for its implementation can be very large. It can be determined as a function $\Omega_1 = f(L, I, T_l)$, which depends on the number of nodes *L*, the number of measurements I, and time T_l needed to process one scalar measurement in the KF. To reduce the total amount of calculations, we propose the heuristic two-stage estimation algorithm described below.

3. TWO-STAGE ESTIMATION ALGORITHM

To clarify the two-stage algorithm for estimating Δ, we present measurements (4) as

$$y_i^s = \phi_i\left(\Delta\right) + \varepsilon_i = g_i + \varepsilon_i, \ i = \overline{1 \ldots I}. \quad (16)$$

The idea underlying the suboptimal algorithm is that measurements (16) are preprocessed in order to improve the accuracy of the sensor output. Thus, the estimate of Δ is calculated only after preprocessing, aimed at obtaining field estimates g_i. After processing they can be represented as

$$\hat{y}_k^s = \phi\left(\mathbf{y}_k^{NS} - \Delta\right) + \vartheta_k, \ k = \overline{1 \ldots K}, \quad (17)$$

Here ϑ_k is the field estimation error, which, after preprocessing, is much lower than the initial error ε_i.

There are different approaches to designing preprocessing algorithms. In this paper, we assume that $g_i = \phi_i\left(\Delta\right)$ are the values of a stationary random sequence with known stochastic properties. Hence, we can state and solve the smoothing problem on a fixed interval using *I* measurements and the Rauch-Tung-Striebel (RTS) procedure (Gelb et al., 1974; Sarkka, 2013). The dimension of the state vector being estimated will be *l* + *p,* where *l* and *p* are dimensions of the state vectors used to describe the measurement errors and the useful signal correspondingly.

Note that preliminary processing in itself does not make the algorithm for solving the problem of Δ estimation simpler.

Moreover, if we use the whole set of measurements $\hat{\mathbf{Y}}_k^s = \left[\hat{y}_1^s, \hat{y}_2^s, ...\hat{y}_k^s\right]^T$ after the smoothing procedure and take a proper account of the resulting estimation error statistics, we can show that the estimation accuracy of $\mathbf{\Delta}$ will remain at the same level. However, since preliminary processing significantly reduces the field estimation errors it becomes possible to reduce the number of measurements used to estimate $\mathbf{\Delta}$. This can be done by subsampling the estimates choosing a certain time interval $\Delta\tilde{t}$ that is a multiple of Δt.

One of the simplest ways of choosing such interval is to ensure that after the constant error component (if any) is eliminated, the remaining errors can be appropriately approximated by discrete white noise. In this case, if the discrete white noise variance remains invariable, we can use a model for errors ϑ_k consist of constant (time-invariant) error with variance σ_c^2 and discrete white noise with variance σ_r^2 to design nonlinear algorithm. Constant error can be determined in nonlinear part of algorithm.

For such a model, instead of a bank of L KFs of l dimension, we will need a bank of L one-dimensional KFs to process K measurements, the number of which is $\Delta\tilde{t}/\Delta t$ times less than the number of initial I measurements. Thus, the required amount of computation can be written as a function $\Omega_2 = f\left(L, K, T_1\right) + T_{l+p}$, where T_1 is the time needed to process one scalar measurement in one-dimensional KF, T_{l+p} is the time needed to solve the smoothing problem for the state vector of l+p dimension with the use of I measurements. Hence, the gain in the amount of calculations should be expected if $I >> K$. It makes sense to discuss the computational advantages only in the case when the accuracy of the suboptimal two-stage algorithm is close to the potential accuracy. This condition can be verified using the procedure proposed in the next section.

4. TWO-STAGE ALGORITHM EFFICIENCY ANALISIS

To evaluate the efficiency of the two-stage suboptimal algorithm, it is proposed to use the unconditional covariance matrix (7), which characterizes the potential accuracy corresponding to the accuracy of the optimal algorithm. This matrix can be calculated using the Monte-Carlo simulations. It characterizes the estimation accuracy for the used map on average for all sets of measurements:

$$\widehat{G}_{\mathbf{\Delta}} \approx \frac{1}{M}\sum_{j=1}^{M}\left(\mathbf{\Delta}^{(j)} - \hat{\mathbf{\Delta}}^{(j)}\left(\mathbf{Y}_i^{s(j)}\right)\right)\left(\mathbf{\Delta}^{(j)} - \hat{\mathbf{\Delta}}^{(j)}\left(\mathbf{Y}_i^{s(j)}\mathbf{Y}_i^{s(j)}\right)\right)^T, \quad (18)$$

where M is the number of Monte-Carlo runs, $\Delta^{(j)}$, $\hat{\Delta}^{(j)}$ are the true values and estimates for j-th run respectively.

Note that the unconditional covariance matrix can also be calculated using the conditional covariance matrices computed in the algorithm, i.e.:

$$\breve{G}_{\mathbf{\Delta}} \approx \frac{1}{M}\sum_{j=1}^{M} P_{\mathbf{\Delta}}\left(\mathbf{Y}_i^{s(j)}\right). \quad (19)$$

Matrices calculated using formulas (18), (19) will be called the real and calculated covariance matrices. Their pproximity allows us to judge the consistency of the results obtained. When comparing the algorithms, the matrices introduced above are calculated for the I initial measurements using the optimal algorithm and K subsampled measurements using the two-stage suboptimal algorithm.

In the next section we present the results of evaluating the effectiveness of two-stage algorithm as applied to the gravity-aided navigation.

5. EXAMPLE

The application of the proposed suboptimal algorithm is illustrated by the example of solving marine map-aided navigation problem using a gravity anomaly (GA) map and relative gravimeter. Without loss of generality, we assume that we need to estimate one component of vessel coordinate, that is, Δ is assumed scalar. This statement is justified by the fact that usually, when inertial navigation systems are used on sea vessels, the main problem consists in refining the longitude of the vessel.

Let us specify useful signal and measurement errors for this problem. For the gravity anomaly, we use the models that are typically used in the field survey. Here, the Jordan model in the form of a stationary process with a correlation function (20) is used to describe the gravity anomaly profiles $g(t)$ along a rectilinear trajectory (Jordan, 1972):

$$K_g\left(\rho\right) = \sigma_{\tilde{g}}^2\left(1 + \alpha\rho - \left(\alpha\rho\right)^2/2\right)e^{-\alpha\rho}. \quad (20)$$

The corresponding shaping filter is written as (Peshekhonov & Stepanov, 2017):

$$\begin{cases} \dot{\xi}_1^g = -\beta\xi_1^g + \xi_2^g, \\ \dot{\xi}_2^g = -\beta\xi_2^g + \xi_3^g, \\ \dot{\xi}_3^g = -\beta\xi_3^g + q_g w_g, \end{cases} \quad (21)$$

where $g = -\beta\zeta\xi_1^g + \xi_2^g$; $\beta = V\sigma_{\partial g/\partial\rho}/\sqrt{2}\sigma_g$; V is the vessel speed; ρ is a distance along the trajectory; $\sigma_{\partial g/\partial\rho}$ is the parameter defining GA spatial variability; w_g is forcing white noise of unit power-spectrum density (PSD); $q_g = \sqrt{10\beta^3\sigma_g^2}$; σ_g^2 is the GA variance; and $\zeta = \left(\sqrt{5}-1\right)/\sqrt{5}$ is the dimensionless coefficient.

When describing the errors of GA measurements on a sea vessel, we consider the model consist of vertical accelerations due to heaving, constant error (random bias) and white-noise error (Peshekhonov & Stepanov, 2017):

$$\begin{cases} \dot{\xi}_1^s = \xi_2^s, \\ \dot{\xi}_2^s = \xi_3^s, \\ \dot{\xi}_3^s = -a_3\xi_1^s - a_2\xi_2^s - a_1\xi_3^s + q_s w_s, \\ \dot{\xi}_4^s = 0. \end{cases} \quad (22)$$

Components $\xi_1^s - \xi_3^s$ describe the model of vehicle vertical displacements ξ_1^s generating vertical accelerations ξ_3^s. The component ξ_4^s with variance σ_c^2 describes the systematic error. In (22) $a_3 = \left(\lambda^2 + \mu^2\right)\gamma$; $a_2 = \lambda^2 + \mu^2 + 2\mu\gamma$; $a_1 = 2\mu + \gamma$; $q_s = \sigma_s\sqrt{2a_3\left(a_1a_2 - a_3\right)/a_1}$; w_s, w_m are forcing white noise of unit PSD; σ_s is the RMS value of vertical displacements ξ_1^s; λ is the predominant pitching frequency; μ is the coefficient of irregularity of waves; γ is the dimensionless coefficient.

For the models introduced, the dimension of the vector $\xi(t)$ used to form measurement errors is 4, that is, l=4, and for the useful signal, p=3. In this case, the gravimeter measurements can be written as

$$y_i = -\beta\zeta\xi_1^g\left(t_i\right) + \xi_2^g\left(t_i\right) + \xi_3^s\left(t_i\right) + \xi_4^s\left(t_i\right) + v_i, \qquad (23)$$

where v_i is the discrete white noise measurement error.

The values of the parameters used in the simulation in the example under consideration are given in Table 1.

Table 1. Simulation parameters

Parameter	Notation	Value
Sampling period	Δt	0.1 s
A priori positioning RMS errors	σ_Δ	700 m
RMS value of the gravity anomaly	σ_g	30 mGal
RMS value of the gravity variability	$\sigma_{\partial g/\partial\rho}$	3 mGal/km
Speed along the trajectory	V	10 m/s
Gravity profile length	ρ	30 km
RMS value of vertical displacements	σ_s	0.2 m
Prevailing heaving frequency	λ	2π/7 rad/s
Coefficient of heaving irregularity	μ	0.01 rad/s
Systematic RMS error of the sensor	σ_c	5 mGal
White-noise RMS error of the sensor	σ_v	0.5 mGal

Gravimeter measurements in the form (23) were formed on a fixed section of each GA profile with coordinates [5000 20000] m. Thus, the number of original GA measurements was N=15001 with a spatial interval of 1 m. An example of the GA profile and its measurements are presented in Fig. 2.

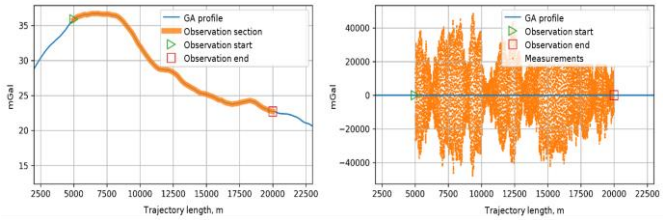

Fig. 2. An example of the GA profile and its measurements.

To estimate the five-dimensional vector $x_i = \left[\Delta \quad \xi_i^{sT}\right]^T$ and the conditional covariance matrix with the use of the optimal algorithm we used the point-mass method with the number of nodes $L = 3000$, which determines the number of four-dimensional Kalman filters. Equation (13) was made more concrete by taking into account the type of models (22)–(22), where matrix $H = \begin{bmatrix}0 & 0 & 1 & 1\end{bmatrix}$, and $\boldsymbol{\Delta} \equiv \Delta$ is a scalar zero-mean Gaussian random variable with variance σ_Δ^2.

Within the two-stage scheme, the above-mentioned smoothing problem for the state vector (21)–(22) using measurements (23) was solved at the preprocessing stage. In the steady-state mode, the RMS error of preliminary smoothing was 0.36 mGal. After preprocessing, the resulting estimates were subsampled, that is, the number of measurements used at the second stage was significantly reduced. In this case, instead of the model (22)–(22), we used the model consist of random bias and white noise, the variance of which corresponds to the GA smoothing RMSE. Below are the results for the case where the values of $\Delta\tilde{t}$ were chosen in such a way that only 21 measurements with a spatial discreteness of 710 m were used. The choice of interval requires special consideration out of scope of the paper.

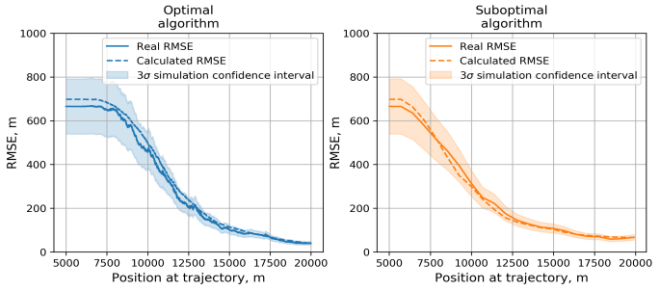

Fig. 3. Real and calculated unconditional RMS errors of the optimal and suboptimal algorithms.

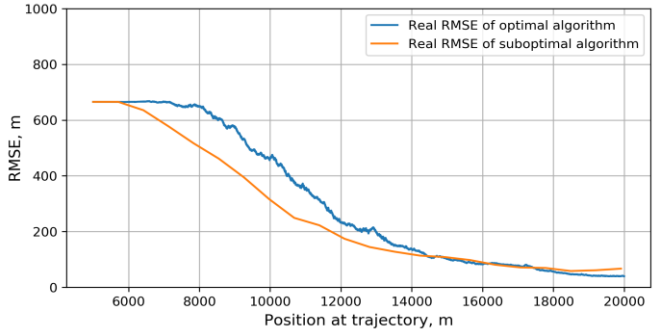

Fig. 4. Real unconditional RMS errors of the optimal and suboptimal algorithms.

From the plots it is obvious that the two-stage suboptimal algorithm with preliminary smoothing is comparable to the optimal algorithm in accuracy: when it was applied, the unconditional real RMS error of both algorithms ware in 50-70 m range. In addition, it should be noted that the calculated accuracy characteristics provided by the suboptimal algorithm were close to the real values.

As for the amount of calculations, we determined that it took 30 s to process all measurements with the optimal algorithm on a test computer and an order of magnitude less, that is, 3 s, for the new two-stage suboptimal algorithm.

6. CONCLUSIONS

An optimal, in the mean-square sense, algorithm has been described to estimate unknown parameters with the use of nonlinear measurements in solving problems of comparing the measured and reference samples.

A suboptimal two-stage algorithm has been proposed to solve estimation problems. The new algorithm consists of preliminary processing of measurements, subsampling, and simplification of the model of measurement errors in the nonlinear part of an algorithm.

The procedure for the analysis of possible losses in accuracy of estimation problem solution with use of the suboptimal two-stage algorithm as compared with the optimal one is described.

The application of the two-stage suboptimal algorithm is illustrated by an example of gravity-aided navigation. It has been shown that in this example the accuracy of the suboptimal algorithm is close to the potential one while the amount of calculations is reduced by an order of magnitude.

REFERENCES

Anonsen, K. B. (2010). *Advances in Terrain Aided Navigation for Underwater Vehicles*. Norwegian University of Science and Technology.

Anonsen, K. B., & Hallingstad, O. (2006). Terrain Aided Underwater Navigation Using Point Mass and Particle Filters. *2006 IEEE/ION Position, Location, And Navigation Symposium*, 1027–1035. https://doi.org/10.1109/PLANS.2006.1650705

Bergman, N. (1999). *Recursive Bayesian estimation: Navigation and tracking applications*. Linkoping University.

Bucy, R. S., & Senne, K. D. (1971). Digital synthesis of non-linear filters. *Automatica*, *7*(3), 287–298. https://doi.org/10.1016/0005-1098(71)90121-X

Candy, J. V. (2016). *Bayesian Signal Processing: Classical, Modern, and Particle Filtering Methods* (2nd. ed.). Wiley. https://doi.org/10.1002/9780470430583

Chen, J., Benesty, J., & Huang, Y. (2006). Time Delay Estimation in Room Acoustic Environments: An Overview. *EURASIP Journal on Advances in Signal Processing*, *2006*(1), 1–19. https://doi.org/10.1155/ASP/2006/26503

Chen, Z. (2003). Bayesian filtering: From Kalman filters to particle filters, and beyond. *Statistics*, *182*(1), 1–69.

Doucet, A., Freitas, N., & Gordon, N. (2001). *Sequential Monte Carlo Methods in Practice*. Springer New York. https://doi.org/10.1007/978-1-4757-3437-9

Gao, W., Zhao, B., Zhou, G., Wang, Q., & Yu, C. (2014). Improved Artificial Bee Colony Algorithm Based Gravity Matching Navigation Method. *Sensors*, *14*(7), 12968–12989. https://doi.org/10.3390/s140712968

Gelb, A., Kasper, J. F. Jr., Nash, R. A. Jr., Price, C. F., & Sutherland, A. A. Jr. (1974). *Applied Optimal Estimation*. The MIT Press.

Groves, P. D. (2013). *Principles of GNSS, inertial, and multisensor integrated navigation systems* (2nd Ed.). Artech house.

Jordan, S. K. (1972). Self-consistent Statistical Models for Gravity Anomaly and Undulation of the Geoid. *Journal of Geophysical Research*, *77*(20), 3660–3670.

Krasovskii, A. A., Beloglazov, I. N., & Chigin, G. P. (1979). *Teoriya korrelyatsionno-ekstremal'nykh navigatsionnykh sistem (Theory of Correlation-Extreme Navigation Systems)*. Nauka.

Lainiotis, D. (1971). Optimal adaptive estimation: Structure and parameter adaption. *IEEE Transactions on Automatic Control*, *16*(2), 160–170. https://doi.org/10.1109/TAC.1971.1099684

Li, P., Sheng, G., Zhang, X., Wu, J., Xu, B., Liu, X., & Zhang, Y. (2018). Underwater terrain-aided navigation system based on combination matching algorithm. *ISA Transactions*, *78*, 80–87. https://doi.org/10.1016/j.isatra.2017.12.018

Peshekhonov, V. G., & Stepanov, O. A. (Eds.). (2017). *Sovremennye metody i sredstva izmerenija parametrov gravitacionnogo polja Zemli*. Elektropribor, JSC.

Quazi, A. (1981). An overview on the time delay estimate in active and passive systems for target localization. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, *29*(3), 527–533. https://doi.org/10.1109/TASSP.1981.1163618

Sarkka, S. (2013). *Bayesian Filtering and Smoothing*. Cambridge University Press.

Stepanov, O. A. (1998). *Primenenie teorii nelineinoi fil'tratsii v zadachakh obrabotki navigatsionnoi informatsii (Application of Nonlinear Filtering Theory for Processing Navigation Information)*. Elektropribor.

Stepanov, O. A., & Toropov, A. B. (2015). Nonlinear filtering for map-aided navigation. Part 1. An overview of algorithms. *Gyroscopy and Navigation*, *6*(4), 324–337. https://doi.org/10.1134/S2075108715040148

Stepanov, O. A., & Toropov, A. B. (2016). Nonlinear filtering for map-aided navigation Part 2. Trends in the algorithm development. *Gyroscopy and Navigation*, *7*(1), 82–89. https://doi.org/10.1134/S2075108716010132

Vaman, D. (2012). TRN history, trends and the unused potential. *2012 IEEE/AIAA 31st Digital Avionics Systems Conference (DASC)*, 1A3-1-1A3-16. https://doi.org/10.1109/dasc.2012.6382278