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Combined Cooperative Adaptive Cruise Control using Collective Initial Excitation based Distributed Parameter Estimator

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Abstract: In cooperative adaptive cruise control (CACC), autonomous vehicles are grouped into a string of platoon and, the main objective is to automatically adapt their speed using on-board sensors and communication with the preceding vehicle to maintain a desired inter-vehicle distance. Cruise control is achieved in the presence of parametric uncertainty in the vehicle dynamics using principles of adaptive control. This work proposes a novel combined CACC strategy for an uncertain homogeneous platoon with guaranteed parameter convergence and asymptotic string stability. A novel distributed consensus-based parameter estimator is proposed in conjunction with a model reference adaptive control (MRAC) algorithm using a direct control-gain update law. The algorithm ensures exponential parameter estimation error convergence to zero as well as asymptotic convergence of tracking-error to zero. Conventional CACC protocols require a condition of persistence of excitation (PE) for parameter convergence, which is required for better transient performance in converging to a string stable configuration. The PE condition is highly restrictive in the context of cruise control since velocity profiles which are demanded in the platoon model do not typically satisfy the PE condition. In contrast, the proposed scheme can ensure parameter convergence under a significantly milder condition, coined as collective initial excitation (C-IE). The C-IE condition is an extension of the concept of initial excitation (IE), which is recently proposed in the context of adaptive control of single agent system. Unlike IE, the C-IE condition caters to distributed estimation in the context of multi-agent systems. As far as the authors are aware, this is the first work on CACC framework, which ensures exponential convergence of parameter estimation error of each vehicle under the mild condition of C-IE, which further leads to asymptotic convergence of the entire vehicle platoon to a string stable configuration. Simulation study dictates that the proposed CACC architecture outperforms the existing CACC algorithms in terms of tracking and estimation performance.

Keywords: C-IE, C-PE, CACC, Exponential Plant Parameter Convergence, string stability.

1. INTRODUCTION

Constrained highway capacities lead to traffic congestion, which is increasing over the years concerning both the number of traffic congestion and their lengths. An effective scheme to enhance the road capacity is to maintain the inter-vehicle distance between individual vehicles in the platoon. To achieve this cooperative adaptive cruise control (CACC), an improved version of adaptive cruise control (ACC) (Marsden et al., 2001), can be employed as an automated vehicle-follower system based on inter-vehicle wireless communication (Vahidi and Eskandarian, 2003; Shladover, 2005).

CACC architectures are better than ACC architectures in terms of superior string stability properties (Jia et al., 2015). The concept of string stability is coined in such a way that the disturbances, which are introduced into a traffic platoon by emergency braking and accelerating vehicles, will not be amplified in the upstream direction. While string stability in ACC schemes cannot be ensured for inter-vehicle time gaps smaller than 1 s, CACC is shown to ensure string stability for time gaps significantly smaller than 1 s (Ploeg et al., 2013). In general, CACC has various advantages like improved road throughput, reduced aerodynamic drag, and reduced fuel consumption over ACC architectures (Van Arem et al., 2006; Shladover, 2005).

Both the schemes ACC and CACC are fundamentally inspired from classical adaptive control, which is a systematic technique of simultaneous estimation and control. Since classical adaptive control typically suffers from poor transient performance in the absence of parameter convergence (see [(Narendra and Annaswamy, 2012), Ch.6]) due to lack of persistence of excitation (PE), similar phenomenon is also observed in ACC and CACC architectures. The PE condition demands richness of information content regarding the unknown parameters for all time-span, which is stringent in nature due to lack of practical viability and online verifiability (Loria, 2004). Hence, a practical solution to the problem of parameter convergence and transient response improvement is an active research issue in adaptive control (Loria, 2004; Krstić et al., 1993; Datta and Ioannou, 1994; Cao and Hovakimyan, 2008). It has been validated that parameter convergence enhances the overall stability and robustness properties of the closed-loop adaptive systems (Lin and Kanellakopoulos, 1998). In cruise control applications, where majority of nominal operating conditions demand a constant speed of the entire vehicle platoon, the PE condition is certainly not satisfied due to scarcity of consistent variation (richness) of the measured signals (velocity, acceleration etc.).

In contrast to PE-based results, some recent works (Roy et al., 2016, 2017a,b; Roy and Bhasin, 2018; Jha et al., 2019, 2018) have proposed a relaxed condition, called initial excitation (IE), which is shown to be sufficient for parameter convergence in the developed composite adaptive control architectures. In comparison to PE, the IE condition can be checked online (Roy et al., 2017b). The IE condition is milder than PE since it requires the excitation/richness of the signal only in the initial time-window of finite length.

CACC based platooning has a similar analytical structure as distributed adaptive control of multi-agent systems, where information is shared via wireless communication using network graph topology. The work in (Papusha et al., 2014) develops a collaborative system identification with consensus-based parameter update law, while proposing a new term called, collective persistence of excitation (C-PE) on the regressor signal to claim parameter convergence even if no single agent has PE input. Since it is not practical to achieve PE as well as C-PE condition in most of the practical scenarios (like CACC, adaptive coverage control (Schwager et al., 2009) etc.), a further relaxed condition C-IE is recently coined (Garg and Roy, 2019) to be sufficient for consensus parameter convergence. The C-IE condition does not require individual agents to satisfy the IE condition necessarily; rather the IE condition can be satisfied cooperatively through information sharing over the communication graph and it is already been proved that the C-IE condition is the most relaxed condition in comparison to PE, IE, and C-PE condition (Garg and Roy, 2019).

The proposed work designs a combined CACC architecture for uncertain homogeneous vehicle platoon. The term "combined" is borrowed from combined MRAC literature, which is a combination of direct and indirect MRAC (Narendra and Annaswamy, 2012). The combined CACC architecture is composed of a distributed parameter estimator of the uncertain vehicle dynamics parameters and a MRAC control law with a differential control parameter update routine. The control parameter estimator uses information from the vehicle dynamics parameter estimator making the design analogous to combined MRAC. The distributed parameter estimator of the vehicle dynamics is designed based on a two-layer filtering mechanism (Jha et al., 2019) and a consensus-based component using information from immediate preceding and following vehicles' instantaneous estimation. This distributed estimator can ensure exponentially fast parameter convergence using the newly defined condition of C-IE and thereby relaxes the need for excitation (information content regarding the unknown parameters) to persist for all time. The C-IE condition implies that the IE condition is satisfied by all the agents cooperatively instead of individually. So the information content is distributed among all the vehicles' regressors in the initial time-window, which is strategically captured in the distributed estimator dynamics leading to parameter convergence. Further the designed MRAC law along with the distributed estimator ensures asymptotic convergence of the vehicle platoon to a string stable reference platoon, thus maintaining smooth and safe operation.

2. PRELIMINARIES

2.1 *Signal Excitation Definitions.*

This sub-section introduces several excitation definitions, which are used to claim the parameter convergence properties of the estimator dynamics.

The extended version of PE condition in multi-agent related architecture called as collective persistence of excitation (C-PE), based on (Papusha et al., 2014), is defined as follows.

Definition 1. A set of bounded signals $p_i(t) \in \mathbb{R}^n$, where $t \in [t_0, \infty)$, $t_0 \geq 0$ for all $i = 1, \ldots, n$, is collectively persistently exciting if $\exists\, T > 0$ and $\gamma > 0$ such that the following inequality holds:

$$\int_{t}^{t+T} \sum_{i=1}^{n} p_i(r) p_i(r)^T dr \geq \gamma I, \quad \forall t \geq t_0. \tag{1}$$

In contrast to C-PE condition, a further slackened condition **collective initial excitation (C-IE)**, based on (Garg and Roy, 2019), is defined subsequently.

Definition 2. A set of bounded signals $p_i(t) \in \mathbb{R}^n$, where $t \in [t_0, \infty)$, $t_0 \geq 0$ for all $i = 1, \ldots, n$, is collectively initially exciting if $\exists\, T > 0$ and $\gamma > 0$ such that the following inequality holds:

$$\int_{t_0}^{t_0+T} \sum_{i=1}^{n} p_i(r) p_i(r)^T dr \geq \gamma I \tag{2}$$

where I denotes the identity matrix of dimension n.

Remark 1. The IE condition requires the excitation/richness (Roy et al., 2016, 2017a) only in the initial finite time window, unlike PE, where the excitation is needed for the entire time span (Tao, 2003). It has been claimed that (Roy et al., 2017a, 2016) the IE condition is less stringent than PE since it is online verifiable and it does not rely on the future behavior of the signals. On the other hand, C-PE condition implies that each regressor signal $p_i(t)$ need not be individually PE, rather the set of signals can collaboratively satisfy the PE condition. Along the similar idea, the introduced concept of the C-IE condition shows the possibility of cooperatively satisfying the IE condition without requiring the individual signals to be IE. Hence, it can be conclude that the C-IE condition is milder than PE, C-PE and IE conditions.

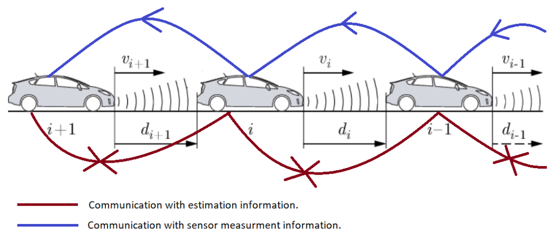

Fig. 1. CACC based homogeneous vehicle platoon.

3. MODEL DESCRIPTION

3.1 *Model description for Vehicle Platoon architecture.*

Consider a homogeneous platoon with **n** no of vehicles. Fig.1, shows the platoon where $v_i(t) \in \mathbb{R}$ denotes the velocity (m/s)

of vehicle i, and $d_i(t) \in \mathbb{R}$ is the distance (m) between vehicle i and its preceding vehicle $i-1$. This distance is measured using a radar or lidar mounted on the front bumper of each vehicle. Furthermore, each vehicle in the platoon string can communicate with its preceding vehicle via wireless communication. The main task of every vehicle in the platoon, except the Leader (virtual leader in the proposed model), is to maintain some desired inter-vehicle distance $d_{r,i}(t) \in \mathbb{R}$ between itself and its preceding vehicle. Define the set $S_n = \{i \in \mathbb{N} | 1 \le i \le n\}$ with the index $i = O$ is fixed for the virtual leader. To regulate the inter-vehicle distance, a constant time headway (CTH) spacing policy is chosen, which is based on (Rajamani and Zhu, 2002). The CTH is formulated by defining the $d_{r,i}(t)$ as

$$d_{r,i}(t) = r_i + hv_i(t) \quad \forall i \in S_n \tag{3}$$

where $r_i \in \mathbb{R}$ is the standstill distance (meters) and $h > 0$ is the time headway (seconds). Hence the spacing error (meters) of the i^{th} vehicle is defined as

$$\begin{aligned} e_i(t) =& d_i(t) - d_{r,i}(t) \\ =& (q_{i-1}(t) - q_i(t) - L_i) - (r_i + hv_i(t)) \; \forall i \in S_n \end{aligned} \tag{4}$$

where $q_i(t) \in \mathbb{R}$ and $L_i \in \mathbb{R}$ representing the rear-bumper position (m) and length (m) of vehicle i, respectively. The desired behavior of the string of vehicle platoon is coined in terms of string stability, which captures the notion of attenuation of disturbances like emergency braking (Ploeg et al., 2013). An established definition of string stability is as follows.

Definition 3 (String Stability (Ploeg et al., 2013)). Consider the acceleration of vehicle i is denoted by $a_i(t) \in \mathbb{R}$. Then, a platoon can be considered as string stable if

$$\sup_w |X_i(jw)| = \sup_w \left| \frac{a_i(jw)}{a_{i-1}(jw)} \right| \le 1 \quad \forall i \in S_n \tag{5}$$

where $a_i(jw)$ is the Laplace transform of the acceleration of vehicle i.

The dynamics of the i^{th} vehicle is represented by the following model.

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} v_{i-1} + \begin{pmatrix} 0 \\ 0 \\ \frac{\Omega}{\tau} \end{pmatrix} u_i \; \forall i \in S_n \tag{6}$$

where $u_i(t) \in \mathbb{R}$ is the control input (m/s^2) of vehicle i and τ denotes each vehicle's unknown driveline time constant (seconds) and Ω denotes the engine's performance. Engine's performance is effected by the different type of disturbances such as wind gust, slope of road, etc. Based on Model (6) proposed in (Ploeg et al., 2013; Harfouch et al., 2017) and considering the ideal engine's performance, the virtual leader vehicle model is defined as

$$\begin{pmatrix} \dot{e}_O \\ \dot{v}_O \\ \dot{a}_O \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_O} \end{pmatrix} \begin{pmatrix} e_O \\ v_O \\ a_O \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{\Omega_O}{\tau_O} \end{pmatrix} u_O \tag{7}$$

4. CACC FOR UNKNOWN HOMOGENEOUS PLATOON

A CACC architecture is constructed for the homogeneous vehicle platoon in the presence of unknown parameters-vehicle engine performance and driveline time-constant. The proposed CACC scheme is composed of a distributed online parameter estimation algorithm combined with a suitable control law to ensure closed-loop system stability. The control law has two components - baseline controller, which is used to ensure string stability of the nominal model and an MRAC controller to ensure asymptotic convergence of the uncertain model to the string stable model.

4.1 Baseline Controller and CACC Reference model.

By considering various baseline conditions such as ideal engine performance, persistent communication availability between consecutive vehicles, the authors in (Ploeg et al., 2013) derived a controller and a spacing policy, which ensures string stability of the platoon. The CACC baseline controller is defined as

$$\dot{u}_{bl,i} = \frac{1}{h}\Big(- u_{bl,i} + K_p e_i + K_d \dot{e}_i + u_{bl,i-1} \Big), \; u_{bl,i}(t_0) = 0 \tag{8}$$

where K_p and K_d are the tuning parameters for controller. The term, $u_{bl,i-1}$ introduces information from the precedent vehicle $(i-1)$, which makes CACC a powerful scheme in contrast to ACC. Further the control input of the virtual leader is designed as

$$\dot{u}_O = \frac{1}{h}(-u_O + u_r) \tag{9}$$

where $u_r(t) \in \mathbb{R}$ is an external input acting as the desired acceleration (m/s^2) of the virtual leader.

To design the adaptive component of the controller based on MRAC approach, the CACC reference model is defined subsequently as provided in (Harfouch et al., 2017).

$$\begin{aligned} \underbrace{\begin{pmatrix} \dot{e}_{i,r} \\ \dot{v}_{i,r} \\ \dot{a}_{i,r} \\ \dot{u}_{i,r} \end{pmatrix}}_{\dot{x}_{i,r}} =& \underbrace{\begin{pmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_O} & \frac{\Omega_O}{\tau_O} \\ \frac{K_p}{h} & -\frac{K_d}{h} & -K_d & -\frac{1}{h} \end{pmatrix}}_{A_r} \underbrace{\begin{pmatrix} e_{i,r} \\ v_{i,r} \\ a_{i,r} \\ u_{i,r} \end{pmatrix}}_{x_{i,r}} \\ &+ \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_d}{h} & \frac{1}{h} \end{pmatrix}}_{B_{w,r}} \underbrace{\begin{pmatrix} v_{i-1} \\ u_{bl,i-1} \end{pmatrix}}_{w_i} \quad \forall i \in S_n \end{aligned} \tag{10}$$

where $x_{i,r}(t) \in \mathbb{R}^4$ and $w_i(t) \in \mathbb{R}^2$ are i^{th} vehicle's reference state vector and input vector, respectively; and $A_r \in \mathbb{R}^{4\times4}$, $B_{w,r} \in \mathbb{R}^{4\times2}$ are the system matrix and input matrix, respectively. Further, combining (9) with (7), the virtual leader dynamics become

$$\underbrace{\begin{pmatrix} \dot{e}_O \\ \dot{v}_O \\ \dot{a}_O \\ \dot{u}_O \end{pmatrix}}_{\dot{x}_O} = \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_O} & \frac{\Omega_O}{\tau_O} \\ 0 & 0 & 0 & -\frac{1}{h} \end{pmatrix}}_{A_{lr}} \underbrace{\begin{pmatrix} e_O \\ v_O \\ a_O \\ u_O \end{pmatrix}}_{x_O} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{h} \end{pmatrix}}_{B_{lr}} u_r \tag{11}$$

It has been proved in (Ploeg et al., 2013) that, the reference model (10) is asymptotically stable around the equilibrium point

$$x_{i,r,eq} = (0 \; v_O \; 0 \; 0)^T, \text{for } x_O = x_{i,r,eq} \text{ and } u_r = 0 \tag{12}$$

where v_O is a constant velocity, provided that the following Routh–Hurwitz conditions are satisfied:

$$h > 0, \; K_p, K_d > 0, \; K_d > \tau_O K_p. \tag{13}$$

To invoke string stability of the CACC reference platoon dynamics (10), the following transfer function model is considered,

$$X_i(s) = \frac{1}{hs+1} \quad \forall i \in S_n \tag{14}$$

which satisfies the string stability condition (5) based of Definition 3 for any $h > 0$.

4.2 MRAC in conjunction with Baseline Controller.

In this section, CACC reference model (10) will be used to design the control input $u_i(t)$, $\forall i \in S_n$, such that the uncertain platoon's dynamics described by (6) and (7) converge to the string stable nominal dynamics. To achieve this, the baseline controller is augmented with an adaptive controller as

$$u_i(t) = u_{bl,i}(t) + u_{ad,i}(t) \quad \forall i \in S_n \tag{15}$$

where $u_{ad,i}(t) \in \mathbb{R}$ is the adaptive controller to be constructed subsequently. Now substituting (15) in (6) and exploiting (8), yields

$$\underbrace{\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \\ \dot{u}_{bl,i} \end{pmatrix}}_{\dot{x}_i} = \underbrace{\begin{pmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau} & \frac{\Omega}{\tau} \\ \frac{K_p}{h} & -\frac{K_d}{h} & -K_d & -\frac{1}{h} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} e_i \\ v_i \\ a_i \\ u_{bl,i} \end{pmatrix}}_{x_i}$$
$$+ \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_d}{h} & \frac{1}{h} \end{pmatrix}}_{B_w} \underbrace{\begin{pmatrix} v_{i-1} \\ u_{bl,i-1} \end{pmatrix}}_{w_i} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \frac{\Omega}{\tau} \\ 0 \end{pmatrix}}_{B_u} u_{ad,i} \quad \forall i \in S_n \tag{16}$$

where $x_i \in \mathbb{R}^4$, $A \in \mathbb{R}^{4\times4}$, $B_u \in \mathbb{R}^4$, and $u_{ad,i}(t)$ is defined as

$$u_{ad,i} = \hat{K}^T x_i \quad \forall i \in S_n \tag{17}$$

where $\hat{K}(t) \in \mathbb{R}^4$.

As compared to conventional CACC architectures (Harfouch et al., 2017), this work modifies the reference model (10) by incorporating actual state information in the reference model dynamics as

$$\dot{x}_{i,c} = A_r x_{i,c} + B_w w_i + l\Big(x_i(t) - x_{i,c}(t)\Big) \; \forall i \in S_n \tag{18}$$

where $l > 0$ is the free design parameter. In (Gibson et al., 2013), this type of reference model modification is denoted as closed-loop reference model. The parameter l plays a crucial role in the proposed CACC architecture as revealed in the subsequent stability analysis. Moreover, note that if asymptotic convergence of $x_i(t)$ to $x_{i,c}(t)$ is satisfied, $x_{i,c}(t)$ will also tend to $x_{i,r}(t)$, which implies that the fundamental objective of following the open-loop string stable model (10) is not hampered in the proposed closed-loop modification.

To facilitate the design objective of making system (16) respond as the chosen reference model of (18), the following matching condition is introduced (Gibson et al., 2013).

Assumption 1. There exist constant matrix $K^* \in \mathbb{R}^4$ such that

$$A_r = A + B_u K^{*T} \tag{19}$$

The tracking-error (between actual and closed-loop reference model) is defined as

$$\zeta_i(t) \triangleq x_i(t) - x_{i,c}(t) \quad \forall i \in S_n \tag{20}$$

using (20), (19), (18), (17), and (16), the tracking-error dynamics $\dot{\zeta}_i(t)$ can be expressed as

$$\dot{\zeta}_i(t) = A_r \zeta_i - l\zeta_i + B_u \tilde{K}^T x_i \quad \forall i \in S_n \tag{21}$$

where $\zeta_i(t) \in \mathbb{R}^4$ and $\tilde{K}(t) \triangleq \hat{K}(t) - K^*$. The standard direct projection based adaptive update law for $\hat{K}$ inspired from (Gibson et al., 2013) is, $\dot{\hat{K}}(t) = Proj_\Omega(-\Gamma \zeta_i^T x_i B_u^T P, K)$. Since in present context B_u is unknown, the control parameter update law is designed as

$$\dot{\hat{K}}(t) = Proj_\Omega(-\Gamma \zeta_i^T x_i \hat{B}_u^T P, K) \quad \forall i \in S_n \tag{22}$$

where $\Gamma > 0$ and $P = P^T \in \mathbb{R}^{4\times4}$ is positive definite solution of following algebraic Lyapunov equation

$$A_r{}^T P + P A_r = -Q \tag{23}$$

where $Q \in \mathbb{R}^{4\times4}$ is a chosen positive definite matrix.

The quantity $\hat{B}_u(t)$ is an online estimate of the unknown input matrix. The following subsection develops the distributed platoon parameter estimator, which supplies $\hat{B}_u(t)$ for the control parameter update.

4.3 Online Identification for unknown platoon Parameters

From the structure of (16), it can be conclude that the dynamics

$$\dot{a}_i = -\frac{1}{\tau} a_i + \frac{\Omega}{\tau} u_{bl,i} + \frac{\Omega}{\tau} u_{ad,i} \quad \forall i \in S_n \tag{24}$$

only makes (16) uncertain, otherwise, all other parameters are known. Hence taking advantage of that structure, the linear parameterization of (24) is obtained as

$$\dot{a}_i = [-a_i \; u_i] \begin{bmatrix} \frac{1}{\tau} \\ \frac{\Omega}{\tau} \end{bmatrix} = y_i^T(a_i, u_i)\theta \quad \forall i \in S_n \tag{25}$$

where $y_i(a_i, u_i) \in \mathbb{R}^2$ is the known regressor and $\theta \in \mathbb{R}^2$ is the unknown platoon parameter needs to be estimated.

Assumption 2. $||\theta|| < \delta_1$, for some known constant $\delta_1 > 0$.

To handle the unavailability of the acceleration measurement $\dot{a}_i(t)$, the following filter equations are designed as

$$\dot{z}_i = -k z_i + y_i, \quad z_i(t_0) = 0 \quad \forall i \in S_n \tag{26}$$

$$\dot{g}_i = -k g_i + \dot{a}_i, \quad g_i(t_0) = 0 \quad \forall i \in S_n \tag{27}$$

where $z_i(t) \in \mathbb{R}^2$ denotes the filtered regressor matrix and $g_i(t) \in \mathbb{R}$ denotes the filtered version of $\dot{a}_i(t)$ and k is a positive scalar introduced to stabilize the above filter equations.

Analytically solving (26) and (27) and utilizing (25), the following relation can be deduced.

$$g_i(t) = z_i^T(t)\theta \quad \forall i \in S_n \tag{28}$$

From (27), $g_i(t)$ cannot be explicitly computed since $\dot{a}_i(t)$ is unknown. However, after analytically solving (27) and applying the by parts rule of integration, it can be shown that

$$g_i(t) = a_i(t) - e^{-kt} a_i(t_0) - k h_i(t), \; a_i(t_0) = 0 \quad \forall i \in S_n \tag{29}$$

where $h_i(t) \in \mathbb{R}$ is the output of the subsequently designed filter dynamics.

$$\dot{h}_i = -k h_i + a_i, \; h_i(t_0) = 0 \quad \forall i \in S_n \tag{30}$$

Since $a_i(t)$ is measurable, (29) and (30) can be utilized to obtain $g_i(t)$ online. Hence, it can be argued that the above filter equations (26) and (27) converts the differential equation in (25) to an algebraic one in (28), leading to the omission of $\dot{a}_i(t)$ information. A gradient-based law using (28) can be designed to estimate the system parameter θ $\forall i \in S_n$. However, this type of law requires the stringent PE condition on $z_i(t)$ for parameter convergence (Narendra and Annaswamy, 2012). To overcome this restriction, another pair of projection-based integral law is introduced, inspired by (Basu Roy et al., 2018).

$$\dot{M}_i = proj(z_i z_i^T), \quad M_i(t_0) = 0 \quad \forall i \in S_n \tag{31}$$

$$\dot{w}_i = proj(z_i g_i), \quad w_i(t_0) = 0 \quad \forall i \in S_n \tag{32}$$

where the square matrix $M_i(t) \in \mathbb{R}^{2\times 2}$ denotes the integrated filtered regressor and $w_i(t) \in \mathbb{R}^2$ can be thought of as integrated filtered version of $\dot{a}_i$ (although dimensionally w_i is different from $\dot{a}_i$). Unlike (Adetola and Guay, 2008; Roy et al., 2016), the use of $proj(.)$ (Basu Roy et al., 2018), which denotes projection operator, restrict the variables $M_i(t)$ and $w_i(t)$ within a compact set.

Preposition 1. Integrating (31) and (32) and using (28), it can be shown that

$$w_i(t) = M_i(t)\theta, \quad \forall t \geq t_0 \quad \forall i \in S_n \tag{33}$$

Proof. For proof refer the (Basu Roy et al., 2018).

The matrices $M_i(t)$'s have the following properties:

Property 1: $M_i(t)$ is a positive semi-definite function of time i.e. $M_i(t) \geq 0, \forall t \geq t_0$.

Property 2: $M_i(t)$ is a non-decreasing function of time in the sense of matrix inequality i.e. $M_i(t_2) \geq M_i(t_1)$ for $t_2 > t_1$.

By exploiting these two properties and above filtering and $proj(.)$ based arguments, a novel distributed consensus-based parameter estimation law is proposed as follows (Garg and Roy, 2019).

$$\dot{\hat{\theta}}_i(t) = \underbrace{k_\theta z_i(g_i - z_i^T\hat{\theta}_i)}_{P} + \underbrace{\Gamma_\theta(w_i - M_i\hat{\theta}_i)}_{I} + \underbrace{\sum_{j\in\mathbb{N}_i}(\hat{\theta}_j - \hat{\theta}_i)}_{C} \tag{34}$$

where $\hat{\theta}_i(t) \in \mathbb{R}^2$ is an online estimate of the unknown platoon parameter vector θ $\forall i \in S_n$, and $k_\theta > 0$ and $\Gamma_\theta > 0$ are two positive scalar gains used to tune the rate of convergence and the neighbors sets are defined as $\mathbb{N}_1 = \{2\}, \mathbb{N}_i = \{(i-1),(i+1)\}, \forall i = 2,3,..,n-1, \mathbb{N}_n = \{n-1\}$. Here, the first term P of (34) is a proportional-like component, the component I is an integral-like term and the last term C is a term based on the neighbor's current estimates according to chosen network topology. Together I and C circumvents the C-PE restriction and leads to parameter convergence under the C-IE condition as revealed subsequently.

Assumption 3. The set of filtered regressors $z_i(t)$, $\forall i \in S_n$, satisfy the C-IE condition as per Definition 2.

4.4 Compact representation for parameter estimation error dynamics

The parameter estimation error dynamics for all the **n** no of unknown follower vehicles in the platoon can be compactly represented as

$$\Delta\dot{\theta} = -(L \otimes I_2)\Delta\theta - k_\theta\Phi(t)\Delta\theta - \Gamma_\theta\Phi_I(t)\Delta\theta \tag{35}$$

where $L \in \mathbb{R}^{n\times n}$ denotes the laplacian matrix, which is used to represent the estimation information sharing phenomena over the given network topology as in (Garg and Roy, 2019), $\otimes$ denotes the kronecker product and $I_2 \in \mathbb{R}^{2\times 2}$ is the identity matrix, column vectors $\hat{\theta} = [\hat{\theta}_1,...,\hat{\theta}_n]^T \in \mathbb{R}^{n2}$ and $\Delta\theta = [\Delta\theta_1,...,\Delta\theta_n]^T \in \mathbb{R}^{n2}$ by stacking the components $\hat{\theta}_i \in \mathbb{R}^2$ and $\Delta\theta_i = \hat{\theta}_i - \theta \in \mathbb{R}^2, \forall i \in S_n$. And $\Phi(t), \Phi_I(t) \in \mathbb{R}^{n2\times n2}$ are block diagonal matrices, which are defined as

$$\Phi(t) = \begin{bmatrix} z_1 z_1^T & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & z_n z_n^T \end{bmatrix}$$

and

$$\Phi_I(t) = \begin{bmatrix} proj(z_1(r)z_1^T(r)) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & proj(z_n(r)z_n^T(r)) \end{bmatrix}.$$

Theorem 1. The origin of the estimation error dynamics (35) is Lyapunov stable, in addition if the Assumption 3 is satisfied, then $||\Delta\theta(t)||$ exponentially converges to zero for $t \geq t_0 + T$ i.e.,

$$||\Delta\theta(t)|| \leq \gamma_1 e^{-\gamma_2 t}, \quad t \geq t_0 + T \tag{36}$$

for some positive scalars γ_1 and γ_2.

Proof. Consider the following Lyapunov candidate

$$V(\Delta\theta) = \frac{1}{2}\Delta\theta^T\Delta\theta \tag{37}$$

Taking the time derivative of (37) along the dynamics (35) yields

$$\dot{V}(\Delta\theta) = -\Delta\theta^T\Big((L\otimes I_2) + k_\theta\Phi(t) + \Gamma_\theta\Phi_I(t)\Big)\Delta\theta \leq 0 \tag{38}$$

which implies $V(\Delta\theta) \in \mathscr{L}_\infty$ and it is non-increasing in time $\forall t \geq t_0$, i.e., the origin of the dynamics of $\Delta\theta(t)$ is Lyapunov stable.

Let's assume the matrix $J(t)$, which is defined as

$$J(t) \triangleq (L\otimes I_2) + \Phi_I(t) \triangleq (L\otimes I_2) + proj(\Phi(t)) \tag{39}$$

then by referring the proof of Theorem 1 from (Garg and Roy, 2019), it can be concluded that $J(t_0+T) > 0$, $\forall t \geq t_0 + T$.

Hence, (38) can be upper bounded as

$$\dot{V}(\Delta\theta) \leq -\Delta\theta^T J(t)\Delta\theta \leq -\lambda_{min}(J(t))||\Delta\theta||^2 \tag{40}$$

using the same argument as in property 2, $J(t) \geq J(t_0+T) > 0$ $\forall t \geq t_0 + T$, which implies that $\lambda_{min}(J(t)) \geq c > 0$, where the λ_{min} is the minimum eigen value of matrix $J(t)$ and c is positive real constant.

Using (37), (40) can be expressed as

$$\dot{V}(\Delta\theta) \leq -2cV(\Delta\theta),\ \forall t \geq t_0 + T \tag{41}$$

This differential inequality leads to the following exponentially convergent bound on $V(\Delta\theta)$

$$V(\Delta\theta(t)) \leq V(\Delta\theta(t_0+T))e^{-2c(t-t_0-T)}, \quad \forall t \geq t_0 + T \tag{42}$$

From (37), $||\Delta\theta(t)|| = \sqrt{2V(\Delta\theta(t))}$, which implies that $||\Delta\theta(t)||$ is exponentially convergent to zero for $t \geq t_0 + T$, i.e., (36) holds true. Since $V(\Delta\theta(t))$ in (37) is radially unbounded, the mentioned result is globally valid.

Remark 3. From classical control literature, it is well established that an integral action in conjunction with a proportional control improves the steady-state accuracy. Motivated by the power of integral action, the integral-like component I is introduced in the update law, which reduces the steady state parameter estimation error. In fact, the integral term in conjunction with the cooperative term C circumvents the restrictive C-PE condition for parameter convergence, while requiring a milder condition of C-IE.

5. TRACKING-ERROR STABILITY/CONVERGENCE ANALYSIS

The tracking-error dynamics $\zeta(t)$ for all **n** no of vehicles in the platoon are compactly represented as

$$\dot{\zeta}(t) = \Big(I_n \otimes (A_r - lI_n)\Big)\zeta(t) + (\mathbf{1}_n^T \otimes B_u)\tilde{K}^T(t)x(t) \quad (43)$$

where $\zeta(t) \in \mathbb{R}^{n4}$, $x(t) \in \mathbb{R}^{n4}$, $\tilde{K}(t) \in \mathbb{R}^{n4}$ and $\mathbf{1}_n = \{1, ..., 1\} \in \mathbb{R}^{1\times n}$.

The compact representation for $\hat{K}$ dynamics from (22) is given as

$$\dot{\hat{K}}(t) = Proj_\Omega(-\Gamma(\mathbf{1}_n \otimes \hat{B}_u^T)(I_n \otimes P)\zeta x^T, K) \quad (44)$$

where the Proj operator in (44) ensures that $\hat{K}(t)$ remains within a compact set for all time (Lavretsky and Wise, 2013).

Theorem 2. For the system (16) along with control input (17) with the parameter update laws (35) and (44), the overall error dynamics $\eta(t) = [\zeta^T(t), \Delta\theta^T(t), \tilde{K}^T(t)]^T$ is Lyapunov stable $\forall t \geq t_0 + T$, provided Assumption 3 holds. In addition, the tracking-error $\zeta(t)$ tends to zero asymptotically with asymptotic string stability i.e., $\lim_{t\to\infty}[x_i(t) - x_{i,r}(t)] = 0$, $\forall i \in S_n$.

Proof. Consider the following Lyapunov candidate.

$$\begin{aligned} V =& \frac{1}{2}\zeta^T(I_n \otimes P)\zeta + \frac{1}{2}Tr(\tilde{K}^T\Gamma^{-1}\tilde{K}) \\ &+ \frac{1}{2}\Delta\theta^T\Delta\theta \end{aligned} \quad (45)$$

Taking the time derivative of (45) along the system trajectories, yields

$$\begin{aligned} \dot{V} =& \frac{1}{2}\zeta^T(I_n \otimes P)\dot{\zeta} + \frac{1}{2}\dot{\zeta}^T(I_n \otimes P)\zeta + Tr(\tilde{K}^T\Gamma^{-1}\dot{\tilde{K}}) \\ &+ \Delta\theta^T\Delta\dot{\theta} \end{aligned} \quad (46)$$

after putting (43), (44) and using the compact representation of Lyapunov equation (23) and resulting argument from proof of Theorem 1, the relation in (45) can be modified as

$$\begin{aligned} \dot{V} \leq& -\frac{1}{2}\zeta^T(I_n \otimes Q)\zeta - l\zeta^T(I_n \otimes P)\zeta - k_1||\Delta\theta||^2 \\ &+ \zeta^T(I_n \otimes P)(\mathbf{1}_n^T \otimes \tilde{B}_u)\tilde{K}^T x \quad \forall t \geq t_0 + T \end{aligned} \quad (47)$$

where k_1 is the $\lambda_{min}(J(t_0 + T))$ and the inequality is due to (44). Further (47) can be upper-bound as

$$\begin{aligned} \dot{V} \leq& -\frac{1}{2}\lambda_{min}(I_n \otimes Q)||\zeta||_2^2 - l\lambda_{min}(I_n \otimes P)||\zeta||_2^2 \\ &- k_1||\Delta\theta||_2^2 + \zeta^T(I_n \otimes P)(\mathbf{1}_n^T \otimes \tilde{B}_u)\tilde{K}^T x \quad \forall t \geq t_0 + T \end{aligned} \quad (48)$$

where $\lambda_{min}(\cdot)$ is the minimum eigen value of the specified matrix. From (20), (44) and by considering the proof of Theorem 1, (48) can be further modified as

$$\begin{aligned} \dot{V} \leq& -\frac{1}{2}\lambda_{min}(I_n \otimes Q)||\zeta||_2^2 - l\lambda_{min}(I_n \otimes P)||\zeta||_2^2 \\ &- k_1||\Delta\theta||_2^2 + ||(I_n \otimes P)||_F||\Delta\theta||_2||\tilde{K}^T||_2||\zeta||_2^2 \\ &+ ||(I_n \otimes P)||_F||\Delta\theta||_2||\tilde{K}^T||_2||\zeta||_2||x_{i,c}||_2 \quad \forall t \geq t_0 + T \end{aligned} \quad (49)$$

where $||\cdot||_F$ denotes the Forbenious norm of a matrix and $||\cdot||_2$ denotes the 2-norm, which is used for vectors.

Since $||I_n \otimes P||_F \in \mathscr{L}_\infty$, $||\Delta\theta||_2 \in \mathscr{L}_\infty$ based on proof of Theorem 1 and $||\tilde{K}||_2 \in \mathscr{L}_\infty$ using $proj(.)$ operator, (49) can be restructured as

$$\begin{aligned} \dot{V} \leq& -\frac{1}{2}\lambda_{min}(I_n \otimes Q)||\zeta||_2^2 - l\lambda_{min}(I_n \otimes P)||\zeta||_2^2 \\ &- k_1||\Delta\theta||_2^2 + \delta_3||\Delta\theta||_2||\zeta||_2^2 + \delta_4||\Delta\theta||_2||\zeta||_2 \quad \forall t \geq t_0 + T \end{aligned} \quad (50)$$

where $\delta_3, \delta_4 > 0$. If the tracking-error $\zeta(t)$ fulfills the following condition

$$||\zeta||_2 \leq \frac{m - \delta_4}{\delta_3} \quad (51)$$

where $m > \delta_4$, the following inequality can be written.

$$\delta_3||\zeta||_2^2 + \delta_4||\zeta||_2 \leq m||\zeta||_2. \quad (52)$$

Hence, based on (52), inequality (50) can be further simplified as

$$\begin{aligned} \dot{V} \leq& -\frac{1}{2}\lambda_{min}(I_n \otimes Q)||\zeta||_2^2 - l\lambda_{min}(I_n \otimes P)||\zeta||_2^2 \\ &+ m||\zeta||_2||\Delta\theta||_2 - k_1||\Delta\theta||_2^2 \quad \forall t \geq t_0 + T \end{aligned} \quad (53)$$

It can be deduced that if the subsequent gain condition

$$k_1 > \frac{m^2}{4l\lambda_{min}(I_n \otimes P)} \quad (54)$$

is satisfied, then $\dot{V}$ is negative semidefinite. Therefore, $V(t) \in \mathscr{L}_\infty$ which imply that the overall error dynamics $\eta(t) \in \mathscr{L}_\infty$. Further, the tracking-error $\zeta(t)$ can be upper-bounded by the following inequality

$$||\zeta(t)|| \leq \sqrt{\frac{2V(t)}{\Lambda_{min}(I_n \otimes P)}} \quad \forall t \geq t_0. \quad (55)$$

Since $\dot{V} \leq 0$, $\forall t \geq t_0$, the inequality in (55) can be alternatively expressed as

$$||\zeta(t)|| \leq \sqrt{\frac{2V(t_0 + T)}{\Lambda_{min}(I_n \otimes P)}} \quad \forall t \geq t_0 + T. \quad (56)$$

The Lyapunov function $V(t)$ can be further upper-bounded as

$$\begin{aligned} V(t) \leq& \frac{1}{2}\Big(\lambda_{max}(I_n \otimes P)||\zeta(t)||_2^2 + \lambda_{min}^{-1}(\Gamma)||\tilde{K}^T(t)||^2 \\ &+ ||\Delta\theta(t)||_2^2\Big) \quad \forall t \geq t_0. \end{aligned} \quad (57)$$

Selecting $||\hat{\theta}(0)|| \leq \delta_1$ and considering assumption 2, it can be claimed that

$$||\Delta\theta(t)|| \leq 2\delta_1 \quad \forall t \geq t_0. \quad (58)$$

Therefore, using (57), (58) and (44), an upper bound of $\zeta(t)$ can be calculated analytically from (56) as

$$\zeta(t) \leq \underbrace{\sqrt{\frac{\lambda_{max}(I_n \otimes P)||\zeta(t_0 + T)||_2^2 + 2\lambda_{min}^{-1}(\Gamma)\delta_2^2 + 2\delta_1^2}{\lambda_{min}(I_n \otimes P)}}}_{\nu} \quad \forall t \geq t_0 + T. \quad (59)$$

Where δ_2 is the upper bound $\tilde{K}(t)$, based on $proj$ operator. Since the stability proof requires (51) to be satisfied, it implies that the error bound ν should be less than $\frac{m-\delta_4}{\delta_3}$. Thus, m is chosen in such a way that, it should satisfying the following inequality

$$m > \delta_4 + \delta_3\nu \quad (60)$$

where a crude estimate of ν is utilized using $\zeta(t_0)$. The choice of m, which satisfies (60) is finally used in (50) to derive the sufficient gain condition for Lyapunov stability. Furhter using Barbalat's Lemma (Slotine et al., 1991) on (53), it can be conclude that the tracking-error $\zeta(t)$ is asymptotically converging to zero.

Remark 4. Since the actual system (16) is a linear system with right hand side to be globally Lipschitz, using Global Existence and Uniqueness Theorem (Slotine et al., 1991), it can be claimed that the system dynamics will remain bounded in the initial time-window $[t_0, t_0 + T)$. Moreover, note that there is a crucial difference between the gain condition (54) and a similar gain condition obtained in (Roy et al., 2017a) for a single agent linear MRAC problem. Unlike Roy et al. (2017a), the gain condition is shared between l and k_1. Hence, due to the introduction of closed-loop reference model, the burden of gain condition is divided between the closed-loop gain l and the distributed estimator gain k_1.

6. SIMULATION RESULTS

The proposed algorithm is simulated by considering the protocol as in Fig.1, where 3 unknown homogeneous vehicles are forming a vehicle platoon using a virtual leader, which have following system parameters.

$\tau_0 = 0.1$, $\Omega_0 = 1$, $\tau = 0.4$, $\Omega = 0.8\ \forall i \in \{1,2,3\}$. The time gap $h = 0.7s$. The baseline controllers' gains are chosen as $K_p = 0.2$ and $K_d = 0.7$ in order to maintain both string stability conditions (13) and (14). The desired acceleration is selected as $u_r(t) = 80exp(-2t)$; the design parameter l is chosen as $l = 5.$, the adaptation gains are chosen as $k_\theta = 5$, $\Gamma_\theta = 5$.

Fig.2 shows the convergence of norm of tracking error $\zeta(t)$ to zero and Fig.3 shows the norm of controller parameter estimation error $\tilde{K}(t) \in \mathscr{L}_\infty$. Fig.4 shows the norm of error between CACC reference model (10) and CACC closed-loop reference model (18). From Fig.4 it can be concluded that $\lim_{t\to\infty}[x_i(t) - x_{i,r}(t)] = 0\ \forall i \in \{1,2,3\}$, which is the primary design objective. Fig.5 represents the comparison of velocity profile, which portrays velocity synchronization of the entire platoon. Fig.6 represents the comparison of norm of parameter estimation-error $\Delta\theta(t)$ for various cases, like P: means parameter estimator consist only proportional error like term, $P+C$: means only proportional and consensus terms are used in estimator, $P+I+C$: means all terms including integral term are used. Now from Fig.6, it can be inferred that integral and consensus terms are significant to achieve the exponential convergence under C-IE condition. The C-IE condition is satisfied approximately after some finite time as verified in the simulation by checking the determinant of the matrix $J(t)$.

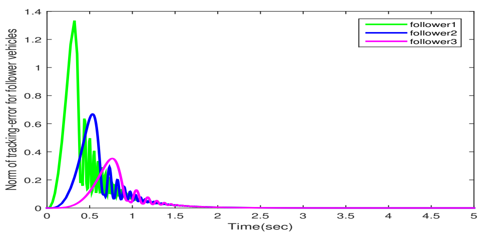

Fig. 2. Norm of tracking-error $\zeta(t)$ for uncertain vehicles in the platoon.

7. CONCLUSION

This paper proposes a combined-CACC architecture using a closed-loop reference model based MRAC algorithm for a homogeneous platoon, without knowledge of the platoon vehicle dynamics parameters Ω (engine performance) and τ (driveline constant). The method is composed of a novel distributed consensus-based plant-parameter estimator in conjunction with a differential adaptive update law for the control-parameters. Provided the set of filtered regressors $z_i(t)$, $\forall i \in S_n$, is C-IE,

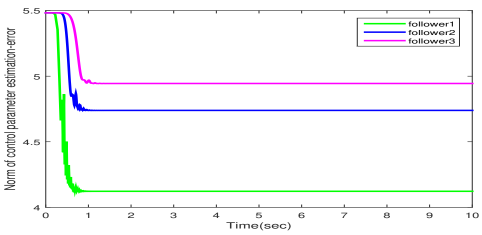

Fig. 3. Norm of controller parameter estimation-error $\tilde{K}(t)$ for uncertain vehicles in the platoon.

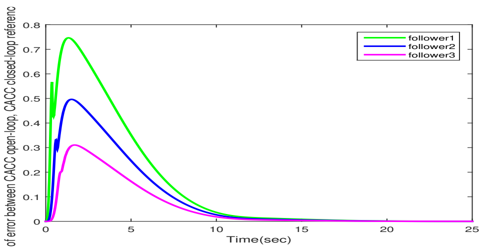

Fig. 4. Norm of error between CACC open-loop, CACC closed-loop reference model for uncertain vehicles in the platoon.

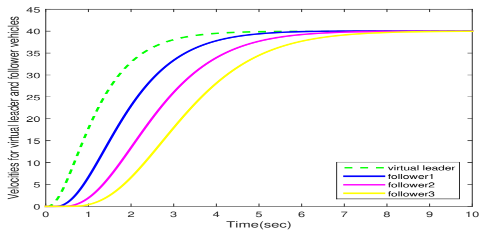

Fig. 5. Comparison of velocities for virtual leader as well as uncertain vehicles in the platoon.

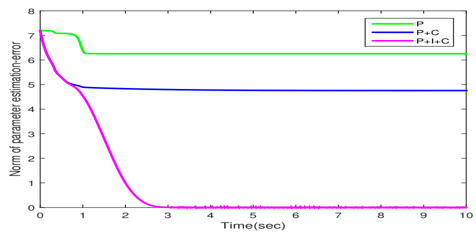

Fig. 6. Comparison of norm of Parameter estimation error $\Delta\theta(t)$.

the algorithm guarantees exponential convergence of parameter estimation error $\Delta\theta(t)$ as well as asymptotic convergence of tracking error $\zeta(t)$ to zero. The C-IE condition is milder than all the conditions for parameter convergence available in literature like PE, C-PE and IE etc. The use of closed-loop reference model instead of the open-loop reference model in MRAC protocol provides an additional design freedom in the CACC algorithm, which is used to achieve better transient as well as stability guarantee.

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