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Collision Risk Assessment based on Line of Sight

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Abstract: Collision avoidance of vehicles is an essential safety feature of modern-day vehicles. The widely used Time to Collision (TTC) approach for collision risk assessment provides a false alarm in many situations like road turning, traffic intersection, and near-miss. Therefore, this risk assessment approach cannot be applied to many realistic scenarios where knowledge of future trajectory plays an important role in collision risk assessment. After evaluating conventionally used techniques, this paper proposes a novel probabilistic approach of collision risk assessment utilizing Line of Sight (LOS) concept for front-to-front end forward-collision situation. This approach does not require high computational power during online execution and is expected to reduce false alarm rates to a significant level. For the implementation of this approach, a large number of forward-collision scenarios are generated, and various motion parameters are characterized. Further, Bayesian learning is used to update the risk at every sampling instant for each scenario followed by a risk threshold generation based on Receiver Operating Characteristic (ROC) plot. Finally, a decision is made by predicting the collision risk at certain distances and then comparing them with the threshold of risk. Simulations using relevant industry-standard software and realistic assumptions have been performed, which produces results ensuring the effectiveness of the proposed methodology.

Keywords: Line of Sight, Collision Avoidance, Head-on Collision, Bayesian Learning, ROC

1. INTRODUCTION

Vehicles play a very important role in human lives. However, they are prone to accidents and can lead to fatalities and therefore, road safety is a very serious issue. Passive safety systems in vehicles like seat-belts, airbags, etc. can reduce the severity of the effect of accidents but their effect is limited to a certain extent. The challenge of avoiding collisions to reduce road fatalities leads to the development of active safety systems like Automatic Emergency Braking (AEB), Automatic Cruise Control (ACC), Forward Collision Warning (FCW), Lane departure Warning and Assist, Blind-spot Monitoring and Assist, Parking Assist, Pedestrian Detection. Still, the main challenge is to avoid collision to reduce road fatalities which has led the way for development of collision avoidance systems. The general architecture of the collision avoidance system consists of four modules- Sensing or Perception module, Risk assessment module, Motion planning and re-planning module and Control and Actuation module (Pendleton et al. (2017)). Among these, Risk assessment plays a crucial role in collision avoidance. There are various risk assessment techniques present in literature. Among these, TTC (Van Der Horst and Hogema (1993)) is the most common method for risk assessment. It is a deterministic approach and assumes that the velocity of each vehicle will remain constant in future driving scenarios. The advantage of this strategy is that it is computationally cheap. However, it's assessment is not robust, since it doesn't take into consideration the uncertainty in future motions which results into a false alarm in some situations, for example – at road turning, road intersections or near-miss. Therefore, probabilistic approaches have been explored for risk assessments. These consider the uncertainties in the motion parameters of vehicles. Risk assessment based on the probabilistic approach has been reported in many literatures. In Houénou et al. (2014), Monte Carlo simulation has been used at each sampling instant to calculate the risk of collision online. The advantage of this method is that it deals with any type of scenario. However, implementing this requires high computational power. On the other hand, Hidden Markov Model (HMM) and Monte Carlo simulations have been used in Laugier et al. (2011) to calculate the risk of collision. By using HMM, it considers a limited number of future maneuvers such as-turn right, turn left, etc. to improve the risk estimation. However, the computational complexity associated with this online Monte Carlo based simulation makes this approach computationally inefficient. Model Predictive Control (MPC) based decision-making Gray et al. (2013) has also been explored for risk assessment. MPC based Collision avoidance problems are generally nonconvex and therefore, computationally expensive. Recently, machine learning methods have also been used for collision risk assessments. In Motamedidehkordi et al. (2017), a large number of features has been generated using sufficiently large training data. Dogan et al. (2011) propose a neural network with many neurons along with lots of features for collision risk assessment. However, many features, lots of hidden layers and complex activation function make this algorithm computationally intensive. A comparative study of various risk assessment methods is provided in Dahl et al. (2018). In this paper, a simple probabilistic approach for forward-collision risk assessment based on LOS (Yanushevsky (2018)) has been proposed. The proposed method does not require high computational power as compared to mentioned probabilistic and optimization-based approaches theoretically. Also, the threshold generation using ROC (Fawcett (2006)) requires only one feature which is very less as compared to data-driven approaches mentioned earlier. This paper deals with the head-on collision in place of conventional front-to-rear end collision. Further, such a technique based on LOS which also generates risk assessment metrics is expected to suitably deal with various types of maneuvers as experienced in practical scenarios.

2. BACKGROUND AND PROBLEM STATEMENT

Currently, the TTC-based approach for measuring the risk of collision is widely implemented. This strategy does not take into account the potential (future) vehicle trajectory. In particular, for time horizon in seconds, TTC is effectively used to access hazards on a straight road. However, in a collision scenario where two vehicles are approaching and are very near (Fig. 1), the TTC approach gives false alarm before a near miss. Thus, this method may not be suitable for application in such practical situations. One common aspect in these types of situations

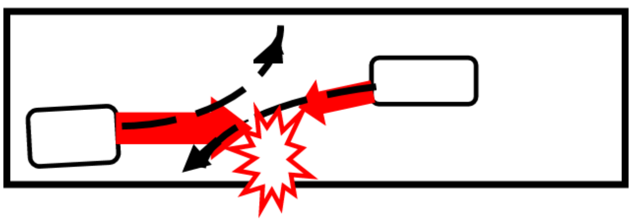

Fig. 1. Overestimation of risk by TTC

that can be observed is that the LOS angle increases with a decrease in relative displacement between vehicles, indicating that the chance of collision is decreasing. The LOS concept is widely used for target interception or capture in the missile guidance field and also for collision avoidance of Unmanned Aerial Vehicles (UAVs) (Cetin et al. (2013)) and Unmanned Surface Vehicles(USVs) (Naeem et al. (2012)). This concept can also be extended to the case of conventional vehicles.

It would be helpful to characterize collision scenarios and future vehicle trajectory relative to each other using the concept of LOS in determining the risk of collision. This characterization can be achieved by understanding the interception kinematics of 2 point objects as in Fig. 2 where M is the ego object and T is the target object. LOS separation between them is R and LOS angle is θ. $\overrightarrow{V_M}$ & $\overrightarrow{V_T}$ are velocities of M and T respectively. The equations

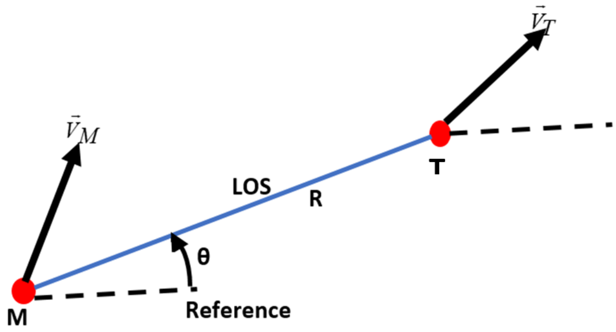

Fig. 2. Engagement of 2 point objects

of motions, assuming constant velocities of both M and T, can be written as in (1).

$$\overrightarrow{V_R} = \overrightarrow{V_{TR}} - \overrightarrow{V_{MR}} \qquad \overrightarrow{V_\theta} = \overrightarrow{V_{T\theta}} - \overrightarrow{V_{M\theta}} \tag{1}$$

where, $\overrightarrow{V_R}$ is the relative velocity of T w.r.t M in direction of LOS, $\overrightarrow{V_{TR}}$ & $\overrightarrow{V}_{MR}$ are velocities of T and M along LOS respectively, $\overrightarrow{V_\theta}$ is the relative velocity of T w.r.t M in the direction perpendicular to LOS direction, while $\overrightarrow{V_{T\theta}}$ & $\overrightarrow{V_{M\theta}}$ are velocities of T and M perpendicular to LOS, respectively. The condition of a collision given in (2) is obtained by assuming that the relative velocity of T w.r.t M is constant and the relative displacement between them is small.

$$\overrightarrow{V_R} < 0 \qquad \& \qquad \overrightarrow{V_\theta} = 0 \tag{2}$$

where, $\overrightarrow{V_R} < 0$ implies that the LOS separation is shrinking at a constant rate and $\overrightarrow{V_\theta} = 0$ implies LOS is not rotating in space with time. The condition (2) leads to the collision triangle condition. Since, vehicles are not point objects but 3D rigid bodies, the concept of lethal radius and miss distance (Ghose (2012)) (used in missile guidance context) can be used to modify the condition of collision. The modified collision condition is given by (3) and (4).

$$|\overrightarrow{V_\theta}| < \sqrt{\frac{R^2_{lethal}}{R^2 - R^2_{lethal}}} |\overrightarrow{V_R}| \tag{3}$$

$$\overrightarrow{V_R} < 0 \tag{4}$$

Equation (3) implies that the velocity of another vehicle w.r.t ego vehicle will be such that the trajectory of former (assuming constant relative velocity) intersects the circular region with radius R_{lethal} (refer to Fig. 4 and 6) for visualization) around the ego-vehicle. The radius R_{lethal} should be properly chosen so that if condition (3) is satisfied along with the another condition (4) then, there is complete certainty that the vehicles will collide otherwise collision will be avoided. The way LOS angle changes in the head-on collision scenario with a decrease in relative displacement between two vehicles, while they are approaching each other ($\overrightarrow{V_R} < 0$), is shown in Fig. 3. It is evident from Figure 3 that the LOS angle is very small (tends to zero) and almost constant when the vehicles are far. On the other hand, the LOS angle increases in magnitude rapidly to avoid the collision when vehicles are

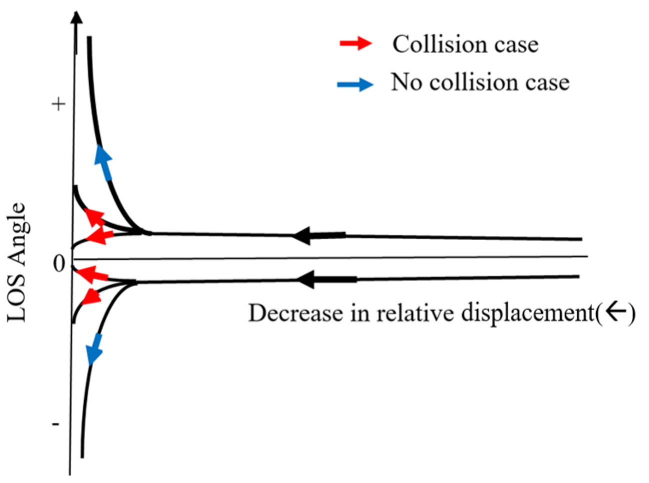

Fig. 3. Variation of LOS angle of another vehicle relative to ego vehicle in a head-on collision case.

very close.Further, the high magnitude of the LOS rate and $\overrightarrow{V_\theta}$ implies that (3) is not satisfied. In the case of collision, either the LOS angle does not increase rapidly in magnitude or may even decrease in magnitude.Also, the LOS rate with a small magnitude implies $\overrightarrow{V_\theta}$ is small in magnitude which further may result in the satisfaction of (3). Based on the analysis using the LOS concept, it seems favourable to utilize the same to improve the reliability of the collision risk assessment in the head-on collision scenario which can further be applied to various realistic scenarios.

3. PROPOSED RISK ASSESSMENT METHODOLOGY

For head-on collision avoidance of vehicles, a simple probabilistic approach of risk assessment that incorporates the LOS concept is proposed. To assess risk, it is necessary to know how other vehicle's path and LOS angle changes concerning the ego vehicle and whether the chance of collision is increasing or decreasing as these change. Two types of cases are assumed in the head-on collision scenarios: one in which collision will occur, and others in which collision will not occur, and the distinction between these cases should be known. A threshold of risk must be identified to differentiate between the two cases. Therefore, it is necessary to analyze a large number of head-on collision scenarios to characterize this. While implementing this method, it is necessary to predict the path of another vehicle w.r.t ego vehicle before the other vehicle arrives as it is not advisable to decide whether a collision will occur or not when both vehicles are very close to each other.
The proposed methodology consists of 3 parts: Multiple Generation of scenarios, Collision Risk Threshold Generation and Decision making.

3.1 Multiple Generation of scenarios

The present work focuses on the head-on collision situations assuming that the speed of the another vehicle as well as ego-vehicle is constant and the driver takes cooperative action to avoid the collision. Acceleration and deceleration during vehicle motion have not been considered and therefore the vehicles must be moved far from each other in the lateral direction. Steering action helps in taking counteraction for avoiding collision, especially when the vehicles have approached very close to each other. Steering lets other vehicles drive away from ego vehicle laterally and thus plays an important role in avoiding the collision. Steering actions also lead to higher acceleration causing higher jerk and resulting in loss of comfort. However, comfort has to be compromised with safety. A large number of trajectories considering steering effect have been simulated.The data of states ($\overrightarrow{x_r}$, $\overrightarrow{y_r}$ ψ_r $\overrightarrow{v_r}$ $\overrightarrow{\omega_r}$) are obtained from the perception module, which has been modeled for this purpose. $\overrightarrow{V_\theta}$ and $\overrightarrow{V_R}$ are calculated from these data as per (5-7).

$$\overrightarrow{V_{xr}} = \overrightarrow{v_r} \cos\psi_r, \quad \overrightarrow{V_{yr}} = \overrightarrow{v_r} \sin\psi_r \tag{5}$$

$$\theta = \tanh \frac{\overrightarrow{y_r}}{\overrightarrow{x_r}} \tag{6}$$

$$\begin{bmatrix} \overrightarrow{V_R} \\ \overrightarrow{V_\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -sin\theta \\ sin\theta & \cos\theta \end{bmatrix}^{-1} \begin{bmatrix} \overrightarrow{V_{xr}} \\ \overrightarrow{V_{yr}} \end{bmatrix} \tag{7}$$

where, $\overrightarrow{x_r}$ = Relative displacement of another vehicle w.r.t ego vehicle along the x-axis.
$\overrightarrow{y_r}$ = Relative displacement of another vehicle w.r.t ego vehicle along the y-axis.
ψ_r = Yaw angle of another vehicle relative to ego vehicle.
$\overrightarrow{v_r}$= Velocity of another vehicle relative to ego vehicle.
$\overrightarrow{\omega_r}$ =Yaw rate of another vehicle relative to ego vehicle
$\overrightarrow{V_{xr}}$ and $\overrightarrow{V_{yr}}$ = Velocity component of relative velocity along the x-axis and y-axis respectively.

3.2 Collision Risk Threshold Generation

There is always a risk of collision associated with every scenario (collision as well as the no-collision case). Unless a data-based threshold characterizing risk is obtained to differentiate between collision and non-collision data, risk assessment cannot be achieved. Therefore, developing a threshold for collision risk is very crucial.For characterizing the collision risk, a suitable measure such as the probability density function (pdf) of collision probability is employed. Based on the condition of collision when the other vehicle follows its trajectory w.r.t ego vehicle, collision pdf changes. It gets updated at each sampling step (decreasing relative displacement) as the vehicle approaches ego vehicle.Now, $\overrightarrow{V_R}$ and $\overrightarrow{V_\theta}$ are random variables. Therefore, Equation (3) and (4) can be seen as a function of random variables (RV) which can take value '0' if the condition is not satisfied and '1' otherwise.At each sampling instant, conditions given by (3) and (4) are checked repeatedly. These outcomes at each sampling instant can be assumed to be independent of the outcomes obtained at previous instants. Therefore, these RVs can be considered to be having Bernoulli distribution with parameter ϕ. Let this RV be represented as 'A' as given in (8)

$$A \sim Ber(\phi), p(A) = \begin{cases} \phi, A = 1 \\ 1-\phi, A = 0 \end{cases}, where 0 \le \phi \le 1 \tag{8}$$

where, ϕ denotes probability of collision. It is a RV having Beta distribution (Heckerman (2008)) with hyperparameters α and β as explained in (9).

$$\phi \sim Beta(\alpha,\beta), \quad p(\phi) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\phi^{\alpha-1}(1-\phi)^{\beta-1} \quad (9)$$

The dependency between RVs (A and ϕ) is represented by a very simple Bayesian network (Heckerman (2008)) and Bayesian learning is applied to update collision probability as given in (10). .

$$p(\phi|A_{1:n}) \propto p(A_{1:n}|\phi)p(\phi) \sim Beta(\alpha{+}m, \beta{+}n{-}m) \quad (10)$$

where,m=number of times condition takes value 1 & n= number of times condition is checked. The probability is being updated at each sampling instant as per (11).

$$p(\phi|A) \sim Beta(\alpha+m, \beta+1-m) = Beta(\alpha^{'}, \beta^{'}) \quad (11)$$

$\alpha^{'}$ increases when the collision condition is satisfied and the pdf tends to skew towards the right. Whereas, $\beta^{'}$ increases when the collision condition is not satisfied and the pdf tends to skew towards left. Probability is updated until another vehicle reaches the ego-vehicle. Mode of the beta distribution at each sampling instant is calculated using (12).

$$mode = \frac{\alpha^{'}-1}{\alpha^{'}+\beta^{'}-1} \quad (12)$$

The data obtained from the perception module are used to check the collision condition at each sampling instant for collision and no-collision cases. To illustrate this, two scenarios, one corresponding to each to no-collision and collision case, has been shown in Fig. 4 and Fig. 6, respectively. Fig. 4 shows a no-collision situation in which the other vehicle approaches the ego vehicle and the latter is at rest. Initially, the other vehicle drives parallel to the reference path and then starts shifting laterally as the relative displacement between them decreases due to cooperative action. Evaluation of various parameters corresponding to this scenario is shown in Fig. 5. From the plot, it can be observed that initially LOS angle is near to 0 and then its magnitude starts to increase as the vehicle moves away from the reference line. Similarly, $\overrightarrow{V_\theta}$ and LOS rate are initially close to 0 and then increases in magnitude. It is also to be noted that $\overrightarrow{V_R}$ is constantly negative (another vehicle is approaching) and its magnitude starts to decrease as the relative displacement decreases. Since, $\overrightarrow{V_\theta}$ is 0 initially (from 10m to 8.5m) and $\overrightarrow{V_R}$ is negative, the collision condition is satisfied. Because of this, the pdf becomes more skewed toward the right as relative displacement decreases from 10m to 6.5m and mode of the pdf also increases. However, as relative displacement decreases further, the collision condition is not satisfied due to a large increase in the magnitude of the LOS angle. This is visible from the change in pdf skewness towards left as displacement decreases from 6.5m to 3m and mode also decreases. From Fig. 6, it is observed that the LOS angle of another vehicle with respect to the reference, is small. The other vehicle shifts laterally causing LOS to move from one side to the other side of the reference with a decrease in the relative displacement between other vehicle and the ego vehicle. Similar to Fig. 5, plots in Fig. 7 corresponds to the scenario in Fig. 6. The plot shows that the LOS angle is decreasing from positive to negative values as relative displacement decreases due to traversal of another vehicle from one side to the other side of the reference path. Due to this change in LOS, $\overrightarrow{V_\theta}$ and LOS rate increase in magnitude and $\overrightarrow{V_R}$ decreases with a decrease in relative displacement.

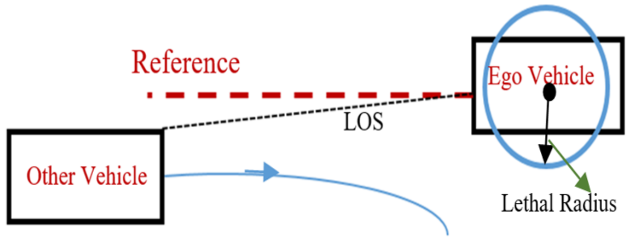

Fig. 4. No-collision case in a head-on collision scenario

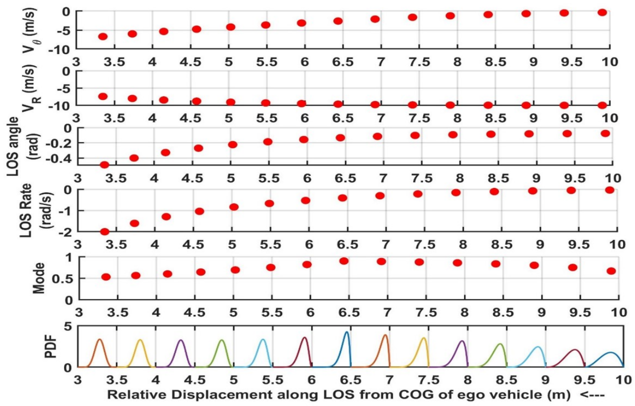

Fig. 5. Variation of various parameters for the scenario given in 4. (Plots should be read from right to left)

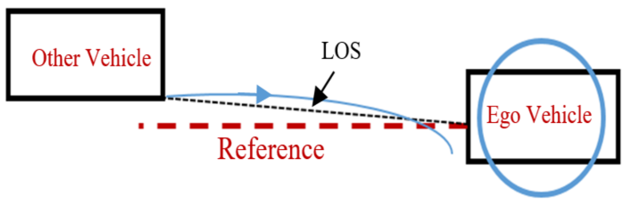

Fig. 6. Collision case in a head-on collision scenario

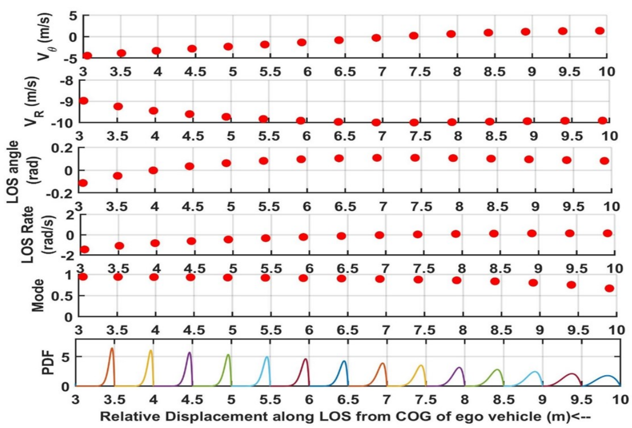

Fig. 7. Variation of various parameters for the scenario given in Figure 5a. (Plots should be read from right to left)

The pdf becomes more and more right-skewed till relative displacement decreases to 3m because the increase in the magnitude of is less and the collision condition is satisfied continuously. Corresponding to this change in pdf, the mode also changes.
Since pdf characterizes risk, it is possible to use the threshold of pdf mode to distinguish between collision and no collision cases. This threshold is obtained by plotting ROC using the mode data obtained for each scenario when other vehicle approaches the ego vehicle and reaches a specific relative displacement named as threshold displacement. The threshold is to be chosen such that the Probability of Detection (P_D) is greater than 0.85 and the Probability of False Alarm (P_{FA}) is less than 0.1. The generated risk threshold can be used to determine whether or not collision would occur when other vehicle reaches within the defined threshold displacement from the ego vehicle for any head-on collision path.

3.3 Decision Making

The ultimate goal of collision risk assessment is to avoid the collision. If the threshold of displacement decreases, the risk assessment accuracy increases. But, both vehicles need enough time to take counteractions to avoid the collision. It would be beneficial if it is possible to predict the states of another vehicle w.r.t ego vehicle before reaching the threshold displacement. To avoid the collision, both will have enough time to maneuver. The Constant Turn Rate and Velocity (CTRV) model (Schubert et al. (2008)) is used to predict the states of another vehicle w.r.t ego vehicle. The states ($\overrightarrow{x_r}$, $\overrightarrow{y_r}$ ψ_r $\overrightarrow{v_r}$ $\overrightarrow{\omega_r}$) are obtained from the perception module of the ego vehicle. All three components of the suggested approach for risk assessment have been explained. The approach has been evaluated by numerous sets of simulation results in the next section.

4. SIMULATION RESULTS AND ANALYSIS

To implement the proposed approach, MATLAB 2019b Simulink platform is used. Automated Driving Toolbox (Takahama and Akasaka (2018)) is used to generate and simulate scenarios. The relative constant speed of another vehicle with respect to ego-vehicle is taken as 10m/s. The probability of collision updation starts once the other vehicle is at 10m distance w.r.t the ego vehicle's Centre of Gravity (COG) and continue until it reaches a relative displacement of 3m from COG. In each simulation, the probability of collision is updated at a sampling time of 0.05s. Multiple simulations of scenarios are performed and mode data for each simulation is recorded when another vehicle is within 2m from the front end of the ego vehicle or 5.2m from the ego vehicle's COG. Hereafter, the stored data is used to plot the ROC curve in order to choose a threshold such that (P_D) is high and (P_{FA}) is low. Fig. 8 shows the ROC plot based on thresholds of the mode of the probability of collision, from which it is seen that P_D is high (0.9) and P_{FA} is low (<0.1) at threshold value 0.64 (approx.). Therefore, 0.64 is an optimum threshold which characterizes the risk of collision in a head-on collision scenario. The obtained threshold is then being used for decision making. To test the effectiveness of the proposed approach, different test scenarios are generated again. States of another vehicle relative to ego vehicle at threshold displacement is predicted in terms of 2 sampling times (0.05*2 = 0.1s) before reaching that threshold displacement.

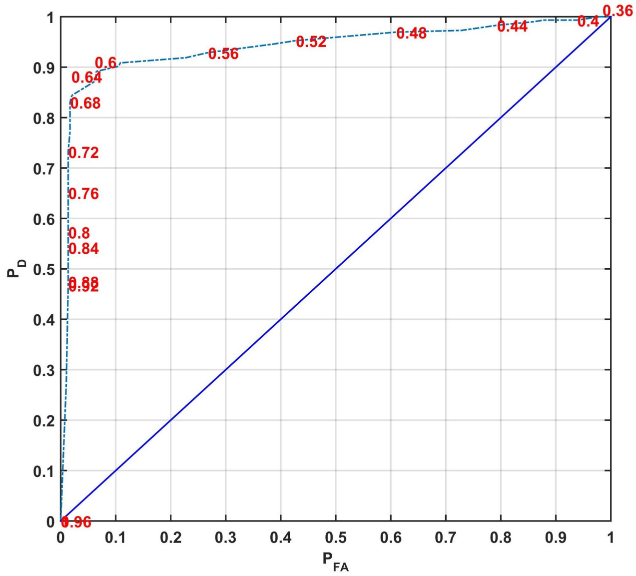

Fig. 8. ROC plot for different mode thresholds.

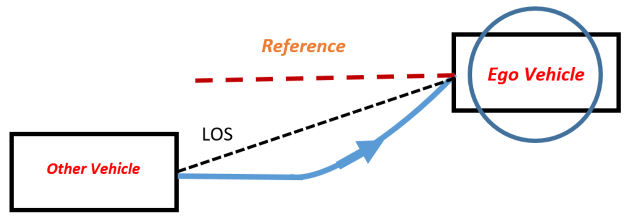

Fig. 9. A Test Scenario

The accuracy of decision making depends on the accuracy of the prediction of states and the motion model used. A head-on collision test scenario, shown in Fig. 9, is generated. The scenario is simulated and the probability of collision till relative displacement 6.13m with mode=0.60 is obtained in the simulation. The various parameters are predicted at 6.13m from the COG of ego vehicle for the next two sampling instants. Fig. 10 shows the variation of various predicted parameters (based on states predicted using the CTRV motion model) with a decrease in relative displacement. It can be seen that the actual and predicted data differ very slightly. Therefore, the accuracy of prediction is good. Based on the comparison of predicted mode (0.667) with the threshold (0.64), it is concluded that the collision might take place, which is indeed the taken scenario.
The efficiency of the proposed approach and the TTC approach is checked by generating 168 forward-collision scenarios corresponding to the no-collision case. Table. 1 shows a comparative study of false alarm rates using these two approaches under similar scenarios. It is obvious from Table 1 that the proposed approach reduces the chances of false alarm than that of the TTC based approach.

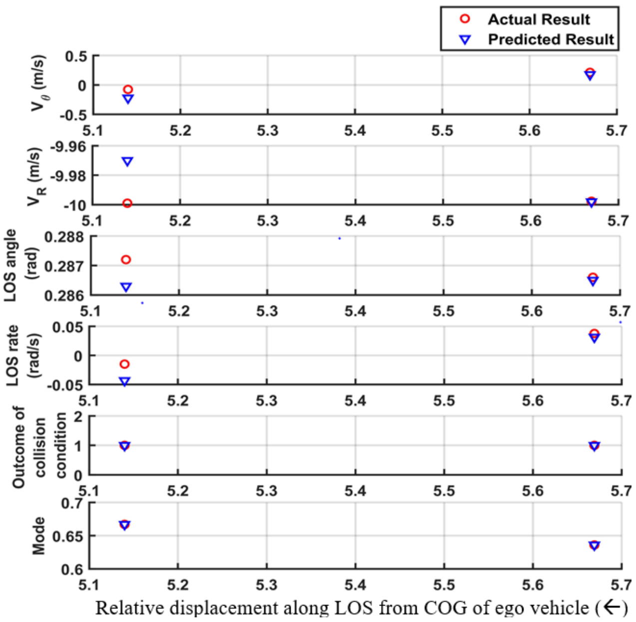

Fig. 10. : Variation of predicted data with a decrease in relative displacement. (Plots should be read from right to left)

Table 1. Comparison of False Detection Rates

Risk Assessment Approach	No. of false detection	% of false detection
TTC	96	57.14
Proposed Approach	28%	16.67%

5. CONCLUSION

This paper addressed the risk assessment for front-to-front end head-on collision situations or scenarios of vehicles using the LOS concept. The threshold of mode for the collision pdf was obtained by plotting ROC which reduces the chances of false alarm and helps to improve safety. A closed-form probability update using Bayesian learning was used for that which does not demand heavy online computational power and therefore this methodology is also computationally less intensive. However, the approach considers collision avoidance in front-to-front end head-on collision scenario only. Further, this approach includes simplifying assumptions like the constant relative speed of vehicles, cooperative actions of the vehicle drivers. This methodology can be extended and improved by incorporating the effect of relative displacement, velocity, and acceleration of the vehicles to deal with complex collision scenarios occurring in the real world.

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