Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Constant Time-Headway Spacing Policy with Limited Communication Range for Discrete Time Platoon Systems ⋆

Andrs A. Peters * **Alejandro J. Rojas** **

* *Departamento de Electricidad, Facultad de Ingeniería, Universidad Tecnológica Metropolitana, Santiago, Chile (e-mail: a.petersr@utem.cl).*
** *Departamento de Ingeniería Eléctrica, Universidad de Concepción, Concepción, Chile (e-mail: arojasn@udec.cl)*

Abstract: We present a study of the scaling properties of the interconnection of n agents (e.g. vehicles) through an r-lookahead network. These networks are considered as a possible implementation for vehicle platooning, although we do not make any assumptions on what the agents represent, and we assume them to be linear time invariant (LTI) discrete time systems, locally controlled by an LTI controller. In particular, we show that the r-lookahead topology gives rise to dynamics which can be studied from the roots of polynomials with transfer functions as their coefficients. Through numerical simulations, we study aspects relating the use of lookahead measurements and their effect on the value of a time headway constant needed for the scalability property known as string stability.

Keywords: Discrete-time systems, Platooning, String stability, Interconnected systems, Linear control systems

1. INTRODUCTION

The study of unidirectionally interconnected vehicular agents has been of interest for many years (Middleton et al, 2010; Knorn et al, 2016; Stüdli et al., 2017; Feng et al., 2019). Applications can be found in such diverse areas as irrigation channels (Cantoni et al, 2007), supply chains (Perea et al, 2000), harmonic oscillators (Yu et al, 2015) and vehicle platooning (Flores et al, 2018; Darbha et al, 2018).

The unidirectionality and low information flow between agents is known to enable instances for the amplification of disturbances along the string of agents. Topologies that allow to compensate for this phenomenon are said to achieve *String Stability*. Many results focus on achieving string stability for homogeneous agents, (Flores et al, 2018; **?**), whilst other results extend the discussion to heterogeneous agents, (Bian et al, 2019; Rodonyi, 2019). Most of the results have been derived for the case where the agents are continuous time systems. Recently, Vargas et al (2018) considered extensions of String Stability results for the discrete time case, motivated by the fact that Cooperative Adaptive Cruise Control Systems (CACC) rely on wireless communications that can be usually more easily treated in a discrete time setting.

A possible analysis approach for continuous time interconnected agents in a chain is to make use of the Laplace transform in the time domain and the $\mathcal{Z}$-transform in the spatial domain to define a *string stability transfer function* as

$$\Gamma_i(s) = \frac{y_i(s)}{y_{i-1}(s)}$$

where y_i is the position of the i-th agent.

In this work, we aim to extend results from Konduri (2017); Bian et al (2019) where, for the continuous time case, extending the range of communications and their impact on the needed time headway for string stability were studied. In both works, the agents considered had fixed simple dynamics, and it was assumed that the agents had measurements of the position, velocity and acceleration of the nearest r predecessors ahead of them. Our contribution is to perform a first approach to the discrete time case with general models for the agents (without specifying their nature a priori), and only assuming that they can measure the output, and the rate of change of the outputs, of the r nearest predecessors. The latter is a key departure from the analysis of continuous time systems in a platooning setting. We provide formulae for the resulting dynamics in a general setting of the interconnection and we also consider the use of a constant time-headway spacing policy in a platooning setting.

Notation: In this work all matrix entries not explicitly stated are assumed to be zero and, for simplicity in the exposition, all transfer functions will have omitted arguments (the frequency domain variable z) unless needed.

⋆ The authors thankfully acknowledge the support from the Advanced Center for Electrical and Electronic Engineering, AC3E, Basal Project FB0008, and from the Grant FONDECYT 1190196, CONICYT Chile.

2. PRELIMINARIES

We will consider n agents, which could be vehicles, with the same frequency domain description given by

$$Y_i = H(U_i + D_i), \tag{1}$$

where Y_i is the Laplace transform of the output of the i-th agent ($y_i(k)$), D_i is the Laplace transform of its input disturbance, H is a transfer function representing the agent LTI dynamics and U_i is the Laplace transform of its control action.

For the considered setting, the control signals U_i at every agent are defined as

$$U_1 = 0, \tag{2}$$

$$U_2 = K(Y_1 - Y_2 - \Delta_2), \tag{3}$$

$$U_i = K\left(\sum_{k=2}^{i-1} B_{k-2}Y_{k-1} + AY_{i-1} - Y_i - \Delta_i\right), \quad 2 < i \le r, \tag{4}$$

$$U_i = \left(\sum_{k=0}^{r-2} B_k Y_{i-k-2} + AY_{i-1} - Y_i - \Delta_i\right), \quad r < i \le n, \tag{5}$$

where A and B_i, $i = 0, \ldots, r-2$, are proper and stable transfer functions, K is a controller that stabilizes H in closed loop and Δ_i reflects a certain *reference policy* or *desired inter-agent spacing*. Note that Δ_i could be a constant but it could also contain other terms such as the rate of change of the i-th agent in platooning applications. In such cases, it would be possible that A and B_i have to be modified accordingly.

Remark 1. Having $A \neq 0$ for all z implies that the i-th agent has available information from its immediate neighbour with index $i-1$. In the platooning setting this implies that the vehicle senses its distance from the nearest front neighbour, i.e. its predecessor.

Having $B_i \neq 0$ for all z implies that the i-th agent also has information from the agents up to the one with index $i = 1$ if $i \le r$ or up to the agent with index $i - r$ otherwise. In a platooning setting this would correspond with the availability at the i-th agent of the lead vehicle information if $i \le r$ or of the $(i-r)$-th agent otherwise. The latter would imply that there exists a limitation on the range of the inter-agent communications, which is a reasonable assumption in real applications.

The input of the controller is then a weighted sum of the outputs of the predecessor within the communication reach.

We will now consider as an illustrative example the case $r = 2$ and $\Delta_i = 0$ for all i. That is, the agents possess information of their two nearest predecessors. Moreover, since the dynamics are LTI, we will consider zero initial conditions for the agents, that is $y_i(0) = 0$, for all i.

Now, the dynamics of the agents can be written in matrix form as

$$\underline{Y} = (\boldsymbol{I} - KH\boldsymbol{G})^{-1}H\underline{D}, \tag{6}$$

where $\underline{Y} = [Y_1 \ \cdots \ Y_n]^\top$, $\underline{D} = [D_1 \ \cdots \ D_n]^\top$, $\boldsymbol{I}$ is the $n \times n$ identity matrix and $\boldsymbol{G} \in \mathbb{C}^{n \times n}$ is given by

$$\boldsymbol{G} = \begin{bmatrix} 0 & & & & \\ 1 & -1 & & & \\ B_0 & A & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & B_0 & A & -1 \end{bmatrix}. \tag{7}$$

Now, in order to obtain closed form expressions for every element of the matrix $(\boldsymbol{I} - KH\boldsymbol{G})^{-1}HD$ we can write $(\boldsymbol{I} - KH\boldsymbol{G})^{-1}$ in the following way

$$(\boldsymbol{I} - KH\boldsymbol{G})^{-1} = \begin{bmatrix} 1 & \zeta_{n-1}^\top \\ -KH(e_1 + Be_2) & KH\boldsymbol{\Phi} \end{bmatrix}^{-1} \tag{8}$$

$$= \begin{bmatrix} 1 & \zeta_{n-1}^\top \\ \boldsymbol{\Phi}^{-1}(e_1 + Be_2) & \frac{1}{KH}\boldsymbol{\Phi}^{-1} \end{bmatrix}, \tag{9}$$

where $\zeta_{n-1}^\top \in \mathbb{R}^{n-1}$ is a vector of zeros, $e_k \in \mathbb{R}^{n-1}$ is a canonical vector of $\mathbb{R}^{n-1}$ (only the k-th entry is non zero and equal to 1) and $\boldsymbol{\Phi}$ is the $n-1 \times n-1$ lower triangular matrix

$$\boldsymbol{\Phi}^{-1} = \begin{bmatrix} T^{-1} & & & \\ -A & \ddots & & \\ -B_0 & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & -B_0 & -A \ T^{-1} \end{bmatrix}^{-1}, \tag{10}$$

$$= \begin{bmatrix} f_{1,1} & & & \\ f_{2,1} & \ddots & & \\ \vdots & \ddots & \ddots & \\ f_{n-1,1} & \cdots & f_{n-1,n-2} & f_{n-1,n-1} \end{bmatrix}, \tag{11}$$

with T, the usual complementary sensitivity function of the local closed loops, given by

$$T = \frac{KH}{1 + KH}. \tag{12}$$

The inverse $\boldsymbol{\Phi}^{-1}$ is also a lower triangular matrix with constant diagonals, whose elements are given by

$$f_{i,j} = \begin{cases} F_{i-j} & \text{if } i \ge j \\ 0 & \text{if } i < j, \end{cases} \tag{13}$$

and the sequence $\{F_k\}$ is such that F_0 corresponds to the main diagonal and F_1 to the first sub-diagonal, etc. As $\boldsymbol{\Phi}$ only has three non-zero diagonals, it is straightforward to note that the sequence $\{F_k\}$ satisfies the recursion

$$F_{k-1} = ATF_{k-2} + B_0TF_{k-3}, \tag{14}$$

with initial conditions $F_0 = T$ and $F_1 = AT^2$. We can now solve the recursion (14) using any method. Using the $\mathcal{Z}$-transform we have that

$$F_k = \alpha_1\lambda_1^{k-1} + \alpha_2\lambda_2^{k-1}, \tag{15}$$

where $\lambda_{1,2}$ are the roots of $\lambda^2 - AT\lambda - B_0T = 0$, that is

$$\lambda_{1,2} = \frac{AT \pm \sqrt{(AT)^2 + 4B_0T}}{2}, \tag{16}$$

and

$$\alpha_1 + \alpha_2 = T, \tag{17}$$

$$\alpha_1\lambda_1 + \alpha_2\lambda_2 = AT^2. \tag{18}$$

Solving for α_1 yields

$$\alpha_1\sqrt{(AT)^2+4B_0T}+\frac{AT^2}{2}-\frac{T\sqrt{(AT)^2+4B_0T}}{2}=AT^2 \quad (19)$$

and therefore

$$\alpha_{1,2}=\frac{T}{2}\pm\frac{AT^2\sqrt{(AT)^2+4B_0T}}{2((AT)^2+4B_0T)}. \quad (20)$$

It is to be expected that all the terms with radicals disappear in the final expressions for F_n, since we are just inverting a matrix with rational functions. However, the roots of the polynomial describing the recursion given in (14) contain implicit information about the limiting behaviour of F_k/F_{k-1}, much like for the Fibonacci sequence. For example, if $|\lambda_2|<1$ on the unit circle, we should have

$$\lim_{k\to\infty}\left|\frac{F_k}{F_{k-1}}\right|=|\lambda_1|=\frac{|AT+\sqrt{(AT)^2+4B_0T}|}{2}. \quad (21)$$

Remark 2. In platooning applications, it is common to consider the study of the inter-vehicle spacings errors $e_{fi}(t)=y_{i-1}(t)-y_i(t)-\delta_i$, $i>1$, where δ_i is the desired inter-vehicle spacing. The frequency domain equivalents can be easily built from the formulae for $\underline{Y}$ in (6), already obtained.

Therefore, it is of interest to study the behaviour of F_k and $F_{k-1}-F_k$ as k grows large. For platooning applications, it would be especially important to characterise conditions for the parameters that will ensure that the sequence of transfer functions $\{F_k\}$ is bounded uniformly with k under certain norms.

3. ARBITRARY COMMUNICATION RANGE

It is well known that having information from the leader is enough to ensure good scalability in platooning applications (see for example Seiler et al. (2004)). It would then make sense, in the case that the communication range is restricted, to have every agent reaching for the agents closest to the leader that it can. We consider now the block matrix

$$\boldsymbol{G}=\begin{bmatrix}\boldsymbol{G_0} & \\ \boldsymbol{G_2} & \boldsymbol{G_1}\end{bmatrix}, \quad (22)$$

where

$$\boldsymbol{G_0}=\begin{bmatrix}0 & & & & & \\ 1 & -1 & & & & \\ B_0 & A & -1 & & & \\ B_0 & B_1 & A & -1 & & \\ \vdots & \vdots & \ddots & \ddots & \ddots & \\ B_0 & B_1 & \cdots & B_{r-1} & A & -1\end{bmatrix}, \quad (23)$$

is an $(r+1)\times(r+1)$ matrix, $\boldsymbol{G_1}$ is the $(n-r-1)\times(n-r-1)$ lower triangular Toeplitz matrix with non zero diagonals given by the vector $(-1,A,B_{r-2},B_{r-3},\dots,B_0)$ (that is, the main diagonal is all -1, the first sub-diagonal is all A, and so on.) and $\boldsymbol{G_2}$ is given by

$$\boldsymbol{G_2}=\left.\begin{bmatrix}0 & B_0 & B_1 & \cdots & A\\ \vdots & \ddots & \ddots & \ddots & \vdots\\ \vdots & & \ddots & \ddots & B_1\\ 0 & \cdots & \cdots & 0 & B_0\\ 0 & \cdots & \cdots & \cdots & 0\\ \vdots & & & & \vdots\\ 0 & \cdots & \cdots & \cdots & 0\end{bmatrix}\right\}(n-r-1 \text{ rows}). \quad (24)$$

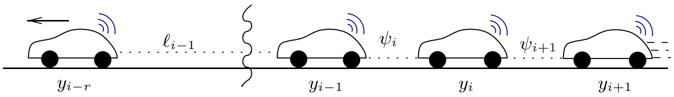

Fig. 1. A platoon of vehicles with r lookahead communications.

Analogously to the case $r=2$, we must have

$$(\boldsymbol{I_n}-KH\boldsymbol{G})^{-1}= =\begin{bmatrix}(\boldsymbol{I_{r+1}}-KH\boldsymbol{G_0})^{-1} & \\ \Theta & (\boldsymbol{I_{n-r-1}}-KH\boldsymbol{G_1})^{-1}\end{bmatrix}, \quad (25)$$

where

$$\Theta=KH(\boldsymbol{I_{n-r-1}}-KH\boldsymbol{G_1})^{-1}\boldsymbol{G_2}(\boldsymbol{I_{r+1}}-KH\boldsymbol{G_0})^{-1}. \quad (26)$$

In other words, we must study the inverse matrices $(\boldsymbol{I_{r+1}}-KH\boldsymbol{G_0})^{-1}$ and $(\boldsymbol{I_{n-r-1}}-KH\boldsymbol{G_1})^{-1}$. However, the inverse $(\boldsymbol{I_{r+1}}-KH\boldsymbol{G_0})^{-1}$ is well known and studied (leader following schemes in platooning). We must focus in computing and studying the inverse $(\boldsymbol{I_{n-r-1}}-KH\boldsymbol{G_1})^{-1}$. Again, analogously to the case $r=2$, we can see that there is a recursion for obtaining the elements of the matrix

$$(\boldsymbol{I_{n-r-1}}-KH\boldsymbol{G_1})^{-1} \quad (27)$$

which is given by (understanding again that this inverse is a Toeplitz matrix, i.e. has constant diagonals, see (13))

$$\mathcal{G}_k=AT\mathcal{G}_{k-1}+B_{r-2}T\mathcal{G}_{k-2}+\cdots+B_0T\mathcal{G}_{k-r}, \quad (28)$$

with initial conditions $\mathcal{G}_1=T$, $\mathcal{G}_i=(AT)^{i-1}T$ for $i=2,\dots,r$. The characteristic polynomial is then given by

$$z^r-ATz^{r-1}-B_{r-2}Tz^{r-2}-\cdots-B_0T=0. \quad (29)$$

Analogous to the case $r=2$, we would be interested in studying the sequence of transfer functions from disturbances to inter-vehicle spacings.

4. PLATOONING AND THE CONSTANT TIME HEADWAY SPACING POLICY

In 1D platooning applications, when the leader state is not available to the followers, the spacing policy is relaxed, in order to avoid scalability issues such as string stability. This is a key compromise between safety and performance, as the relaxation will decrease the usage of the capacity of the traffic network. Each follower will keep a desired distance to its predecessor given by a fixed distance δ (now assumed to be equal for all agents) plus a distance proportional to its instantaneous speed, through the use of a time headway constant h.

For example, in Fig. 1, we can see a graphical representation of the agents in a platoon configuration. In such case, the signal ψ_i represents the measured distance to the $(i-1)$-th agent made by the i-th agent. The signal ℓ_{i-1} represents the measured distance to the $(i-r)$-th agent made by the $(i-1)$-th agent.

We will assume that the agents start at rest in the desired formation (when all the agents have zero velocity) and have the following control signals in the frequency domain

$$U_1 = 0, \tag{30}$$

$$U_2 = K\left(Y_1 - Y_2 - \delta - h\left(Y_2 - \frac{Y_2}{z}\right)\right), \tag{31}$$

$$U_i = K\left(\eta\left(Y_1 - Y_i - (i-1)\delta - h\sum_{k=2}^{i}\left(Y_k - \frac{Y_k}{z}\right)\right)\right.$$
$$\left. + (1-\eta)\left(Y_{i-1} - Y_i - \delta - h\left(Y_i - \frac{Y_i}{z}\right)\right)\right), \tag{32}$$

for $3 < i \leq r$, and

$$U_i = K\left(\eta\left(Y_{i-r} - Y_i - (i-1)\delta - h\sum_{k=i-r}^{i}\left(Y_k - \frac{Y_k}{z}\right)\right)\right.$$
$$\left. + (1-\eta)(Y_{i-1} - Y_i - \delta - h\left(Y_i - \frac{Y_i}{z}\right)\right), \tag{33}$$

for $r < i \leq n$, where $\eta \in [0,1]$ is a design parameter that weights how relevant are the available measurements in the control action.

Remark 3. In this case, every follower builds the input to their controller by comparing their distance to the immediate predecessor and their distance to either the leader or the $(i-r)$-th follower and averaging them through η. Moreover, the desired spacing between vehicles corresponds to a constant space δ plus a variable constant time headway space, proportional to the rate of change of the output of each agent. It is important to note that in order for a follower to compare their distance to the leader/farthest reachable predecessor to the desired inter-spacing, they must have access to the *speed* of every predecessor between them. This requirement increases the communication network complexity and demands extra steps for new agents to merge into the platoon, especially when the agents use different spacing policies.

For simplicity in the exposition, in the following derivation we will consider that $\delta = 0$. With this, it is possible to write the control signals as

$$U_2 = K\left(Y_1 - \left(1 + h\frac{z-1}{z}\right)Y_2\right), \tag{34}$$

$$U_i = K\left(\eta Y_1 - \eta h\sum_{k=2}^{i-2}\frac{z-1}{z}Y_k\right.$$
$$\left. + \left((1-\eta) - \eta h\frac{z-1}{z}\right)Y_{i-1} - \left(1 + h\frac{z-1}{z}\right)Y_i\right), \tag{35}$$

for $2 < r < i \leq n$, and

$$U_i = K\left(\eta Y_{i-r} - \eta h\sum_{k=i-r-1}^{i-2}\frac{z-1}{z}Y_k\right.$$
$$\left. + \left((1-\eta) - \eta h\frac{z-1}{z}\right)Y_{i-1} - \left(1 + h\frac{z-1}{z}\right)Y_i\right), \tag{36}$$

Recalling Fig. 1, each agent is using the measurements ψ_i and ℓ_i in order to build the input to the controller K by comparing them to the desired inter-vehicle spacings and the leader-follower spacings. If we consider the transfer function

$$W = (1+h) - h/z, \tag{37}$$

we have that the current control scheme is equivalent to the interconnection studied in the previous section with

$$A = \frac{1-\eta W}{W}, \quad B_0 = \frac{\eta}{W}, \quad B_i = \eta\frac{1-W}{W}, \tag{38}$$

for $i = 1, \ldots, r-2$.

Remark 4. For achieving the main goal of platooning, that is, every agent reaching the speed of the leader, while maintaining the desired inter-vehicle spacings, it is required that the product HK possesses two integrators (see for instance Seiler et al. (2004)). We will assume that this is indeed the case.

5. CONDITIONS FOR STRING STABILITY

The following Bode-like integral Lemma, taken from Seron et al. (2012), is key in determining the aspects of the local closed loops at each agent that could produce disturbance amplifications.

Lemma 5. Let T be a real rational scalar function of $z \in \mathbb{C}$. Suppose that $T(1) = 1$ and also that T is stable. Then

$$\int_0^{\pi} \ln\left|T(e^{j\theta})\right| \frac{d\theta}{1-\cos(\theta)} \geq \pi T'(1). \quad \blacksquare$$

Lemma 5 can be used to establish that the complementary sensitivity function T, defined in (12), satisfies $||T||_\infty > 1$, since the product HK possesses two integrators, as stated in Remark 4 (Vargas et al, 2018).

According to results in Konduri (2017); Bian et al (2019) the string stability of the r-lookahead interconnection is completely determined by the spectral radius of the polynomial in (29). In general we cannot obtain analytical expressions for the roots of this polynomial, however, we can make some initial simple observations.

Proposition 6. The interconnected system (6) is string unstable for any arbitrary but finite communication range r if $||B_0 T||_\infty > 1$.

Proof. $B_0 T$ corresponds to the constant term of the polynomial in (29), which also corresponds to the product of all the roots of the polynomial (recall Vieta's formulas). If there exists any $\theta \in [0, 2\pi]$ such that $|B_0(e^{j\theta})T(e^{j\theta})| > 1$, we must have that the product of the roots of the polynomial is greater than 1 at said frequency, which implies string instability. □

Although this result is straightforward, it provides an initial design restriction of the filters used to interconnect the agents. In particular, we have that $B_0 = \eta/W$. Both η and h can be selected to ensure that $||B_0 T||_\infty < 1$.

6. NUMERICAL EXAMPLES

We will consider a collection of agents modelled by double integrator dynamics, that is, in (1)

$$H = \frac{1}{(z-1)^2}, \tag{39}$$

and the control structure and spacing policy (30)-(33) with local controllers given by

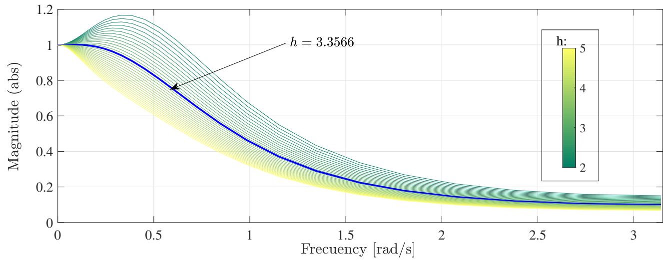

Fig. 2. Bode plots for $|T(e^{j\theta})/W(e^{j\theta})|$ for varying $h \in [2, 5]$

$$K = \frac{1.1548(z-0.7832)}{W(z+0.8306)} = \frac{1.1548(z-0.7832)}{((1+h)-h/z)(z+0.8306)}, \tag{40}$$

where h is the time headway constant for the used spacing policy. Since the product HKW has two integrators, Lemma 5 implies that $||T||_\infty > 1$. In particular, for the data above $||T||_\infty \approx 1.856$.

This selection for the local controllers, with W as a factor in the denominator, is made by many works that consider a constant time headway spacing policy. In particular Knorn et al (2013) computes the infimal value of the constant h for the continuous time case in order to achieve string stability of a nearest neighbour communication topology. The discrete time case was first reported in Vargas et al (2018). In particular, the infimal value of the time headway constant for string stability when $r = 1$, h_{inf} is given by the largest root of

$$2h(1+h) - c = 0, \tag{41}$$

where c is computed as

$$c = \sup_{\theta\in(0,\pi)} \left\{ \left(\left| T\left(e^{j\theta}\right) \right|^2 - 1 \right) / (1-\cos\theta) \right\}. \tag{42}$$

For the considered data we have that $h_{inf} \approx 3.3566$.

For the time simulations, we will focus on the signals

$$e_{fi}(t) = y_{i-1}(t) - y_i(t) - h(y_i(t) - y_i(t-1)), \tag{43}$$

that is, the inter-vehicle errors from the desired spacings.

6.1 Effect of the time headway constant

In Fig. 2 we have the Bode plots of the transfer function T/W for different values of the time headway constant h. For $r = 1$ this is the transfer function that defines the string stability of the interconnection, as it is straightforward to note that the polynomial (29) has a single root at T/W. As computed before we see that for values of h greater than 3.3566 the magnitude peak of T/W occurs at $\theta = 0$ and equals unity. Otherwise, the peak is greater than one and string instability will occur.

In Fig. 3 we see the time response of 50 agents when $r = 1$ and the leader travels at 1 unit per sample. For $h = 3.8 > h_{inf}$, the inter-vehicle errors e_f do not grow along the platoon. However, the opposite occurs when $h = 2.8 < h_{inf}$. Note that Blue corresponds to agents closer to the leader and Red corresponds to agents farther away from the leader.

6.2 Effect of increasing the communication range

As given by (16), the eigenvalues of the interconnection can be obtained explicitly. For $\eta = 0.3$ in Fig. 4 we can see

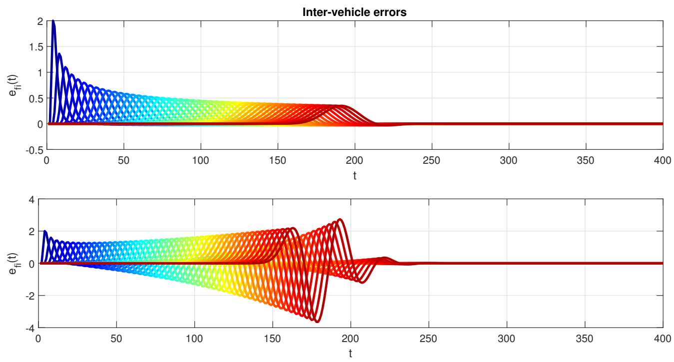

Fig. 3. Inter-vehicle errors for $h = 3.8$ (top) and $h = 2.8$ (bottom) with nearest neighbour communication ($r = 1$)

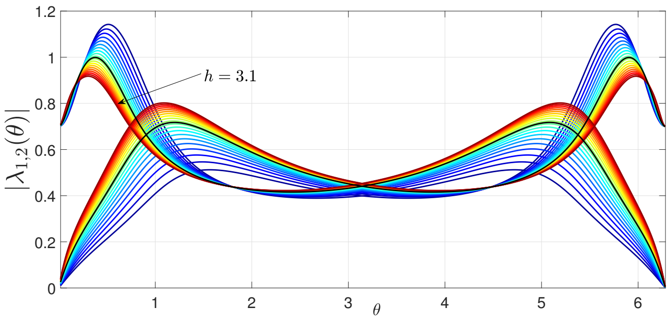

Fig. 4. Magnitudes of the roots $\lambda_{1,2}$ in (16) for $r = 2$ when $\eta = 0.3$. Blue: $h = 1.1$, Red: $h = 5$, Black $h = 3.1$

the magnitudes of these eigenvalues for different values of the time headway constant h. We can observe that when $h = 3.1 < h_{inf}$ both eigenvalues have magnitude less than 1. Therefore, amplification of disturbances should not be observed.

In Fig. 5 we see the time response of 50 agents for the same leader trajectory as before when h is fixed at $3.1 < h_{inf}$. For $r = 1$, as predicted before, the transient amplifies along the string. For $r = 2$ and $\eta = 0.3$, we have that the platoon becomes string stable. This highlights the potential for increasing the platoon performance in the inter-vehicle spacings, at the cost of increased communication range.

6.3 Effect of varying the parameter η

In Fig. 6 we observe the effect of the parameter η with fixed $h = 3.2$ and $r = 3$ with $N = 100$. We can see that increasing η has a negative impact on the string stability of the platoon. When $r \to \infty$ (that is, when every agent communicates with the leader) it is known that $\eta > ||T||_\infty^{-1}$ is sufficient to ensure string stability, even with $h = 0$, (see for instance Seiler et al. (2004)). It is unclear what is the trade-off between η and h when r is finite.

According to results from Bian et al (2019), increasing r yields smaller necessary values of the time-headway for string stability. However, this result was obtained with every follower possessing measurements of the position, velocity and acceleration of its predecessors. In the current setting, it is not clear whether this same conclusion is true and more research is required.

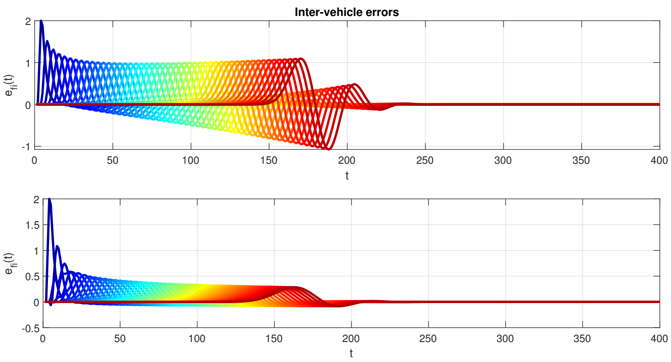

Fig. 5. Inter-vehicle errors for $r = 1$ (top) and $r = 2$ (bottom) with fixed $h = 3.1$ and $\eta = 0.3$.

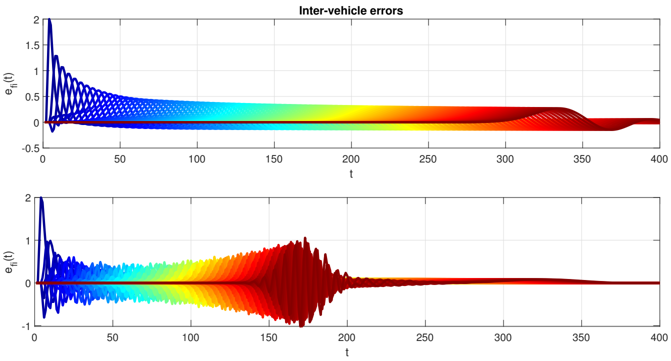

Fig. 6. Inter-vehicle errors for $\eta = 0.1$ (top) and $\eta = 0.45$ (bottom) for fixed range $r = 3$ and $h = 3.2$.

7. CONCLUSION

We have studied a platooning problem with limited range communications, where the agents are modelled by discrete time LTI plants. By using numerical simulations and a direct method for obtaining the resulting dynamics of the interconnected system we have demonstrated that increasing the forward communication range of an agent may allow for the relaxation of the necessary time headway constant value for string stability. More work is needed to characterize the relationship between the increase in the range and the control scheme parameters. We noted that if the parameters are not selected properly, the range increase may deteriorate the string stability property. Future works will be directed at understanding this and related issues.

REFERENCES

Y. Bian, Y. Zheng, W. Ren, S.E. Li, J. Wang, and K. Li. Reducing time headway for platooning of connected vehicles via V2V communication *Transportation Research Part C* 102:87-105, 2019.

M. Cantoni, E. Weyer, Y. Li, S.K. Ooi, I. Mareels, M. Ryan. Control of Large-Scale Irrigation Networks *Proceeding of the IEEE* 95:75–91, 2007.

S. Darbha, S. Konduri, and P.R. Pagilla. Benefits of V2V Communication for Autonomous and Connected Vehicles *IEEE Transactions on Intelligent Transportation Systems* 20(5):1954–1963, 2018.

C. Flores, and V. Milanés. Fractional-order-based ACC/CACC algorithm for improving string stability *Transportation Research Part C* 95:381–393, 2018.

S. Knorn, and R.H. Middleton. String Stability Analysis of a Vehicle Platoon with Communication Range 2 Using the Two-Dimensional Induced Operator Norm 2013 European Control Conference (ECC), July 17-19, 2013, Zurich, Switzerland.

S. Knorn, Z. Chen and, R.H. Middleton. Overview: Collective Control of Multiagent Systems *IEEE Transactions on Control of Network Systems* 3:334–347, 2016.

S. Konduri. *Robust String Stability of Vehicle Platoons with Communication* Ph.D. dissertation, Texas A&M University, 2017.

R.H. Middleton and J.H. Brasslavksy. String Instability in Classes of Linear Time Invariant Formation Control With Limited Communication Range *IEEE Transactions on Automatic Control* 55:1519–1530, 2010.

E. Perea, I. Grossmann, E. Ydstie, and T. Tahmassebi. Dynamic modelling and classical control theory for supply chain management *Computers and Chemical Engineering* 24:1143–1149, 2000.

G. Rodonyi. Heterogenous String Stability of Unidirectionally Interconnected MIMO LTI Systems *Automatica* 103:354–362, 2019.

B. Sturmfels. *Solving Systems of Polynomial Equations* CBMS, 2002.

B. Yu, J. Freunderberg, R. Gillespie, and R.H. Middleton. Beyond synchronization: String instability in coupled harmonic oscillator systems. *International Journal of Robust and Nonlinear Control* 25:2745–2769, 2015.

F. Vargas, A. Maass, and A. Peters. String stability for predecessor following platooning over lossy communication channels. International Symposium on Mathematical Theory of Networks and Systems, July 16-20, 2018, Hong-Kong.

Seiler, P., Pant, A., and Hedrick, K. (2004). Disturbance propagation in vehicle strings. *IEEE Transactions on automatic control*, 49(10), 1835–1842.

Seron, M.M., Braslavsky, J.H., and Goodwin, G.C. (2012). *Fundamental limitations in filtering and control.* Springer Science & Business Media.

Stüdli, S., Seron, M.M., and Middleton, R.H. (2017). From vehicular platoons to general networked systems: String stability and related concepts. *Annual Reviews in Control*, 44, 157–172.

Feng, S., Zhang, Y., Li, S.E., Cao, Z., Liu, H.X., and Li, L. (2019). String stability for vehicular platoon control: Definitions and analysis methods. *Annual Reviews in Control.*