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An Improved Gramian-based Interaction Measure for Time-Delay Systems

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Abstract: In this paper, a modified Gramian based control configuration selection (CCS) method for linear multi-input multi-output (MIMO) plants with time delays in input-output channels is proposed. In contrast to the typical approach of approximating the delayed system, the time delay is directly integrated in the method by using the finite-time H_2 norm for the time- delay system (TDS). The methodology is based on an explicit formula for computing the finite-time H_2 norm for stable SISO systems. Gramian-based CCS methods are either insensitive to time delays or favor channels with large delays, while the proposed method suggests configuration which are more reasonable. A numerical examples is used to discuss and benchmark the method. It is concluded that the proposed methods provides adequate configuration suggestions and circumvents a well-known shortcoming.

Keywords: Gramians, time delays, control configuration selection

1. INTRODUCTION

Control Configuration Selection (CCS) and its sub problem input-output (I-O) pairing is an essential step in the design of multivariable and decentralized control systems, and over the years numerous CCS methodologies have been proposed. Generally, there are two main classes of indicators that are used in the configuration selection: relative gain based approaches inspired by the pioneering work of Bristol (1966), and Gramian based interaction measures Khaki-Sedigh and Moaveni (2009). Common to both classes is the desire to consider the system dynamics in the analysis and to provide guarantees for the closed loop performance of a selected control configuration for a wide range of systems. The indicators that are used in the selection process are usually referred to as interaction measures and it is common practice to use a combination of interaction measures to mitigate shortcomings of the individual interaction measure, like the inability to cope with time delays.

The first Gramian-based interaction measure, the Participation Matrix (PM) was suggested in (Salgado and Conley, 2004) and a way to consider the effect of time delays in CCS was introduced. The main issue of the approach is that the selection of channels with large delays are favored, which is not reasonable. To address this issue Salgado and Yuz (2007) proposed to consider the delays as input or output delays. The Gramian-based measure suggested by Birk and Medvedev (2003), later compared in Halvarsson (2008) and further developed by Castano and Birk (2012) are completely insensitive to time-delays, which also poses problems as the time delay in channels is ignored. While this could be seen as a virtue, practitioners are left without an indication on how time delays would affect the performance of a selected control configuration.

The contribution of this paper now lies in proposing a modified Gramian based interaction measure providing a more adequate behaviour when dealing with TDSs. For this end, we propose the use of the finite-time H_2 norm, as derived by Jarlebring et al. (2010), and the approach is further extended to consider input-output delays of a system. Introducing the notion of finite-time H_2 norm together with a specification of the desired closed loop response time yields a proper treatment of time-delays.

The paper is arranged as follows. First, some notation and the original H_2 norm are reviewed in Preliminaries. Then, the finite-time H_2 norm and its theoretical foundation for single-input single-output (SISO) systems with input/output time delay is presented, followed by the suggestion of an I-O pairing methodology based on the finite-time H_2 norm is introduced. The methods is then discussed and benchmarked in a numerical example. Finally some conclusions and outlook are given in section 6.

2. PRELIMINARIES

First some of the needed background and terminology is summarized. For the sequel, a linear strictly proper SISO stable system $g(s)$ is defined as

$$g(s) : \begin{cases} \dot{x}(t) = Ax(t) + bu(t) \\ y(t) = cx(t) \end{cases} \tag{1}$$

where $g(s)$ is the transfer function for a corresponding state space realization.

2.1 Controllability and Observability Gramians

The general form of the controllability Gramian is defined as

$$W_c(t) = \int_0^t e^{A\tau} bb' e^{A'\tau} d\tau \quad (2)$$

which is the solution of the differential equation

$$\frac{d}{dt} W_c(t) = AW_c(t) + W_c(t)A' + bb' \quad (3)$$

If $W_c = \lim_{t\to\infty} W_c(t)$ exists, then the steady state controllability Gramian, W_c, is satisfying the Lyapunov equation (4):

$$AW_c + W_c A' + bb' = 0 \quad (4)$$

Also, the general form of the observability Gramian is defined as

$$W_o(t) = \int_0^t e^{A'\tau} c'c e^{A\tau} d\tau \quad (5)$$

which is the solution of the differential equation

$$\frac{d}{dt} W_o(t) = A'W_o(t) + W_o(t)A + c'c \quad (6)$$

Similarly, if $W_o = \lim_{t\to\infty} W_o(t)$ exists, then the steady state observability Gramian, W_o, is satisfying the Lyapunov equation (7):

$$A'W_o + W_o A + c'c = 0 \quad (7)$$

2.2 Definition of the H_2 norm

The H_2 norm of the $g(s)$ is the energy of its impulse response, $h(t)$, and it can be computed as:

$$\|g\|_2^2 = \int_0^\infty h^2(t)dt = \frac{1}{\pi}\int_0^\infty g^*(j\omega)g(j\omega) \quad (8)$$

where $h(t)$ and $g(j\omega)$ denote the impulse response and frequency response of the $g(s)$, respectively. Fig. 1 shows the interpretation of he H_2 norm of the $g(s)$ based on its definition. Based on the definition of controllability and observability Gramians, Eqs.(2) and (5), and definition of H_2 norm, (8), it is clear that the H_2 norm $g(s)$ can be computed using Gramian matrices

$$\begin{aligned} \|g\|_2^2 &= trace(cW_c c') = cW_c c' \\ \|g\|_2^2 &= trace(b'W_o b) = b'W_o b \end{aligned} \quad (9)$$

2.3 H_2 norm for SISO systems with input/output delay

Consider the state space model of a linear SISO stable strictly proper system with input-delay $g_d(s)$ as

$$g_d(s) : \begin{cases} \dot{x}(t) = Ax(t) + bu(t - t_d) \\ y(t) = cx(t) \end{cases} \quad (10)$$

and its equivalent transfer function derived from (1) as $g_d(s) = g(s)e^{-t_d s}$

For the sake of simplicity and without loss of generality, we just consider input-delay systems. The same results can be developed for output-delay SISO systems.

Obviously, based on the definition of the H_2 norm, (8) and as shown in Fig. 2, the time-delay has no effect on the value of the H_2 norm, and thus $\|g_d\|_2 = \|g\|_2$.

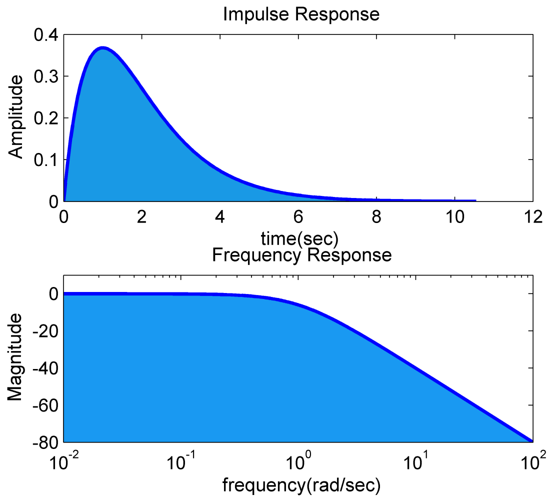

Fig. 1. H_2 norm of the system: (a) based on the energy of its impulse response, (b) based on the energy of the frequency response

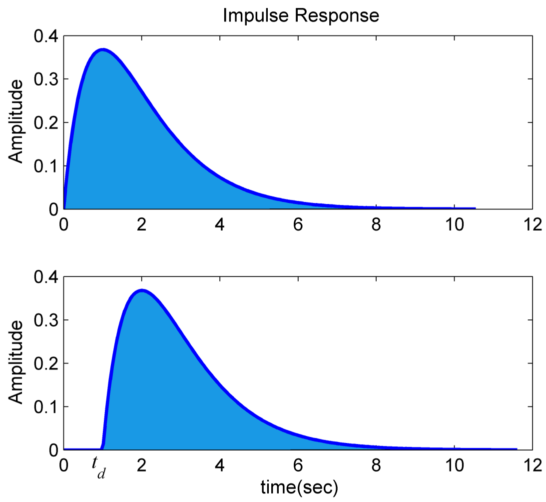

Fig. 2. H_2 norm derivation for system (a) without time-delay, and (b) with input/output-delay t_d

3. H_2 NORM FOR TIME-DELAY SYSTEMS

By introducing the finite-time H_2 norm, denoted $H_{2,t}$, the effect of input/output time-delay on the H_2 norm is reflected in the resulting norm value. In other words, by defining a time-frame as $t \in [0, t_s]$, to compute the $H_{2,t}$ norm as (11), instead of computing it from 0 to ∞, the time-delay affects the $H_{2,t}$. Fig. 3 shows the $H_{2,t}$ norm in the given time-frame $[0, t_s]$.

$$\|g\|_{2,t}^2 = \int_0^{t_s} h^2(t)dt \quad (11)$$

Clearly, the choice of t_s has an immense effect on the $H_{2,t}$ and becomes a design parameter for the engineer. Here, it should be remembered that the designed of a closed loop system is usually pursued on the basis of a performance requirement, which can be mapped into a response or settling time, or equally a desired bandwidth. In this way, t_s captures the notion of performance.

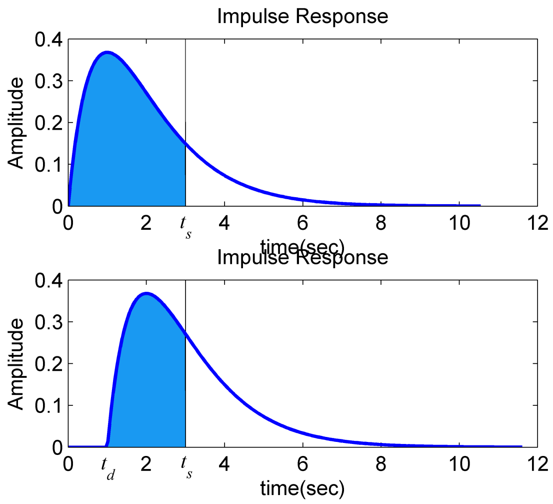

Fig. 3. $H_{2,t}$ norm: (a) normal system, (b) time-delay system

Hence, the value of t_s can be selected based on the desired bandwidth of the closed loop system. $H_{2,t}$ norm can then be seen as the amount of energy which is transferred from input to the output during a certain period of time and it is clear that the $H_{2,t}$ of g_d is less than or equal to its value of g in a same time frame.

$$\|g_d\|_{2,t} \leq \|g\|_{2,t} \tag{12}$$

Using the additive property of integrals, (11) for the time-delay system g_d can be rewritten as (13).

$$\|g_d\|^2_{2,t} = \int_0^{t_s - t_d} h^2(t)dt \tag{13}$$

and using (2), (5) and (9) the finite-time H_2 norm of the system in a time frame can be computed as:

$$\|g_d\|^2_{2,t} = cW_c(t_s - t_d)c' = b'W_o(t_s - t_d)b \tag{14}$$

In the following some needed explicit formulas for computing the finite-time H_2 norm of time delay systems are derived.

1^{st} order SISO systems Clearly, for a 1^{st} order input-delay SISO system, (10) can be simplified to

$$g_d(s) : \begin{cases} \dot{x}(t) = \lambda x(t) + bu(t - t_d) \\ y(t) = cx(t) \end{cases} \tag{15}$$

Then (3) for computing the controllability Gramian of the 1^{st} order system becomes

$$\frac{d}{dt} W_c(t) = 2\lambda W_c(t) + b^2 \tag{16}$$

Resultingly, by the use of (16) and (14) the H_2 norm of (15) renders

$$\|g_d\|^2_{2_t} = -\frac{b^2 c^2}{2\lambda}\left(1 - e^{\lambda(t_s - t_d)}\right) \tag{17}$$

2^{nd} order SISO systems The $H_{2,t}$ norm for 2^{nd} order SISO systems can be determined using the Jordan form of the state space model. For the Jordan form, we determine an explicit formula for computing the finite-time H_2, where two cases need to be considered.

Case 1: Distinct eigenvalues
Consider the diagonal form of a 2^{nd} order time-delay system as (10):

$$g_d(s) : \begin{cases} \dot{x}(t) = \Lambda x(t) + b_d u(t - t_d) \\ y(t) = c_d x(t) \end{cases} \tag{18}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, b_d = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, c_d = [c_1 \;\; c_2] \tag{19}$$

Using (18) and (19), the differential equation (3) for delay-free systems can be rewritten as

$$\begin{bmatrix} \dot{w}_{11} & \dot{w}_{12} \\ \dot{w}_{12} & \dot{w}_{22} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} w_{11} & w_{12} \\ w_{12} & w_{22} \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} \\ w_{12} & w_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} [b_1 b_2] \tag{20}$$

The matrix differential equation (20) results in 3 differential equations

$$\begin{cases} \dot{w}_{11} = 2\lambda_1 w_{11} + b_1^2 \\ \dot{w}_{22} = 2\lambda_2 w_{22} + b_2^2 \\ \dot{w}_{12} = (\lambda_1 + \lambda_2) w_{12} + b_1 b_2 \end{cases}, \tag{21}$$

and we get

$$\begin{cases} w_{11}(t) = -\dfrac{b_1^2}{2\lambda_1}\left(1 - e^{2\lambda_1 t}\right) \\ w_{22}(t) = -\dfrac{b_2^2}{2\lambda_2}\left(1 - e^{2\lambda_2 t}\right) \\ w_{12}(t) = -\dfrac{b_1 b_2}{\lambda_1 + \lambda_2}\left(1 - e^{(\lambda_1 + \lambda_1)t}\right) \end{cases} \tag{22}$$

Consequently, the $H_{2,t}$ norm of the time-delay system with time-delay t_d in the time-frame $[0, t_s]$ is derived from

$$\|g_d\|^2_{2_t} = \begin{bmatrix} -\dfrac{b_1^2 c_1^2}{2\lambda_1} & -2\dfrac{b_1 b_2 c_1 c_2}{\lambda_1 + \lambda_2} & -\dfrac{b_2^2 c_2^2}{2\lambda_2} \end{bmatrix} \begin{bmatrix} 1 - e^{2\lambda_1(t_s - t_d)} \\ 1 - e^{(\lambda_1 + \lambda_2)(t_s - t_d)} \\ 1 - e^{2\lambda_2(t_s - t_d)} \end{bmatrix} \tag{23}$$

Case 2: Repeated eigenvalues
Again, consider the Jordan form of a 2^{nd} order time-delay system with repeated eigenvalues

$$g_d(s) : \begin{cases} \dot{x}(t) = \Lambda x(t) + b_J u(t - t_d) \\ y(t) = c_J x(t) \end{cases} \tag{24}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_1 \end{bmatrix}, b_J = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, c_J = [c_1 \;\; c_2] \tag{25}$$

Similarly, using (24) and (25), the differential equation (3) for delay-free sytems can be rewritten as

$$\begin{bmatrix} \dot{w}_{11} & \dot{w}_{12} \\ \dot{w}_{12} & \dot{w}_{22} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 1 \\ 0 & \lambda_1 \end{bmatrix} \begin{bmatrix} w_{11} & w_{12} \\ w_{12} & w_{22} \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} \\ w_{12} & w_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 1 & \lambda_1 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} [b_1 b_2] \tag{26}$$

The matrix differential equation (26) can be rewritten to

$$\begin{bmatrix}\dot{w}_{11}\\ \dot{w}_{12}\\ \dot{w}_{22}\end{bmatrix} = \begin{bmatrix}2\lambda_1 & 2 & 0\\ 0 & 2\lambda_1 & 1\\ 0 & 0 & 2\lambda_1\end{bmatrix}\begin{bmatrix}w_{11}\\ w_{12}\\ w_{22}\end{bmatrix} + \begin{bmatrix}b_1^2\\ b_{12}b_2\\ b_2^2\end{bmatrix} \tag{27}$$

Based on (27), we have:

$$\begin{cases} w_{11}(t) = -\dfrac{b_1^2}{2\lambda_1}\left(1-e^{2\lambda_1 t}\right)\\ -\dfrac{b_2^2}{8\lambda_1^3}\left(2-e^{2\lambda_1 t}\left(2\lambda_1 t-1\right)^2+e^{2\lambda_1 t}\right)\\ +\dfrac{2b_1b_2}{4\lambda_1^2}\left(e^{2\lambda_1 t}\left(2\lambda_1 t-1\right)+1\right)\\ w_{12}(t) = \dfrac{b_2(b_2-2b_1\lambda_1)}{4\lambda_1^2}\left(1-e^{2\lambda_1 t}\right)+\dfrac{b_2^2}{4\lambda_1^2}2\lambda_1 t e^{2\lambda_1 t}\\ w_{22}(t) = -\dfrac{b_2^2}{2\lambda_1}\left(1-e^{2\lambda_1 t}\right)\end{cases} \tag{28}$$

Consequently, the $H_{2,t}$ norm of the time-delay system with time-delay t_d in the time-frame $[0, t_s]$ is given by

$$\|g_d\|_{2,t}^2 = c_1^2 w_{11}(t_s-t_d) + 2c_1c_2w_{12}(t_s-t_d) + c_2^2 w_{22}(t_s-t_d) \tag{29}$$

4. I/O PAIRING USING FINITE-TIME H_2 NORMS

As it was explained in Section 2, the H_2 norm of a system shows the amount of energy which has been transferred from the input to the output. Consequently, the H_2 norm can be a good choice for input-output pairing in MIMO plants, by evaluating and comparing the amount of transferred energy from specific inputs to specific outputs Castano and Birk (2012). On the basis of this new quantification method a CCS method can be stated.

Consider the following $m \times m$ linear time invariant (LTI) multivariable plant, which is assumed to be strictly proper, stable, controllable and observable:

$$\begin{cases}\dot{x} = Ax + Bu\\ y = Cx\end{cases} \tag{30}$$

with $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{m\times n}$, $u \in \mathcal{R}^m$, and $y \in \mathcal{R}^m$. The realization of (30) as a transfer function matrix of size $m \times m$ is given as

$$G(s) = C(sI-A)^{-1}B \tag{31}$$

where I denotes the identity matrix of appropriate size and an elementary transfer function in $G(s)$ is $g_{ij}(s)$. It should be noted that the multivariable system can be seen as a collection of SISO system, which implies that the realization is not minimal.

In the sequel, to express the decomposition of B into column vectors and C into row vectors, the following notation is used

$$B = [b_{*1}, b_{*2}, ..., b_{*m}]$$
$$C^T = [c_{1*}^T, c_{2*}^T, ..., c_{m*}^T]$$

The original version of H_2 norm input-output pairing strategy was proposed by Birk and Medvedev (2003) and it was shown by Halvarsson (2008) that it is unaffected by time delays. Essentially the H_2 norm is then replaced by the $H_{2,t}$ for the calculation of the input-output pairing matrix, and is denoted as $\Sigma_{2,t}$. Using (30) and the transfer function realization of a general state space system from (31), the $\Sigma_{2,t}$ is then defined as follows

$$[\Sigma_{2,t}]_{ij} = \frac{\|g_{ij}\|_{2,t}}{\Sigma\,\|g_{ij}\|_{2,t}}, \tag{32}$$

According to Birk and Medvedev (2003) an appropriate input-output pairing would be constituted by a permutation matrix $\mathcal{P}$ of size $m \times m$ which corresponds to the largest sum of elements of $\Sigma_{2,t}$. Essentially, this selection can be stated as the following optimization problem

$$\mathcal{P}^* := \arg\max_{\mathcal{P}\in\mathbb{P}} ||\Sigma_{2,t}\circ\mathcal{P}||_{sum} \tag{33}$$

where $\mathcal{P}$ is the set of all possible $m \times m$ permutation matrices, and $\circ$ denotes the element-wise Hadamard product.

The most important properties of the $\Sigma_{2,t}$ are:

- $\Sigma_{2,t}$ is frequency scaling dependent, which can been seen as its advantage over the PM Salgado and Conley (2004).
- $\Sigma_{2,t}$ considers the effect of time delays and in contrast to the HIIA, subsystems with larger time delay have less chance to be selected as I-O pair.
- $\Sigma_{2,t}$ does not reflect closed loop properties beside the desired bandwidth in the treatment of time delays which is common for all Gramian based methods.

5. NUMERICAL EXAMPLE

As numerical example the continuous time counter-part of the 2 inputs-2 outputs transfer function matrix from Salgado and Conley (2004) is used to benchmark $\Sigma_{2,t}$. In the benchmarking, other Gramian-based measures alongside with some of the most used relative gain based measures, namely RGA (Bristol, 1966), dynamic RGA (DRGA) (Mc Avoy et al., 2003), and effective RGA (ERGA) (Xiong et al., 2005). Differences and similarities in the indications will be discussed and analyzed. Since the PM can not be derived for time delayed system, the following example will use the fourth order Padé approximation of the time-delay transfer functions for the calculation of the PM.

The transfer function matrix of the examples is given as follows

$$G_1(s) = \begin{bmatrix}\dfrac{0.7}{s+0.7} & \dfrac{0.17e^{-t_d s}}{(s+0.22)}\\ \dfrac{0.15}{(s^2+0.92s+0.15)} & \dfrac{0.36}{s+0.36}\end{bmatrix} \tag{34}$$

The main challenge in this example is the effect of the internal time delay on the I-O pairing, where the following two cases are assessed: $t_d = 0$ sec and $t_d = 10$ sec.

The pairing analysis using RGA, ERGA, HIIA, mHIIA, $\Sigma_{2,t}$ and DRGA is summarized in Table 1. There, it can be seen that RGA and ERGA result in the same matrices for $t_d = 0$ sec and $t_d = 10$ sec, since these two indicators do not consider the effect of time delays in the pairing analysis. In Fig. 2 the DRGA is displayed and recommends the diagonal I-O pairing in the bandwidth range for both $t_d = 0$ sec and $t_d = 10$ sec. It is worthwhile noting that the DRGA is minorly affected by the internal time delays. The HIIA and PM results in two different matrices each

Table 1. Summary of the indicator results

Method	$t_d = 0$	$t_d = 10$
RGA	$\Lambda = \begin{bmatrix} 4.00 & -3.00 \\ -3.00 & 4.00 \end{bmatrix}$	$\Lambda = \begin{bmatrix} 4.00 & -3.00 \\ -3.00 & 4.00 \end{bmatrix}$
ERGA	$\Gamma = \begin{bmatrix} 1.15 & -0.15 \\ -0.15 & 1.15 \end{bmatrix}$	$\Gamma = \begin{bmatrix} 1.15 & -0.15 \\ -0.15 & 1.15 \end{bmatrix}$
HIIA	$\Sigma_H = \begin{bmatrix} 0.29 & 0.18 \\ 0.28 & 0.25 \end{bmatrix}$	$\Sigma_H = \begin{bmatrix} 0.263 & 0.251 \\ 0.253 & 0.232 \end{bmatrix}$
mHIIA	$\Delta = \begin{bmatrix} 1.07 & 0.42 \\ 0.54 & 1.23 \end{bmatrix}$	$\Delta = \begin{bmatrix} 0.84 & 0.57 \\ 1.00 & 0.79 \end{bmatrix}$
PM	$PM = \begin{bmatrix} 0.25 & 0.15 \\ 0.34 & 0.25 \end{bmatrix}$	$PM = \begin{bmatrix} 0.16 & 0.457 \\ 0.22 & 0.16 \end{bmatrix}$
DRGA	see Fig. 4	see Fig. 4
Σ_2	$\Sigma_2 = \begin{bmatrix} 0.66 & 0.09 \\ 0.10 & 0.15 \end{bmatrix}$	$\Sigma_2 = \begin{bmatrix} 0.66 & 0.09 \\ 0.10 & 0.15 \end{bmatrix}$
$\Sigma_{2,t}$	see Fig. 5	see Fig. 5

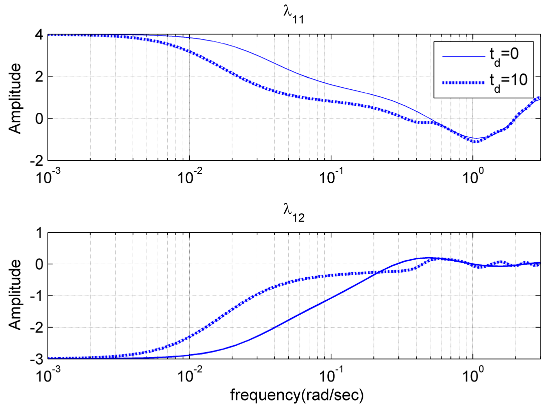

Fig. 4. Real part of the frequency responses of the elements of DRGA for G_2 in the two cases

one of them showing the main deficiency of the HIIA and PM, as elements with large time delays are usually preferred for I-O pairing, which is not desirable from a closed loop perspective. As for the Σ_2, it can be seen that the time delay is not affecting the measure, as it already has been discussed. In Fig. 5 the results of $\Sigma_{2,t}$ is shown for a range of t_s, and obviously, the effect of the time delay is considered, and $\Sigma_{2,t}$ clearly recommends diagonal pairing for all values of t_s. Fig. 5 then shows for t_s less than 10 sec, or the equivalent bandwidth (BW) greater than 0.1 Hz, that $\|g_{12}\|_{2t}$=0 and the diagonal pairing is the only possible choice.

6. CONCLUSIONS

This paper proposes a method to compute the finite-time H_2 norm, $H_{2,t}$ norm, of linear time invariant (LTI), stable systems which include input/output delay. The $H_{2,t}$ norm, in contradiction to conventional H_2 norm can consider the effect of time delay. Explicit equations for computing the $H_{2,t}$ norm for 1^{st} adn 2^{nd} order input/output delay LTI systems have been presented. Further research can be done to present explicit formula for computing the finite time $H_{2,t}$ norm. Also, by using the $H_{2,t}$ norm and Σ_2 input-output pairing, a new I-O pairing strategy, $\Sigma_{2,t}$, which can consider the effect of I-O delay of subsystems has been introduced. The effectiveness of the $\Sigma_{2,t}$ has been evaluated on three well known bench marking examples. It can be concluded that the proposed modification of the

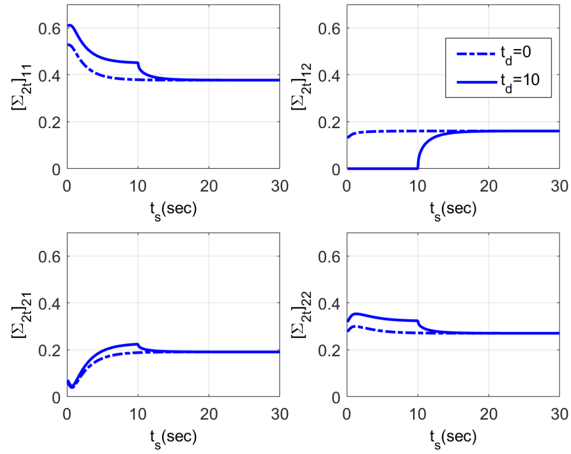

Fig. 5. $\Sigma_{2,t}$ of G_1 in two cases: $t_d = 0$ sec (dash-dotted line) and $t_d = 10$ sec (solid line).

Σ_2 measure enables a more conscious decision making on the control configurations in the presence of time delays.

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