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A New Reference Governor Strategy For Union of Linear Constraints

Raffaele Romagnoli * **Luis D. Couto** ** **Emanuele Garone** **

* *Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213-3815 USA (e-mail: rromagno@andrew.cmu.edu).*
** *Department of Control Engineering and System Analysis, Université libre de Bruxelles, B-1050 Brussels, Belgium (e-mail: lcoutome,egarone@ulb.ac.be)*

Abstract: Classical scalar Reference Governor (RG) schemes require a convex admissible region. Recently, a novel scalar RG approach has been proposed for the case of nonconvex constraints that can be approximated as union of polyhedral sets. This new method, specifically developed for the charge control of lithium-ion batteries, shows good performance and the capability of handling these kind of constraints while keeping a very low computational footprint. However, this method can guarantee that the system will reach the desired set point only under very specific properties of the constraints. In this paper, we analyze these limitations and propose a solution that ensures *convergence* of the RG scheme under much milder conditions on the topology of the constraints.

Keywords: Constrained Control, Nonconvex constraints, Reference Governor, Tracking, UAVs.

1. INTRODUCTION

Reference Governor (RG) is an add-on control scheme that implements a receding horizon-based technique that acts on the reference input of a controlled system in order to guarantee constraints fulfillment (Gilbert et al., 1995). These kind of schemes take advantage of the stability property of the existing control system in order to generate a feasible and convergent reference input that solves the constrained control problem. A factor that makes this scheme appealing among other receding horizon methods (e.g. model predictive control (MPC)), is the reduced computational costs associated to it (Gilbert and Kolmanovsky, 1999; Garone et al., 2017).

In fact, considering the scalar RG (SRG) scheme, the admissible reference input is computed by maximizing a scalar parameter instead of computing a sequence of inputs along the prediction horizon as for MPC schemes. In the SRG case, the optimization problem can be solved without using any solver, which translates in low computational cost algorithms (e.g. see Kolmanosky and Gilbert (1998)). One of the key aspects of RG schemes is that the whole admissible region that considers all the future predictions of the system, namely the *maximum output admissible set*, can be computed using a finite number of predictions (Gilbert and Tan, 1991). Hence, constraints fulfillment can be ensured for all time instants. A very efficient computation of the maximum output admissible set can be carried out under the assumption that the admissible set is a polyhedral convex region.

In general, real systems are often associated with nonlinear constraints. SRG schemes for linear systems with nonlinear constraints have been studied in Kalabić et al. (2011), where the special case of concave constraints was considered. In this case, dynamically reconfigurable linear constraints were used to model the concave constraints. In the same vein, a modified version of the Explicit Reference Governor (ERG) (Nicotra and Garone, 2018) has been proposed in Romagnoli et al. (2017) for concave constraints characterizing the charging problem of lithium-ion batteries. Other ERG able to work with union and intersection of concave constraints have been presented in Hosseinzadeh and Garone (2019) and Hosseinzadeh et al. (2019).

Another RG technique for piecewise affine systems has been proposed in Falcone et al. (2009). There, the presence of nonconvex constraints is managed by multiparametric solvers. In order to reduce the computational cost, this approach considers a suboptimal smaller region of attraction for finding an admissible solution resorting to invariant sets and reachable set theory. Other RG/command governor approaches (Bemporad et al., 1997; Bemporad, 1998) developed for linear and nonlinear systems generally use convex sets, and they require the use of specific solvers that can be too computationally expensive for certain applications.

One way to tackle the complexity of nonconvex contraints is by representing the nonconvex regions as the union of a finite number of polyhedral sets. This idea is not new and it has been used to solve many engineering problems. In Pérez et al. (2011), such representation is used to compute the maximal closed-loop admissible set, which is used as

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terminal region in order to ensure the stability of MPC controllers. In the same spirit, the standard SRG was extended in Romagnoli et al. (2019) for a specific type of nonconvex constraints in order to manage the charge operations of lithium-ion batteries. That work resorts to the logic operator OR to express the union of sets as OR conditions between constraints, which ultimately define the nonconvex region. This OR-SRG strategy resulted in a simple algorithm that bypasses the use of optimization solvers.

The main limit of the OR-SRG presented in Romagnoli et al. (2019) is that the admissible reference inputs must be chosen at each instant on the line segment that joins the initial admissible reference input and the final desired point. This line segment has to be in the admissible region for the OR-SRG to generate a reference input that converges to the desired one, otherwise the reference gets stuck to the constraint boundary. In this paper, we extend the seminal OR-RG scheme presented in Romagnoli et al. (2019) by using standard waypoints that guarantee the *convergence* of the algorithm under less conservative conditions.

2. PRELIMINARIES: THE OR-SRG

The goal of this section is twofold. First, the system to be controlled is introduced together with the generalized OR constraints to which it is subject to. Secondly, the SRG with OR constraints (referred to as 'OR-SRG') presented in Romagnoli et al. (2019) is explained in order to familiarize the reader with the existing context.

Consider the dynamics of an asymptotically stable system given by the following discrete-time linear time-invariant model

$$x(t+1) = Ax(t) + Br(t) \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $t \in \mathbb{Z}^+$ is the discrete-time variable, $x \in \mathbb{R}^n$, $r \in \mathbb{R}^p$ and $y \in \mathbb{R}^q$ are the state, reference input and output vectors, respectively, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$ and $C \in \mathbb{R}^{q\times n}$ are the state, input and output matrices, respectively. The matrix A is Schur and the system (1) can represent a controlled system.

System (1),(2) is subject to a general set of constraints taking the form

$$y(t) \in \mathcal{Y} \subset \mathbb{R}^n \;\; \forall t \in \mathbb{Z}^+. \tag{3}$$

In this paper we assume that the admissible set $\mathcal{Y}$ can be well approximated by the following union of polyhedral sets:

$$\hat{\mathcal{Y}} = \bigcup_{k=1}^{n_r} \mathcal{Y}_k \subseteq \mathcal{Y}, \tag{4}$$

where $\mathcal{Y}_i$ are polyhedral sets represented by linear constraints in the form

$$\mathcal{Y}_k = \{x : F_k x - b_k \leq 0\}, \tag{5}$$

n_r is the number of convex sets used to approximate the nonconvex region, $F_k \in \mathbb{R}^{n_{c,k}\times n}$, and $b_k \in \mathbb{R}^{n_{c,k}}$. For convenience, we define the set of indices $\mathcal{K} = \{1, \cdots, n_r\}$.

To derive the general representation (4), let us start considering two adjacent nodes $\mathcal{Y}_{\overline{k}}$ and $\mathcal{Y}_{\underline{k}}$. Each one can be represented through the AND logical operator ($\wedge$) as

$$\bigwedge_{m=1}^{n_{c,\overline{k}}} F_{m,\overline{k}}x \leq b_{m,\overline{k}}, \tag{6}$$

where $F^T_{m,\overline{k}} \in \mathbb{R}^n$, and $b_{m,\overline{k}} \in \mathbb{R}$. The union of those two sets is expressed using the OR logical operator ($\vee$) as

$$\left(\bigwedge_{m=1}^{n_{c,\overline{k}}} F_{m,\overline{k}}x \leq b_{m,\overline{k}}\right) \vee \left(\bigwedge_{m=1}^{n_{c,\underline{k}}} F_{m,\underline{k}}x \leq b_{m,\underline{k}}\right). \tag{7}$$

The set (7) is given by the operator OR between two sets described by the operator AND of a set of constraints. For the implementation of the OR-SRG we need to represent this set as the AND of a set of constraints in OR. By analogy, set (7) can be described by

$$(\underline{A} \cap \underline{B}) \cup (\underline{C} \cap \underline{D}), \tag{8}$$

Defining $\hat{A} \triangleq (\underline{A} \cap \underline{B})$, we can apply the distributive property to the set $\hat{A} \cup (\underline{C} \cap \underline{D}) = (\hat{A} \cup \underline{C}) \cap (\hat{A} \cup \underline{D})$. Then, using first the commutative property and then the distributive one to the sets $((\underline{A}\cap\underline{B})\cup\underline{C})$ and $((\underline{A}\cap\underline{B})\cup\underline{D})$ we obtain $(\underline{C} \cup \underline{A}) \cap (\underline{C} \cup \underline{B})$ and $(\underline{D} \cup \underline{A}) \cap (\underline{D} \cup \underline{B})$, i.e.

$$(\underline{C} \cup \underline{A}) \cap (\underline{C} \cup \underline{B}) \cap (\underline{D} \cup \underline{A}) \cap (\underline{D} \cup \underline{B}), \tag{9}$$

which represents the AND of a set of constraints in OR. Applying the above results to the general case, we can write the admissible region that describes $\hat{\mathcal{Y}}$ as

$$\bigwedge_{j=1}^{c_c} \bigvee_{i=1}^{n_{c,j}} S_{j,i}x \leq s_{j,i}, \tag{10}$$

where c_c is the total number of constraints to be considered in AND, and $n_{c,j}$ is the number of constraints in OR for each j-th constraint in AND. $S^T_{j,i} \in \mathbb{R}^n$ is used to even out the notation for the RG framework and it is a vector that contains an instantiation of the vectors representing the same vector $F^T_{m,\overline{k}}$ and $F^T_{m,\underline{k}}$.

In order to control the closed-loop system (1),(2) subject to constraints (10), we resort to Reference Governors. Note, however, that while RGs are computationally efficient to handle *linear* systems subject to *convex* constraints, they are not able to manage nonconvex admissible regions. This fact motivates the use of the SRG with OR constraints (Romagnoli et al., 2019).

The RG philosophy states that, at each time instant, an applied reference $v(k)$ is computed such that, if $v(k)$ were to be kept constant from k onward, the future trajectory of the state would never violate the constraints, i.e.

$$\bigwedge_{j=1}^{c_c} \bigvee_{i=1}^{n_{c,j}} S_{j,i}\hat{x}(\ell|x(k), v(k)) \leq s_{j,i}, \quad l = 0, ..., \infty, \tag{11}$$

where $\hat{x}(\ell|x, v) = A^\ell x + (I - A)^{-1}(I - A^\ell)Bv$ is the ℓ step-ahead prediction given the initial state x and applying the constant reference v. The set of all the initial states x and applied references v for which (11) is satisfied is denoted as the *maximal output admissible set* O_∞. The computation of O_∞ from Eq. (11) implies an infinite number of constraints. Nevertheless, if the constraints (10) define a compact set, it is possible to find a inner approximation of O_∞ (see Gilbert and Tan (1991)). This approximation, denoted as $\tilde{O}_\infty$, is obtained as

$$\tilde{O}_\infty = O_\infty \cap O_\epsilon, \tag{12}$$

where

$$O_\varepsilon = \left\{ (x_v, v) | \bigwedge_{j=1}^{c_c} \bigvee_{i=1}^{n_{c,j}} S_{j,i}(I-A)^{-1}Bv \leq (1-\varepsilon)s_{j,i} \right\} \quad (13)$$

represents the set of the admissible references v and the associated states x_v computed at steady state that satisfy the constraints reduced of a quantity $\epsilon > 0$. In this way $\tilde{O}_\infty$ can be computed using a finite number of predictions ℓ^* (see Romagnoli et al. (2019)). Hence, (11) can be expressed as

$$\bigwedge_{\ell=0}^{\ell^*} \bigwedge_{j=1}^{c_c} \bigvee_{i=1}^{n_{c,j}} S_{j,i}\hat{x}(\ell|x(t), v(t)) \leq s_{j,i}. \quad (14)$$

At this point, using the same ideas as the standard SRG, the reference to be applied at time k is

$$v(t) = v(t-1) + \kappa(t)(r(t) - v(t-1)) \quad (15)$$

where $\kappa(t) \in [0, 1]$. The scalar κ takes the zero value if there is a risk of constraint violation (i.e. the applied reference is kept constant) whereas the value of one is taken otherwise (i.e. the applied reference equals the desired reference). By updating v using (15), it is ensured that $(x(t), v(t)) \in \tilde{O}_\infty$. The variable $\kappa(t)$ is the solution to the following optimization problem

$$\kappa(t) = \max_{\kappa\in[0,1]} \kappa \quad \text{subject to } (13) - (15). \quad (16)$$

This optimization problem defines a linear program, and therefore it can be efficiently solved by checking some inequalities. A computationally cheap algorithm to solve this problem was introduced in Romagnoli et al. (2019). From (13)-(14), (12) can be expressed as

$$\tilde{O}_\infty = \bigcup_{k=1}^{n_r} O_\infty^k \cap O_\epsilon^k = \bigcup_{k=1}^{n_r} O_\infty^k \cap \bigcup_{k=1}^{n_r} O_\epsilon^k, \quad (17)$$

where O_∞^k and O_ϵ^k are the O_∞ and O_ϵ associated to the k-th convex set $\mathcal{Y}_k$ respectively.

3. PROBLEM STATEMENT

Although the OR-SRG presented in the previous section has the merit of handling constraints in OR, it also exhibits the following main limitations:

- the scheme has been introduced for a specific application, such as the control of lithium-ion batteries. The problem now has to be extended to the general formulation of constraints proposed in the previous section (4);
- the scheme is able to steer the state of the closed-loop system from an initial condition to the targeted final condition if the line segment connecting these two points is obstacle-free. Otherwise, the system state hits a constraint and stays stuck there.

This contribution aims at tackling these issues, providing an algorithm and the conditions that guarantee the *convergence* of the OR-SRG scheme, which is a notion introduced in the next section. In general terms, the convergence property represents the capability of v to converge to the reference input $\bar{r}$ (Gilbert et al., 1995). In the case of command governors, this characteristic is described as the viability property (Bemporad et al., 1997). In the classical SRG with a convex admissible region, every initial condition $x_0 \in \mathcal{Y}$ is viable, then it represents a global property of the SRG. In the case of OR-SRG, convergence refers to the couple $(x_0, \bar{r})$. In fact, not all $x_0 \in \mathcal{Y}$ can be said *convergent* for a specific $\bar{r}$. In this paper we take the following assumption.

Assumption 1: the approximation $\hat{\mathcal{Y}}$ can be described by an undirected graph $\hat{G} = (\hat{V}, \hat{E})$ where each $\mathcal{Y}_k$ is a node, and there exists an edge connecting two nodes if the steady state

$$\mathcal{Y}_{\overline{k}} \cap \mathcal{Y}_{\underline{k}} \neq \emptyset, \quad (18)$$

where $\overline{k} \neq \underline{k}$ and $\overline{k}, \underline{k} \in \mathcal{K}$. Hence the node set is $\hat{V} = \{\mathcal{Y}_1, \cdots, \mathcal{Y}_{n_r}\}$ and the edge set is $\hat{E} = \{\{\mathcal{Y}_{\overline{k}}, \mathcal{Y}_{\underline{k}}\}\}$ for all $\overline{k}$ and $\underline{k}$ that satisfy (18). We assume that (4) generates a connected graph $\hat{G}$. We also assume that for each node $\mathcal{Y}_{\overline{k}}$ there is a correspondence with the set $O_\epsilon^{\overline{k}}$, and any adjacent node $\mathcal{Y}_{\underline{k}}$ follows that

$$O_\epsilon^{\bar{k}} \cap O_\epsilon^{\underline{k}} \neq \emptyset. \quad (19)$$

In other words, if we take as nodes all the sets $O_\epsilon^{\bar{k}}$ and we build the edge set considering (19), the obtained graph has the same topology of $\hat{G}$.

4. CONVERGENCE

The *convergence* property represents the capability of the SRG scheme to generate an admissible sequence of reference inputs v that drives the system from a given x_0 to $\bar{x}_r \triangleq (I-A)^{-1}B\bar{r} \in \hat{\mathcal{Y}}$, which can be defined as:

Definition 1. Given the system (1)-(2), the admissible region $\mathcal{Y}$ and the steady state $\bar{x}_r$, the initial condition $x_0 \in \hat{\mathcal{Y}}$ is said *convergent* with respect to the SRG scheme (16) if the sequence of reference inputs $v(t)$ converges to r and the state of the system $x(t)$ converges to $\bar{x}_r$.

Considering the case of a constant reference $\bar{r}$, for any initial condition $x_0 \in \hat{\mathcal{Y}}$ for which v_0 is admissible, there exists a sequence of applied references v that converges to r, where v belongs to the line segment co $\{v_0, \bar{r}\}$ (Gilbert et al., 1995), where co$\{\cdot\}$ denotes the convex hull. Since we consider linear systems, for each v, $\bar{x}_v = (I-A)^{-1}Bv$ and then $\bar{x}_v \in \text{co}\{\bar{x}_{v_0}, \bar{x}_r\}$. Hence we define the following notation

$$\text{co}\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\} \triangleq \text{co}\{v_0, \bar{r}\} \times \text{co}\{\bar{x}_{v_0}, \bar{x}_r\} \quad (20)$$

that denotes the generalized line segment in the domain of the couples $(\bar{x}_v, v) \in \mathbb{R}^n \times \mathbb{R}^p$ where O_ϵ is defined.

Theorem 2. (Convergence). Consider a linear system (1), (2) subject to constraints (10) under the usual RG assumptions that $A \in \mathbb{R}^{n\times n}$ is Schur, that at time $t = 0$ the applied reference v_0 is such that $(\bar{x}_{v_0}, v_0) \in O_\epsilon$, and for the initial condition x_0, $(x_0, v_0) \in \tilde{O}_\infty$. Then if the applied reference is (15) where $\kappa(t)$ is computed using (16), for a given constant desired reference $\bar{r}$ such that $(\bar{x}_r, \bar{r}) \in O_\epsilon$, the initial condition x_0 is said *convergent* if

$$\text{co}\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\} \subseteq O_\epsilon. \quad (21)$$

Proof. An alternative way to show the convergence of the SRG scheme is to show that for any admissible $v \neq \bar{r}$ there exists a time instant when the SRG algorithm (16) generates v' closer to $\bar{r}$. This means that for v', the state x never leaves $\mathcal{Y}$ if v' is kept for all the times.

Suppose that

$$\mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\right\} \subseteq O_\epsilon,$$

and for all the elements in $\mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\right\}$ there exists a Lyapunov level set

$$\mathcal{E}(v) \triangleq \left\{(x - \bar{x}_v)^T P (x - \bar{x}_v) \leq \beta\right\} \tag{22}$$

such that $\mathcal{E}(v) \subseteq \mathcal{Y}$. Given $0 < \beta' < \beta$, we can also find a smaller Lyapunov level set

$$\mathcal{E}'(v) \triangleq \left\{(x - \bar{x}_v)^T P (x - \bar{x}_v) \leq \beta'\right\} \tag{23}$$

such that $\mathcal{E}'(v) \subset \mathcal{E}(v)$. Hence for a given $(\bar{x}_v, v) \in \mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\right\}$, there exists

$$(\bar{x}'_v, v') \in \mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\right\}$$

such that

$$\mathcal{E}'(v) \subset \mathcal{E}(v'). \tag{24}$$

where $\|\bar{r} - v\| > \|\bar{r} - v'\|$. Starting from any $x_0 \in \mathcal{E}(v)$, since $\mathcal{E}'(v)$ is also a Lyapunov level set, x_t is converging in a finite time to $\mathcal{E}'(v)$ (Kalman and Bertram, 1960). Once $x \in \mathcal{E}'(v)$, since $\mathcal{E}'(v) \subset \mathcal{E}(v')$, the trajectory of x_t never leaves $\mathcal{Y}$ if v can switch to v'. This means that there exists a $\kappa > 0$ that generates the admissible reference v'. Hence, for any $v \neq \bar{r}$, there exists a finite time instant where (16) generates a $\kappa > 0$.

Corollary 3. Given a convex set $\mathcal{Y}$, then for any initial condition x_0 and an admissible initial reference v_0 such that $(x_{v_0}, v_0) \in O_\epsilon$, and for any desired reference $(\bar{x}_r, \bar{r}) \in O_\epsilon$, x_0 is said convergent.

Proof. If $\mathcal{Y}$ is convex then all $n_{c,j}$ of (13)-(14) are equal to zero. Then, O_ϵ is a convex set, hence (21) holds for all $(\bar{x}_0, v_0) \in O_\epsilon$.

Corollary 4. If $\mathcal{Y}$ is not convex, for a given $\bar{r}$ there exists an initial condition x_0 with an admissible v_0 such that it is not convergent.

Proof. The proof follows directly the definition of non-convex sets. Since $\mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\right\}$ is a segment, then, if $\mathcal{Y}$ is not convex, there exists $(\bar{x}_{v_0}, v_0) \in O_\epsilon$ such that

$$\mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\right\} \nsubseteq O_\epsilon.$$

Hence, from Theorem 2 follows that we cannot find $\mathcal{E}(v) \in \mathcal{Y}$ for all v, and therefore v cannot converge to $\bar{r}$.

Corollary 4 shows the main limitation of the proposed approach. Considering the approximation $\hat{\mathcal{Y}}$, for two adjacent nodes $\mathcal{Y}_{\overline{k}}$ and $\mathcal{Y}_{\underline{k}}$, there are two corresponding convex sets $O_\epsilon^{\overline{k}}$ and $O_\epsilon^{\underline{k}}$. Under Assumption 1, their union is nonconvex but it defines a compact set (see Fig. 1).

Now we consider two adjacent nodes of $\hat{G}$, $\mathcal{Y}_{\overline{k}}$ and $\mathcal{Y}_{\underline{k}}$ as represented in Fig. 1. For each one we have $O_\epsilon^{\overline{k}}$ and $O_\epsilon^{\underline{k}}$, then we can state the following proposition.

Proposition 5. For a given reference $\bar{r}$ such that $(\bar{x}_r, \bar{r}) \in O_{\underline{k}}$, we consider an initial condition $x_0 \in \mathcal{Y}_{\overline{k}}$, with an admissible v_0 such that $(\bar{x}_{v_0}, v_0) \in O_\epsilon^{\overline{k}}$, if

$$O_\epsilon^{\overline{k}} \cap O_\epsilon^{\underline{k}} \cap \mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\right\} \neq \emptyset, \tag{25}$$

then x_0 is convergent.

Proof. From Assumption 1, $O_\epsilon^{\overline{k}} \cap O_\epsilon^{\underline{k}} \neq \emptyset$. For the convexity of the sets $O_\epsilon^{\overline{k}}$ and $O_\epsilon^{\underline{k}}$, if there is an element $(\bar{x}_v, v)$ of the segment $\mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_r, \bar{r})\right\}$ in the intersection $O_\epsilon^{\overline{k}} \cap O_\epsilon^{\underline{k}}$, then (21) is satisfied (see P.1 in Fig. 1).

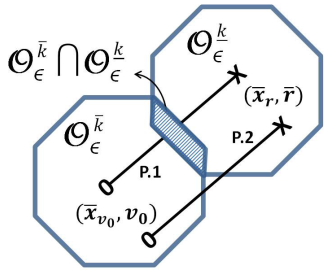
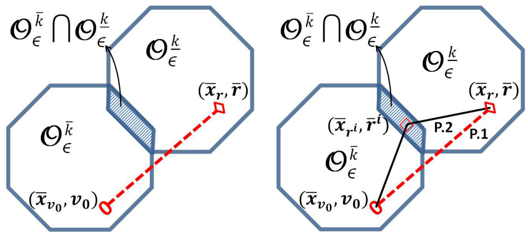


Fig. 1. Diagram of the union of two convex sets $\mathcal{O}_\epsilon^{\overline{k}}$ and $\mathcal{O}_\epsilon^{\underline{k}}$ resulting in a nonconvex set, where the initial admissible reference input corresponds to the element $(\bar{x}_{v_0}, v_0) \in \mathcal{O}_\epsilon^{\overline{k}}$, the final reference $(\bar{x}_r, \bar{r}) \in \mathcal{O}_\epsilon^{\underline{k}}$ and there are two possible types of path P.1 and P.2 to go from $\mathcal{O}_\epsilon^{\overline{k}}$ to $\mathcal{O}_\epsilon^{\underline{k}}$.

In the case of not convergent x_0 (see P.2 in Fig. 1), the idea is to use suitable waypoints $(\bar{x}_{r^i}, \bar{r}^i)$ such that (15) uses $\bar{r}^i$, which makes x_0 convergent with respect to the new reference and leads the state of the system to a point such that it is convergent for the reference $\bar{r}$.

Proposition 6. Suppose to be in the same conditions of Proposition 5, then selecting a waypoint $(\bar{x}_{r^i}, \bar{r}^i)$ such that $(\bar{x}_{r^i}, \bar{r}^i) \in O_\epsilon^{\overline{k}} \cap O_\epsilon^{\underline{k}}$, for any $(\bar{x}_r, \bar{r}) \in O_\epsilon^{\underline{k}}$, any initial condition $x_0 \in \mathcal{Y}_{\overline{k}}$ is convergent. Then, for any $(\bar{x}_r, \bar{r}) \in O_\epsilon$, any $x_0 \in \hat{\mathcal{Y}}$ can be made convergent using a proper choice of waypoints $\bar{r}^i$.

Proof. If x_0 is not convergent, then we select a waypoint $(\bar{x}_{r^i}, \bar{r}^i)$ such that $(\bar{x}_{r^i}, \bar{r}^i) \in O_\epsilon^{\overline{k}} \cap O_\epsilon^{\underline{k}}$, the union of the two segments $\mathrm{co}\left\{(\bar{x}_{v_0}, v_0), (\bar{x}_{r^i}, \bar{r}^i)\right\} \cup \mathrm{co}\left\{(\bar{x}_{r^i}, \bar{r}^i), (\bar{x}_r, \bar{r})\right\}$ is inside $O_\epsilon^{\overline{k}} \cup O_\epsilon^{\underline{k}}$. Hence for Theorem 2, there exists an admissible v such that $(\bar{x}_v, v) \in O_\epsilon^{\overline{k}} \cup O_\epsilon^{\underline{k}}$, which drives the state x in a point which is convergent for the original reference $\bar{r}$.



(a) (b)

Fig. 2. Based on the diagram of Fig. 1, (a) represents a division of intersecting convex sets $\mathcal{O}_\epsilon^{k}$ and $\mathcal{O}_\epsilon^{k+1}$, whereas in (b) it is represented the roadmap from $(\bar{x}_{v_0}, v_0)$ to $(\bar{x}_r, \bar{r})$ that crosses the waypoint $(\bar{x}_{r^i}, r^i)$.

In Fig. 2 is represented the solution to the problem of not convergent initial condition between two adjacent nodes of $\hat{G}$. In the next corollary we prove that it is possible to generate a sequence of waypoints, i.e. a *roadmap*, that makes any admissible initial condition convergent. The proof of the corollary describes the algorithm that we use to generate this roadmap.

Corollary 7. For any $(\bar{x}_r, \bar{r}) \in O_\epsilon$, any $x_0 \in \hat{\mathcal{Y}}$ can be made convergent using a proper choice of waypoints $\bar{r}^i$.

Proof. To prove this corollary, we define $\bar{r} \triangleq \bar{r}^n$ for notation convenience, and we refer to the nodes of $\hat{G}$ with the index k^i. Take an initial condition $x_0 \in \mathcal{Y}_{k^0}$ with an admissible v_0 such that $(\bar{x}_{v_0}, v_0) \in O_\epsilon^{k^0}$ with $\bar{x}_{v_0} \in \mathcal{Y}_{k^0}$, and a desired reference $\bar{r}^n$ such that $(\bar{x}_{r^n}, \bar{r}^n) \in O_\epsilon^{k^n}$ with $\bar{x}_{r^n} \in \mathcal{Y}_{k^n}$, $k^0, k^n \in \mathcal{K}$ with $k^0 \neq k^n$, and $\mathcal{Y}_{k^0}$, $\mathcal{Y}_{k^n}$ are not adjacent nodes of $\hat{G}$. Since $\hat{G}$ is connected, we can find a path (Nicotra, 2016; Nicotra et al., 2017)

$$P = \mathcal{Y}_{k^0}, \mathcal{Y}_{k^1}, \cdots, \mathcal{Y}_{k^n}$$

where each couple $(\mathcal{Y}_{k^i}, \mathcal{Y}_{k^{i+1}})$ is made by adjacent nodes of $\hat{G}$, where $k^i \in \mathcal{K}$. Using Assumption 1, we can find for $i = 1, \cdots, n-1$

$$(\bar{x}_{r^i}, \bar{r}^i) \in O_\epsilon^{k^i} \cap O_\epsilon^{k^{i+1}}.$$

Hence, given the sequences of references $\bar{r}^1, \cdots, \bar{r}^n$ and switching from $\bar{r}^i$ to $\bar{r}^{i+1}$ when $x(t) \in O_\epsilon^{k^i} \cap O_\epsilon^{k^{i+1}}$, x_0 converges to $\bar{x}_{r^n}$

Remark 8. It is important to note that the set of waypoints can be generated a priori based on the topology of the graph $\hat{G}$, assigning each edge of the set $\hat{E}$, $\bar{r}^i$ such that $(\bar{x}_{r^i}, \bar{r}^i) \in O_\epsilon^{k^i} \cap O_\epsilon^{k^{i+1}}$. Then at the beginning of the mission and given an initial state x_0 and a final desired reference $\bar{r}$, a shortest path search algorithm (Dijkstra) can be used to generate a sequence of nodes associated to the different waypoints to be used as roadmap.

5. QUADROTOR EXAMPLE

This section shows the effectiveness of the proposed OR-SRG control scheme to handle other problems than the one for which it was originally proposed in Romagnoli et al. (2019). This new benchmark corresponds to a quadrotor system navigating inside a room with walls and ceiling constraints. We propose to solve this constrained control problem by designing a controller with two loops, namely (i) a linear quadratic regulator (LQR) that pre-stabilizes the system and has v as a desired set-point; and (ii) the generalized OR-SRG that computes a virtual set-point v from the desired reference r and uses it as a suitable input to the closed-loop system in order to ensure constraint satisfaction. For details on the quadrotor model as well as the closed-loop system the reader is referred to Romagnoli et al. (2019); Beard (2008).

The quadrotor model was implemented in simulation and used to design the RG control scheme for OR constraints. Four cases with different kinds of possible violation of condition (25) were considered, namely:

- case 1: no violation, i.e. clear path from initial state to desired reference;
- case 2: ceiling violation;
- case 3: wall violation;
- case 4: both ceiling and wall violation.

The results for each case are shown in Fig. 3(a)-3(d), respectively, through four plots for each spatial position combination, i.e. x-y, x-z, y-z and x-y-z plots. The wall constraints (in x-y plane) and ceiling constraints (in z plane) are marked with solid blue lines. All the constraints are shown in every plot, with dashed blue lines marking not active constraints that are kept as reference (see e.g. the x-z plot in Fig. 3(a)). The initial state position (x_0) is denoted with a red circle while the desired reference ($\bar{r}$, coinciding with the final state position) is denoted with a red diamond. When a waypoint is required, it is marked with a red square. The dashed red lines depict the line segment connecting x_0 and $\bar{r}$ (see the x-y plot in Fig. 3(c) for instance), whereas the solid black line is the actual path followed by the quadrotor state. The x-y-z plot portrays the 3D trajectory of the quadrotor state with a solid black line, and in solid red, green and blue lines are the state projections of each coordinate.

Fig. 3(a) shows the benchmark case 1. From this figure follows the successful stabilization and reference tracking of the OR-RG scheme to control the quadrotor inside a room. Cases 2 and 3 in Figs. 3(b) and 3(c), respectively, show the quadrotor capabilities to avoid ceiling and wall constraints. On the one hand, ceiling constraints force the quadrotor to change its natural curvy dynamics in the z-direction produced by its inertia, but it keeps the straight line path in the x-y plane (see Fig. 3(b)). On the other hand, the proposed RG algorithm is able to circumvent wall constraints by deviating the quadrotor straight path towards the waypoint at the intersection of the two rooms, until it overcomes the obstacle and it can arrive safely to the destination (see Fig. 3(c)). The last case 4 portrays the effectiveness of the proposed OR-RG method when the previous two cases are combined (see Fig. 3(d)).

6. CONCLUSION

This paper presented a general SRG control scheme with capabilities of handling nonconvex constraints in the form of OR logical conditions. An admissible solution is provided even when such constraints interfere in the path between the current position and the desired position. This contribution improves upon previously reported OR-SRG algorithms by making it a general framework for optimization-free control under nonconvex constraints. The effectiveness of the proposed approach was validated through numerical studies considering the constrained control problem of steering a quadrotor from an initial point to a final point within a nonconvex room. Future work will cover the experimental validation of the proposed control strategy.

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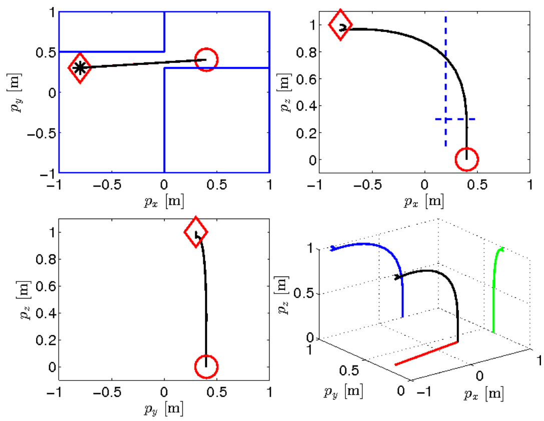
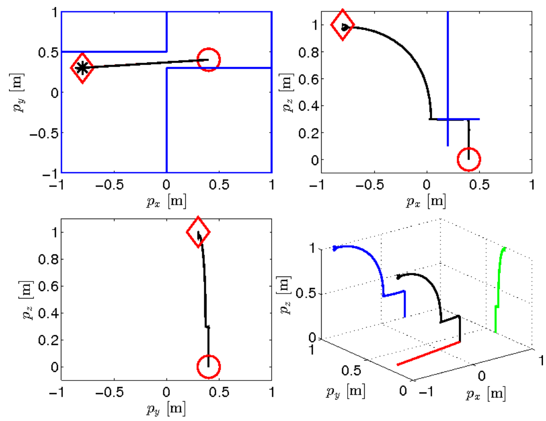
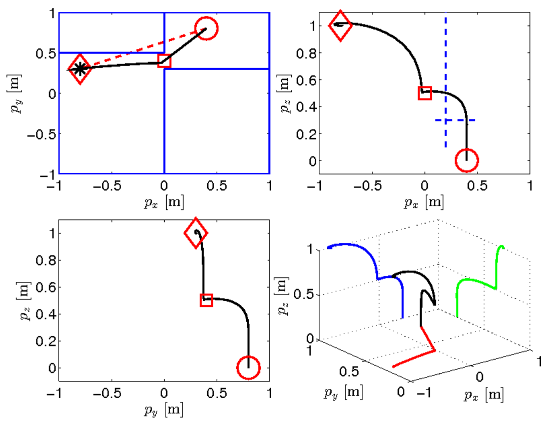


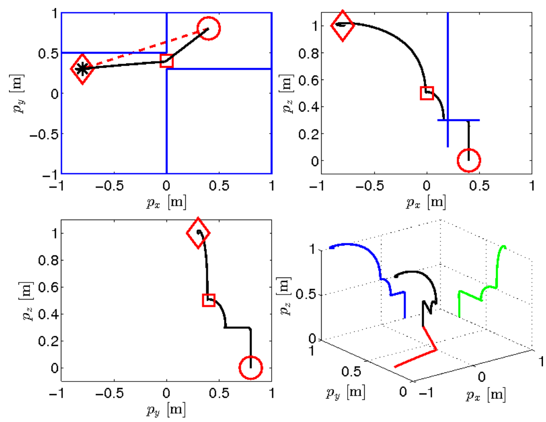
(a) Case 1: no violation.



(b) Case 2: ceiling violation.



(c) Case 3: wall violation



(d) Case 4: ceiling and wall violation.

Fig. 3. Results of an indoor quadrotor controlled through an RG scheme with OR-constraints handling capabilities. Each case shows four plots, one for each spatial position combination, namely x-y (upper left), x-z (upper right), y-z (lower left) and x-y-z (lower right) plots.

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