Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Multivariable Correlation-based Tuning for Load Disturbance Rejection ⋆

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Abstract: In many industrial processes, the setpoint signals present few changes and variations in the output are due mainly to the disturbances that enter the closed-loop and may be understood as load disturbances. Even though this is a common problem, the response to load disturbances is a topic not well covered in the direct data-driven control literature. This work seeks to fill that gap presenting a direct data-driven method to tune a multivariable controller in order to achieve certain load disturbance response described by some reference model. This work extends a previous one dealing with the monovariable case and uses the same correlation approach employed before in the Correlation-based Tuning (CbT).

Keywords: Data-based control, model following control, disturbance rejection, linear multivariable systems, parametric optimisation.

1. INTRODUCTION

There are many approaches to adjust the parameters of a feedback controller, most of them may be classified in model-based or direct data-driven approaches. Model-based approaches rely on a process model previously crafted from an identification procedure or directly from first principles equations. That model is then used to obtain one controller, usually solving some kind of optimisation problem. On the other hand, direct data-driven approaches use process data and an optimisation procedure to adjust the controller parameters directly, without the need of the process model (Bazanella et al., 2011).

There are two main disadvantages of the model-based approaches: the first one is that the model must be identified, what is usually a hard task; the second one is that usually the controller obtained with model-based approaches posses high order that makes it difficult to implement or it needs to be reduced, what degrades the closed-loop performance. The direct approaches, on the other hand, compute directly the parameters of low order controllers yielding better results, as shown by Campestrini et al. (2017).

Within the direct data-driven control literature, the reference tracking problem is well explored as indicated by the number of methods developed. These methods include the Iterative Feedback Tuning (IFT) (Hjalmarsson et al., 1998), the Virtual Reference Feedback Tuning (VRFT) (Campi et al., 2002), the Correlation-base Tuning (CbT) (van Heusden et al., 2011), the Optimal Controller Identification (OCI) (Campestrini et al., 2017), among others (Bazanella et al., 2011; Hou and Wang, 2013). Conversely, the problem of load disturbance rejection does not seem to share the same amount of interest, as proven by the amount of publications about that matter, or the lack thereof. The only examples the authors found are the Virtual Disturbance Feedback Tuning (VDFT) (Eckhard et al., 2018) and our previous work, the Disturbance Correlation-based Tuning (DCbT) (da Silva and Eckhard, 2019), both dealing only with the monovariable case. The latter work shows that DCbT does not need multiple experiments to deal with noise, unlike VDFT, and already presents good performance in the monovariable case. Therefore, that work was selected to be extended here to the multivariable case.

Another issue concerning data-driven methods is the closed-loop stability. One approach to deal with that problem is to add constraints to the optimisation problem as done by van Heusden et al. (2011). However, it is already known that a poor load disturbance reference model selection may be connected to poor closed-loop performance or even instability, as shown in Bordignon and Campestrini (2018). Therefore, the present work opted for a flexible reference model as shown in Bazanella et al. (2011) and Bordignon and Campestrini (2018), for example.

The remaining of this paper is organised as follows: Section 2 introduces the notation and the load disturbance problem; Section 3 presents the correlation approach proposed; Section 4 presents the least squares solution that emerges from the linear parametrisation of the controller; while Section 5 proposes a solution using a flexible reference model; Section 6 validates the proposal through simulation of a quadruple-tank water level process; and finally, Section 7 draws some conclusions from the research and presents some open issues to be tackled in the future.

⋆ This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. This work was supported in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

2. PROBLEM DEFINITION

Consider a Discrete-Time (DT) Multi-Input Multi-Output (MIMO) Linear Time-Invariant (LTI) system described by

$$y(t) = G(q)u(t) + v(t), \quad (1)$$

where $u(t) \in \mathbb{R}^n$ is the input signal, $y(t) \in \mathbb{R}^n$ is the output signal and $v(t) \in \mathbb{R}^n$ is zero-mean random noise. The process matrix $G(q)$ has dimension $n \times n$ and it is composed of rational functions of q which is the time-forward operator $qx(t) = x(t+1)$.

The system operates in closed-loop with a DT MIMO LTI feedback controller $C(q, \rho)$ with dimension $n \times n$ and that it is also composed of rational functions of q. The controller has a set of parameters to be adjusted which are grouped in a vector $\rho \in \mathbb{R}^p$. The set of all possible controllers is called the controller class and is defined as

$$\mathcal{C} = \{C(q, \rho) \mid \rho \in \mathcal{K} \subseteq \mathbb{R}^p\},$$

where $\mathcal{K}$ is a set possibly representing constraints on the parameter's values. To present an easily implementable solution, in this work it is assumed that the controller is linearly parametrised, or equivalently, each element of the controller matrix is given by

$$C_{i,j}(q, \rho) = \rho_{i,j}^T \beta_{i,j}(q), \quad (2)$$

where $\rho_{i,j} \in \mathbb{R}^{p_{i,j}}$ is the respective subvector containing the $p_{i,j}$ parameters for this element, and $\beta_{i,j}(q)$ is a vector of $p_{i,j}$ rational functions of q.

The closed-loop connections of the system with the controller are shown in Fig. 1 and are given by

$$\begin{bmatrix} u(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} U(q,\rho) & \tilde{S}(q,\rho) & -U(q,\rho) \\ T(q,\rho) & Q(q,\rho) & S(q,\rho) \end{bmatrix} \begin{bmatrix} r(t) \\ d(t) \\ v(t) \end{bmatrix}, \quad (3)$$

where $r(t) \in \mathbb{R}^n$ is the reference input and $d(t) \in \mathbb{R}^n$, while

$$\begin{aligned} S(q,\rho) &= [I + G(q)C(q,\rho)]^{-1} \\ \tilde{S}(q,\rho) &= [I + C(q,\rho)G(q)]^{-1} \\ T(q,\rho) &= I - S(q,\rho) \\ U(q,\rho) &= S(q,\rho)C(q,\rho) \\ Q(q,\rho) &= S(q,\rho)G(q) \end{aligned}$$

are the system's sensitivity functions and I is the identity matrix with appropriate dimensions. In particular, $Q(q, \rho)$ is the load disturbance sensitivity that this paper proposes to shape by tuning the controller's parameters.

It is assumed that the noise that enters the system is uncorrelated with other external inputs, which means that, when performing an experiment in open-loop,

$$\mathrm{E}\left[v(t)u(s)\right] = 0, \qquad \forall t, s,$$

while, when performing an experiment in closed-loop,

$$\mathrm{E}\left[v(t)d(s)\right] = \mathrm{E}\left[v(t)r(s)\right] = 0, \qquad \forall t, s.$$

where $\mathrm{E}\left[\cdot\right]$ is the expected value operator.

In this work, the main objective of the controller is to attenuate the effect of the disturbance signal $d(t)$ on the output $y(t)$. The relation between this two signals is given by $Q(q, \rho)$ which ideally should present fast dynamics and small norm, in order to reduce the effect of disturbance signals on the outputs. In a model reference control approach, a desired behaviour for the closed-loop system is specified by the **reference model** $M(q)$ and the parameters of the controller are obtained as the solution of an optimisation problem:

$$\rho_{\mathrm{MR}}^\star = \arg\min_{\rho \in \mathcal{K}} \|M(q) - Q(q, \rho)\| \quad (4)$$

When the class of controllers $\mathcal{C}$ is flexible enough, it is possible to achieve perfectly the desired performance such that the objective criterion is null. This is a special case that can be characterised.

Condition 1. (Matching controller). There is a parameter vector $\rho^\star \in \mathcal{K}$ such that

$$C(q, \rho^\star) = M^{-1}(q) - G^{-1}(q),$$

such that $M(q) = Q(q, \rho^\star)$.

When the process model $G(q)$ is known, the *ideal controller* $C(q, \rho^\star)$ can be computed using the above expression. However, in practice the controller class $\mathcal{C}$ is not flexible enough to include the *ideal controller* $C(q, \rho^\star)$, for instance when the controller class is restricted to PID controllers. Also, depending on the choice of the reference model $M(q)$ this ideal controller may not be causal, and therefore cannot be implemented even when the process model $G(q)$ is known, and the best controller that can be achieved is given by $C(q, \rho_{\mathrm{MR}}^\star)$.

It may be very difficult to solve the optimisation problem (4) because it is non-convex and depends on the knowledge of the process model $G(q)$. In data-driven control methods, instead of knowing the process model, the user collects batches of input-output data from the process which contains information about the dynamics of the system. These signals feed an optimisation problem which delivers an estimate for the optimal controller parameters $\rho_{\mathrm{MR}}^\star$. Some methods use non-convex data-driven optimisation criteria as IFT, the iterative version of CbT and OCI, while others solve a convex one such as VRFT and the noniterative CbT. The objective of this work is to present a data-driven control method, which solves a convex optimisation problem based on the correlation approach to obtain the parameters of the controller that attenuate the effect of disturbance signals on the output.

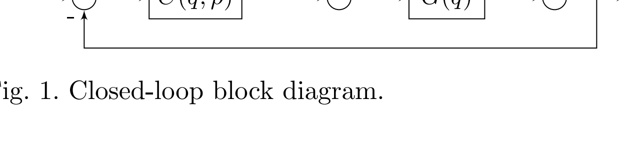

Fig. 1. Closed-loop block diagram.

3. CORRELATION APPROACH

Within the data-driven control methods, the user collects a batch of data from an open- or closed-loop experiment in order to obtain knowledge about the system. When the experiment runs in open-loop the user collects input and output data

$$\mathcal{Z}_o = \{u(1), y(1), \ldots, u(N), y(N)\}.$$

On the other hand, when the experiment runs in closed-loop it is possible to excite the reference signal $r(t)$ or to inject fictitious disturbance signals $d(t)$ in the closed-loop system. In this work we assume that only one of these external closed-loop signals is excited during the experiment while the other is kept null. For experiments exciting the reference signal the user should collect the set

$$\mathcal{Z}_r = \{r(1), u(1), y(1), \ldots, r(N), u(N), y(N)\},$$

while for fictitious disturbance experiments the user should collect the set

$$\mathcal{Z}_d = \{d(1), u(1), y(1), \ldots, d(N), u(N), y(N)\}.$$

The **correlation approch** consists of solving an optimisation problem where the correlation between two signals is minimised. These two signals are filtered versions of the signals collected from the experiment and the filters should be carefully chosen to ensure that the correlation between them reflects the control objective criterion. In this work, one of those signals is the external input while the other one being an error signal that depends on the data and the decision variables, i.e. the controller's parameters.

3.1 SISO Case

In the SISO case, the controller parameters are obtained as the solution of an optimisation problem

$$\rho_s = \arg\min_{\rho \in \mathcal{K}} \|R(\rho)\|^2 \tag{5}$$

where the correlation between the two signals is defined as

$$R(\rho) = E\left[\varepsilon(t,\rho)\zeta(t)\right]$$

and $\varepsilon(t,\rho)$ is a signal that must depend on the controller's parameters ρ and ideally (if there is no noise) should be null when $\rho = \rho^\star_{\mathrm{MR}}$. A previous work (da Silva and Eckhard, 2019) proposes the following signals:

$$\varepsilon(t,\rho) \triangleq M(q)u(t) + [M(q)C(q,\rho) - I]y(t), \tag{6}$$

for the monovariable problem, i.e. $I = 1$, and

$$\zeta(t) \triangleq \left[x(t-l) \ \cdots \ x(t) \ \cdots \ x(t+l)\right]^T, \tag{7}$$

where $x(t)$ is the external excitation signal used in the experiment, i.e $x(t)$ is either $u(t)$, $d(t)$, or $r(t)$ and l is a design parameter related to the length of the correlation.

Observe that there are three cases:

(1) Experiment performed in open-loop, and set $\mathcal{Z}_o$ is collected. Then $x(t) = u(t)$.
(2) Experiment performed in closed-loop, with reference excited and set $\mathcal{Z}_r$ is collected. Then $x(t) = r(t)$.
(3) Experiment performed in closed-loop, while the reference is kept null and a fictitious disturbance signal excites the system, while set $\mathcal{Z}_d$ is collected. Then $x(t) = d(t)$.

Now we can state some properties of the controller parameters ρ_s obtained minimising the correlation between $\varepsilon(t,\rho)$ and $\zeta(t)$.

Theorem 2. If Condition 1 is met then $\rho_s = \rho^\star$.

Proof. Note that using (1), after some algebraic manipulations we may rewrite (6) in terms of the the process' input and the noise:

$$\begin{aligned}\varepsilon(t,\rho) = &\left[M(q) - Q(q,\rho)\right]\tilde{S}^{-1}(q,\rho)u_x(t)\\ &+ \left[M(q)C(q,\rho) - I\right]v(t).\end{aligned} \tag{8}$$

From Condition 1, replacing $M(q)$ with $Q(q,\rho^\star)$ and after more algebraic manipulations, the error (8) calculated with the ideal parameters becomes only

$$\begin{aligned}\varepsilon_{\mathrm{ol}}(t,\rho^\star) &= \left[Q(q,\rho^\star)C(q,\rho^\star) - I\right]v(t)\\ &= \left[T(q,\rho^\star) - I\right]v(t)\\ &= -S(q,\rho^\star)v(t),\end{aligned}$$

which is filtered noise and hence is assumed to be uncorrelated with signal $x(t)$ for the three cases. Therefore $\varepsilon(t,\rho^\star)$ is not correlated with $\zeta(t)$ and $\rho^\star$ is the parameter vector that minimises the correlation between $\varepsilon(t,\rho)$ and $\zeta(t)$ and it is the solution of the optimisation problem (5). ■

3.2 MIMO Case

In the MIMO case, both $\varepsilon(t,\rho)$ and $x(t)$ are vectors, such that we can define the correlation between an element of each vector as

$$R_{i,j}(\rho) = E\left[\varepsilon_i(t,\rho)\zeta_j(t)\right], \tag{9}$$

where $\varepsilon_i(t,\rho)$ is the i-th element of $\varepsilon(t,\rho)$ and

$$\zeta_j(t) \triangleq \left[x_j(t-l) \ \cdots \ x_j(t) \ \cdots \ x_j(t+l)\right]^T, \tag{10}$$

where $x_j(t)$ is the j-th element of $x(t)$.

In this case, we may define the optimisation problem to be solved as the sum of all possible correlations:

$$\rho_m = \arg\min_{\rho \in \mathcal{K}} \sum_{i=1}^{n}\sum_{j=1}^{n} \|R_{i,j}(\rho)\|^2. \tag{11}$$

Observe that for $n = 1$, (11) is equivalent to (5).

Theorem 3. If Condition 1 is met then $\rho_m = \rho^\star$.

Proof. Using the same arguments of Theorem 2 it is possible to show that $\varepsilon(t,\rho^\star)$ is filtered noise which is assumed to be uncorrelated with the external excitation signal $x(t)$. Therefore, the correlation between $\varepsilon_i(t,\rho^\star)$ and $\zeta_j(t)$ is null for all i,j and $\rho^\star$ is the minimum of the optimisation problem (11). ■

Remark 4. Even when Condition 1 is not met (underparametrization), the resulting controller forces a small norm in (4), much like CbT (van Heusden et al., 2011).

4. LEAST SQUARES ESTIMATE

In order to obtain an estimate of the controller parameters using only finite input-output data, it is necessary to obtain an approximate optimisation problem which depends only on that data. An estimate of the correlation function (9), is given by:

$$\hat{R}_{i,j}(\rho) = \frac{1}{N}\sum_{t=1}^{N} \varepsilon_i(t,\rho)\zeta_j(t) \tag{12}$$

and the approximate optimisation problem is defined as:

$$\hat{\rho} = \arg\min_{\rho \in \mathcal{K}} \sum_{i=1}^{n}\sum_{j=1}^{n} \|\hat{R}_{i,j}(\rho)\|^2. \tag{13}$$

Note that the i-th component of the error variable (6) may be split in two terms:

$$\varepsilon_i(t,\rho) = \underbrace{M_{i,*}(q)u(t) - y_i(t)}_{\xi_i(t)} + \underbrace{M_{i,*}(q)C(q,\rho)y(t)}_{f_i(t,\rho)}, \tag{14}$$

where $M_{i,*}(q)$ is the i-th row of $M(q)$ and $y_i(t)$ is the i-th component of the output $y(t)$. Now, further developing the last term of (14) results in

$$\begin{aligned}f_i(t,\rho) &= \sum_{j=1}^{n}\sum_{k=1}^{n} C_{j,k}(q,\rho)M_{i,j}(q)y_k(t) &(15)\\ &= \sum_{j=1}^{n}\sum_{k=1}^{n} \rho_{j,k}^T \underbrace{\beta_{j,k}(q)M_{i,j}(q)y_k(t)}_{\phi_{j,k}^{(i)}(t)} &(16)\\ &= \rho^T\phi^{(i)}(t), &(17)\end{aligned}$$

where (15) uses the fact that scalar transfer functions commute, (16) uses the linear parametrization (2), and (17) uses the definitions of the following vectors:

$$\rho \triangleq \begin{bmatrix} \rho_{1,1} \\ \rho_{2,1} \\ \vdots \\ \rho_{n,1} \\ \vdots \\ \rho_{n,n} \end{bmatrix} \qquad \phi^{(i)}(t) \triangleq \begin{bmatrix} \phi^{(i)}_{1,1}(t) \\ \phi^{(i)}_{2,1}(t) \\ \vdots \\ \phi^{(i)}_{n,1}(t) \\ \vdots \\ \phi^{(i)}_{n,n}(t) \end{bmatrix}$$

Observe that the only restriction is that each subvector $\phi^{(i)}_{j,k}(t)$ must appear in vector $\phi^{(i)}(t) \in \mathbb{R}^p$ at the same position as the related subvector $\rho_{j,k}$ appears in the parameters vector $\rho \in \mathbb{R}^p$.

From (14) and (17), each component of the error may be written as

$$\varepsilon_i(t,\rho) = \xi_i(t) + [\phi^{(i)}(t)]^T\rho, \tag{18}$$

which indicates that the error variable is affine in the controller's parameters. Then the estimate of the correlation function becomes

$$\hat{R}_{i,j}(\rho) = Y_{i,j}\rho + Z_{i,j}, \tag{19}$$

with

$$Y_{i,j} \triangleq \frac{1}{N}\sum_{t=1}^{N}\zeta_j(t)[\phi^{(i)}(t)]^T \qquad Z_{i,j} \triangleq \frac{1}{N}\sum_{t=1}^{N}\zeta_j(t)\xi_i(t).$$

Using the above definitions, the following theorem may be stated along with its proof.

Theorem 5. The solution of the problem (13) is given by:

$$\hat{\rho} = -\left(\sum_{i=1}^{n}\sum_{j=1}^{n}Y_{i,j}^TY_{i,j}\right)^{-1}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}Y_{i,j}^TZ_{i,j}\right) \tag{20}$$

Proof. The problem (13) may be written as

$$\hat{\rho} = \arg\min_{\rho\in\mathcal{K}}\sum_{i=1}^{n}\sum_{j=1}^{n}\hat{R}^T_{i,j}(\rho)\hat{R}_{i,j}(\rho). \tag{21}$$

Replacing (19) in (21), taking its gradient, equating it to zero, and solving for ρ results in (20). ■

5. FLEXIBLE REFERENCE MODEL

One of the challenges faced by the methods that use the direct approach and a reference model to design controllers is the choice of the reference model itself. As mentioned before, choosing a model that is too far from what can be achieved may result in poor performance and even unstable systems (Bazanella et al., 2011). In that sense, employing a flexible reference model relieves the designer of that burden, letting the task of selecting the most achievable model to the optimisation method.

Considering a linear parametrised reference model with s parameters where each element is given by

$$M_{i,j}(q,\eta) = \eta_{i,j}^T\gamma_{i,j}(q), \tag{22}$$

where $\eta_{i,j} \in \mathbb{R}^{s_{i,j}}$ is a vector of parameters to be estimated and $\gamma_{i,j}(q) \in \mathbb{R}^{s_{i,j}}(q)$ is a vector of $s_{i,j}$ fixed rational functions of q. Notice that all the poles of $M_{i,j}(q,\eta)$ are pre-specified in the function $\gamma_{i,j}(q)$, such that the user still specifies the "velocity" of the closed-loop system using a flexible reference model. Also, some of the zeros of $M_{i,j}(q,\eta)$ may also be chosen, for instance it is usual to include a zero at 1 to ensure null error for constant disturbances.

Then, the correlation function may be written as

$$\hat{R}_{i,j}(\rho,\eta) = \frac{1}{N}\sum_{t=1}^{N}\zeta_j(t)\varepsilon_i(t,\rho,\eta) \tag{23}$$

where it is explicit that the correlation depends on both ρ and η, which both can be obtained as the solution of

$$\hat{\rho},\hat{\eta} = \arg\min_{\rho,\eta}\sum_{i=1}^{n}\sum_{j=1}^{n}\hat{R}^T_{i,j}(\rho,\eta)\hat{R}_{i,j}(\rho,\eta). \tag{24}$$

Observe that using (22) the i-th component of the error variable (14) may be written

$$\begin{aligned}\varepsilon_i(t,\rho,\eta) &= M_{i,*}(q,\eta)[u_x(t) + C(q,\rho)y_x(t)] - y_i(t)\\ &= \sum_{j=1}^{n}M_{i,j}(q)[u_j(t) + C_{j,*}(q,\rho)y_x(t)] - y_i(t)\\ &= \sum_{j=1}^{n}\eta_{i,j}\underbrace{\gamma_{i,j}(q)[u_j(t) + C_{j,*}(q,\rho)y_x(t)]}_{\psi^{(i)}_{i,j}(t,\rho)} - y_i(t)\\ &= [\psi^{(i)}(t,\rho)]^T\eta - y_i(t),\end{aligned} \tag{25}$$

where the vectors η and $\psi^{(i)}(t,\rho)$ are defined in a manner similar to the one the vectors ρ and $\phi^{(i)}(t)$ were defined before. Note that the same order restriction presented before also applies here for these vectors. Also, observe that

$$\psi^{(i)}_{k,j}(t,\rho) = \begin{cases}\gamma_{i,j}(q)[u_j(t) + C_{j,*}(q,\rho)y_x(t)], & \text{if } k = i, \text{ and}\\ 0, & \text{otherwise.}\end{cases}$$

Considering (25), the correlation may be rewritten as

$$\hat{R}_{i,j}(\rho,\eta) = A_{i,j}(\rho)\eta - B_{i,j}, \tag{26}$$

with

$$A_{i,j}(\rho) \triangleq \frac{1}{N}\sum_{t=1}^{N}\zeta_j(t)[\psi^{(i)}(t,\rho)]^T, \quad B_{i,j} \triangleq \frac{1}{N}\sum_{t=1}^{N}\zeta_j(t)y_i(t).$$

Notice that, for a fixed $\eta = \eta_0$, the solution of the optimisation problem (24) is given by (20). Also, for a fixed $\rho = \rho_0$, it is easy to show that the solution of the optimisation problem (24) is given by

$$\hat{\eta} = \left(\sum_{i=1}^{n}\sum_{j=1}^{n}A^T_{i,j}(\rho_0)A_{i,j}(\rho_0)\right)^{-1}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}A^T_{i,j}(\rho_0)B_{i,j}\right) \tag{27}$$

which was obtained equating the gradient of the criterion to zero.

Now, we propose the following iterative procedure to obtain a local minimum of the optimisation criterion:

$$\hat{\eta}^{(i)} = \begin{cases}\eta^{(0)}, & \text{if } i = 1\\ \hat{\eta} \text{ from (27) using } \rho = \hat{\rho}^{(i-1)}, & \text{otherwise}\end{cases}$$

$$\hat{\rho}^{(i)} = \hat{\rho} \text{ from (20) using } \eta = \hat{\eta}^{(i)},$$

where i indicates the iteration and $\eta^{(0)}$ is some initial educated guess for the parameters of the reference model.

Since each step reduces the norm of the correlation this iterative algorithm converges to a minimum of the criterion when the number of iterations tends to infinity. Note that in pratical cases a stop criterion must be included. Also, the flexible solution is iterative, but all steps use the same dataset from a single experiment —the method is one-shot.

6. CASE STUDY

A simulation example to validate the solution will be presented next. This example considers the quadruple-tank process described in Johansson (2000). That process consists of four water tanks (T1 to T4) interconnected, two electric pumps (P1 and P2) and two valves (V1 and V2) as shown in Fig. 2.

The valves only split the water flow between the upper and the bottom tanks in some predefined and fixed fractions. The input of the system is the electric tension applied to each pump and the output is the height of the water column in the bottom tanks. Some nonlinear dynamics are present, but the model employed represents the system after linearisation around some operating point. Depending on the position of the valves this model may have minimum- or nonminimum-phase characteristics.

For this validation, the linearised minimum-phase model from Johansson (2000) was chosen. After the discretization, using a zero-order holder and a sampling time of 5 s, the resulting model is given by:

$$G(q) = \begin{bmatrix} \dfrac{0.2014}{q - 0.9225} & \dfrac{0.01192q + 0.01079}{q^2 - 1.727q + 0.7423} \\ \dfrac{0.006022q + 0.005592}{q^2 - 1.792q + 0.8007} & \dfrac{0.1513}{q - 0.9460} \end{bmatrix}$$

Note that this model will be employed only to generate the data from which a controller will be estimated directly and, then, to validate that controller.

In order to simulate a closed-loop experiment, the following decentralised PI controller is used with the model:

$$C^{(0)}(q) = \begin{bmatrix} 0.1 + 0.01\dfrac{q}{q-1} & 0 \\ 0 & 0.1 + 0.01\dfrac{q}{q-1} \end{bmatrix}, \qquad (28)$$

The system is excited through the reference inputs $r_1(t)$ and $r_2(t)$ using two square waves with periods of 100

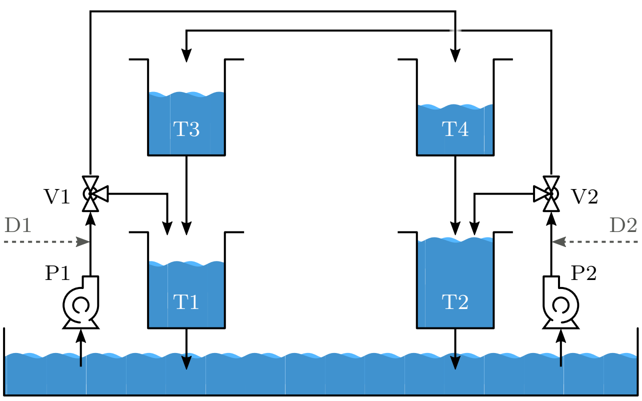

Fig. 2. The quadruple-tank process.

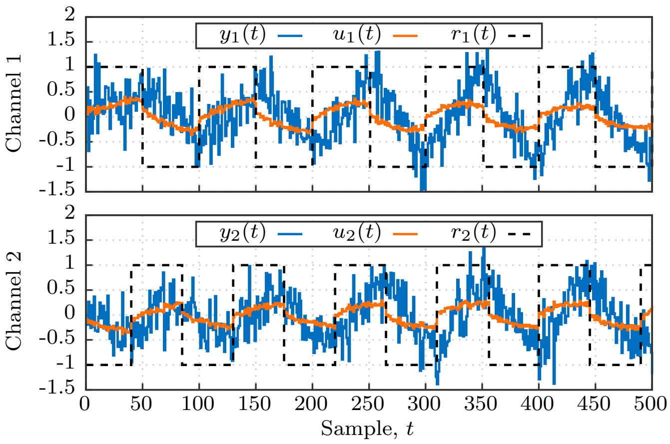

Fig. 3. Example of data collected.

and 90 samples, respectively. The simulation considers a zero-mean white measurement noise $v(t)$ with variance $\sigma^2 = 0.1$. All the signals have a length of 5000 samples and the first 500 samples of the input and output data collected are presented in Fig. 3 to give an idea of the signals and the noises involved.

The original model did not consider disturbances. However, considering additive disturbances (D1 and D2 in Fig. 2) affecting the flow in the pumps, the closed-loop load sensitivity $Q(q)$ is easily calculated from (3). The controller (28) yields the initial load sensitivity $Q^{(0)}(q)$ whose step response is represented by the red dotted lines in Fig. 4. Constant disturbances are already rejected because of the PI controllers, however the peak amplitude and the settling time are large. Also, a disturbance in one channel affects the other channel significantly. To counteract those effects, a parametrised reference model is crafted. For a shorter settling time, the poles of the flexible reference model are chosen faster than in open-loop:

$$M_{1,1}(q,\eta) = \frac{\eta_1(q-1)}{(q-0.87)(q-0.9)}, \qquad (29)$$

$$M_{1,2}(q,\eta) = \frac{(\eta_2 q + \eta_3)(q-1)}{(q-0.95)(q-0.92)(q-0.9)}, \qquad (30)$$

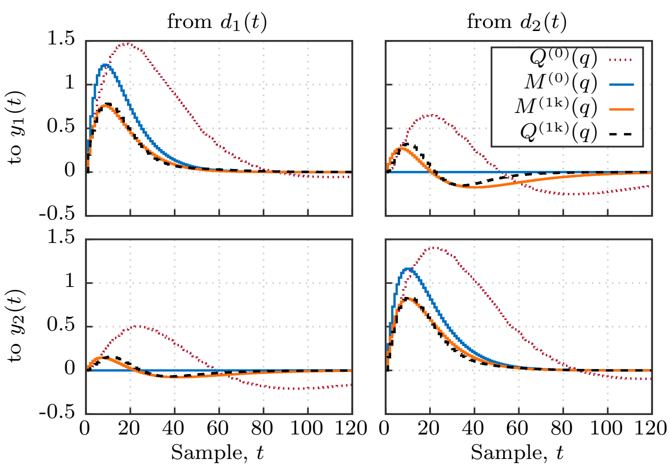

Fig. 4. Responses of the closed-loop systems to step disturbances.

$$M_{2,1}(q,\eta) = \frac{(\eta_4 q + \eta_5)(q-1)}{(q-0.95)(q-0.92)(q-0.9)}, \text{ and} \quad (31)$$

$$M_{2,2}(q,\eta) = \frac{\eta_6(q-1)}{(q-0.9)^2}. \quad (32)$$

To reduce the influence of the disturbance, the initial model $M^{(0)}(q)$ was constructed considering $\eta_1^{(0)} = 0.36$, $\eta_6^{(0)} = 0.3$, and all the other parameters zero, trying to force decoupling the channels' disturbance response. The step response of this initial model is also presented in Fig. 4 as the blue line. A new centralised controller structure was chosen comprising two PID controllers in the diagonal and two PD in the off-diagonal, that is,

$$C_{1,1}(q,\rho) = \rho_1 + \rho_2 \frac{q}{q-1} + \rho_3 \frac{q-1}{q}, \quad (33)$$

$$C_{1,2}(q,\rho) = \rho_4 + \rho_5 \frac{q-1}{q}, \quad (34)$$

$$C_{2,1}(q,\rho) = \rho_6 + \rho_7 \frac{q-1}{q}, \text{ and} \quad (35)$$

$$C_{2,2}(q,\rho) = \rho_8 + \rho_9 \frac{q}{q-1} + \rho_{10} \frac{q-1}{q}. \quad (36)$$

This case study evaluated the results obtained with the flexible approach. Therefore, the inputs are the data ($r_x(t)$, $u_x(t)$, and $y_x(t)$), the reference model structure (29)–(32), the controller structure (33)–(36), the initial guess for the model $M^{(0)}(q)$, and the number of lags l, in this case set to 200 samples. After 1000 iterations, the proposed method identified the following controller $C^{(1k)}(q)$:

$$C_{1,1}^{(1k)}(q,\rho) = 0.4844 + 0.0530\frac{q}{q-1} + 0.2638\frac{q-1}{q},$$

$$C_{1,2}^{(1k)}(q,\rho) = -0.0692 - 0.3195\frac{q-1}{q},$$

$$C_{2,1}^{(1k)}(q,\rho) = 0.0369 - 0.0922\frac{q-1}{q}, \text{ and}$$

$$C_{2,2}^{(1k)}(q,\rho) = 0.4153 + 0.0463\frac{q}{q-1} - 0.9284\frac{q-1}{q},$$

along with the following reference model $M^{(1k)}(q)$:

$$M_{1,1}(q,\eta) = \frac{0.2267(q-1)}{(q-0.87)(q-0.9)},$$

$$M_{1,2}(q,\eta) = \frac{(0.0481q - 0.0484)(q-1)}{(q-0.95)(q-0.92)(q-0.9)},$$

$$M_{2,1}(q,\eta) = \frac{(0.0891q - 0.0907)(q-1)}{(q-0.95)(q-0.92)(q-0.9)}, \text{ and}$$

$$M_{2,2}(q,\eta) = \frac{0.2117(q-1)}{(q-0.9)^2},$$

which should be close to what could be attainable. To see that, observe the step response of the estimated best reference model $M^{(1k)}(q)$, represented by the orange line in Fig. 4. Replacing the decentralised controller $C^{(0)}(q)$ with the new centralised controller $C^{(1k)}(q)$ yielded a load disturbance behaviour $Q^{(1k)}(q)$ whose step response is presented as the black dashed line in Fig. 4. Compare the orange and the dashed lines, observe that the estimated behaviour $M^{(1k)}(q)$ and the actual behaviour $Q^{(1k)}(q)$ are very close to each other, while distant from the initial model $M^{(0)}$, represented by the blue line. Also, comparing the old response (dotted line) and the new one (dashed line), note how the disturbance effects have been reduced and how the settling time is also shorter than before. A more aggressive disturbance rejection is easily achieved, for example by changing the poles of the reference model structure, at the likely cost of poor reference tracking and a jumpy control action.

7. CONCLUSION

Despite being a very common problem, load disturbance rejection has not been well covered by the direct data-driven control literature. In an effort to increase the research concerning that subject, a previous work extended a well known correlation-based data-driven method allowing it to be employed in the load disturbance rejection problem. The previous work was developed to deal with monovariable systems while the present work introduced a set of modifications that, as far as the authors know, makes this method the first one-shot direct data-driven method to deal with load disturbances in the multivariable case. The proposed method was validated through simulation, working as expected. Nevertheless, in the future the authors intend to develop a filter to improve the accuracy of the results and also intend to provide guidance as how to choose a good reference model, which is still an open issue for the multivariable case.

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