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Disassembly Lot Sizing Problem with Disposal Decisions for Multiple Product Types with Parts Commonality

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Abstract: Disassembly lot sizing problem is one of the important operational problems in disassembly systems. It can be defined as the problem of determining the disassembly quantity and timing of the used-products to fulfill the demand of their parts over a finite planning horizon. This paper considers the case of multiple product types with parts commonality and the objective is to minimize the sum of setup, disassembly operation, and inventory holding costs. High inventory holding cost can be generated: because of disparity between independent and unbalanced demands, and the disassembly of one unit of used-product generates all the parts with different ratios. Aggregate formulation (AGG) can be used to model this problem by considering disposal decisions. Linear-Programming (LP) relaxation of this model doesn't give very good lower bound, especially for the large-sized instances. We aim to improve lower bound of the problem. Facility Location-based formulation (FAL) is developed which can obtain optimal or near optimal solution by using LP relaxation approach. A two-phase heuristic is proposed which constructs an initial solution by using LP relaxation approach, and then improves by a dynamic programming based heuristic. Computational experiments are conducted on randomly generated test problems which show that the models and methods can give optimal or near-optimal solutions in very short computational times.

Keywords: Reverse logistics; Disassembly lot sizing; Modeling; Inventory; Disposal; Heuristics.

1. INTRODUCTION

In recent years, disassembly problems in reverse logistics has attained a lot of research interests because of the growing environmental concerns and legislation obligations such as End-of-Life Vehicles (ELVs) directive (2000/53/EC). Disassembly aims to decrease environmental impacts of End-Of-Life (EOL) products by separating dangerous or valuable materials and parts. Although, there is an important challenge to provide economic balance in disassembly systems, because of several costs related to disassembly process. This motivated some researchers to provide more effective and appropriate models to increase opportunities for cost savings and make disassembly systems more profitable (Tafti et al. (2019b); Ji et al. (2016)).

Among various decision problems considered in the literature, this paper considers disassembly lot sizing problem which is the problem of determining the disassembly quantity and timing of the EOL products to satisfy the demand of their parts over a finite planning horizon. The research done on disassembly lot sizing can be classified according to the number of EOL product, parts commonality, number of level on the EOL product structure and capacity limitation. Here, parts commonality indicate that different EOL products share the same parts.

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Tafti et al. (2019b) mention that due to yields (number of parts obtained after disassembly one unit of EOL product) and multiple demand sources of parts, unnecessary surplus inventories of parts can be accumulated during planning horizon. These surplus inventories can be led to the significant inventory holding cost. Lost sales and external purchasing decisions can be considered to handle the issue of surplus inventory accumulation (Hrouga et al. (2016), Ji et al. (2016)). Disposal decisions are considered for the single product disassembly lot sizing to handle surplus inventory and make disassembly systems more profitable. In this paper, we aim to develop the models and methods for the Multi-Product with Parts Commonality Disassembly Lot Sizing Problem with Disposal (MP-DLSPD).

Disassembly lot sizing problem is somewhat similar to ordinary lot sizing problem. However, it has specific characteristics that make it challenging for planning decisions: a) The EOL product diverges into multiple independent demand sources of parts; b) The disassembly of one unit of EOL product simultaneously generates all the parts with different ratios (Tafti et al. (2019a); Kim and Xirouchakis (2010); Kim et al. (2006)). The proposed models and methods in the literature of ordinary lot sing problem cannot be directly applied; hence, the new planning models and algorithms should be developed.

Limited number of the researches are studied disassembly scheduling problem. Kim et al. (2007) review the existing researches on the disassembly problem with its generalizations and classify the disassembly scheduling problem. Gupta and Taleb (1994) consider the basic case, i.e. single product type without parts commonality, and propose a reverse version of Material Requirement Planning (R-MRP). They extend this work by including parts commonality for disassembly of multiple product types (Taleb et al. (1997)). Lee and Xirouchakis (2004) introduce a heuristic algorithm that improves the solutions obtained by the algorithm of Gupta and Taleb with the objective of minimizing various costs related to disassembly processes. An Integer-Programming (IP) model can be used to formulate disassembly scheduling problem as in Lee et al. (2002). They consider the case of single product with capacity constraints with various cost factors in the objective function. Some researches consider setup and inventory cost together in the objective function so that lot sizing decisions should be considered. In this case, a lot sizing heuristic study considering related disassembly operations costs is addressed to improve the solutions of R-MRP algorithm (Barba-Gutiérrez et al. (2008)). Kim et al. (2006) propose a Mixed-Integer Programming (MIP) model for the problem with parts commonality. A multi-product problem with a Linear-Programming (LP) relaxation based heuristic that gives the good solutions in reasonable times is considered in Kim et al. (2003). Kim et al. (2009) address a single product type without parts commonality suggest a branch and bound algorithm that incorporates a Lagrangian heuristic. Recently, a capacited single-item multi-period disassembly scheduling problem with random parameters which is formulated as a mixed-integer nonlinear program, is studied in Liu and Zhang (2018). These researches don't consider the decisions on the management of surplus inventory.

Different formulations are proposed and compared in the ordinary lot sizing problem (Brahimi et al. (2006)). There are not a lot of works who consider different formulations for the problem of disassembly lot sizing (with or without disposal). To the best of authors' knowledge, there is no work who considers disposal decisions for multi-product disassembly lot sizing problem with parts commonality. Tafti et al. (2019b) consider a single product disassembly lot sizing problem with disposal decision (DLSPD) and propose different formulations which consider disposal decisions. They suggest an original algorithm to calculate the amount of consumed and non-consumed surplus inventory and two heuristics for the DLSPD without parts commonality. Afterward, they develop a Facility-Location based (FAL) model for the DLSPD so that it LP relaxation can improve lower bound (LB) of the problem in a very short computational time (Tafti et al. (2019a)). In Pour-Massahian-Tafti et al. (2019), they show that adding valid inequalities constraints to the AGG model can improve LB of the problem. Since the existing disassembly lot sizing models with parts commonality do not consider disposal decision, we are interested to developed different formulations which consider disposal decisions.

Motivated by above discussion, in this paper, we study a multi-product with parts commonality disassembly lot sizing problem with decisions on the surplus inventory. The contribution of the research is threefold: First, we develop two MIP models considering disposal decisions for the MP-DLSPD with parts commonality. Second, we aim to improve the lower bound of the considered problem by using LP relaxation of the FAL model. Third, A two-phase heuristic of Kim et al. (2006) is adapted for the MP-DLSPD with parts commonality. The MIP models are solved by CPLEX solver to obtain optimal solutions. The two-phase heuristic constructs an initial solution by using LP relaxation of the AGG model, and improves the solutions by changing them and considering cost trade-offs, iteratively. In the computational results, we report and compare the performance of the proposed models and methods.

Next section presents the problem considered in this research with the proposed models are presented. Resolution methods i.e. exact using CPLEX solver, LP relaxation approach, and two-phase heuristic to solve the problem are presented in Section 3. The test results on the new randomly generated instances are reported in Section 4. Finally Section 5 concludes this study with a summary and future works.

2. PROBLEM STATEMENT

A new disassembly lot sizing problem with disposal decisions for the multi-product and two-level disassembly structure with parts commonality is modeled in this section. Figure 1 presents an example of this structure. The number in parenthesis is the yield of a given part when one unit of its root item (1 ,2, 3) is disassembled. The first level represents leaf items, while the second level represents root items (EOL Products). The parts 6 and 7 can be obtained by several root items which implies parts commonality.

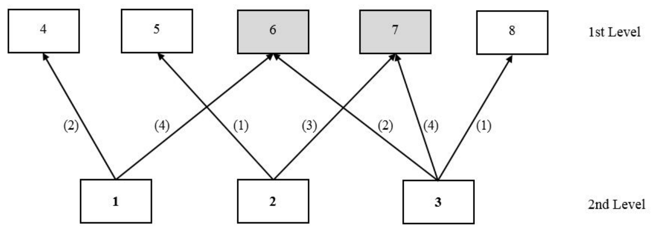

Fig. 1. Two-level and multi-product with parts commonality structure

The assumptions made in this paper are summarized as follows: *a*) EOL products can be obtained whenever they are needed and there is no holding cost for them; *b*) backlogging and lost sales are not allowed, and hence demands should be satisfied on time; *c*) demand of parts are given and deterministic; *d*) the disassembled parts have the same quality; *e*) we assume, without loss of generality, that the stock of the root and leaf items at the beginning of the planning horizon are zero. *f*) there is no disposal cost for the surplus leaf items and they will be dispose of after disassembly operation. In the models, without loss of generality, all items are numbered with integer numbers: 1, 2, . . . , i_r, i_l, . . . , N. i_r indicates the index for the root items and the numbers that are $\geq i_l$ represent leaf items. The following notations are used in this paper:

Index and Parameters

- i Index for items ($1, 2 \dots i_r, i_l \dots N$)
- t Index for periods ($1, 2 \dots T$)
- M_{it} Arbitrary big number considered for root item i in period t
- s_{it} Setup cost of disassembling root item i in period t
- p_{it} Disassembly operation cost of root item i in period t
- a_{il} Number of unit of item l obtained by disassembly of one unit of root item i
- h_{it} Inventory holding cost of leaf item i in period t
- H_{igt} Cumulative holding cost of leaf item i from period g to t ($g \leq t$)
- d_{it} Demand of leaf item i in period t
- $\Phi(i)$ Parents of item i
- $\eta(i)$ Children of root item i

Decision variables

- Y_{it} 1 if there is a setup in period t for root item i, and 0 otherwise
- X_{it} Disassembly quantity of root item i in period t
- Z_{ikjt} Quantity of leaf item i disassembled in period j from root item k to satisfy demand of period t
- E_{it} Disposed quantity of leaf item i in period t
- I_{it} Inventory level of leaf item i at the end of period t

2.1 Aggregate formulation (AGG)

A natural formulation of the problem (MP-DLSPD with parts commonality) can be represented as follows:

$$[\mathbf{P1}]\ Min\{\sum_{i=1}^{i_r}\sum_{t=1}^{T} s_{it}\cdot Y_{it} + \sum_{i=1}^{i_r}\sum_{t=1}^{T} p_{it}\cdot X_{it} + \sum_{i=i_l}^{N}\sum_{t=1}^{T} h_{it}\cdot I_{it}\} \tag{1}$$

Subject to

$$I_{it} = I_{it-1} + \sum_{k\in\Phi(i)} a_{ki}\cdot X_{kt} - E_{it} - d_{it} \quad \forall i = i_l \dots N \ \ \&\ \ t = 1\dots T \tag{2}$$

$$X_{it} \leq M_{it}\cdot Y_{it} \qquad \forall i = 1, 2\dots i_r \ \&\ t = 1\dots T \tag{3}$$

$$X_{it} \geq 0 \ \&\ integer \qquad \forall i = 1, 2\dots i_r \ \&\ t = 1\dots T \tag{4}$$

$$I_{it}, E_{it} \geq 0 \qquad \forall i = i_l \dots N \ \&\ t = 1\dots T \tag{5}$$

$$Y_{it} = 0\, or 1 \qquad \forall i = 1, 2\dots i_r \ \&\ t = 1\dots T \tag{6}$$

Objective function (1) is the sum of setup, disassembly operation, and inventory holding costs over a T-period planning horizon. Constraints (2) express the inventory balance equations for the leaf items. Constraints (3) guarantee that a setup cost is performed in period t if any disassembly operation is done in that period. Constraints (4-6) impose the non-negativity and binary restrictions on the variables. Note that equation (7) is used to calculate the value of M_{it} in equations (3) which can improve LB of the problem.

$$M_{it} = \max_{j\in\eta(i)}\left\{\left\lceil \frac{\sum_{k=t}^{T} d_{jk}}{a_{ij}} \right\rceil\right\} \qquad \forall i = 1\dots i_r \ \&\ t = 1\dots T \tag{7}$$

2.2 Facility Location-based formulation (FAL)

The below formulation is called the disaggregate formulation or facility location-based formulation (FAL). This formulation is commonly used for ordinary lot sizing problem, because its LP relaxation for the uncapacited problem provides an optimal solution with integer setup variables and it has stronger lower bounds for capacited lot sizing problem (Brahimi et al. (2017)). An additional disassembly variable Z_{ikjt} is considered, which corresponds to the quantity of leaf items i disassembled from root item k in period j to satisfy demand of period t. The product disassembly variable X_{it} cannot however be removed since each part can be received after a disassembly operation in a period. In this model, the variables X_{it} need to be integer but the variables Z_{ikjt} can be set as real. The disaggregate formulation for the MP-DLSPD with parts commonality can be represented as follows:

$$[\mathbf{P2}]\quad Min\ \{\sum_{i=1}^{i_r}\sum_{t=1}^{T}(s_{it}\cdot Y_{it} + p_{it}\cdot X_{it}) + \sum_{i=i_l}^{N}\sum_{k\in\Phi(i)}\sum_{t=1}^{T}\sum_{j=1}^{t} H_{ikt-1}\cdot Z_{ikjt}\} \tag{8}$$

Subject to

$$\sum_{j=1}^{t}\sum_{k\in\Phi(i)} Z_{ikjt} = d_{it} \qquad \forall i = i_l \dots N \ \ \&\ \ t = 1\dots T \tag{9}$$

$$Z_{ikjt} \leq d_{it}\cdot Y_{kj} \quad \forall i = i_l \dots N \ \ \&\ \ k \in \Phi(i) \ \&\ \ t = 1\dots T \ \ \&\ \ j \leq t \tag{10}$$

$$a_{ki}\cdot X_{kt} \geq \sum_{j=t}^{T} Z_{iktj} \quad \forall i = i_l \dots N \ \ \&\ \ k \in \Phi(i) \ \&\ \ t = 1\dots T \tag{11}$$

$$X_{it} \geq 0 \ \&\ integer \quad \forall i = 1, 2\dots i_r \ \ \&\ \ t = 1\dots T \tag{12}$$

$$Z_{ikjt} \geq 0 \quad \forall i = i_l \dots N \ \ \&\ \ k \in \Phi(i) \ \ \&\ \ t = 1\dots T \ \&\ j \leq t \tag{13}$$

$$Y_{it} = 0\, or 1 \qquad \forall i = 1, 2\dots i_r \ \ \&\ \ t = 1\dots T \tag{14}$$

Objective function (8) is to minimize the sum of setup, disassembly operation, and inventory holding costs over the whole T-period horizon. Constraints (9) represent that the demands of leaf items should be satisfied. Constraints (10) relate the disassembled quantity of leaf items to the binary setup variables. Constraints (11) express that the total quantity of leaf item i obtained by root item k in period t, after disassembly of product, will be delivered to satisfy the demand or will be disposed of. Constraints (12-14) define the domains of decision variables.

3. RESOLUTION METHOD

Three methods for the formulation of the problem are developed in this section and will be compared in section 4. The first one is an exact method by using CPLEX solver. Then, we propose a two-phase heuristic which improves iteratively the constructed initial solution by LP relaxation approach. Finally, we apply LP relaxation of the FAl model in order to improve LB of the problem.

3.1 Exact

Since the proposed models are MIPs, so they can be applied to obtain the optimal solution of the generated problem instances by using CPLEX solver. But, we cannot guarantee that CPLEX solver will be efficient for all the problems. We propose a two-phase heuristic with the advantage that it can be programmed into a code via simple applications in real industrial cases. It constructs an initial solution, and then improves the solutions iteratively by changing them using a dynamic programming based algorithm with forward-looking check.

3.2 Two-phase heuristic

We adapt the two-phase heuristic method suggested by Kim et al. (2006) for the MP-DLSPD with parts commonality. The heuristic algorithm consists of first phase in which an initial solution obtained from LP relaxation of the model P1 is constructed. Then, it is improved by a forward-looking algorithm based on dynamic programming approach.

Phase 1. Solution construction: the solution obtained by solving LP relaxation of the model P1 (real values) using CPLEX solver will be rounded down. The rounded-down solution is modified so that all the constrains of the model P1 are satisfied. The balance quantity (BL_{it}) defined by the equation (15) is used to check the feasibility of the rounded-down solution.

$$BL_{it} = I_{it} - I_{it-1} - \sum_{r\in\Phi(i)} a_{ri} \cdot X_{rt} + E_{it} + d_{it}$$
$$\forall i = i_l \dots N \;\; \& \;\; t = 1 \dots T \tag{15}$$

If $BL_{it} \neq 0$, the corresponding rounded-down solution should be modified by increasing or decreasing the decision variables, while considering cost changes. For the case with $BL_{it} > 0$, If $I_{it} \geq BL_{it}$, we calculate new inventory level by the equation 16. Otherwise, if $E_{it} \geq BL_{it}$, we update the disposed quantity by the equation (17):

$$I^{'}_{it} = I_{it} - BL_{it} \tag{16}$$
$$E^{'}_{it} = E_{it} - BL_{it} \tag{17}$$

Where $I^{'}_{it}$ and $E^{'}_{it}$ are the changed inventory and disposed quantity of a given leaf item, respectively. If both cases above result in infeasible solution, we consider the case of increasing root item r; X_{rt}, $r \in \Phi(i)$ so that $X^{'}_{rt} = X_{rt} + \Delta_r$. The amount of Δ_r for a root item $r \in \eta(i)$ is $\lceil BL_{it}/a_{ri}\rceil$. Where $\lceil * \rceil$ represents the smallest integer value greater than or equal to *. This change can be resulted in the violation of constraints 2 of child items $k \in \eta(r)$, $k \neq i$. The modification of the inventory level of the child items can be done as follows:

$$I^{'}_{kt} = I_{kt} + a_{rk} \cdot \Delta_r \qquad \forall k \in \eta(r), k \neq i$$
$$I^{'}_{it} = I_{it} + (a_{ri} \cdot \Delta_r - BL_{it}) \tag{18}$$

This increment of disassembly quantity of a given root item r results in cost increasing. A_r represents the cost change when increasing the disassembly quantity of root item $r \in \Phi(i)$:

$$A_r = p_{rt} \cdot \Delta_r + \sum_{k\in\eta(r),k\neq i} h_{kt} \cdot a_{rk} \cdot \Delta_r + s_r \cdot \{1 - \delta(X_{rt})\} + h_{it} \cdot (a_{ri} \cdot \Delta_r - BL_{it}) \tag{19}$$

where $\delta(*) = 1$ if and only if $* > 0$, and 0 otherwise. Equation (19) represents the increase in: disassembly operation cost of root item r, inventory holding cost of child items ($\neq i$), setup cost, and excess inventory holding cost incurred by item i after satisfying the balance BL_{it}. Then, we choose the best root item candidate with the minimum increasing cost . For the case with $BL_{it} < 0$, we consider increasing the quantity of disposed E_{it} as follow:

$$E^{'}_{it} = E_{it} + \mid BL_{it} \mid \tag{20}$$

Phase 2. Solution improvement: the solutions are improved by using dynamic programming based algorithm which is applied to each root item i.e. by starting from first root item to the last one. If last setup occurs in period j ($1 \leq j \leq t$) for a t-period sub-problem of a given root item r, the change in the current disassembly lot sizing is as follow:

$$X^{'}_{ru} = \sum_{k=j}^{t} X_{rk} \qquad u = j$$
$$X^{'}_{ru} = 0 \qquad u = j+1, j+2 \dots t \tag{21}$$

Where X_{ru} and $X^{'}_{ru}$ represent the current and new disassembly lot sizing for a given root item r, respectively. This change can decrease setup cost, while increasing inventory holding cost. Note that we don't consider the change in the disposed quantity of leaf items (E_{iu}) in the adapted two-phase heuristic. This means that when changing the current disassembly lot sizing using the equations (21), the disposed quantity will be held to be disposed as the current disassembly lot sizing. This can be improved in the future work. $B(j,t)$ represents the decrease in the total cost for a given root item r when the last setup occurs in period j for a t-period sub-problem:

$$B(j,t) = \max\{0, \sum_{k=j}^{t} p_{rk} \cdot X_{rk} - p_{rj} \cdot X^{'}_{rj}\} + \sum_{u=j+1}^{t} s_{ru} \cdot \delta(X_{ru}) \tag{22}$$

Where $\delta(*)$ =1, if $X_{ru} > 0$, otherwise, $\delta(*)$ =0. $C(j,t)$ represents the increase in the total cost as follow:

$$C(j,t) = \max\{0, p_{rj} \cdot X^{'}_{rj} - \sum_{k=j}^{t} p_{rk} \cdot X_{rk}\} + \sum_{u=j}^{t-1} \sum_{i\in\eta(i)} h_{iu} \cdot a_{ri} \cdot (X^{'}_{rj} - \sum_{k=j}^{u} X_{rk}) + s_{rj} \cdot (1 - \delta(X_{rj})) \tag{23}$$

Formulation 24 represents recursive cost saving function for the t-period sub-problem of a given root item r (Where $F_r(0) = 0$):

$$[\mathbf{H}] \; F_r(t) = \max_{1\leq j\leq t}\{\max(0, B(j,t) - C(j,t) + F_r(j-1))\} \tag{24}$$

Note that the cost saving function for all root items given above has a polynomial CPU time bound. It can be calculated in $\mathbf{O}(R \cdot T^2)$: includes calculating $B(j,t)$ and $C(j,t)$ when a last setup occurs in period j in a t-period sub-problem. Then, the best cost saving and last setup can be calculated in $\mathbf{O}(R \cdot T)$.

3.3 LP relaxation of the FAL model

Tafti et al. (2019a) propose LP relaxation approach for the single product DLSPD. They mention that LP relaxation of the FAL model can obtain optimal or near optimal solutions and has a very strong lower bounds of the problem. We adapt their formulation for MP-DLSPD with parts commonality. LP relaxation of the model P2 is solve by removing integrality constraints 12 and 13 and using the CPLEX solver. Its solution quality and computational time will be compared with other models and methods.

4. COMPUTATIONAL EXPERIMENTS

Computational tests are performed on randomly generated problem instances and the models and methods are compared regarding to the percentage deviation from the optimal solution. The proposed models, methods, and generated data can be applied in different real industrial cases. An example can be the ELVs recycling sector, where EOL vehicles will be disassembled into their parts such as engines, doors, seats, tyres, etc. The obtained parts are used to satisfy their demands and the dangerous, unusable, and excess parts/ materials will be disposed of to the specialized channels such as recycling, material sources for the energy recovery.

The tests are done on a laptop with an Intel Core i5-3210M 2.5 GHz and 8 Go RAM on windows 7 and the optimal solution are obtained by solving the MIP models directly using CPLEX solver v.12.8.

4.1 New benchmark

The various instances are generated with different problem sizes and different values of parameters. The Benchmark of Kim and Xirouchakis (2010) is adapted to obtain an average of cycle Time Between Orders equal to 2. We generate 225 problems i.e. 25 problems for each combination of three levels of number of items (N) (10, 20, 30)and three levels of number of periods (T)(10, 20, 30). For each levels of the number of items, 5 different disassembly structure are randomly generated. The number of leaf items for each Root item are generated from DU(2, 5), DU(5, 10) and DU(10,15) for each level of the number of items. In the generated disassembly structure, the number of root items are generated from (2, N/5). Also, the number common leaf items are generated from (1,⌊N/3⌋).

For each disassembly structure, 5 problems with different data are generated for each level of the number of periods. Table 1 provides the generated parameters. Note that DU(m_1, m_2) means the discrete uniform distribution with a rage of $[m_1, m_2]$.

4.2 Numerical results

Table 2 summarizes the test results of the proposed MIP models which shows that both models P1 and P2 can obtain optimal solution in very short computational times.

Table 3 and 4 show the performance of the LP relaxation of model P2. It can obtain optimal solutions of the problem in 46.6% and the overall average of gap is only 0.06%. LP relaxation of the model P1 is almost faster than LP relaxation of the model P2 with the overall average CPU of 0.11 but its overall average of gap is 8.94%. The significant performance of the model P2 is that its LP relaxation can obtain a very high lower bound of the problem.

Table 1. Generated parameters

Parameter	**value**
items (N)	(10, 20, 30)
Period (T)	(10, 20, 30)
Holding cost (h_{it})	N=10⇒ DU(0.3, 0.5)
	N=20⇒ DU(0.1, 0.3)
	N=30⇒ DU(0.1, 0.16)
Demand (d_{it})	DU(50, 250)
Yield (a_{il})	DU(1, 4)
Disassembly cost (p_{it})	DU(38, 62)
Setup cost (s_{it})	DU(2500, 3500)

Table 2. CPU (s) of P1 & P2

N	T	P1	P2
		Mean (Min, Max)	Mean (Min, Max)
10	10	0.18(0.15, 0.23)	0.19(0.15, 0.27)
	20	0.29(0.21, 0.73)	0.30(0.21, 0.76)
	30	0.66(0.27, 1.89)	0.67(0.28, 1.88)
20	10	0.23(0.19, 0.23)	0.23(0.20, 0.28)
	20	0.57(0.34, 0.83)	0.50(0.33, 0.77)
	30	1.25(0.66, 2.60)	0.94(0.54, 1.91)
30	10	0.32(0.22, 0.51)	0.29(0.20, 0.46)
	20	0.82(0.46, 1.36)	0.65(0.35, 1.22)
	30	2.07(0.94, 5.18)	1.38(0.62, 2.63)
Avg.		**0.71(0.38, 1.51)**	**0.57(0.32, 1.13)**

Table 3. Gap (%) of LP Relaxation of P1 & P2

N	T	LP-R of P1	LP-R of P2
		Mean (Min, Max)	Mean (Min, Max)
10	10	10.75(3.57, 25.38)	0.25(0.00, **2.52**)
	20	11.40(5,59, 28.59)	0.11(0.00, 1.50)
	30	12.99(6.58, 21.15)	0.14(0.00, 0.97)
20	10	8.18(3.41, 16.81)	0.02(0.00, 0.12)
	20	9.22(5.29, 14.67)	0.01(0.00, 0.14)
	30	8.50(4.14, 14.63)	0.02(0.00, 0.14)
30	10	6.76(3.90, 12.66)	0.02(0.00, 0.23)
	20	6.45(4.27, 9.36)	0.01(0.00, 0.03)
	30	6.24(4.83, 9.34)	0.00(0.00, 0.02)
Avg.		**8.94(4.62, 16.95)**	**0.06(0.00, 0.63)**

Table 4. CPU (s) of LP Relaxation of P1 & P2

N	T	LP-R of P1	LP-R of P2
		Mean (Min, Max)	Mean (Min, Max)
10	10	0.13(0.12, 0.14)	0.17(0.16, 0.18)
	20	0.14(0.13, 0.14)	0.20(0.18, 0.22)
	30	0.14(0.14, 0.15)	0.29(0.33, 0.41)
20	10	0.09(0.08, 0.10)	0.17(0.17, 0.19)
	20	0.10(0.09, 0.10)	0.28(0.24, 0.32)
	30	0.10(0.10, 0.12)	0.50(0.40, 0.72)
30	10	0.09(0.09, 0.10)	0.21(0.18, 0.28)
	20	0.10(0.09, 0.11)	0.44(0.31, 0.65)
	30	0.12(0.11, 0.19)	0.89(0.52, 1.66)
Avg.		**0.11(0.11, 0.13)**	**0.35(0.28, 0.51)**

The results of two-phase heuristic are summarized in Table 5. Two-phase heuristic can obtain the solutions for the generated problem instances in very short computational times. Second phase has a very efficient improvement on the initial solution obtained from the first phase. This implies that the forward-looking check dynamic program approach suggested is very effective in improving the initial constructed solutions. For example, in the case with 30 items and 30 periods, second phase can improve the overall average of gap from 20.55% to 3.21%. The the overall computational time of the two-phase heuristic are significantly shorter than models P1 and P2 solving by CPLEX solver.

Table 5. Gap (%) of two-phase heuristic

N	T	Phase 1	Phase 2
		Mean (Min, Max)	Mean (Min, Max)
10	10	19.56(7.08, **38.82**)	2.71(0.21, 6.73)
	20	18.55(8.83, 38.70)	3.00(0.71, 7.35)
	30	20.55(11.20, 33.99)	3.21(1.50, 5.57)
20	10	13.25(5.95, 26.30)	1.81(0.18, 5.45)
	20	16.96(9.06, 28.10)	2.71(0.90, 4.89)
	30	16.11(10.00, 28.53)	2.82(1.04, 6.43)
30	10	10.29(4.13, 20.53)	2.28(0.15, **9.17**)
	20	12.72(5.87, 21.96)	2.32(0.08, 3.94)
	30	12.40(8.34, 19.82)	2.78(1.06, 5.98)
Avg.		**13.60(7.83, 28.53)**	**2.63(0.65, 6.17)**

We also analyze the impact of considering disposal decisions on the total cost for the case with 30 items and 30 periods. The result show that a maximum cost reduction of around 58% can be let by allowing disposal.

5. CONCLUSION

This paper addresses disassembly lot sizing problem for the multi-product and two-level product structure with parts commonality. The objective is to minimize the sum of setup, disassembly operation and inventory holding costs. Disposal decisions are applied to handle the issue of surplus inventory in disassembly systems, which can make a maximum cost reduction of around 58%, for the tested problem instances. Two new MIP formulations (AGG and FAL) with considering disposal decisions are proposed. For real industrial cases, a two-phase heuristic is suggested in which an initial solution is obtained by using LP relaxation approach, and it is improved by using a forward-looking check dynamic programming based algorithm. The improvement is made by changing the current solution and considering cost changes efficiently. The two-phase heuristic gives near optimal solutions in very short computational times and it has an efficient performance to improve the initial solution obtained by the first phase. Also, we apply LP relaxation approach for the FAL model to improve lower bound of the problem. As a future work, we aim to develop the proposed model and method for more complex product structure with multi-level and parts commonality. Also, it is necessary to consider the problem with resource capacity constraint for the real industrial applications.

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