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Decentralised Sliding Mode Control for Nonlinear Interconnected Systems with Unknown Interconnections ⋆

Nan Ji * **Xing-Gang Yan** * **Zehui Mao** ** **Dongya Zhao** *** **Bin Jiang** **

* *Instrumentation and Control Research Group, University of Kent, Canterbury, CT2 7NT, United Kingdom (e-mail: nj219@kent.ac.uk; x.yan@kent.ac.uk).*
** *College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P. R. China (email: zehuimao@nuaa.edu.cn; binjiang@nuaa.edu.cn)*
*** *College of New Energy, China University of Petroleum, Qingdao 266555, P. R. China (e-mail: dyzhao@upc.edu.cn)*

Abstract: In this paper, a novel decentralised robust state feedback sliding mode control is presented to stabilise a class of nonlinear interconnected systems with matched uncertainty and unknown interconnections. A composite sliding surface is designed, and a set of conditions are developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable. Then, a decentralised state feedback sliding mode control is proposed to drive interconnected systems to the designed sliding surface in finite time, and sliding motion occurs thereafter. The bounds on the uncertainties and interconnections are known nonlinear functions, which are employed in the control design to reject the effects of uncertainties and unknown interconnections to enhance the robustness. Finally, a numerical simulation example is given to demonstrate the effectiveness of the proposed control strategy.

Keywords: Nonlinear interconnected systems, decentralised control, sliding mode control, state feedback, uniform asymptotical stability.

1. INTRODUCTION

With the advancement of scientific technology, many industrial and commercial systems are interconnected systems with high complexity and large scale, such as modern power systems, transportation systems, aircraft and robots. In detailed description, interconnected systems can be described as a set of composite objects with different sorts of physical, natural, and artificial dynamics, which are called as subsystems (Yan et al. (1998)). In reality, the existence of nonlinearities, uncertainties and interconnections increases the difficulty of control design and analysis for interconnected systems. Besides that, practical systems are affected by internal and external disturbances such as modelling errors, parameter variation, temperature, pressure and mechanical loss. In terms of the control of interconnected systems, centralised control and decentralised control are two different approaches. Decentralised control only adopts local information of each subsystem, which does not need information transfer between subsystems and can reduce the computation burden. Therefore, decentralised control is convenient for practical implementation. To be specific, decentralised control law consists of many local controllers which only use their local information of the corresponding subsystems. So the structure of decentralised controllers is simpler and more effective than centralised controllers. In last few decades, decentralised control has received great attention and many results have been achieved (Misgeld et al. (2015)).

Sliding mode control (SMC), which has the high robustness against uncertainties, has been recognised as an effective method for control systems with matched uncertainties. Yan et al. (2005) proposed a modified SMC which was able to deal with mismatched uncertainties where dynamic feedback was employed. Khan et al. (2011) proposed a novel dynamic integral sliding mode controller for state dependent matched and mismatched uncertainties. A neural network fuzzy SMC presented by Chiang and Chen (2017) was applied to pneumatic muscle actuators, where an adaptive training used neural network was able to establish a fuzzy SMC controller, and an integrator could minimize the tracking error. It should be noted that all of the results mentioned above only considered centralised systems.

Recently, many researchers have focused on the decentralised SMC to cope with the issue of the interconnected systems. A kind of novel decentralised state-feedback adaptive SMC was imposed by Mirkin et al. (2011) to large-scale interconnected systems with nonlinear interconnections and time-delay. The global decentralised discrete SMC for interconnected systems based on output

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feedback was employed by Mahmoud et al. (2012). These two strategies achieved good results for specified interconnected systems where only the interconnections were nonlinear while all the isolated subsystems were linear. A decentralised integral SMC combined with PID was proposed in Thien and Kim (2018) for unmanned aerial vehicles, where the control sensitivity with respect to the network topology was analysed, but the mismatched uncertainties were not considered. Although many researchers have obtained the remarkable achievements of decentralised SMC, few people concentrated on the nonlinear interconnected systems with mismatched uncertainties and unknown interconnections at same time. Due to the completeness of nonlinear systems, the technology of SMC combined with decentralised control for nonlinear interconnected systems with unknown interconnections is challenging and significant.

In this paper, a decentralised SMC using state feedback control for nonlinear interconnected systems is presented. The considered interconnected systems possess both nonlinear interconnections and nonlinear isolated subsystems. A coordinate transformation is applied to transform all the isolated subsystems into the regular forms. Then, a composite sliding surface is designed, and a set of conditions are developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable based on the Lyapunov theory. A state feedback SMC law is established to drive the system to the sliding surface in finite time and keep the sliding motion after that. The bounds on all uncertainties and interconnections have general nonlinear forms and are nonlinear functions of the system states, which are employed in decentralised control design to reduce the effects of uncertainties. It is shown that under certain conditions, the effect of the unknown interconnections can be completely cancelled by an appropriate designed decentralised controllers with regard to the reachability analysis. At last, a numerical simulation example is provided to demonstrate the effective of the proposed control strategy.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

For simply statement in the following, a few concepts are introduced.

Definition 1 (Khalil (2002)): A continuous function $\alpha : [0, a) \longrightarrow [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

Definition 2 (Yan et al. (2014)): A class $\mathcal{K}$ function is called as class $\mathcal{KC}^1$ if it is continuously differentiable.

Take the consideration of nonlinear time-varying interconnected systems with matched disturbances and unknown interconnections consisted of n interconnected subsystems,

$$\begin{aligned}\dot{x}_i &= f_i(t, x_i) + g_i(t, x_i)(u_i + \varphi_i(t, x_i)) + h_i(t, x) \\ & i = 1, 2, \cdots, n\end{aligned} \tag{1}$$

where $x_i \in \Omega_i \subset \mathcal{R}^{n_i}$ (Ω_i denotes a neighbourhood of the origin, $x := \text{col}(x_1, x_2, \cdots, x_n) \in \Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n$) and $u_i \in \mathcal{R}^{m_i}$ are, respectively, the state variables and inputs with $m_i < n_i$. It is assumed that the matrix function $g_i(\cdot) \in \mathcal{R}^{n_i \times m_i}$ is known and has full column rank; the nonlinear vector $f_i(\cdot) \in \mathcal{R}^{n_i}$ is known. The term $\varphi_i(\cdot)$ denotes the matched disturbance, and $h_i(\cdot)$ represents the unknown interconnection. All the nonlinear functions are assumed to be continuous in their arguments in the considered domain.

For simply description, the considered domain may not be pointed out in the subsequent analysis in this paper. The following basic definitions are introduced.

Definition 3: Consider the system (1). The system

$$\dot{x}_i = f_i(t, x_i) + g_i(t, x_i)(u_i + \varphi_i(t, x_i)), i = 1, 2, \cdots, n \tag{2}$$

is called the i-th isolated subsystem of system (1). The system

$$\dot{x}_i = f_i(t, x_i) + g_i(t, x_i)u_i, i = 1, 2, \cdots, n \tag{3}$$

is called the i-th nominal isolated subsystem of system (1).

It is widely known that one of the major issues for interconnected systems is to design a controller such that the interconnected system (1) has the desired performance if all nominal isolated subsystems (3) exhibit the required performance. Compared with centralised control, one of the important issues for interconnected systems is how to deal with interconnections. And as for decentralised case, each controller can only be allowed to use its own state information.

The definition of the decentralised control is given as follows:

Definition 4: Consider system (1). If the controller u_i for the i-th subsystem depends on the time t and states x_i only, that is,

$$u_i = u_i(t, x_i), \qquad i = 1, 2, \cdots, n \tag{4}$$

Then, (4) is called as the decentralised state feedback controller.

Consider a nonlinear transformation

$$z_i = T_i(x_i), \quad i = 1, 2, \cdots, n \tag{5}$$

which is a diffeomorphism, i.e. $\partial T_i / \partial x_i \neq 0$ in the considered domain. Then, it defines a new coordinate z_i. In new coordinates z_i, system (1) can be described by

$$\begin{aligned}\dot{z}_i =& \left[\frac{\partial T_i}{\partial x_i}\dot{x}_i\right]_{x_i = T_i^{-1}(z_i)} \\ =& \left[\frac{\partial T_i}{\partial x_i}\big(f_i(t, x_i) + g_i(t, x_i) \right. \\ & \left. \cdot (u_i + \varphi_i(t, x_i)) + h_i(t, x)\big)\right]_{x_i = T_i^{-1}(z_i)} \\ i =& 1, 2, \cdots, n\end{aligned} \tag{6}$$

It is assumed that the system (1) in new coordinate z_i can be described by

$$\dot{z}_{i1} = F_{i1}(t, z_{i1}, z_{i2}) + H_{i1}(t, z) \tag{7}$$

$$\begin{aligned}\dot{z}_{i2} =& F_{i2}(t, z_{i1}, z_{i2}) + G_i(t, z_{i1}, z_{i2}) \\ & \cdot (u_i + \Phi_i(t, z_{i1}, z_{i2})) + H_{i2}(t, z)\end{aligned} \tag{8}$$

where $z_{i1} \in \mathcal{R}^{n_i - m_i}$, $z_{i2} \in \mathcal{R}^{m_i}$, $z = \text{col}(z_1, z_2, \cdots, z_n)$ and $z_i = \text{col}(z_{i1}, z_{i2})$, $i = 1, 2, \cdots, n$,

$$\begin{bmatrix} F_{i1}(\cdot) \\ F_{i2}(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} f_i(t, x_i) \right]_{x_i = T_i^{-1}(z_i)} \tag{9}$$

$$H_i(\cdot) := \begin{bmatrix} H_{i1}(\cdot) \\ H_{i2}(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} h_i(t, x) \right]_{x_i = T_i^{-1}(z_i)} \tag{10}$$

$$\begin{bmatrix} 0 \\ G_i(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} g_i(t, x_i) \right]_{x_i = T_i^{-1}(z_i)} \tag{11}$$

$$\Phi_i(\cdot) := [\varphi_i(t, x_i)]_{x_i = T_i^{-1}(z_i)} \tag{12}$$

where $G_i(\cdot) \in \mathcal{R}^{m_i \times m_i}$ is nonsingular in the considered domain Ω_{T_i} for $i = 1, 2, \ldots, n$.

In the new coordinate (z_{i1}, z_{i2}), the considered domain Ω_{T_i} is defined as

$$\begin{aligned} \Omega_{T_i} &:= \Omega_{z_{i1}} \times \Omega_{z_{i2}} \\ &:= \{(z_{i1}, z_{i2}) \mid (z_{i1}, z_{i2}) = T_i(x_i), x_i \in \Omega_i\} \end{aligned}$$

where $z_{i1} \in \Omega_{z_{i1}}$ and $z_{i2} \in \Omega_{z_{i2}}$. The system (7)−(8) is in the usual regular form for sliding mode design, which is very useful for the constructive application of the sliding mode paradigm.

Remark 1: It should be pointed out that there is no general way to find a coordinate transformation (5) to transfer system (1) to the regular form (7)-(8). But the work in Marino (1995) and Yan et al. (2014) can be referred to construct the transformation.

In this paper, nonlinear interconnected systems (7)-(8) are to be focused. However, it should be emphasised that the results developed in this paper can be easily extended to all the interconnected systems (1) which can be transformed to system (7)−(8) by a nonsingular transfomation.

The objective of this paper is to construct a state feedback decentralised SMC law, such that the controlled system (7)−(8) is uniformly asymptotically stable irrespective of disturbances and unknown interconnections.

3. SLIDING MOTION ANALYSIS AND CONTROL SYNTHESIS

In this section, the sliding surface is designed and the corresponding sliding motion is analysed. Then, the novel decentralised state feedback SMC is proposed.

3.1 Stability of Sliding Motion

Based on the SMC theory, the switching function for the i-th subsystem can be selected as

$$s_i(z_i) = z_{i2}, \qquad i = 1, 2, \cdots, n \tag{13}$$

Then, the composite sliding function for the interconnected system (7)−(8) is given as

$$\begin{aligned} S(z) &= \operatorname{col}(s_1(z_1), s_2(z_2), \cdots, s_n(z_n)) \\ &= \operatorname{col}(z_{12}, z_{22}, \cdots, z_{n2}) \end{aligned} \tag{14}$$

So, the composite sliding surface is written as

$$\{\operatorname{col}(z_1, z_2, \ldots, z_n) \mid s_i(z_i) = z_{i2} = 0, i = 1, 2, \ldots, n\} \tag{15}$$

When the system is limited on the sliding surface (15), $z_{i2} = 0$ for $i = 1, 2, \ldots, n$, and thus the sliding mode dynamics can be described by

$$\dot{z}_{i1} = F_{i1s}(t, z_{i1}) + H_{i1s}(t, z_{11}, z_{21}, \cdots, z_{n1}) \tag{16}$$

where $z_{i1} \in \Omega_{z_{i1}} \subset \mathcal{R}^{n_i - m_i}$ denotes the state of the sliding mode dynamics,

$$F_{i1s}(\cdot) := F_{i1}(t, z_{i1}, z_{i2})|_{z_{i2}=0} \tag{17}$$

$$H_{i1s}(\cdot) := H_{i1}(t, z)|_{z_{12}=0, \cdots, z_{n2}=0} \tag{18}$$

where $F_{i1}(\cdot)$ and $H_{i1}(\cdot)$ are defined in (7). From (10), it is clear to see that the term $H_{i1s}(\cdot)$ comes from $h_i(t, x)$, which represents the unknown interconnection of the i-th subsystems in (16) for $i = 1, 2, \ldots, n$.

In order to analyse the sliding motion of system (16) related to the composite sliding surface (15), the following assumptions are needed.

Assumption 1: There exists the continuously differentiable functions $V_i(t, z_{i1}) : \mathcal{R}^+ \times \mathcal{R}^{n_i - m_i} \mapsto \mathcal{R}^+$ for $i = 1, 2, \ldots, n$ and class $\mathcal{K}$ functions $p_{il}(\cdot)$ for $l = 1, 2, 3, 4$, such that for any $z_{i1} \in \Omega_{z_{i1}}$ the following inequalities hold:

(i) $p_{i1}^2(\|z_{i1}\|) \le V_i(t, z_{i1}) \le p_{i2}^2(\|z_{i1}\|)$.

(ii) $\frac{\partial V_i(\cdot)}{\partial t} + \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}}\right)^T F_{i1s}(t, z_{i1}) \le -p_{i3}^2(\|z_{i1}\|)$.

(iii) $\left\| \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}}\right)^T \right\| \le p_{i4}(\|z_{i1}\|)$.

Remark 2: Assumption 1 implies that the nominal system of the interconnected system (16), ie. the dynamics $\dot{z}_{i1} = F_{i1s}(t, z_{i1})$, is uniformly asymptotically stable. It should be mentioned that, the fact that $\dot{z}_{i1} = F_{i1s}(t, z_{i1})$ is uniformly asymptotically stable does not mean that the nominal system (7) is uniformly asymptotically stable.

Assumption 2: The interconnection term $H_{i1s}(\cdot)$ in system (16) satisfies

$$\begin{aligned} &\|H_{i1s}(t, z_{11},\ z_{21}, \cdots, z_{n1})\| \\ &\qquad \le \beta_i(t, z_{11}, z_{21}, \cdots, z_{n1}) \sum_{j=1}^{n} \|z_{j1}\| \end{aligned} \tag{19}$$

where $\beta_i(\cdot)$ for $i = 1, 2, \cdots, n$ are known continuous functions.

Assumption 2 ensures that the interconnection in (16) is bounded by a known function of states of the system (16).

It is obvious if the functions $p_{il}(\cdot)$ for $l = 1, 2, 3, 4$ in Assumption 1 are strengthened to class $\mathcal{KC}^1$ functions, then from the Definition 2, there are continuous functions $\varsigma_{il}(\cdot)$ such that for any $z_{i1} \in \Omega_{z_{i1}}$, $p_{il}(\cdot)$ can be decomposed as

$$p_{il}(\|z_{i1}\|) = \varsigma_{il}(\|z_{i1}\|)\|z_{i1}\|, \qquad l = 1, 2, 3, 4 \tag{20}$$

where $\varsigma_{il}(\cdot)$ is a continuous function in $\mathcal{R}^+$ for $i = 1, 2, \ldots, n$ and $l = 1, 2, 3, 4$.

The following result is ready to be presented.

Theorem 1. Suppose that the functions $p_{il}(\cdot)$ for $l = 1, 2, 3, 4$ in Assumption 1 are class $\mathcal{KC}^1$ functions. Then, under Assumptions 1 and 2, the system (7)−(8) has a uniformly asymptotically stable sliding motion associated with the sliding surface (15) if the function matrix

$$M^T(\cdot) + M(\cdot) > 0$$

where $M = (m_{ij}(\cdot))_{n \times n}$ is a $n \times n$ function matrix with its entries defined by

$$m_{ij} = \begin{cases} \varsigma_{i3}^2(\cdot) - \varsigma_{i4}(\cdot)\beta_i(\cdot), & i = j \\ -\varsigma_{i4}(\cdot)\beta_i(\cdot), & i \neq j \end{cases} \tag{21}$$

where $\varsigma_{i3}(\cdot)$ and $\varsigma_{i4}(\cdot)$ are given in (20), $\beta_i(\cdot)$ is defined in (19) for $i, j = 1, 2, \ldots, n$.

Proof. From the analysis above, it is clear to see that system (16) is the sliding mode dynamics related to the composite sliding surface (15). Under the condition that $p_{il}(\cdot)$ is class $\mathcal{KC}^1$ function, the equations in (20) hold. Consider the Lyapunov candidate function

$$V(t, z_{11}, z_{21}, \cdots, z_{n1}) = \sum_{i=1}^{n} V_i(t, z_{i1}) \tag{22}$$

where $V_i(\cdot)$ is defined in Assumption 1. The time derivative of $V(\cdot)$ along the trajectory of system (16) based on Assumptions 1 and 2 is described as

$$\begin{aligned} &\dot{V}(t, z_{11}, z_{21}, \cdots, z_{n1}) = \sum_{i=1}^{n} \dot{V}_i(t, z_{i1}) \\ &= \sum_{i=1}^{n} \left(\frac{\partial V_i(\cdot)}{\partial t} + \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}}\right)^T (F_{i1s}(\cdot) + H_{i1s}(\cdot)) \right) \\ &\leq \sum_{i=1}^{n} \left(-p_{i3}^2(\|z_{i1}\|) + p_{i4}(\|z_{i1}\|) \cdot \beta_i(\cdot) \sum_{j=1}^{n} \|z_{j1}\| \right) \end{aligned} \tag{23}$$

According to equation (20), it follows that

$$\begin{aligned} &\dot{V}(t, z_{11}, z_{21}, \cdots, z_{n1}) \\ &\leq -\sum_{i=1}^{n} \left(\varsigma_{i3}^2(\|z_{i1}\|) - \varsigma_{i4}(\|z_{i1}\|)\beta_i(\cdot)\right) \|z_{i1}\|^2 \\ &\quad - \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \varsigma_{i4}(\|z_{i1}\|) \cdot \beta_i(\cdot) \|z_{i1}\| \cdot \|z_{j1}\| \\ &= -\frac{1}{2} Z^T \left(M^T + M\right) Z \end{aligned} \tag{24}$$

where $Z := \text{col}(\|z_{11}\|, \|z_{21}\|, \cdots, \|z_{n1}\|)$, M is the $n \times n$ matrix defined in (21). Hence, $M^T + M > 0$. □

3.2 Reachability Analysis

The stability conditions have been presented in Theorem 1 above. The objective now is to design a decentralised state feedback SMC such that the interconnected system (7)−(8) is driven to the sliding surface (15).

For the interconnected system (7)−(8), the corresponding reachability condition is described by

$$S^T(z)\dot{S}(z) \leq -\eta \|S(z)\| \tag{25}$$

where $S(z)$ is defined by (14), η is a positive constant.

Consider system (7)−(8). It should be pointed out that there has not been any limitation imposed on the terms $\Phi_i(t, z_{i1}, z_{i2})$ and $H_{i2}(t, z)$ so far. It is necessary to introduce the following assumption for further control design.

Assumption 3: The uncertainties $\Phi_i(t, z_{i1}, z_{i2})$ and $H_{i2}(t, z)$ in (8) satisfy

$$\|\Phi_i(t, z_{i1}, z_{i2})\| \leq \xi_{i1}(t, z_{i1}, z_{i2}) \tag{26}$$

$$\|H_{i2}(t, z)\| \leq \sum_{j=1}^{n} \epsilon_{ij}(t, z_j) \tag{27}$$

where $\xi_{i1}(t, z_{i1}, z_{i2})$ and $\epsilon_{ij}(t, z_j)$ are known continuous functions.

Construct the control law

$$\begin{aligned} &u_i = -G_i^{-1}(t, z_{i1}, z_{i2}) F_{i2}(t, z_{i1}, z_{i2}) - \\ &G_i^{-1}(t, z_{i1}, z_{i2}) \Big(\|G_i(t, z_{i1}, z_{i2})\| \xi_{i1}(t, z_{i1}, z_{i2}) \text{sgn}(z_{i2}) \\ &+ \frac{n}{2} z_{i2} + \frac{1}{2} \frac{z_{i2}}{\|z_{i2}\|^2} \sum_{j=1}^{n} \epsilon_{ji}^2(t, z_i) \Big) \\ &- G_i^{-1}(t, z_{i1}, z_{i2}) k_i \cdot \text{sgn}(z_{i2}) \end{aligned} \tag{28}$$

where $F_{i2}(\cdot)$ is given in (9), $\xi_{i1}(\cdot)$ and $\sum_{j=1}^{n} \epsilon_{ij}(t, z_j)$ are given in (26) and (27), respectively, $\text{sgn}(\cdot)$ is the usual signum function, and k_i is the control gain which is a positive constant.

Theorem 2. Under Assumption 3, the nonlinear interconnected system (7)−(8) can be driven to the sliding surface (15) in finite time by the designed controller in (28) and maintains a sliding motion on it thereafter.

Proof. From the definition of $S(z)$ in (14), system equation (8), and the control u_i in (28), it follows that

$$\begin{aligned} &S^T(z)\dot{S}(z) \\ &= \sum_{i=1}^{n} \Big(z_{i2}^T G_i(\cdot)\Phi_i(\cdot) - \|G_i(\cdot)\| \xi_{i1}(\cdot) z_{i2}^T \text{sgn}(z_{i2}) \Big) \\ &\quad + \Big(\sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot) - \sum_{i=1}^{n} \frac{n}{2} z_{i2}^T z_{i2} \\ &\quad - \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2} \epsilon_{ji}^2(\cdot) \Big) - \sum_{i=1}^{n} k_i z_{i2}^T \text{sgn}(z_{i2}) \end{aligned} \tag{29}$$

Based on (26), (27) and the fact that $s^T \text{sgn}(s) \geq \| s \|$ for any vectors s (see Lemma 1 in Yan and Edwards (2008)), it follows that:

$$\begin{aligned} &\sum_{i=1}^{n} \left(z_{i2}^T G_i(\cdot)\Phi_i(\cdot) - \|G_i(\cdot)\| \xi_{i1}(\cdot) z_{i2}^T \text{sgn}(z_{i2}) \right) \\ &\leq \sum_{i=1}^{n} \left(\|z_{i2}\| \cdot \|G_i(\cdot)\| \cdot \|\Phi_i(\cdot)\| - \|z_{i2}\| \cdot \|G_i(\cdot)\| \cdot \xi_{i1}(\cdot) \right) \\ &\leq 0 \end{aligned} \tag{30}$$

By similar reasoning as in (30), and from (27)

$$\begin{aligned} &\sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot) - \sum_{i=1}^{n} \frac{n}{2} z_{i2}^T z_{i2} - \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2} \epsilon_{ji}^2(\cdot) \\ &\leq \sum_{i=1}^{n} \|z_{i2}\| \cdot \|H_{i2}(\cdot)\| - \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \|z_{i2}\|^2 \\ &\quad - \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \epsilon_{ij}^2(t, z_j) \end{aligned} \tag{31}$$

From the fact that $\frac{a^2+b^2}{2} \geq | a | | b |$, it follows that

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\|z_{i2}\|^2+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\epsilon_{ij}^2(t,z_j) \geq \sum_{i=1}^{n}\|z_{i2}\|\sum_{j=1}^{n}\epsilon_{ij}(t,z_j) \geq \sum_{i=1}^{n}\|z_{i2}\|\cdot\|H_{i2}(\cdot)\| \quad (32)$$

From (32) and (31),

$$\sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot)-\sum_{i=1}^{n}\frac{n}{2}z_{i2}^T z_{i2}-\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2}\epsilon_{ji}^2(\cdot) \leq 0 \quad (33)$$

Substituting (30) and (33) into (29) yields

$$S^T(z)\dot{S}(z) \leq -\sum_{i=1}^{n} k_i z_{i2}^T \mathrm{sgn}(z_{i2}) \leq -\eta\sum_{i=1}^{n} z_{i2}^T\mathrm{sgn}(z_{i2}) \leq -\eta\|S\| \quad (34)$$

where $\eta := \max\{k_1, k_2, \cdots, k_n\}$.

The inequality (34) shows that the reachability condition (25) is satisfied, and thus the interconnected systems (7)−(8) can be driven to the sliding surface (15) in finite time and maintain a sliding motion on it thereafter. Hence, the result follows. □

According to sliding mode control theory, Theorem 1 and Theorem 2 show that the closed-loop system formed by applying control law (28) into system (7)-(8) is uniformly asymptotically stable.

4. SIMULATION RESULTS

Consider the interconnected system which is composed of two third-order subsystems

$$\dot{x_1} = \begin{bmatrix} -6x_{12}^2x_{13}^2-4x_{12}^2-2x_{11} \\ -3x_{12}x_{13}^2-3x_{12}+\frac{1}{16}(x_{12}^2-x_{11})^2 \\ 3x_{12}^2x_{13}-3x_{13}-\frac{1}{4}(x_{12}^2-x_{11})\exp\{-t\}\cos(x_{13}t)\end{bmatrix} + \begin{bmatrix} -4(x_{13}^2\sin^2 t+1) \\ 0 \\ 0\end{bmatrix}(u_1+\varphi_1(t,x_1))+h_1(t,x) \quad (35)$$

$$\dot{x_2} = \begin{bmatrix} -8x_{21}+x_{23} \\ -7x_{22}+x_{23} \\ x_{21}\end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1\end{bmatrix}(u_2+\varphi_2(t,x_2))+h_2(t,x) \quad (36)$$

where $x_i = \mathrm{col}(x_{i1}, x_{i2}, x_{i3}) \in \mathcal{R}^3$ and $u_i \in \mathcal{R}$ are, respectively, the state variables and inputs of the i-th subsystem for $i = 1, 2$. The terms $\varphi_i(\cdot)$ and $h_i(\cdot)$ for $i = 1, 2$ are matched disturbances and unknown interconnections.

Consider the transformation T_1 and T_2 defined by

$$T_1: \begin{cases} z_{11}^a = x_{12} \\ z_{11}^b = x_{13} \\ z_{12} = \frac{1}{4}(x_{12}^2 - x_{11}) \end{cases} \quad \text{and} \quad T_2: \begin{cases} z_{21}^a = x_{21} \\ z_{21}^b = x_{21}+x_{22} \\ z_{22} = x_{23} \end{cases}$$

It is easy to find that the Jacobian matrices of T_1 and T_2 are

$$J_{T_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(1/4) & (1/2)x_{12} & 0 \end{bmatrix} \text{ and } J_{T_2} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which are nonsingular in whole state space. By direct calculation, system (35)-(36) in the new coordinates is given by

$$\dot{z}_{11} = \begin{bmatrix} -3z_{11}^a\left(z_{11}^b\right)^2 - 3z_{11}^a + z_{12}^2 \\ 3\left(z_{11}^a\right)^2 z_{11}^b - 3z_{11}^b - z_{12}\exp\{-t\}\cos\left(z_{11}^b t\right) \end{bmatrix} + H_{11}(\cdot) \quad (37)$$

$$\dot{z}_{12} = -2z_{12} + \frac{1}{2}z_{11}^a z_{12}^2 + (1+\left(z_{11}^b\right)^2\sin^2 t)(u_1+\Phi_1(\cdot)) + H_{12}(\cdot) \quad (38)$$

$$\dot{z}_{21} = \begin{bmatrix} -8z_{21}^a + z_{22} \\ -7z_{21}^b - z_{21}^a + 2z_{22} \end{bmatrix} + H_{21}(\cdot) \quad (39)$$

$$\dot{z}_{22} = z_{21}^a + (u_2+\Phi_2(\cdot)) + H_{22}(\cdot) \quad (40)$$

where $H_{i1}(\cdot) \in \mathcal{R}^2$ and $H_{i2}(\cdot) \in \mathcal{R}^1$, $i = 1, 2$. In order to demonstrate the theoretical results obtained in this paper, it is assumed that the uncertainties in (37)-(40) satisfy that

$$|\Phi_1(\cdot)| \leq (\|z_{11}^b\|+1)\exp\{-t\} \quad (41)$$

$$\|H_{11}\| \leq \|z_{11}^a\|\sin^2 t + \|z_{12}\| + \|z_{22}\| \quad (42)$$

$$\|H_{12}\| \leq 0.25(\|z_{11}^a\|\sin^2 t + \|z_{12}\| + \|z_{22}\|) \quad (43)$$

$$|\Phi_2(\cdot)| \leq \|z_{21}^b\|\sin^2 z_{22} \quad (44)$$

$$\|H_{21}\| \leq 1.618(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2\sin^2 z_{22} \quad (45)$$

$$\|H_{22}\| \leq 0.40(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2\sin^2 z_{22} \quad (46)$$

Select the switching function $S(z) := \mathrm{col}\,(z_{12}, z_{22})$. When in the sliding surface, $z_{12} = z_{22} = 0$. It can be obtained that the sliding mode dynamics are written as follows

$$\dot{z}_{11} = \begin{bmatrix} -3z_{11}^a\left(z_{11}^b\right)^2 - 3z_{11}^a \\ 3\left(z_{11}^a\right)^2 z_{11}^b - 3z_{11}^b \end{bmatrix} + H_{11s}(\cdot) \quad (47)$$

$$\dot{z}_{21} = \begin{bmatrix} -8z_{21}^a \\ -7z_{21}^b - z_{21}^a \end{bmatrix} + H_{21s}(\cdot) \quad (48)$$

It is clear to see from (42) and (45) that

$$\|H_{11s}(\cdot)\| \leq \|z_{11}^a\|\sin^2 t \leq \|z\| \quad (49)$$

$$\|H_{21s}(\cdot)\| = 0 \quad (50)$$

and thus $\beta_1 = 1$ and $\beta_2 = 0$.

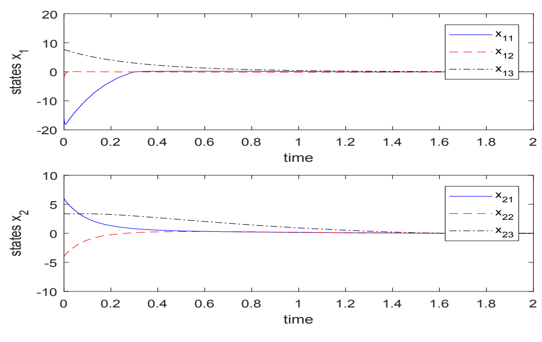

Fig. 1. Time response of subsystem (35) (Upper); time response of subsystem (36) (Bottom).

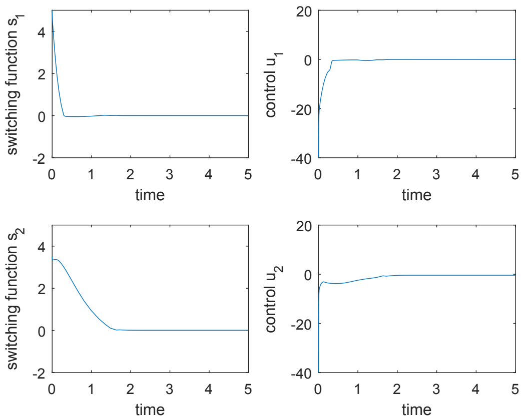

Fig. 2. The switching function s_1 and the control signal u_1 (Upper); the switching function s_2 and the control signal u_2 (Bottom).

For system (37)-(40), consider the candidate Lyapunov function as $V = V_1 + V_2$ where $V_1 = (z_{11}^a)^2 + (z_{11}^b)^2$ and $V_2 = (z_{21}^a)^2 + (z_{21}^b)^2$. By direct calculation,

$$p_{il}(\|z_{i1}\|) = \tau_{il}\|z_{i1}\|, \quad i = 1, 2, \quad l = 1, 2, 3, 4 \tag{51}$$

where τ_{il} for $i = 1, 2$, $l = 1, 2, 3, 4$ are the positive constants. It is easy to find $M^T + M > 0$. According to Theorem 1, it presents that the designed sliding mode is asymptotically stable.

Based on (28), the designed control is

$$\begin{aligned} u_1(\cdot) = & \frac{-2z_{12} + 0.5z_{11}^a z_{12}^2}{1 + (z_{11}^b)^2 \sin^2 t} \\ & - \Bigg((\|z_{11}^b\| + 1) \exp\{-t\} \text{sgn}(z_{12}) + z_{12} \\ & + \frac{0.03z_{12}(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|)^2}{\|z_{12}\|^2 \left(1 + (z_{11}^b)^2 \sin^2 t\right)} \Bigg) \\ & - \frac{k_1 \text{sgn}(z_{12})}{1 + (z_{11}^b)^2 \sin^2 t} \end{aligned} \tag{52}$$

$$\begin{aligned} u_2(\cdot) = & z_{21}^a - \Bigg(\|z_{21}^b\| \sin^2 z_{22} \text{sgn}(z_{22}) + z_{22} \\ & + \frac{0.16z_{22}}{\|z_{22}\|^2} \left(z_{11}^a + (z_{11}^a)^2 - 4z_{12} \right)^4 \cdot \sin^4 z_{22} \Bigg) \\ & - k_2 \text{sgn}(z_{22}) \end{aligned} \tag{53}$$

where constants k_1 and k_2 are chosen as $k_1 = 0.2$, $k_2 = 1.5$. From Theorem 2, it follows that the controller (52)-(53) can stabilise the interconnected system (37)-(40) uniformly asymptotically.

For the simulation purposes, the initial states are chosen as $x_{10} = (-2, 7.5, 5)$ and $x_{20} = (6, 2, 3.5)$. The simulation results in Fig. 1 and Fig. 2 show that the closed-loop system formed by applying control (52)-(53) to the interconnected system (37)-(40) is uniformly asymptotically stable which is in consistence with the theoretical results.

5. CONCLUSION

A class of nonlinear interconnected systems with unknown nonlinear interconnections has been considered in this paper. A composite sliding surface has been designed, and a set of conditions has been developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable. A novel decentralised state feedback control law is designed for the nonlinear interconnected systems to ensure that the reachability condition is satisfied. The proposed strategy supplies an approach to improve the robustness for nonlinear interconnected systems in that effects of all matched uncertainties and mismatched interconnections can be rejected by the designed decentralised control regarding the reaching phase, which can avoid unnecessary control efforts. Finally, numerical simulation results have been presented to show the effectiveness of the proposed methods. In the future, it is expected to extend the results developed in this paper to time delay nonlinear interconnected systems.

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