Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Notes on Yakubovich's method of recursive objective inequalities and its application in adaptive control and robotics

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Abstract: The purpose of the paper is to introduce to the control community the brilliant but little-known part of Yakubovich's academic heritage — the method of recursive objective inequalities. This method was successfully used by V.A.Yakubovich and his followers in pattern recognition, adaptive control and robotics. The paper deals with the last two topics. The most of surveyed results were published in Russian. A 1975 video about experiments with the first Soviet self-learning robot will be shown.

Keywords: Adaptive control, robotics, recursive inequalities

1. INTRODUCTION

Initially, the problem of solving a system of inequalities emerged in the paper on pattern recognition Yakubovich (1965). A detailed description of these applications is beyond the scope of this report.

In 1968 V.A.Yakubovich published two papers Yakubovich (1968a,b), which laid the foundations of the method of recursive objective inequalities as a new approach to solving problems of adaptive control and control of self-learning robots. In Yakubovich (1968a) he uses the word "robot" for the first time in the mathematical literature. This article gives a strict definition of a self-learning robot (the term "intelligent robot" is used) and gives examples of simple robots: "robot grasshopper" and "robot eye-hand". In the article V.A. Yakubovich proves a theorem describing the behavior of the self-learning robot. Figure 1 shows the formulation of the theorem copied from the paper.

Theorem 1. If the assumptions (I)-(IV) are satisfied, the brain equations can be constructed in such a way that the simple robot thus obtained becomes rational in the class of problems $\mathfrak{M}$.

Fig. 1. The first theorem of robotics.

Apparently, this is the first in the history a rigorous mathematical statement about the intelligent behavior of a robot.

We should note that Yakubovich's use of the term robot in a scientific article was a bold act at that time. As you can see from the video "Shakey the Robot: The First Robot to Embody Artificial Intelligence", presented on YouTube by SRI International, scientists at Stanford Research Center, who worked on the first intelligent robot in the early 70s, did not use the term "robot" in their project. They were afraid that the project related to science fiction would not receive financial support.

This talk is an extended version of the short report made by the authors at the 2017 IFAC Congress in Toulouse that was published as preprint Gusev and Bondarko (2017) but not included in the proceedings of the conference.

2. ALGORITHMS FOR SOLVING INFINITE SYSTEMS OF INEQUALITIES

The paper Yakubovich (1968b) considers two more examples of self-learning robots: "robot hawk" and "robot cyclist". The latter example is particularly interesting, since the robot motion is described by the linearized equation of the actual driving dynamics of the bicycle

$$\chi_{t+1} + \alpha_1\chi_t + \alpha_2\chi_{t-1} = \beta\psi_t + \phi_t, t = 0, 1, 2, \ldots,$$

where χ_t is the deviation of the bicycle frame from the vertical, ψ_t is bicycle steering angle, ϕ_t is the unknown external disturbance, t is the time. It is assumed that the parameters of the system $\alpha_1, \alpha_2, \beta$ are unknown, but the parameter β lies in known interval $0 < \beta \leq \kappa$. The external disturbance is bounded $|\phi_t| \leq \varepsilon$. The aim is to guarantee smallness of the bicycle frame deviation from the vertical, i.e. to fulfill the objective inequality

$$|\chi_t| < e, t = 0, 1, 2, \ldots, \tag{1}$$

for some given $e > 0$. At the beginning of the movement the values χ_0, χ_1 are randomly assigned and satisfy the condition

$$|\chi_0| < \delta, |\chi_1| < \delta, \tag{2}$$

where $0 < \delta < e$. If the inequality (1) is violated, it is considered that the bike has fallen, and a new game with initial data that satisfies (2) begins. The linear feedback $\psi_t = \gamma_1\chi_t + \gamma_2\chi_{t-1}$, is used, the parameters of which γ_1, γ_2 are to be found in the process of robot motion.

⋆ Supported by RFBR, grant 19-08-00865

Substituting the value of χ_{t+1} for closed-loop control system in (1), we obtain the objective recursive inequalities for the vector of unknown parameters $\tau = (\gamma_1, \gamma_2)$

$$|\beta(\tau, \sigma_t) + \zeta_t| < e, t = 0, 1, 2, \ldots, \quad (3)$$

where $\sigma_t = (\xi_1, \xi_{t-1}), \zeta_t = -\alpha_1\chi_t - \alpha_2\chi_{t-1} + \phi_t$, (τ, σ_t) is the scalar product of vectors τ and σ_t. It should be noted that ζ_t is unavailable for measurement. The sequence $\tau_t, t = 0, 1, 2, \ldots,$ is called a solution of the recursive system of inequalities (3) if for all sufficiently large $t, \tau_t = \tau = \text{const}$ and inequalities (3) are satisfied. Thus, to solve the problem of "robot cyclist" control it is sufficient to propose an algorithm for solving the recursive system of inequalities (3).

For simplicity, let $\beta = 1$. The simplest algorithm for solving a system of inequalities (3) is defined as follows Yakubovich (1966):

$$\tau_{t+1} = \begin{cases} \tau_t, & \text{if } |\eta_t| < e, \\ \tau_t - \eta_t |\sigma_t|^{-2} \sigma_t, & \text{if } |\eta_t| \geq e, \end{cases} \quad (4)$$

where discrepancy $\eta_t = \beta(\tau, \sigma_t) + \zeta_t = \chi_{t+1}$ is available for measurement. As proved in Yakubovich (1966) the algorithm (4) solves the system of inequalities (3), when $e > 2\varepsilon$. In Fomin et al. (1981) this algorithm is called "strip." The reason for this name becomes clear from the geometric interpretation of the algorithm depicted in Figure 2.

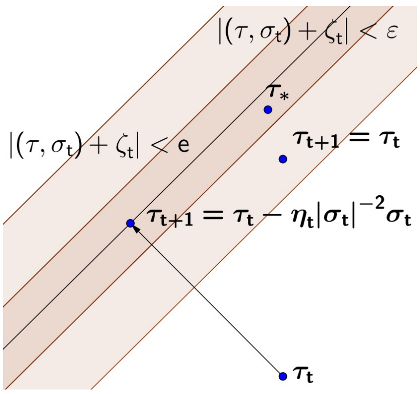

Fig. 2. Algorithm "strip" (simplest version).

From a computational point of view, the algorithm can be considered as the Kaczmarz's algorithm with a dead zone. The algorithm proposed by Kaczmarz (1937) is a well-known iterative method for solving systems of linear equations. However, it should be noted that in the presence of similarities in the computational procedure, the method of recursive objective inequalities is based on a different mathematical formulation of the problem.

This method considers the solution of infinite systems of inequalities, and inequalities are not given in advance, but appear in the process of solving the system as an answer to the solution process itself. Thus, the problem-solving process is a game in which one of the players is a recursive algorithm that solves inequalities one after another, and the other is a controlled object that responds to applied control actions. When the method appeared, this unexpected recursive formulation of the problem did not find understanding. Critics have argued that it is impossible to solve a problem whose conditions change in response to attempts to solve it. Of course, such statements have no basis, the convergence of the method of recursive objective inequalities is rigorously proved.

In order to weaken the condition $e > 2\varepsilon$ Yakubovich proposed an improved version of the algorithm, which has the form

$$\tau_{t+1} = \begin{cases} \tau_t, & \text{if } |\eta_t| \leq e, \\ \tau_t + \mu_t \dfrac{\eta_t - \varepsilon \,\text{sign}\, \eta_t}{|\sigma_t|^2} \sigma_t, & \text{if } |\eta_t| > e, \end{cases} \quad (5)$$

where parameter μ_t may be arbitrary chosen in $[1/2, 1]$. It converges, when $e > \varepsilon$. Figure 3 depicts the geometrical interpretation of the algorithm in the case $\mu_t = 1$.

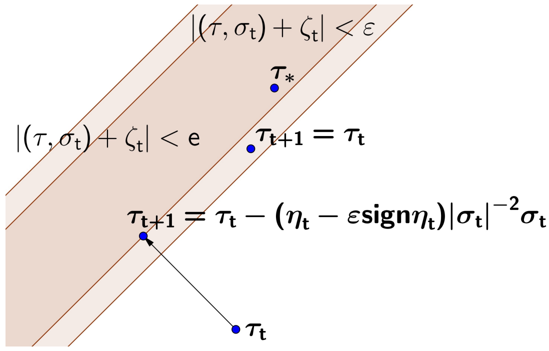

Fig. 3. Algorithm "strip" ($\mu_t = 1$).

3. ADAPTIVE CONTROL OF LINEAR SYSTEMS

Let us briefly describe the further development of the method of recursive objective inequalities and list some of the results obtained. The first results on the adaptive control of linear systems of general form were obtained in Yakubovich and Penev (1971); Lubachevsky (1974). In these papers, the problem of "robot cyclist" control was considered as a test case for the simulation of the proposed adaptive control algorithms for linear plants.

From a practical point of view, the drawback of this approach was the presence of so-called "games", that is, the need to stop the process of control from time to time and start it again, with new initial conditions. The method of excluding such stops was proposed by V.A.Bondarko Bondarko and Yakubovich (1979); Fomin et al. (1981).

In the paper Gusev (1989) an algorithm for solving inequalities of the form (3) is proposed, in which the absolute value is replaced by the sample standard deviation for the whole prehistory of the process. The obtained recursive formulas allow the implementation of the algorithm using finite memory. The method eliminates the requirement of bounded disturbances and can be used in the presence of stochastic disturbances with Gaussian and other distributions.

For a long time, the method of recursive objective inequalities did not allow to control nonminimum-phase systems. This problem was overcome in Fomin et al. (1981); Bondarko et al. (1991); Bondarko (1994).

A more detailed survey of adaptive control of linear systems by the method of recursive objective inequalities is given in Bondarko and Yakubovich (1992).

Sokolov (2015) uses the method of recursive objective inequalities for a generalization of the well-known result of Huang and Guo (2012).

4. ADAPTIVE CONTROL WITH POTENTIALLY INFINITE DIMENSION OF ESTIMATED VECTOR.

Considering problems of optimal adaptive control Bondarko (2006) or adaptive control of sampled infinite-dimensional systems Bondarko (1996) leads to the systems of inequalities (3) with infinite-dimensional vectors τ and σ_t. As a matter of fact, finite-dimensional solution of inequalities with slightly increased right side does exist, and it would be enough for achievement of control objective, but dimension of this solution is unknown. This obstacle was overcome in the paper Bondarko (2006) by means of the algorithm like (5) which increases a dimension of τ_t each time when current inequality $|\eta_t| \le e$ is not fulfilled. Sooner or later these events will stop, thus final dimension of the solution obtained is finite.

The paper Bondarko (2006) considered discrete-time and continuous-time plants with bounded disturbances. In both cases sub-optimal adaptive controller is the same discrete-time system, completed with zero order hold extrapolation in case of continuous-time plant. It should be mentioned that zero order hold sampled model of continuous-time prototype does not satisfy conditions of discrete-time theorem. Namely, disturbance of this model is bounded, but it belongs to another class than was considered in discrete-time case. Nevertheless, the use of an adaptive controller with discrete-time leads to a suboptimal level of the cost function, if sampling rate is sufficiently high. The same effect was described in the early paper Bondarko (1991) which concerns with more restricted class of disturbances.

5. ALGORITHM FOR SOLVING SOME SYSTEMS OF NON-CONVEX INEQUALITIES

A general approach to the transforming of a wide class of nonlinear recursive inequalities to inequalities of the form (3) was proposed in Bondarko (2010). Simplest example is a problem of an induction motor control with known parameters, but without velocity sensor. So called field-oriented controller requires an unobservable values of electromagnetic field flux. In turn, estimation of the flux becomes possible, if we find a 2-vector τ, which is a solution of inequalities

$$\left| |\tau - C_t|^2 - R_t^2 \right| \le \varepsilon, \tag{6}$$

where C_t, R_t, and ε may be evaluated using observable signals. In other words, τ should belongs to the intersection of an infinite sets of rings, while inequalities (3) distinguish intersection of strips, i.e. gaps between a pair of hyperplanes. Unlike a strip, a ring is not a convex set, thus a problem of solving system (6) is much more complicated. Nevertheless, we can combine a pare of inequalities (6) with $t = t'$ and $t = t''$ to obtain one inequality (3) with $\beta = 1$,

$$\sigma_t = 2[C_{t'} - C_{t''}],\ \zeta_t = C_{t'}^2 - C_{t''}^2 + R_{t'}^2 - R_{t''}^2,\ e > 2\varepsilon.$$

This means we replace a pair of the rings by one strip which includes an intersection of those rings. The Figure 4 shows a result of estimating τ by means of the algorithm (5) applied to inequalities (3) obtained from (6). The broken line with small circles shows evolution of the τ_t, rings are depicted as circles, and strips are shown as straight lines.

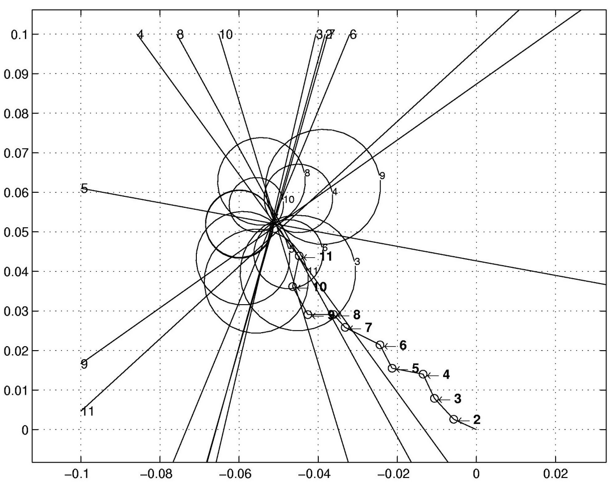

Fig. 4. The process of solving non-convex inequalities.

Let us consider inequalities of the general sort

$$|\zeta_t + (\tau, \sigma_t) + (\varphi(\tau), \phi_t)| < e,\ t = 0, 1, \ldots, \tag{7}$$

with nonlinear term $(\varphi(\tau), \phi_t)$ in left sides. The approach described let us diminish a dimension of vectors $\varphi(\tau)$ and ϕ_t by 1, combining two inequalities (7) with $t = t'$ and $t = t''$. Step by step, nonlinear term would be eliminated at all, and then we could apply the algorithm (5) to solve obtained convex inequalities (3). In a particular, this approach allows to estimate electromagnetic flux of an induction motor with unknown parameters Bondarko (2010).

6. CONTROL AND ESTIMATION IN THE PRESENCE OF UNKNOWN BOUNDED DISTURBANCES

One of the original features of the approach proposed by Yakubovich is the consideration of unknown deterministic bounded disturbances additively acting on the plant Yakubovich (1968b). The consideration of this class of disturbances led to the emergence of a number of new directions in control theory.

Almost simultaneously with the works on adaptive control of Yakubovich, Schweppe (1968) considers the problem of the state estimation of a system with unknown bounded disturbances. Proposed in this article method was further developed and is now known as the set-membership estimation. In Sokolov (1985) this method is applied in the problem of adaptive control of a SISO linear plant of the first order in the presence of bounded disturbances. The control criterion is the upper limit of the absolute value of the plant output. The result obtained in this work is somewhat paradoxical: the proposed adaptive control algorithm can be called super-optimal, because ensures the achievement of control quality no worse than the optimal linear minimax control for the system with known parameters, and in many cases, the quality of adaptive control is better than that of the optimal linear minimax control.

In Yakubovich (1968b), the problem of adaptive sub-optimal control of a first order linear SISO minimum phase plant with a minimax criterion was solved. In parallel with Yakubovich's studies, the general problem of minimax control of a plant with known parameters in the presence of bounded disturbances was considered in Witsenhausen (1968). But the minimax control problem for a non-minimum phase plant remained open. For the SISO discrete time plant the problem was solved in Barabanov and Granichin (1984). Subsequently, this area of research has become known as the theory of l_1-optimization Dahleh and Diaz-Bobillo (1995).

7. ADAPTIVE CONTROL OF ROBOTS.

In parallel with the problems of linear systems control, the techniques were elaborated for adaptive control of nonlinear systems describing the motion of robots. In 1974, two models of the transport robot, controlled by a computer, were developed at the Mathematics and Mechanics Faculty of Leningrad (now St. Petersburg) State University. These models were used for the first implementation of adaptive motion control algorithm in the presence of disturbances that change the parameters of the robot, for example, the supply voltage of the motors, the load on the chassis, the coefficient of adhesion to the ground. In the photo (see Fig. 5), the developers of the robot control algorithm V.A.Yakubovich and S.V.Gusev are next to the robot.

Fig. 5. S.V. Gusev (left) and V.A.Yakubovich with the first robot (1974).

As the chassis of the robot a toy was used - a planet rover. The robot is equipped with two sensors located on the turning vertical axis. The lower sensor is an ultrasonic rangefinder, the upper one is a photosensor, with the help of which the directions to light beams are determined. This sensor was used to determine the coordinates and orientation of the robot. The robot has 4 modes of movement: forward, backward, turn left, and turn right.

The results of the first adaptive control experiments were published in Gusev et al. (1975), a detailed description of the adaptive control algorithm is given in Gusev (1981).

As an illustrative material in the talk, we shall show a video about experiments conducted in 1975.

In 1980, to continue research on adaptive robot control, a more advanced mobile robotic arm was developed, shown in the photo (see Fig. 6). The robot has a six-wheeled chassis, a manipulator with 4 degrees of freedom and a three-fingered grip. It is equipped with a rotating ultrasound rangefinder system and a stereo vision system consisting of two cameras. The control was carried out by a computer via a radio channel.

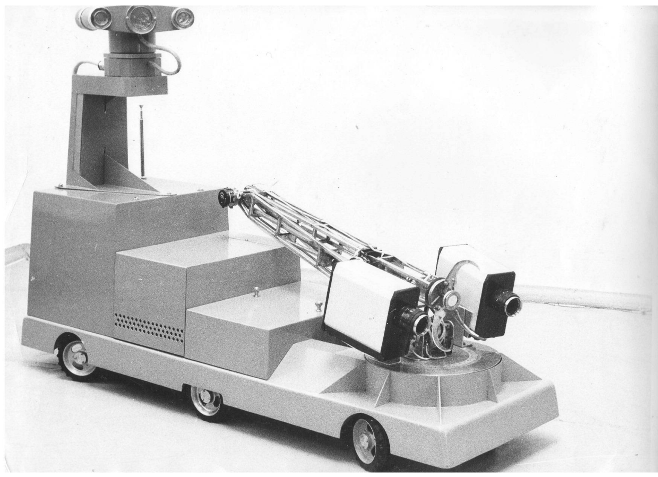

Fig. 6. The mobile robot with a manipulator (1980).

The results of some experiments with the robot are published in Grigor'ev et al. (1982).

In Timofeev and Yakubovich (1976); Gusev and Yakubovich (1980) adaptive control algorithms for robotic manipulators are proposed. It is assumed that the dynamic parameters of the manipulator are unknown. Adaptive control is constructed, which ensures the specified accuracy of desired motion tracking. In Gusev and Yakubovich (1980) the sampled time control and measurement is considered.

In Gusev et al. (1983); Gusev and Shishkin (1999) the problem of self-learning of a biped robot is treated. It is supposed that the dynamic parameters are unknown. The robot control system has no information about desired motion as function of time. The only information that characterizes walking consists of a lower estimate of the robot step length and inequalities that prohibit the robot from touching the surface of any points of the body except for the feet. A method proposed for reducing the robot walking task to the solution of recursive non-convex inequalities of a special kind, and an algorithm is constructed for solving such systems of inequalities.

8. CONCLUSION

V.A.Yakubovich famous as the discoverer of the Kalman-Yakubovich-Popov Lemma. He is widely known for his outstanding results in the theory of nonlinear systems and optimal control. In this article, we highlighted the less known, but important and still relevant area of his extensive scientific interests — the theory of adaptive control and robotics. Along with his personal scientific achievements, it is necessary to note the great contribution of V.A.Yakubovich to the development of science by creating a powerful and successful scientific school, from which many famous scientists came out. Confining ourselves to the topic of our presentation, we will name here only his most striking students, experts in the field of adaptive control and robotics.

Currently, the Department of Theoretical Cybernetics, founded by V.A.Yakubovich in Leningrad (now St. Petersburg) State University, is headed by prof. A.L.Fradkov, who is one of co-authors of Yakubovich in the monograph on adaptive control Fomin et al. (1981). Professor A.S. Matveev, who co-authored with V.A. Yakubovich two monographs on the theory of optimal control, now is actively working in the field of robotics. A.S. Matveev, together with another student of V.A.Yakubovich prof. A.Savkin are co-authors of two monographs on robotics Savkin et al. (2015); Matveev et al. (2016). Now A.Savkin is head of Systems and Control at the University of New South Wales, Sydney. The outstanding results in the field of robotics of another graduate of the Department of Theoretical Cybernetics, professor of the Norwegian University of Science and Technology A.S. Shiryaev are also widely known Shiriaev et al. (2005, 2014).

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