# Composite Adaptive Backstepping Control considering Computational Complexity and Relaxation of Persistent Excitation

**Byoung-Ju Jeon** [*] **Hyo-Sang Shin** [**] **Antonios Tsourdos** [***]

[*] *School of Aerospace, Transport and Manufacturing (SATM), Cranfield University, College Road, Cranfield, United Kingdom (e-mail: B.Jeon@cranfield.ac.uk).*
[**] *School of Aerospace, Transport and Manufacturing (SATM), Cranfield University, College Road, Cranfield, United Kingdom (e-mail: h.shin@cranfield.ac.uk).*
[***] *School of Aerospace, Transport and Manufacturing (SATM), Cranfield University, College Road, Cranfield, United Kingdom (e-mail: a.tsourdos@cranfield.ac.uk).*

**Abstract:** A new composite adaptive backstepping control is proposed in this paper, which achieves parameter estimation convergence without persistent excitation and reduces estimation problem dimension for less computational complexity. A composite adaptation law is utilized to improve estimation and tracking performance. Relaxation of the persistent excitation requirement for parameter convergence is accomplished by making information matrix full rank only with finite excitation. The adaptation law for the proposed composite adaptive backstepping control algorithm estimates parameters in each loop separately by taking an advantage from a cascade control structure of backstepping control. Comparing to the adaptation laws which estimate whole parameters of the dynamic system at once, the designed adaptation law deals with smaller estimation problems, resulting in reduced computational complexity.

*Keywords:* Backstepping Control, Composite Adaptive Control, Finite Excitation, Persistent Excitation, Cascade Control Structure, Computational Complexity

## 1. INTRODUCTION

Backstepping(BKS) control is one of the most widely and successfully applied nonlinear flight control methods Kokotovic (1992); Kim and Kim (2013); Ghommam and Saad (2017). BKS has a cascade control structure, where a state for an inner loop acts on an outer loop as a pseudo input driving a state for an outer loop to its desired value. This implies that a control law design for a system with large dimension can be split into several control law designs for simple systems with smaller dimension in recursive way. The closed-loop system with BKS fulfils a desired system response with known stability and convergence properties under Lyapunov framework. One of the critical issues about BKS is that it is sensitive to model uncertainties, because it requires full model information for implementation of the algorithm. Since it is difficult to get accurate model information in general, it is important to make BKS less dependent on model information.

In this paper, a composite adaptive control approach Slotine et al. (1991); Duarte and Narendra (1989); Ciliz (2009) is introduced to BKS in order to reduce its model dependency by estimating part of model parameters online and utilizing the estimates for controller implementation. In order to improve estimation performance, an estimation error based term is introduced to a tracking error based adaptation law, resulting in a composite adaptation law. A tracking error based adaptation law can be interpreted as a simple integration of a tracking error signal, while a composite adaptation law appears to be a low pass filter on a tracking error signal due to the additionally introduced estimation error based term. This implies that, when adaptation gains are increased to enhance estimation speed, oscillations on estimation and tracking response in transient phase can be amplified with a tracking error based adaptation law. On the other hand, increase of adaptation gains enlarges a bandwidth of a composite adaptation law without excessive amplifications of oscillatory behaviors in estimation and tracking response in transient phase. Thus, a composite adaptation law achieves smoother transient response than a tracking error based adaptation law, resulting in enhanced tracking performance and system robustness.

One of the main issues with the composite adaptation law is that persistent excitation (PE) is required to guarantee parameter convergence. This PE condition results in persistent oscillations of state and control input signals, which is unrealistic for practical applications. There have been previous studies Chowdhary et al. (2013, 2014); Parikh et al. (2018); Cho et al. (2017) on relaxation of PE condition to finite excitation (FE) condition for composite adaptation laws. To achieve convergence of parameter estimation only with FE, they utilize a similar approach as follows. An information matrix is obtained by accumulating regressor data and algorithms to make use of richer regressor signals for the information matrix are introduced. This results in full rank of the information matrix after a certain time, only with FE. Besides, speed in the slowest adaptation direction is maximized since the information matrix which maximizes its minimum eigenvalue is selected. The approaches in Chowdhary et al. (2013, 2014); Parikh et al. (2018) require larger memory than Cho et al. (2017), since they store all regressor matrices for the information matrix calculation. To the best of our knowledge, relaxation of PE condition has not been discussed for the composite ABKS Ciliz (2007). Chowdhary et al. (2013) and Cho et al. (2017) are based on full-state feedback control, and Chowdhary et al. (2014) and Parikh et al. (2018) utilize dynamic inversion control scheme.

For successful design of an adaptation law, it is also important to consider practical issues related to computational complexity induced from the adaptation law structure. A structure of an adaptation law is highly dependent on a structure of a baseline control algorithm. If the baseline control law has a cascade control structure, it is possible to design the adaptation law for each loop of the control system. In previous studies Chowdhary et al. (2013, 2014); Parikh et al. (2018); Cho et al. (2017), baseline control algorithms do not have cascade control structures. Even in Ciliz (2007) with ABKS, an adaptation law design does not fully take advantage of the structural characteristics of BKS. As a result, adaptation laws in relevant literature Ciliz (2007); Chowdhary et al. (2013, 2014); Parikh et al. (2018); Cho et al. (2017) are designed to estimate all the uncertain parameters in the dynamic system at once. Since a dimension of an augmented parameter estimation problem enlarges as the number of dynamic equations and uncertain parameters increases, matrix operations with excessively large matrices are required, resulting in high computational complexity. Thus, a structure of the proposed adaptation law needs to be designed in a way to decrease the estimation problem dimension.

In this paper, a composite ABKS control algorithm is designed, where its estimation problem dimension is reduced and PE requirement is relaxed. The composite adaptation law which enhances both estimation and tracking performance is utilized. The relaxation of PE requirement to FE for parameter convergence is accomplished by utilizing information matrix construction and selection methods based on Cho et al. (2017). One of the main contributions of this paper is that the adaptation problem for the overall dynamic system is divided into smaller estimation problems with the new composite ABKS. By taking advantages from a cascade control structure of BKS, the proposed adaptation law estimates model parameters in each loop separately, rather than estimates whole parameters of the system at once. This results in decreased computational complexity from reduced estimation problem dimension.

This paper is organized as follows. System dynamics with model uncertainty is defined in Section 2. Derivation and stability analysis of the proposed composite ABKS are addressed in Section 3. In Section 4, simulations are conducted to show performance and characteristics of the new composite ABKS. The overall concluding remarks are stated in Section 5.

## 2. SYSTEM DYNAMICS

In this paper, system dynamics with model uncertainty is considered as follows.

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{\Delta}(\boldsymbol{x})$$

where

$$\begin{aligned}
&\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T \\
&\boldsymbol{x_i'} = [x_1, x_2, \cdots, x_i]^T \quad (i = 1, \cdots, n) \\
&\boldsymbol{f}(\boldsymbol{x}) = \left[f_1(\boldsymbol{x_1'}), f_2(\boldsymbol{x_2'}), \cdots, f_n(\boldsymbol{x_n'})\right]^T \\
&\boldsymbol{g}(\boldsymbol{x}) = \operatorname{diag}\left[g_1(\boldsymbol{x_1'}), g_2(\boldsymbol{x_2'}), \cdots, g_n(\boldsymbol{x_n'})\right] \\
&\boldsymbol{u} = [u_1, u_2, \cdots, u_n]^T \text{ with } u_i = \begin{cases} x_{i+1} & (i = 1, \cdots, n-1) \\ \delta & (i = n) \end{cases} \\
&\boldsymbol{\Delta}(\boldsymbol{x}) = \left[\Delta_1(\boldsymbol{x_1'}), \Delta_2(\boldsymbol{x_2'}), \cdots, \Delta_n(\boldsymbol{x_n'})\right]^T
\end{aligned} \tag{1}$$

$\boldsymbol{x} \in \mathbb{R}^{n\times 1}$ indicates a state vector and $\boldsymbol{x_i'} \in \mathbb{R}^{i\times 1}$ is a subset of the state vector $\boldsymbol{x}$. $\boldsymbol{f}(\boldsymbol{x}) \in \mathbb{R}^{n\times 1}$ and $\boldsymbol{g}(\boldsymbol{x}) \in \mathbb{R}^{n\times n}$ represent known model information. $\boldsymbol{u} \in \mathbb{R}^{n\times 1}$ denotes a control input vector. Model uncertainty is expressed as $\boldsymbol{\Delta}(\boldsymbol{x}) \in \mathbb{R}^{n\times 1}$, which satisfies the matching condition in Leitmann (1979). Since a control algorithm based on the backstepping methodology will be proposed, the system dynamics (1) is suggested in a strict-feedback form. First, $f_i(\boldsymbol{x_i'})$ and $g_i(\boldsymbol{x_i'})$ in $\boldsymbol{f}(\boldsymbol{x})$ and a diagonal matrix $\boldsymbol{g}(\boldsymbol{x})$ only depend on $\boldsymbol{x_i'}$. Second, a real control input $\delta$ is applied for the innermost loop and a state becomes a pseudo input for the next outer-loop in recursive way, constructing the control input vector $\boldsymbol{u}$ as (1).

A structured model uncertainty $\boldsymbol{\Delta}(\boldsymbol{x})$ which is linearly parameterized, is utilized in this paper as below.

$$\begin{aligned}
&\Delta_i(\boldsymbol{x_i'}) = \boldsymbol{\theta_i}^T \boldsymbol{\phi_i}(\boldsymbol{x_i'}) \qquad (i = 1, \cdots, n) \\
&\text{where} \\
&\boldsymbol{\theta_i} = \left[\theta_{i_1}, \theta_{i_2} \cdots \theta_{i_{m_i}}\right]^T \\
&\boldsymbol{\phi_i}(\boldsymbol{x_i'}) = \left[\phi_{i_1}(\boldsymbol{x_i'}), \phi_{i_2}(\boldsymbol{x_i'}) \cdots \phi_{i_{m_i}}(\boldsymbol{x_i'})\right]^T
\end{aligned} \tag{2}$$

$\boldsymbol{\theta_i} \in \mathbb{R}^{m_i\times 1}$ is a vector of unique constant true parameters, which is unknown. $\boldsymbol{\phi_i}(\boldsymbol{x_i'}) \in \mathbb{R}^{m_i\times 1}$ represents a known regressor vector which is continuously differentiable.

## 3. COMPOSITE ADAPTIVE BACKSTEPPING CONTROL

### 3.1 Derivation

A control command vector $\boldsymbol{u_c} \in \mathbb{R}^{n\times 1}$ is defined as (3) from (1). Subscript c indicates a command.

$$\begin{aligned}
&\boldsymbol{u_c} = [u_{1_c}, u_{2_c}, \cdots, u_{n_c}]^T \\
&\text{with } u_{i_c} = \begin{cases} x_{i+1_c} & (i = 1, \cdots, n-1) \\ \delta_c & (i = n) \end{cases}
\end{aligned} \tag{3}$$

$u_{i_c}$ is a command which $u_i$ should follow in order to drive $x_i$ to $x_{i_c}$. Under the assumption of ideal actuator, $u_{n_c} = \delta_c = \delta$.

A tracking error $\boldsymbol{z} \in \mathbb{R}^{n\times 1}$ is given as below.

$$\begin{aligned}
&\boldsymbol{z} = [z_1, z_2, \cdots, z_n]^T \\
&\text{where } z_i = x_i - x_{ic}
\end{aligned} \tag{4}$$

The unknown vector $\boldsymbol{\theta_i}$ of true parameters will be estimated as $\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}$ by the adaptation law (7). A vector of parameter estimation errors is given as (5).

$$\hat{\boldsymbol{\theta}}_{\boldsymbol{i}} - \boldsymbol{\theta}_i = \left[\hat{\theta}_{i_1} - \theta_{i_1}, \hat{\theta}_{i_2} - \theta_{i_2}, \cdots, \hat{\theta}_{i_{m_i}} - \theta_{i_{m_i}}\right]^T \quad (5)$$

A recursive design methodology for the control command $\boldsymbol{u_c}$ with an adaptation law is utilized under Lyapunov framework, resulting in (6) with (7). Asymptotic stability is achieved for the closed loop system with (6) and (7), which will be proved under Lyapunov stability analysis framework in following subsection 3.2.

A derived control command $u_{i_c}$ is suggested as follows.

$$\begin{aligned} u_{ic} &= \frac{1}{g_i}\left[-C_i z_i - g_{i-1} z_{i-1} - f_i - \hat{\boldsymbol{\theta}}_{\boldsymbol{i}}^T \boldsymbol{\phi_i} + \dot{x}_{ic}\right] \\ &\text{where } g_0 z_0 \triangleq 0 \end{aligned} \quad (6)$$

$C_i$ is a constant and positive design parameter for the control law to achieve a desired closed-loop response.

$\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}$ in (6) can be obtained from the adaptation law (7).

$$\frac{d}{dt}\hat{\boldsymbol{\theta}}_{\boldsymbol{i}} = z_i \boldsymbol{\Gamma_i} \boldsymbol{\phi_i} - \lambda_i \boldsymbol{\Gamma_i}\left(\boldsymbol{\Omega_i}\hat{\boldsymbol{\theta}}_{\boldsymbol{i}} - \boldsymbol{\eta_i}\right) \quad (7)$$

$\boldsymbol{\Gamma_i} = \text{diag}\left[\gamma_{i_1}, \gamma_{i_2}, \cdots, \gamma_{i_{m_i}}\right] \in \mathbb{R}^{m_i \times m_i}$ is a matrix of constant and positive design parameters for the adaptation law. Constant and positive $\lambda_i$ is a relative weight factor on the estimation error based term to the tracking error based term. $\boldsymbol{\Omega_i} \in \mathbb{R}^{m_i \times m_i}$ denotes an information matrix to be designed. $\boldsymbol{\eta_i} \in \mathbb{R}^{m_i \times 1}$ denotes an auxiliary vector, which can be expressed as $\boldsymbol{\eta_i} = \boldsymbol{\Omega_i}\boldsymbol{\theta}_i$. Since $\boldsymbol{\theta}_i$ is unknown, $\boldsymbol{\eta_i}$ should be calculated from known signals in another way, which will be discussed in later part related to a regressor filtering scheme. The estimation error based term in the composite adaptation law (7) leads to smoother transient estimation and tracking response, resulting in enhanced system performance.

Regressor filtering scheme is utilized to compute the auxiliary vector without state derivative information. For the i-th loop, (1) can be rewritten as (8) with $g_i' \triangleq g_i u_i$.

$$\dot{x}_i = f_i + g_i' + \boldsymbol{\theta_i}^T \boldsymbol{\phi_i} \quad (8)$$

By applying Laplace transform to (8),

$$s x_i(s) = f_i(s) + g_i'(s) + \boldsymbol{\theta_i}^T \boldsymbol{\phi_i}(s) \quad (9)$$

Filtered dynamics (10) can be obtained by multiplying a first order filter $F(s) = \frac{1}{ks+1}$ with the filter parameter $k$ for both sides.

$$sF(s)x_i(s) = F(s)\left\{f_i(s) + g_i'(s) + \boldsymbol{\theta_i}^T \boldsymbol{\phi_i}(s)\right\} \quad (10)$$

Using $sF(s) = \frac{1}{k}(1 - F(s))$, (10) can be rearranged as (11).

$$\begin{aligned} &\frac{1}{k}\left\{x_i(s) - F(s)x_i(s)\right\} \\ &= F(s)f_i(s) + F(s)g_i'(s) + \boldsymbol{\theta_i}^T\left\{F(s)\boldsymbol{\phi_i}(s)\right\} \end{aligned} \quad (11)$$

Filtered dynamics in the time domain (12) is obtained by applying inverse Laplace tranform to (11).

$$\frac{1}{k}\left(x_i - x_{if}\right) = f_{if} + g_{if}' + \boldsymbol{\theta_i}^T \boldsymbol{\phi_{if}} \quad (12)$$

where $(\cdot)_f$ represents a filtered signal by $F(s)$.

Let

$$\zeta_{if} \triangleq \frac{1}{k}x_{if} + f_{if} + g_{if}' \quad (13)$$

(14) is obtained by rearranging (12) in terms of $\boldsymbol{\theta_i}^T \boldsymbol{\phi_{if}}$ and substituting (13) into the rearranged equation.

$$\begin{aligned} &\boldsymbol{\theta_i}^T \boldsymbol{\phi_{if}} = \frac{1}{k}x_i - \zeta_{if} \\ &\text{where} \\ &\dot{\boldsymbol{\phi}}_{\boldsymbol{if}} = \frac{1}{k}\left(\boldsymbol{\phi_i} - \boldsymbol{\phi_{if}}\right) \\ &\dot{\zeta}_{if} = \frac{1}{k}\left(\frac{1}{k}x_i + f_i + g_i' - \zeta_{if}\right) \end{aligned} \quad (14)$$

$\boldsymbol{\theta_i}^T \boldsymbol{\phi_{if}}$ information in (14) will be utilized to calculate the auxiliary vector in (15). Without regressor filtering scheme, $\boldsymbol{\theta_i}^T \boldsymbol{\phi_i}$ information is required to calculate the auxiliary vector instead of $\boldsymbol{\theta_i}^T \boldsymbol{\phi_{if}}$ information, and $\boldsymbol{\theta_i}^T \boldsymbol{\phi_i}$ information can be obtained from (8), resulting in usage of the state derivative information. In general, state derivatives are difficult to be measured, and noise in state measurement signals can be amplified if the state derivatives are calculated via differentiation of the state measurements. Hence, it is advantageous to utilize regressor filtering scheme and prevent usage of the state derivative information in auxiliary vector calculation.

Update laws for the information matrix $\boldsymbol{\Omega_i}$ and the auxiliary vector $\boldsymbol{\eta_i}$ are designed as (15).

$$\begin{aligned} \dot{\boldsymbol{\Omega}}_{\boldsymbol{i}}(t) &= -K(t)\boldsymbol{\Omega_i}(t) + \boldsymbol{\phi_{if}}(t)\boldsymbol{\phi_{if}}^T(t) \\ \dot{\boldsymbol{\eta}}_{\boldsymbol{i}}(t) &= -K(t)\boldsymbol{\eta_i}(t) + \boldsymbol{\phi_{if}}(t)\left(\frac{1}{k}x_i(t) - \zeta_{if}(t)\right)^T \end{aligned} \quad (15)$$

with $\boldsymbol{\Omega_i}(t_0) = 0_{m_i \times m_i}$ and $\boldsymbol{\eta_i}(t_0) = 0_{m_i \times 1}$. $K(t)$ is a forgetting factor to be designed, which is positive and bounded. The information matrix update law in (15) consists of two terms. The first term is defined by introducing the forgetting factor (17) to the current information matrix obtained from accumulated regressor data for previous time interval. The second term represents the effects of the current filtered regressor $\boldsymbol{\phi_{if}}$ on the information matrix update.

$\boldsymbol{\Omega_i}$ and $\boldsymbol{\eta_i}$ are derived by integrating the update laws (15).

$$\begin{aligned} \boldsymbol{\Omega_i}(t) &= \int_{t_0}^{t} e^{-\int_\tau^t K(\nu)d\nu}\boldsymbol{\phi_{if}}(\tau)\boldsymbol{\phi_{if}}^T(\tau)d\tau \\ \boldsymbol{\eta_i}(t) &= \int_{t_0}^{t} e^{-\int_\tau^t K(\nu)d\nu}\boldsymbol{\phi_{if}}(\tau)\left(\frac{1}{k}x_i(\tau) - \zeta_{if}(\tau)\right)^T d\tau \\ &= \boldsymbol{\Omega_i}(t)\boldsymbol{\theta}_i \end{aligned} \quad (16)$$

It can be observed from (16) that the information matrix $\boldsymbol{\Omega_i}$ is positive semi-definite. Besides, (16) implies that the information matrix can have full rank and become positive definite matrix over time, when the regressor signal is excited.

$K(t)$ is a forgetting factor, which is designed as below.

$$K(t) = k_L + (k_U - k_L)\tanh\left(\vartheta\|\dot{\boldsymbol{\phi}}_{\boldsymbol{if}}(t)\|\right) \quad (17)$$

$k_L$ and $k_U$ indicate lower and upper bounds of $K(t)$ with positive constant values, respectively. $\vartheta$ is a constant and positive design parameter for the forgetting factor.

The effects of the forgetting factor on the information matrix are addressed as follows. First, the forgetting factor enables the information matrix to be upper bounded in its norm. Second, the forgetting factor makes richer signal to be reflected more on the information matrix update (15). The forgetting factor (17) becomes larger when $\boldsymbol{\phi_{if}}$ contains richer data with large $\|\dot{\boldsymbol{\phi}}_{\boldsymbol{if}}(t)\|$. Consequently, the information matrix is updated to consider the current filtered regressor signal more and the accumulated data less. On the other hand, when $\boldsymbol{\phi_{if}}$ does not contain rich data with small $\|\dot{\boldsymbol{\phi}}_{\boldsymbol{if}}(t)\|$, the forgetting factor (17) gets smaller. As a result, the current filtered regressor signal is reflected less and the accumulated data affects more to the information matrix update.

After the excitation is finished, the information matrix will be degenerated due to the forgetting design and the incoming filtered regressor signal which is not rich. In order to prevent this phenomena, information matrix selection method is introduced.

$$\begin{aligned}&\boldsymbol{\Omega_{ib}}(t) \triangleq \boldsymbol{\Omega_i}(t_b), \quad \boldsymbol{\eta_{ib}}(t) \triangleq \boldsymbol{\eta_i}(t_b)\\ &t_b \triangleq \max\left\{\underset{\tau\in(t_0,t)}{\operatorname{argmax}}\, \mathcal{F}(\Omega(\tau))\right\}\\ &\text{where } \mathcal{F}(\Omega(\tau)) = \sigma_{min}(\tau)\end{aligned} \tag{18}$$

$\mathcal{F}(\Omega(\tau))$ is selected as a minimum eigenvalue of the information matrix. Eigenvalues of the information matrix are related to speed for the estimation error based term of the adaptation law in corresponding eigenvector directions. This means that maximizing the minimum eigenvalue can be interpreted as maximizing the adaptation speed of the slowest direction. Information matrix becomes positive definite during excitation, but after FE it might become positive semi-definite again with rank deficiency. The selection method (18) has an effect which automatically excludes positive semi-definite matrices with zero eigenvalues. Hence, the condition to guarantee parameter convergence is relaxed from PE to FE through accumulation and selection procedures.

A final adaptation law is suggested as (19), with best information matrix and auxiliary vector from (18).

$$\frac{d}{dt}\boldsymbol{\hat{\theta}_i} = z_i\boldsymbol{\Gamma_i\phi_i} - \lambda_i\boldsymbol{\Gamma_i}\left(\boldsymbol{\Omega_{ib}\hat{\theta}_i} - \boldsymbol{\eta_{ib}}\right) \tag{19}$$

Note that the adaptation law (19) is designed to estimate $\boldsymbol{\theta_i}$ for each loop separately. If all $\boldsymbol{\theta_i}$ are augmented into one parameter estimation problem for whole system, the size of this augmented matrix for the unknown parameters is $\sum_{i=1}^n m_i \times n$, and the size of the corresponding information matrix becomes $\sum_{i=1}^n m_i \times \sum_{i=1}^n m_i$ in maximum. This implies that complex matrix operations, like matrix multiplication and eigenvalue calculation for matrix selection, should be conducted with an excessively large size of a matrix. Since the computational complexity of those matrix operations dramatically increases as the matrix size enlarges, it is beneficial to divide the adaptation problem into smaller ones and conduct estimation for parameters in each loop, as addressed in (19).

*3.2 Stability Proof*

A Lyapunov candidate function $V_n$ considering both tracking and parameter estimation errors, is selected as below.

$$V_n = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{1}{\gamma_{i_j}}\left(\hat{\theta}_{i_j} - \theta_{i_j}\right)^2 \tag{20}$$

$V_n$ is positive definite for all tracking and parameter estimation errors except the origin.

$\dot{V}_n$, derivative of $V_n$, can be derived as (21).

$$\begin{aligned}\dot{V}_n =& \sum_{i=1}^{n} z_i\dot{z}_i + \sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{1}{\gamma_{i_j}}\left(\hat{\theta}_{i_j} - \theta_{i_j}\right)\dot{\hat{\theta}}_{ij}\\ =& \Bigg[z_1\left\{-C_1z_1 + g_1z_2 - \tilde{\boldsymbol{\theta}}_1^T\boldsymbol{\phi}_1\right\}\\ &+ \sum_{i=2}^{n-1} z_i\left\{-C_iz_i - g_{i-1}z_{i-1} + g_iz_{i+1} - \tilde{\boldsymbol{\theta}}_i^T\boldsymbol{\phi}_i\right\}\\ &+ z_n\left\{-C_nz_n + g_{n-1}z_{n-1} - \tilde{\boldsymbol{\theta}}_n^T\boldsymbol{\phi}_n\right\}\Bigg]\\ &+ \sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{1}{\gamma_{i_j}}\left(\hat{\theta}_{i_j} - \theta_{i_j}\right)\dot{\hat{\theta}}_{i_j}\\ =& -\sum_{i=1}^{n} C_iz_i^2 - \sum_{i=1}^{n}\sum_{j=1}^{m_i}\left(\hat{\theta}_{i_j} - \theta_{i_j}\right)\phi_{i_j}z_i\\ &+ \sum_{i=1}^{i}\sum_{j=1}^{m_i}\frac{1}{\gamma_{i_j}}\left(\hat{\theta}_{i_j} - \theta_{i_j}\right)\dot{\hat{\theta}}_{i_j}\\ =& -\sum_{i=1}^{n} C_iz_i^2 - \sum_{i=1}^{n}\lambda_i\left(\boldsymbol{\hat{\theta}_i} - \boldsymbol{\theta_i}\right)^T\left(\boldsymbol{\Omega_{ib}\hat{\theta}_i} - \boldsymbol{\eta_{ib}}\right)\\ =& -\sum_{i=1}^{n} C_iz_i^2 - \sum_{i=1}^{n}\lambda_i\left(\boldsymbol{\hat{\theta}_i} - \boldsymbol{\theta_i}\right)^T\boldsymbol{\Omega_{ib}}\left(\boldsymbol{\hat{\theta}_i} - \boldsymbol{\theta_i}\right)\end{aligned} \tag{21}$$

Since $\boldsymbol{\Omega_{ib}}$ becomes positive definite under FE, $\dot{V}_n$ becomes negative definite for all tracking and parameter estimation errors except the origin. To this end, the asymptotic stability for the closed-loop system is guaranteed.

## 4. SIMULATION

Simulations are carried out to check performance of the proposed composite ABKS. As an illustrative example, simulation results with short period mode dynamics for an aircraft will be suggested in this section.

A short period mode dynamics is considered in this simulation as follows.

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}\end{bmatrix} = \begin{bmatrix}Z_\alpha^* & 1\\ M_\alpha^* & M_q^*\end{bmatrix}\begin{bmatrix}\alpha\\ q\end{bmatrix} + \begin{bmatrix}Z_\delta^*\\ M_\delta^*\end{bmatrix}\delta \tag{22}$$

State variables $\alpha$ and $q$ represent an angle of attack and a pitch rate, respectively. $\delta$ denotes a deflection angle of an elevator. $Z_\alpha^*$, $M_\alpha^*$, $M_q^*$, $Z_\delta^*$ and $M_\delta^*$ are aerodynamic derivatives.

(22) is rewritten in the strict-feedback form in (1) with (2) under the assumption that the effect of $Z_\delta^*$ is negligible McLean (1990).

$$
\begin{aligned}
\boldsymbol{x} &= [\alpha, q]^T \\
\boldsymbol{f}(\boldsymbol{x}) &= [0, 0]^T \\
\boldsymbol{g}(\boldsymbol{x}) &= \mathrm{diag}\,[1, M_\delta^*] \\
\boldsymbol{u} &= [q, \delta]^T \\
\boldsymbol{\Delta}(\boldsymbol{x}) &= \left[\Delta_1(\boldsymbol{x}_1'), \Delta_2(\boldsymbol{x}_2')\right]^T \\
&= \left[\boldsymbol{\theta}_1^T \boldsymbol{\phi}_1(\boldsymbol{x}_1'), \boldsymbol{\theta}_2^T \boldsymbol{\phi}_2(\boldsymbol{x}_2')\right]^T \qquad (23)
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{\theta}_1 &= [Z_\alpha^*] \\
\boldsymbol{\phi}_1(\boldsymbol{x}_1') &= [\alpha] \\
\boldsymbol{\theta}_2 &= \left[M_\alpha^*, M_q^*\right]^T \\
\boldsymbol{\phi}_2(\boldsymbol{x}_2') &= [\alpha, q]^T
\end{aligned}
$$

Note that $Z_\alpha^*$, $M_\alpha^*$ and $M_q^*$ are considered as unknown model parameters to be estimated as $\hat{\boldsymbol{\theta}}_1 = \left[\hat{Z}_\alpha^*\right]$ and $\hat{\boldsymbol{\theta}}_2 = \left[\hat{M}_\alpha^*, \hat{M}_q^*\right]^T$ with the proposed composite ABKS. The estimate $\hat{(\cdot)}$ of the parameter $(\cdot)$ is defined as $\hat{(\cdot)} = (\cdot)(1 + D_{(\cdot)})$, where $D_{(\cdot)}$ is a parameter uncertainty level on $(\cdot)$ in percentage. True values for the model parameters are set to be $\boldsymbol{\theta}_1 = [-1.963]$ and $\boldsymbol{\theta}_2 = [-4.749, -3.933]^T$ and $M_\delta^* = -26.685$.

To have critical understandings about closed-loop characteristics with the proposed composite ABKS, simulation results with BKS will be additionally suggested and investigated for cases with and without $D_{(\cdot)}$, as references. For BKS and the proposed composite ABKS, $C_1$ and $C_2$ are set to be 1.5, and angle of attack command $\alpha_c$ is given as low pass filtered $0° \to 1.5° \to 0° \to -1.5° \to 0°$ with $\frac{1}{s+1}$. Simulation parameters for the proposed composite ABKS can be summarized as Table. 1.

Table 1. Simulation parameters for ABKS

| $\boldsymbol{\Gamma_1}$ | $\lambda_1$ | $\boldsymbol{\Gamma_2}$ | $\lambda_2$ | $k$ | $k_L$ | $k_U$ | $\vartheta$ |
|---|---|---|---|---|---|---|---|
| $10^4$ | 0.5 | $\mathrm{diag}\left[10^4, 10^4\right]$ | 0.5 | $10^{-3}$ | 0.1 | 10 | 1 |

Figure. 1 shows each $\alpha$ response for nominal BKS, BKS with $D_{(\cdot)} = 5\%$ and ABKS.

Magnitudes of tracking errors for each case and estimation errors with the proposed composite ABKS are suggested in Figure. 2.

It is shown in Fig. 1 that the closed-loop system with BKS under true model information tracks a desired response determined by $C_1$ and $C_2$ without any steady state error. However, for BKS, 5% error on model information results in about 30% steady state error. The angle of attack response with new composite ABKS converges to the command without any prior knowledge on model parameters. At the early stage, since the parameter estimation is not converged yet, it is observed in Fig. 1 and Fig. 2 that the tracking performance with the proposed composite ABKS is worse than with BKS for the nominal case. As the parameter estimation converges, the tracking performance of the system with new composite ABKS is enhanced, showing similar performance with nominal BKS, as addressed in Fig. 1 and Fig. 2. These simulation results imply that

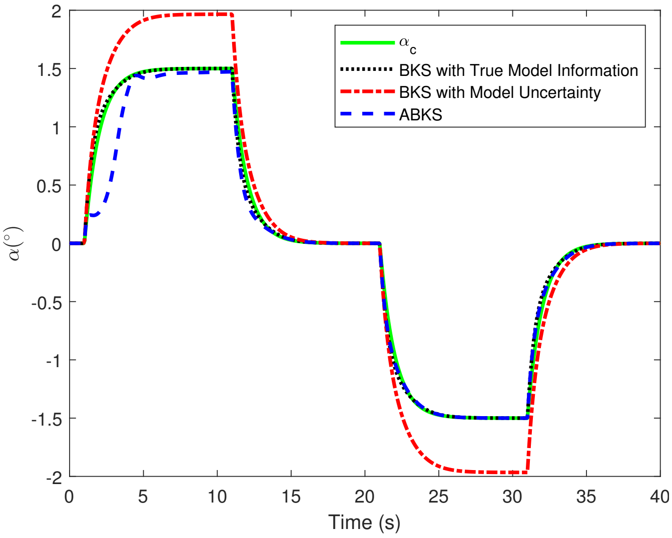


Fig. 1. $\alpha$ Response

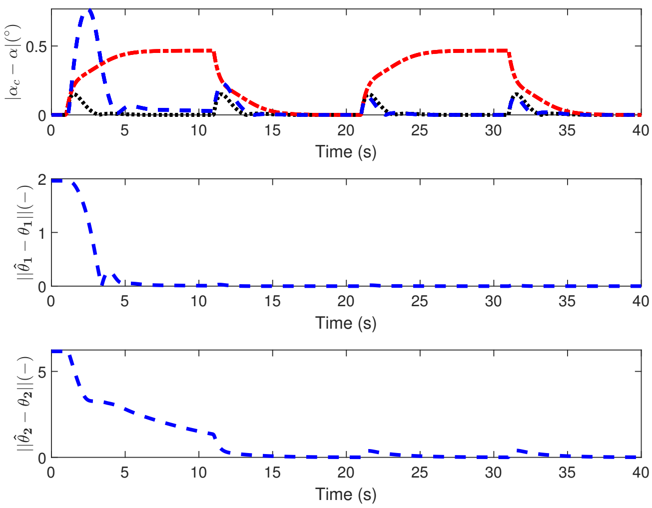


Fig. 2. Tracking and Estimation Error

the information matrix becomes full rank with the finite excitation and the information matrix is maintained to be full rank after the finite exitation. The proposed composite ABKS shows high estimation and tracking performance without persistent excitation.

## 5. CONCLUSION

A new composite ABKS control is successfully suggested with relaxation of PE requirement to FE for parameter convergence and reduction of computational complexity from decreased estimation problem dimension. A composite adaptation method is applied for enhanced estimation and tracking performance. Parameter convergence is accomplished without PE by making the information matrix full rank only with FE. The adaptation law of the proposed composite ABKS is designed by taking advantage of a cascade control structure of BKS. As a result, this adaptation law estimates uncertain model parameters of each loop separately and computational complexity is decreased from the reduced estimation problem dimension. Simulation results are provided to show performance with the proposed composite ABKS under FE.

## REFERENCES

Carrillo, L.G., Dzul, A., and Lozano, R. (2012). Hovering quad-rotor control: A comparison of nonlinear controllers using visual feedback. *IEEE Transactions on Aerospace and Electronic Systems*, 48(4), 3159–3170.

Cho, N., Shin, H.S., Kim, Y., and Tsourdos, A. (2017). Composite model reference adaptive control with parameter convergence under finite excitation. *IEEE Transactions on Automatic Control*, 63(3), 811–818.

Chowdhary, G., Mühlegg, M., and Johnson, E. (2014). Exponential parameter and tracking error convergence guarantees for adaptive controllers without persistency of excitation. *International Journal of Control*, 87(8), 1583–1603.

Chowdhary, G., Yucelen, T., Mühlegg, M., and Johnson, E.N. (2013). Concurrent learning adaptive control of linear systems with exponentially convergent bounds. *International Journal of Adaptive Control and Signal Processing*, 27(4), 280–301.

Ciliz, M.K. (2009). Combined direct and indirect adaptive control for a class of nonlinear systems. *IET Control Theory & Applications*, 3(1), 151–159.

Ciliz, M. (2007). Adaptive backstepping control using combined direct and indirect adaptation. *Circuits, Systems & Signal Processing*, 26(6), 911–939.

Duarte, M.A. and Narendra, K.S. (1989). Combined direct and indirect approach to adaptive control. *IEEE Transactions on Automatic Control*, 34(10), 1071–1075.

Ghommam, J. and Saad, M. (2017). Autonomous landing of a quadrotor on a moving platform. *IEEE Transactions on Aerospace and Electronic Systems*, 53(3), 1504–1519.

Huang, X. and Yan, Y. (2017). Saturated backstepping control of underactuated spacecraft hovering for formation flights. *IEEE Transactions on Aerospace and Electronic Systems*, 53(4), 1988–2000.

Khalil, H.K. (1996). *Nonlinear systems*. Upper Saddle River.

Kim, Y. and Kim, B.S. (2013). Pitch autopilot design for agile missiles with uncertain aerodynamic coefficients. *IEEE Transactions on Aerospace and Electronic Systems*, 49(2), 907–914.

Kokotovic, P.V. (1992). The joy of feedback: nonlinear and adaptive. *IEEE Control Systems Magazine*, 12(3), 7–17.

Lavretsky, E. (2009). Combined/composite model reference adaptive control. *IEEE Transactions on Automatic Control*, 54(11), 2692–2697.

Leitmann, G. (1979). Guaranteed asymptotic stability for some linear systems with bounded uncertainties. *Journal of Dynamic Systems, Measurement, and Control*, 101(3), 212–216.

Lian, B. and Bang, H. (2006). Momentum transfer-based attitude control of spacecraft with backstepping. *IEEE transactions on aerospace and electronic systems*, 42(2), 453–463.

M. Krstic, I.K. and Kokotovic, P. (1995). *Nonlinear and Adaptive Control*. John Wiley and Sons, Inc., New York.

McLean, D. (1990). *Automatic flight control systems*. Prentice Hall, New York.

Narendra, K.S. and Duarte, M.A. (1989). Application of robust adaptive control using combined direct and indirect methods. *International Journal of Adaptive Control and Signal Processing*, 3(2), 131–142.

Parikh, A., Kamalapurkar, R., and Dixon, W.E. (2018). Integral concurrent learning: Adaptive control with parameter convergence using finite excitation. *International Journal of Adaptive Control and Signal Processing*.

Patre, P.M., MacKunis, W., Johnson, M., and Dixon, W.E. (2010). Composite adaptive control for euler–lagrange systems with additive disturbances. *Automatica*, 46(1), 140–147.

Slotine, J.J.E. and Li, W. (1989). Composite adaptive control of robot manipulators. *Automatica*, 25(4), 509–519.

Slotine, J.J.E., Li, W., et al. (1991). *Applied nonlinear control*. Prentice hall Englewood Cliffs, NJ.