Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

The Generalized Internal Model Control Method

O. Taiwo and D. Fasiku

Department of Chemical Engineering, Obafemi Awolowo University, Ile-Ife, Nigeria
(E-mails: femtaiwo@yahoo.com, damfash@gmail.com)

Abstract: The work deals with a generalization of the internal model control method whereby the original process model is suitably simplified to facilitate direct parameterization of feedback controllers. Here, the m/n moment approximant is adopted as the simplified model. The parameterized feedback controller contains a filter that can the tuned for closed loop system acceptable characteristics. The method facilitates cheap computation of controllers satisfying desired performance and stipulated constraints.

Keywords: Generalized internal model control, moment approximants, Structured controllers, controller parameterization, cheap computation of controllers

1. INTRODUCTION

The internal model control (IMC) method has simplified and expedited the design of simple controllers for single input single output (SISO) plants. Its direct generalization to multivariable systems is still a subject of continued research. To appreciate this fact, the reader may confirm the statement of masters in this area in their authoritative book (Morari and Zafiriou, 1989) where on p. 293 they state "except in trivial cases (minimum phase systems) the derivation of the IMC controllers for MIMO systems and the effect of right half plane zeros and poles is quite complex". This work proposes the simplification of the process model to a form which engenders its direct use for rapid feedback controller parameterization. The applicability of the method to small and large plants or plants with or without time delays is equally simple. The method facilitates the design of feedback controllers satisfying desired performance and meeting stipulated constraints. One of the preliminary steps proposed in this generalization is the simplification of plant model as an m/n moment approximant directly facilitating simple feedback controller parameterization. This procedure is directly utilizable in SISO plants thereby further enhancing IMC utility. An exposition of the new method is given in section 2 and its application to various plants is given in section 3. A discussion of the results and conclusions from the work are given in section 4.

2. DESCRIPTION OF THE METHOD

2.1 Internal Model Control Generalization Using Moment Matching

It is assumed that the plant transfer function was originally expressed in any commonly acceptable form. In order to facilitate direct feedback controller parameterization, the m/n moment approximant of the original transfer function G(s) (assumed asymptotically stable and strictly proper) is computed. This is undertaken by expanding G(s) into infinite series:

$$G(s) = \sum_{i=0}^{\infty} G_i s^i \tag{1}$$

Its reduced model R(s) in the right matrix fraction form, without loss of generality, is expressed as:

$$R(s) = (\sum_{i=0}^{m} V_i s^i)(\sum_{i=0}^{n} T_i s^i)^{-1},\ (T_n = I,\ m < n) \tag{2}$$

$R(s)$ is an m/n moment approximant at s=0 if $R(s)$ is asymptotically stable and

$$\sum_{i=0}^{n-1} G_{j-i} T_i = -G_{j-n} \quad (n \le j \le m+n) \tag{3}$$

$$V_j = \sum_{i=0}^{j} G_{j-i} T_i \quad (0 \le j \le m) \tag{4}$$

In (3) and (4), $G_j = 0, j < 0$. A unique solution exists and

$$R(s) = G(s) + O(s^{m+n+1}) \tag{5}$$

where the notation means that the power series expansions not only exist but also agree up to terms of degree (m+n). Note that the existence of the series (1) in a region R is assured if G(s) is analytic at all points s =s_0 in R (Apostol, 1982). However, if expansion about s=0 does not yield a stable m/n approximant, Taiwo and Krebs (1995) have shown how a stable approximant may be obtained by moment matching about more than the single point s = 0.

2.2 Derivation of Feedback Controllers Using Simplified Moment Approximants

Consider the situation where the 0/1 moment approximant is used as the moment approximant. Then

$$R(s) = V_o(Is + T_o)^{-1} \tag{6}$$

hence the (improper) internal model controller $\overline{Q}$, is given by

$$\overline{Q}(s) = R^{-1} = (Is + T_o)V_o^{-1} \tag{7}$$

Properness of (7) is achieved by introducing the filter

$$f = 1/(\lambda s + 1) \tag{8}$$

such that

$$Q(s) = \bar{Q}(s)f \qquad (9)$$

The conventional feedback controller $C(s)$ (Fig 1) is given by

$$C(s) = \frac{1}{\lambda s}\left(V_0^{-1}s + G_o^{-1}\right) \qquad (10)$$

And note that $G_o^{-1} = T_0 V_0^{-1}$

For illustration purposes, suppose G(s) is 2×2, (10) simplifies to

$$C(s) = \frac{1}{\lambda s}\begin{bmatrix} \bar{q}_{11}(s) & \bar{q}_{12}(s) \\ \bar{q}_{21}(s) & \bar{q}_{22}(s) \end{bmatrix} \qquad (11)$$

where

$$\bar{q}_{ij}(s) = \hat{V}_{oij}s + \hat{G}_{oij} \qquad (12)$$

and $\hat{V}_{oij}$, $\hat{G}_{oij}$ respectively denotes the (i,j)th element of V_o^{-1} and G_o^{-1}. It is therefore clear that 0/1 approximant parameterizes a PI controller and λ is the tuning parameter which may be chosen to produce a closed loop system with desirable characteristics.

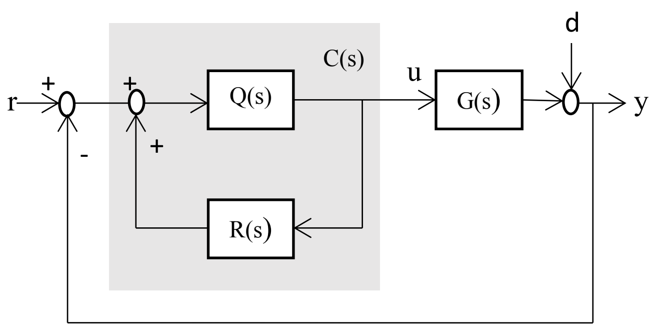

Fig 1: IMC Feedback control structure

Clearly, other values of m and n different from $m = 0$ and $n = 1$ could be used when simplifying the plant model. For example 0/2 and 1/2 moment approximants can be used to parameterize PID controllers. Following exactly the same procedure given above, the 0/2 approximant can be used to parameterize the feedback controller

$$C(s) = \frac{1}{\lambda s}\left(V_0^{-1}s^2 + T_1V_0^{-1}s + T_0V_0^{-1}\right) \qquad (13)$$

While the ½ approximant parameterizes the PID feedback controller

$$C(s) = \frac{1}{\lambda s}\left(\left(V_0^{-1} - T_1V_0^{-1}V_1V_0^{-1} + T_0V_0^{-2}V_1^2V_0^{-1}\right)s^2 + \left(T_1V_0^{-1} - T_0V_0^{-1}V_1V_0^{-1}\right)s + T_0V_0^{-1}\right) \qquad (14)$$

It should be noted that unlike (10) and (13), equation (14) is part of an infinite series and was truncated after three terms to yield a controller possessing PID structure. The coefficient of s and the constant term in the matrix polynomials (10), (13) and (14) are always equal being respectively given by $-G_0^{-1}G_1G_0^{-1}$ and G_0^{-1} whenever these coefficients result from recursions (3) and (4) and moments are matched about s=0 only. Consequently, the same PI controller would be parameterized by using these two coefficients from any of the three simplified models. The caveat to be observed here though is that it is expedient to utilize only matrix polynomials (10), (13) and (14) having their zeros in the left half plane to parameterize the PI or PID controllers. Consequently, the following steps should be followed in parameterizing controllers. After simplifying the original transfer function to a moment approximant, the stability of the latter should be ascertained. Unstable approximants should not be used for controller parameterization. Whenever instability is encountered, stable approximants can be obtained by matching moments about more than a single point as demonstrated by Taiwo and Krebs (1995). Another issue is the choice of λ.

Comments:

1. A suitable value of λ is usually based on acceptable system closed loop characteristics.
2. It may be expedient in certain situations to use different tuning parameters for the different coefficients of the polynomials in (10), (13) or (14) (whichever one pertains to the problem at hand)
3. If the desirable closed loop characteristics cannot be so expeditiously arrived at, it may sometimes be expedient to scale different columns of the coefficients in the matrix polynomials (10), (13) or (14) unequally.
4. Whenever getting closed loop characteristics is not easily amenable to manual trial and error, automatic computation may be resorted to.

The next section gives copious expositions of the new method. μ_{RP} denotes the structured singular value for robust performance. A closed loop system is said to have robust performance when μ_{RP} is less than 1. As usual, ISE and IAE respectively denote integral of squared error and integral of absolute error.

3. ILLUSTRATIVE EXAMPLES

3.1 Wood and Berry column

Consider the Wood and Berry distillation column (1973) given as:

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}\begin{bmatrix} u1 \\ u2 \end{bmatrix} + \begin{bmatrix} \frac{3.8e^{-8.1s}}{14.9s+1} \\ \frac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix}[F] \qquad (15)$$

Where u_1 = Overhead reflux flowrate, u_2 = Bottoms steam flowrate, y_1 = Overhead mole fraction of methanol, y_2 = Bottoms mole fraction of methanol, F= feed flowrate into the column. It is desired to use a PI controller on this column. Consequently, C(s) is given by (10) where

$$V_0^{-1} = a_1 = \begin{bmatrix} 1.7011 & -0.6479 \\ 0.5520 & -1.0913 \end{bmatrix} \qquad (16a)$$

$$G_o^{-1} = T_0V_0^{-1} = a_0 = \begin{bmatrix} 0.1570 & -0.1529 \\ 0.0534 & -0.1036 \end{bmatrix} \qquad (16b)$$

It now remains to choose λ in (10). Through tuning, acceptable closed loop responses were obtained with

$$C(s) = \frac{a_1}{7} + \frac{a_0}{3s} \qquad (17)$$

Three closed loop systems are compared in Fig 2, where the closed loop system designed using the proposed method is generally favorable. In addition, the input uncertainty weight (w_u) and the performance weight (w_p) used for robustness analysis are given by:

$$w_u = \frac{0.5s+0.1}{0.4s+1}, \text{ and } w_p = \frac{5.1s+0.34}{15s} \qquad (18)$$

Unless otherwise stated, only the responses to a unit step change in reference 1 and possibly simultaneous unit step disturbance changes have been displayed in order to conserve space. Nevertheless, the cost functions given in Tables are the cumulative values for responses to step changes in all reference inputs.

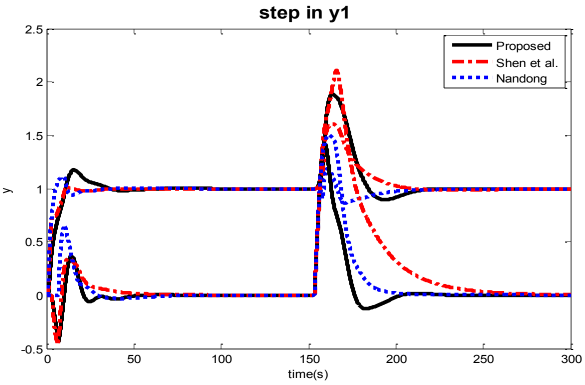

Fig 2: Responses of the Wood and Berry column

Table 1: Performance of the Wood and Berry column

Method	IAE	ISE	μ_{RP}
Proposed	91.03	60.74	0.9463
Shen et. al	157.5	152.3	0.7253
Nandong	81.37	64.77	0.8176

3.2 Alatiqi Column

This model of a distillation column has been taken from Shen et al. (2010). It is given as

$$G(s) = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \qquad (19)$$

Where; $g_{11} = \frac{4.09e^{-1.3s}}{(33s+1)(8.3s+1)}$; $g_{12} = \frac{-6.36e^{-0.2s}}{(31.6s+1)(20s+1)}$; $g_{13} \frac{-0.25e^{-0.4s}}{(21s+1)}$; $g_{14} = \frac{-0.49e^{-5s}}{(22s+1)^2}$; $g_{21} = \frac{-4.17e^{-4s}}{(45s+1)}$;

$g_{22} = \frac{6.93e^{-1.01s}}{(44.6s+1)}$; $g_{23} = \frac{-0.05e^{-5s}}{(34.5s+1)^2}$; $g_{24} = \frac{1.53e^{-2.8s}}{(48s+1)}$;

$g_{31} \frac{-1.73e^{-17s}}{(13s+1)^2}$; $g_{32} \frac{5.11e^{-11s}}{(13.3s+1)^2}$; $g_{33} \frac{4.61e^{-1.02s}}{(18.5s+1)}$;

$g_{34} = \frac{-5.48e^{-0.5s}}{15s+1}$; $g_{41} = \frac{-11.18e^{-2.6s}}{(43s+1)(6.5s+1)}$;

$g_{42} = \frac{14.04e^{-0.02s}}{(45s+1)(10s+1)}$; $g_{43} = \frac{-0.1e^{-0.05s}}{(31.6s+1)(5s+1)}$

$g_{44} = \frac{4.49e^{-0.6s}}{(48s+1)(6.3s+1)}$;

After computing a_1 and a_0 as in the previous example (omitted here to conserve space), the uncertainty and performance weights for this system are given by

$$w_u = \frac{2.5s+0.1}{2.5s+1}, \text{ and } w_p = \frac{s/2.75+0.001}{s} \qquad (20)$$

On observing the responses, two sets of parameters were used, as shown in Table 2, to exemplify the responses obtained. Since, the responses of the system with larger gains have smaller integral error, this may be deemed better. The design here is favourable to that of Shen et al (2010). Fig 3 displays the responses for the larger controller parameters.

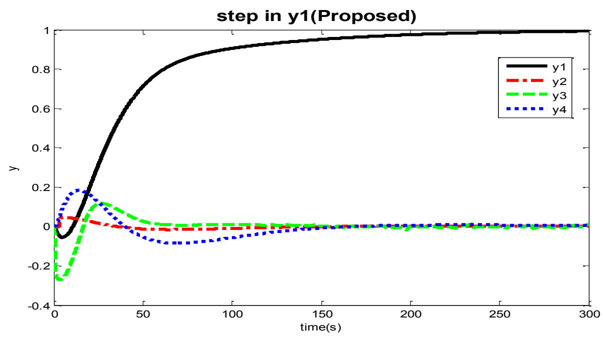

Fig 3: Response of Alatiqi distillation column

Table 2: Performance of the Alatiqi distillation column

Method	IAE	ISE	μ_{RP}
Centralized	247.6472	110.5697	0.6102
C(s)=[a_1+a_0/s]/50			
Centralized	176.1605	70.5200	0.9233
C(s)=[a_1+a_0/s]/25			
Shen et al.	311.3	171.5	16.8005

3.3 Nandong (2015)

This is a model of a two stage extractive five input, five output. alcoholic fermentation process.

$$G(s)=\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{55} \end{bmatrix} \quad (21)$$

Where: $g_{11}=\frac{-0.57e^{-4.2s}}{(33s+1)}$, $g_{12}=\frac{0.25e^{-4.7s}}{(40.9s+1)}$, $g_{13}=\frac{53.6e^{-2.1s}}{(51.1s+1)}$,

$g_{14}=\frac{0.20e^{-2.5s}}{(31.9s+1)}$, $g_{15}=\frac{27.7e^{-3.3s}}{(64.3s+1)}$, $g_{21}=\frac{0.10(1-0.94s)e^{-1.6s}}{(7.72s+1)(1.53s+1)}$,

$g_{22}=\frac{-0.07(1-4.2s)e^{-1.7s}}{(13.2s+1)(1.5s+1)}$, $g_{23}=\frac{-24e^{-2.2s}}{(7.39s+1)}$,

$g_{24}=\frac{-0.04(1+146s)e^{-3.8s}}{(210.2s+1)(1.81s+1)}$, $g_{25}=\frac{0.25(1-0.14s)e^{-1.8s}}{(5.65s+1)(1.54s+1)}$,

$g_{31}=\frac{-0.08(1+23.7s)e^{-4.2s}}{(3.19s+1)(2.42s+1)}$, $g_{32}=\frac{0.06(1+27.2s)e^{-4.4s}}{8s^2+4.98s+1}$,
$g_{33}=\frac{-101e^{-2.9s}}{(10.6s+1)}$, $g_{34}=\frac{-0.28(1+13.4s)e^{-3.1s}}{(9.57s+1)(1.87s+1)}$,

$g_{35}=\frac{-26.2(1-8.62s)e^{-3.3s}}{(3.83s+1)(2.25s+1)}$, $g_{41}=\frac{-0.004(1+264.9s)e^{-1.9s}}{(4.27s+1)(1.95s+1)}$, $g_{42}=\frac{0.02(1+38.3s)e^{-4.5s}}{(3.93s+1)(2.13s+1)}$, $g_{43}=\frac{-4.93(1-33s)e^{-2s}}{(4.34s+1)(1.93s+1)}$,

$g_{44}=\frac{-0.73e^{-1.1s}}{(2.88s+1)}$, $g_{45}=\frac{-10.1(1-12.2s)e^{-1.4s}}{(4.39s+1)(1.93s+1)}$,

$g_{51}=\frac{-0.14e^{-0.6s}}{(4.26s+1)}$, $g_{52}=\frac{0.12e^{-0.7s}}{(4.33s+1)}$, $g_{53}=\frac{26.4e^{-0.6s}}{(4.34s+1)}$,

$g_{54}=\frac{-0.04(1+23.2s)e^{-1.2s}}{(26.6s+1)(4.36s+1)}$, $g_{55}=\frac{24.3e^{-0.8s}}{(4.37s+1)}$.

$$C(s)=\frac{1}{50}\left[a_1+\frac{a_0}{s}\right] \quad (22)$$

The uncertainty and performance weights used for robustness analysis are given as

$$w_u=\frac{0.01s+0.15}{0.0067s+1},\ \text{and}\ w_p=\frac{s/2.75+0.01}{s} \quad (23)$$

The closed loop responses with C(s) given above were deemed acceptable and the closed loop response for a change in reference 1 is given in Fig 4. This design is compared to that obtained by Nandong (2015) using the performance metrics given in Table 3. It is seen that the proposed system's performance is favourable.

Table 3: Performance of the Nandong plant

METHOD	ISE	IAE	μ_{RP}
C(s) (Proposed)	132.8470	309.6825	0.9632
NANDONG	132.8865	404.5993	1.7639

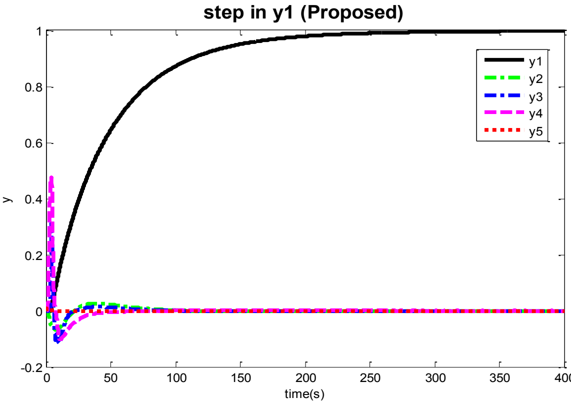

Fig 4. Responses of the Nandong plant with the proposed controller.

3.4 HVAC System

The HVAC is a four input, four output interactive system taken from Garrido et al (2011). It is represented by

$$G(s)=\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \quad (24)$$

Where; $g_{11}=\frac{-0.098e^{-17s}}{(122s+1)}$; $g_{12}=\frac{-0.036e^{-27s}}{(149s+1)}$; $g_{13}=\frac{-0.014e^{-32s}}{(158s+1)}$; $g_{14}=\frac{-0.017e^{-30s}}{(155s+1)}$; $g_{21}=\frac{-0.043e^{-25s}}{(147s+1)}$; $g_{22}=\frac{-0.092e^{-16s}}{(130s+1)}$; $g_{23}=\frac{-0.011e^{-33s}}{(156s+1)}$; $g_{24}=\frac{-0.012e^{-34s}}{(157s+1)}$;
$g_{31}\frac{-0.012e^{-31s}}{(153s+1)}$; $g_{32}=\frac{-0.016e^{-34s}}{(151s+1)}$; $g_{33}\frac{-0.102e^{-16s}}{(118s+1)}$;

$g_{34}=\frac{-0.033e^{-26s}}{146s+1}$; $g_{41}=\frac{-0.013e^{-32s}}{(156s+1)}$; $g_{42}=\frac{-0.015e^{-31s}}{(159s+1)}$;
$g_{43}=\frac{-0.029e^{-25s}}{(144s+1)}$; $g_{44}=\frac{-0.108e^{-18s}}{(128s+1)}$

$$C(s)=\frac{1}{45}\left[a_1+\frac{a_0}{s}\right] \quad (25)$$

The uncertainty and performance weights are given as:

$$w_u=\frac{10s+0.2}{5s+1},\ \text{and}\ w_p=\frac{s/2+0.008}{s} \quad (26)$$

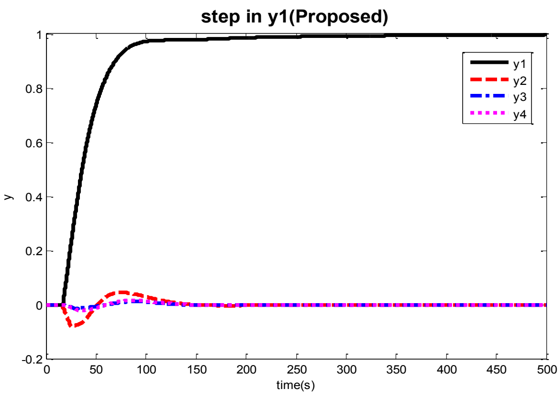

Fig 5. Closed loop responses of the HVAC system with controller (25)

The closed loop responses using the proposed centralized controller (25) are displayed in Fig 5 and compared with that obtained by Garrido et al., who used the method of inverted decoupling, in Table 4.

Table 4: Performance of the HVAC system

METHOD	IAE	ISE	μ_{RP}
Proposed	206.6	123.0	0.998
Garrido et al.	281.5	193.8	0.970

3.5 SISO System

A non-minimum phase, single input single output (SISO) system has been taken from Luyben (2000).This system has also been studied by Chien et al., (2003) , and Kaya and Cengiz (2017). The transfer function is given by (27).

$$G(s) = \frac{(-0.2s+1)e^{-1.6s}}{(s+1)(s+1)} \quad (27)$$

PID controller parameterization was achieved for this system after computing the ½ moment approximant of the model as described under section 2; giving, a_2 =6.24; a_1 = 3.80; a_0 = 1. Where a_2, a_1 and a_0 are the coefficients of the polynomial in (14), from left to right. The parameterized PID controller upon tuning is given by:

$$C(s) = \left[\frac{a_1}{4.5} + \frac{a_0}{2.8s} + \frac{a_2 s}{9}\right] \quad (28)$$

The closed loop response with the proposed controller (28) is compared with the closed loop responses obtained with the PID controllers from Chien et al., (2003), Kaya and Cengiz (2017), and the conventional IMC procedure. The responses are shown in Fig 6 and the performance metrics are given in Table 5.

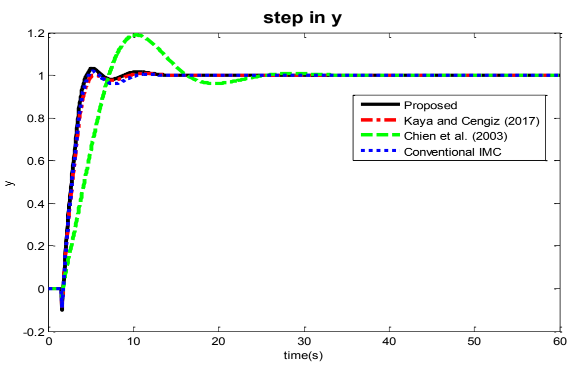

Fig 6. Closed loop responses of the SISO system

Table 5: Performance of the SISO system

METHOD	IAE	ISE
Proposed	2.9656	2.4522
Kaya and Cengiz	3.0537	2.5157
Chien et al.(2003)	5.6138	3.5918
Conventional IMC	3.0491	2.4972

3.6 Heat Integrated Distillation Column

The model in (29) is that of an integrated distillation column which was first studied by Ding and Luyben (1990), and has also been used by Escobar and Trierweiler (2013), where frequency response approximation was proposed for controller parameterization.

$$\begin{bmatrix} XB_1 \\ XD_2 \\ XS_2 \\ XB_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} Q_1 \\ R_2 \\ S_2 \\ Q_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ gd_{21} & gd_{22} \\ gd_{31} & gd_{32} \\ gd_{41} & gd_{42} \end{bmatrix} \begin{bmatrix} Z1 \\ Z2 \end{bmatrix} \quad (29)$$

Where: $g_{11} = \frac{-7.39e^{-s}}{(11s+1)(s+1)}$, $g_{12} = 0$, $g_{13} = 0$, $g_{14} = 0$,

$g_{21} = \frac{-0.11(200s+1)e^{-5s}}{(20s+1)^3}$, $g_{22} = \frac{10.1e^{-s}}{(28s+1)(4s+1)}$,
$g_{23} \frac{1.18e^{-11s}}{(31s+1)(6s+1)}$, $g_{24} \frac{-18.3e^{-s}}{(28s+1)(5s+1)}$, $g_{31} = \frac{1.9e^{-2s}}{(4s+1)^2}$,
$g_{32} = \frac{1.7(200s+1)e^{-1.4s}}{(108s+1)(s+1)^2}$, $g_{33} = \frac{-3.15e^{-s}}{(3s+1)(0.3s+1)}$,

$g_{34} = \frac{-1.27(188s+1)e^{-s}}{(68s+1)(s+1)}$, $g_{41} = \frac{4.9e^{-1.6s}}{(40s+1)(3s+1)}$,
$g_{42} = \frac{-8.21e^{-2.5s}}{(24s+1)(3s+1)}$ $g_{43} = \frac{12e^{-1.5s}}{(29s+1)(3s+1)}$, $g_{44} = \frac{-19.4e^{-s}}{(26s+1)(3s+1)}$

$gd_{11} = gd_{12} = 0$; $gd_{21} = \frac{2.42e^{-5s}}{(3s+1)(26s+1)^2}$, $gd_{22} = \frac{-2.47e^{-5s}}{(3s+1)(22s+1)^2}$; $gd_{31} = \frac{0.592e^{-5s}}{(7s+1)^2}$, $gd_{32} = \frac{1.83e^{-6s}}{(25s+1)(2s+1)}$;
$gd_{41} = \frac{-1.51e^{-19s}}{(45s+1)(5s+1)^2}$, $gd_{42} = \frac{-4.52e^{-8s}}{(50s+1)(7s+1)^2}$

The 0/1 moment approximant of this model was found to be unstable upon expansion about s=0 only. Therefore, expansion was done about s=0 and another point (s=0.0391), obtained by optimization, as explained in Taiwo and Krebs (1995). By tuning,

$$C(s) = \frac{1}{11}\left[a_1 + \frac{a_0}{s}\right] \quad (30)$$

The uncertainty and performance weights for robustness analysis are given as

$$w_u = \frac{s+0.15}{0.5s+1}, \text{ and } w_p = \frac{s/2.25+0.04}{s} \quad (31)$$

However, it was found that closed loop responses did not deteriorate much when all the other elements apart from element (1,1) in column 1 were set equal to zero for a_1 and a_0 in (30). In a bid to improve the overall (servo and regulatory) closed loop responses, the latter controller was used as the starting point in the automatic computation of the final controller given in (32) using MATLAB optimization toolbox.

$$C(s) = \begin{bmatrix} -0.2777 & 0 & 0 & 0 \\ 0 & 0.3965 & 0.1188 & 0.4442 \\ 0 & 0.6300 & -0.1022 & 0.4260 \\ 0 & -0.1741 & -0.0925 & 0.1619 \end{bmatrix} + \frac{1}{s}\begin{bmatrix} -0.0161 & 0 & 0 & 0 \\ 0 & 0.0706 & 0.1312 & 0.0371 \\ 0 & -0.0252 & -0.0930 & 0.0257 \\ 0 & 0.0332 & 0.0899 & 0.0153 \end{bmatrix} \quad (32)$$

The closed loop responses obtained using the proposed controller (32) which are displayed in Fig 7 are compared with those obtained by Escobar and Trierweiler, (2013) in Table 6 respectively for unit step changes in XB_1 reference at t=0, disturbance reference Z1 at t = 200, and disturbance reference Z2 at t=600.

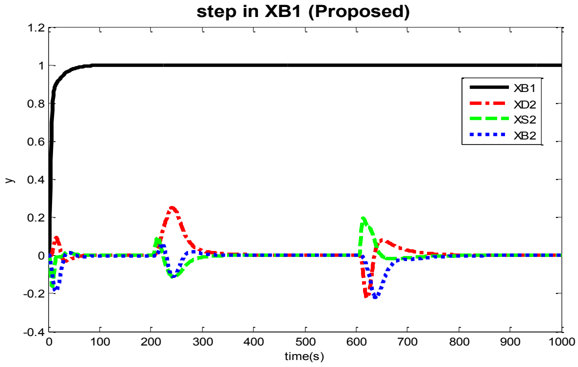

Fig 7. Closed loop responses of the heat integrated column with the proposed controller (32)

Table 6: Performance of the heat integrated column

METHOD	IAE	ISE	μ_{RP}
Proposed	246.2751	42.7055	0.9962
Escobar and Trierweiler,(2013)	375.4781	76.9556	1.3703

4 DISCUSSION AND CONCLUSIONS

The philosophy of IMC is to compute the controller by inverting either the full or suitably modified process model. This direct approach usually encounters difficulties especially in the case of multivariable transfer functions. The approach taken here is to simplify the model using moment matching. Thereafter, the proposed procedure can be used to determine controller type and parameters. Determination of controller parameters is based on choosing a tuning parameter. Easy determination of a tuning parameter that would make the system meet several closed loop performance demands is a welcome development in view of the complexity of the typical multivariable system. This is a general method which is also applicable to SISO systems. Several examples are given in the paper to show how erstwhile difficult feedback control problems can be easily solved by the proposed technique. The new method has been used to design simple PI(D) feedback controllers for MIMO and SISO systems. Although the new method utilizes a single tuning parameter, more than one tuning parameter may sometimes make the procedure more versatile. It may therefore be expedient to use different values of the tuning parameter for the proportional and integral part of, for example, a PI controller in order to ensure flexibility as well as engendering faster and best results. The method also does not preclude the use of optimization when necessary in which case the best tuned values would be initial parameters.

REFERENCES

Apostol, T. M. (1982). Mathematical Analysis, Addison-Wesley, Second Edition.

Bequette, B.W. (2002). Process Control:Modeling,Design and Simulation. Prentice Hall PTR.

Chien, I. L, Chung,Y.C., Chen,B.S., and Chuang,C.Y. (2003).“Simple PID Controller Tuning Method for Processes with Inverse Response Plus Dead Time or Large Overshoot Response Plus Dead Time,” Ind. Eng. Chem.Res., vol. 42, no. 20, pp. 4461–4477

Ding, S., Luyben, W., (1990). Control of a heat integrated complex distillation configuration. Ind. Chem. Eng. Res. 29, 1240–1249.

Escobar, M. and Trierweiler, J.O. (2013). Multivariable PID controller desing for chemical processes by frequency response approximation. *Chem. Eng. Sci*., (88):1-15.

Garrido, J., Vazquez, F. and Morilla, F. (2011). An extended approach of inverted decoupling. *Journal of process control*, (21):55-68.

Kaya, I., Cengiz H., (2017). “Optimal Analytical PI and PID tuning rules for controlling stable processes with inverse response”. Conference Paper.

Luyben, W.L., (2000) “Tuning Proprotional-Integral Controllersfor Processes with Both Inverse Response and Deadtime,” Ind. Eng. Chem. Res., vol. 39, no. 4, pp. 973–976.

Morari, M. and Zafiriou, E. (1989). Robust Process Control. Prentice Hall, New Jersey, Englewood Ciffs.

Nandong, J. (2015). Heuristic-based multi-scale control procedure of simultaneous multi-loop PID tuning for multivariable processes. *Journal of process control*, (35):101-112.

Shen, Y., Cai, W and Li, S. (2010). Multivariable process control: Decentralized, Decoupling, or sparse? *Ind. Eng. Chem. Res*, 49, 761-771.

Taiwo, O. and Krebs, V. (1995) Multivariable system simplification using moment matching and optimization. *IEE Proc. Control Theory Application*, Vol.142 pp103-110.

Wood R and Berry M. (1973). “Terminal composition control of a binary distillation column,” *Chemical Engineering Science*, vol. 28, pp. 1707-1717.