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Fixed-order controller synthesis for monotonic closed-loop responses: a linear programming approach ⋆

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Abstract: We consider the problem of synthesizing dynamic controllers to guarantee monotonic closed-loop step responses. Restricting our attention to controllers which yield positive closed-loop systems, we derive synthesis conditions that are linear in the controller parameters. A linear programming formulation that attempts to optimize the decay rate of the closed-loop system while ensuring asymptotic stability and monotonic step response is developed. An alternative approach which guarantees closed-loop stability and a near-monotonic response is also introduced. Several illustrative examples demonstrate the effectiveness of the approach.

Keywords: linear control systems, discrete-time systems, linear programming, monotonic step response, nonnegative impulse response, positive systems

1. INTRODUCTION

The condition that a transfer function should have a non-negative impulse response appears in many transient performance criteria for control systems. A common example is the desire that a closed-loop step response should not overshoot. Although this problem has been considered several times in the past *e.g.*, (Liu and Bauer, 2008, 2009; Du et al., 2019), it has not yet been fully solved. Many existing results are based on the following lemma: if $H(z)$ is analytic in $|z| \geq 1$, then its impulse response $\{h_t\}$ is non-negative if and only if the sequence of transfer functions

$$\begin{aligned} H_0(z) &= H(z) \\ H_{k+1}(z) &= -z\frac{d}{dz}H_k(z), \quad k \geq 0 \end{aligned} \tag{1}$$

does not have a positive real zero outside the unit disk (Malik et al., 2009). Unfortunately, this result is difficult to use for control synthesis, since the conditions lead to complicated expressions in the transfer function coefficients. In this note we show how simpler conditions on the transfer function coefficients allow to solve the controller synthesis problem using convex optimization.

Many approaches have been proposed for designing monotonic controllers. For example, Malik et al. (2009) used (1) and the Markov-Lukács theorem to develop a two-parameter controller synthesis procedure for a particular class of systems. Mohsenizadeh et al. (2012) built on the work by Malik et al. (2009) and formulated a PID design procedure for achieving monotoic step responses in terms of polynomial matrix inequalities. Schmid and Ntogramatzidis (2010, 2012) investigated the state feedback design problem for systems in state space form with the aim of avoiding overshoot and undershoot in step respones. An LMI approach for monotonic state feedback design was proposed in Garone and Ntogramatzidis (2015). For systems described by transfer functions, it was shown in Darbha and Bhattacharyya (2002) that a combination of cascade, pre-filter and feedback compensators exists that results in a non-negative closed-loop impulse response if and only if the plant does not have positive non-minimum phase zeros. Corresponding results for continuous-time systems were developed in Darbha (2003).

In this paper, we show that enforcing the closed-loop transfer function to have a positive realization allows the monotonic synthesis problem to be solved using linear programming. In contrast to Malik et al. (2009) and Mohsenizadeh et al. (2012), who provide outer approximations for the admissible controller parameter regions, our synthesis method is based on a convex inner approximation, and thus guarantees that the desired characteristics are achieved. Moreover, while the synthesis procedure in Darbha (2003) may yield controllers of very high orders, our method allows to fix the controller order a priori. To deal with the fact that not all systems can be rendered monotonic using cascade compensators, we also propose an approach for designing controllers with a *near-monotonic* closed-loop response. Several numerical examples illustrate the effectiveness of our approach.

Notation The set of real numbers is denoted $\mathbb{R}$. We let $\mathbf{1}_n \in \mathbb{R}^n$ be the vector of all ones, $\mathbf{0}_n \in \mathbb{R}^n$ be the vector of all zeros and e_i be the i^{th} Euclidean basis vector. For a vector $a \in \mathbb{R}^n$, the inequality $a \geq \mathbf{0}_n$ means that all elements of a are non-negative. For a matrix $M \in \mathbb{R}^{n\times m}$, $[M]_{ij}$ denotes its element in row i and column j; If $a \in \mathbb{R}^n$, then its Toeplitz matrix $T(a) \in \mathbb{R}^{(n+m-1)\times m}$ satisfies $[T(a)]_{ij} = a_{i-j}$ if $i - j \in [0, n-1]$ and 0 otherwise.

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2. CONDITIONS FOR A NON-NEGATIVE IMPULSE RESPONSE

Given a discrete-time SISO system with transfer function

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 z^n + b_1 z^{n-1} + \ldots + b_{n-1} z + b_n}{z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n} \quad (2)$$

we are interested in conditions on the coefficients b and a which ensures that the impulse response $\{h_t\}$ is non-negative. If the transfer function describes the input-output relation of the closed-loop system, then the problem is equivalent to finding conditions under which the closed-loop step response is monotonically increasing.

There is no simple way to express the condition that the impulse response should be non-negative using the corresponding transfer function coefficients. Generally speaking, one would need to perform an inverse Z-transform of the transfer function and then impose a sign restriction on the resulting signal in the time domain.

The next theorem introduces an interesting class of transfer functions whose impulse responses $\{h_t\}$ are ensured to be non-negative, *i.e.* $h_t \geq 0$ for $t \geq 0$.

Theorem 1. In (2) assume $b_k \geq 0$ for $0 \leq k \leq n$ and $a_k \leq 0$ for $1 \leq k \leq n$. Then $h_t \geq 0$ holds for all $t \geq 0$.

Proof. The transfer function (2) has the following controllable canonical realization

$$\begin{aligned} x_{t+1} &= A_s x_t + B_s u_t \\ y_t &= C_s x_t + D_s u_t \end{aligned}$$

where

$$A_s = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}, \quad B_s = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (3)$$

$$C_s = [b_n - a_n b_0 \ \cdots \ b_1 - a_1 b_0], \qquad D_s = b_0$$

Under the given assumptions, A_s, B_s, C_s and D_s are all non-negative. Hence, its impulse response

$$h_t = \begin{cases} C_s A_s^{t-1} B_s, & t \geq 1 \\ b_0, & t = 0 \end{cases} \quad (4)$$

is also non-negative.

3. STABILITY

Next, we consider how the conditions for a non-negative impulse response above can be extended to ensure asymptotic stability. It is well-known that the exact stability region in the space of characteristic polynomial coefficients a is not convex (Nurges, 2009; Ackermann, 1993). It is therefore common to find outer and inner convex approximations of this region, leading to necessary and sufficient conditions of stability, respectively. A well-known inner approximation is Cohn's diamond (Ackermann, 2012)

$$\sum_{k=1}^{n} |a_k| < 1 \quad (5)$$

which is simple but conservative. In our context however, under the restrictions already made on the signs of characteristic polynomial coefficients a in Theorem 1, Cohn's criterion becomes both necessary and sufficient and therefore adds no additional conservatism to our problem. This is asserted in the following lemma.

Lemma 2. Consider the LTI system with transfer function (2) and assume that $a_k \leq 0$ for all $1 \leq k \leq n$. This system is Schur stable if and only if (5) is met, i.e. $1+\sum_{k=1}^{n} a_k > 0$.

Proof. Sufficiency is well-established. For necessity, assume that (5) does not hold and consider the characteristic polynomial $A(z)$ on the ray $z \in [0, +\infty)$. Since A is continuous with $A(1) \leq 0$ and $\lim_{z \to +\infty} A(z) > 0$ it has at least one real unstable root in the range $[1, +\infty)$.

4. MONOTONIC CONTROL SYNTHESIS

Based on the results presented so far, a design technique is now provided which ensures a stable and monotonic closed-loop step response. Consider the plant with transfer function (2) with the cascade compensator

$$C(z) = \frac{F(z)}{G(z)} = \frac{f_0 z^l + f_1 z^{l-1} + \ldots + f_{l-1} z + f_l}{g_0 z^l + g_1 z^{l-1} + \ldots + g_{l-1} z + g_l} \quad (6)$$

in a negative feedback loop giving rise to the following closed-loop transfer function

$$H_{\text{cl}}(z) = \frac{B_{\text{cl}}(z)}{A_{\text{cl}}(z)} = \frac{B(z)F(z)}{A(z)G(z) + B(z)F(z)} \quad (7)$$

The following theorem derives conditions on the l^{th}-order compensator (6) which ensure that the closed-loop system H_{cl} is stable and has a monotonic step response.

Theorem 3. Let $a = [1 \ a_1 \ \cdots \ a_n]^T$ and $b = [b_0 \ b_1 \ \cdots \ b_n]^T$ be coefficient vectors of $A(z)$ and $B(z)$ in (2), and let $T(a)$ and $T(b)$ be their corresponding Toeplitz matrices of dimension $(n+l+1) \times (l+1)$. If $f = [f_0 \ f_1 \ \cdots \ f_l]^T \in \mathbb{R}^{l+1}$ and $g = [g_0 \ g_1 \ \cdots \ g_l]^T \in \mathbb{R}^{l+1}$ satisfy

$$\begin{aligned} &T(b)f \geq \mathbf{0} \\ &{e_1}^T(T(a)g + T(b)f) = 1 \\ &W(T(a)g + T(b)f) \geq \mathbf{0} \\ &\mathbf{1}^T(T(a)g + T(b)f) \geq \epsilon \end{aligned} \quad (8)$$

where $W = [\mathbf{0}_{n+l} \ \ -I_{n+l}] \in \mathbb{R}^{(n+l)\times(n+l+1)}$ and $\epsilon > 0$, then the controller (6) ensures that the closed-loop system (7) is Schur stable and has a monotonic step response.

Proof. It follows from (7) that the coefficients $\hat{b}_k$ of $B_{\text{cl}}(z)$ are given by $\hat{b}_k = (b * f)_k$. Similarly, the coefficients $\hat{a}_k$ of $A_{\text{cl}}(z)$ are given by $\hat{a}_k = (a * g)_k + (b * f)_k$. These relationships can be written by means of Toeplitz matrices:

$$\begin{aligned} \hat{b} &= T(b)f \\ \hat{a} &= T(a)g + T(b)f \end{aligned} \quad (9)$$

To ensure monotonicity using Theorem 1, we restrict $\hat{b}$ to be non-negative, $\hat{a}_k$ to be non-positive for $1 \leq k \leq n+l$ and fix $\hat{a}_0 = 1$ so that the closed loop system's denominator is monic. Finally, the stability conditions in Lemma 2, $\sum_{k=0}^{l+n} \hat{a}_k > 0$, are expressed as $\sum_{k=0}^{l+n} \hat{a}_k \geq \epsilon$ with $\epsilon > 0$.

It is worth mentioning that the parameter space introduced in Theorem 3 is convex, and that controller parameters, whenever they exist, can be found by solving a linear programming feasibility problem. In what follows, we will show that it is also possible to formulate linear and convex programs that attempt to optimize the convergence rate of the closed-loop system.

4.1 Strictly proper closed-loop systems

We will first consider the case when the closed-loop system is strictly proper. This is common in practice, since sampling and computational delays effectively render the controller's view of the system strictly proper, even when the physical plant has a direct feed-through. It also simplifies our exposition and makes the conditions for non-negativity of C_s in Theorem 1 both necessary and sufficient.

The decay rate of a positive linear system can be computed by solving a geometric program (Feyzmahdavian et al., 2014). However, this result is not straightforward to use for controller synthesis. In this work, we will therefore explore an alternative approach to formulate the design problem as a linear or convex program. Specifically, we will rely on the following bound on the maximum modulus ρ of the roots of the monic closed-loop characteristic polynomial $A_{\text{cl}}(z) = z^{n+l} + \hat{a}_1 z^{n+l-1} + \cdots + \hat{a}_{n+l}$ derived by Fujii and Kubo (1993):

$$\rho \leq \cos(\pi/(n+l+1)) + \left(|\hat{a}_1| + \sqrt{\sum_{k=1}^{n+l} |\hat{a}_k|^2} \right) /2 \quad (10)$$

By norm equivalence, we also have the bound

$$\rho \leq \cos(\pi/(n+l+1)) + \bar{\rho}/2 \quad (11)$$

where

$$\bar{\rho} = 2|\hat{a}_1| + \sum_{k=2}^{n+l} |\hat{a}_k| \quad (12)$$

Although the right-hand side of (10) is a convex function of the coefficient vector $\hat{a}$, we will first use the bound (12) since it leads to conditions that can be verified using linear programming. We propose the following corollary:

Corollary 4. Let $\epsilon > 0$ and $[f^\star, g^\star]^T$ be the optimal solution to the following linear program

$$\begin{array}{ll} \text{minimize} & [0\ -2\ -1\ \cdots\ -1]\,(T(a)g + T(b)f) \\ \text{subject to} & T(b)f \geq \mathbf{0} \\ & e_1^T(T(a)g + T(b)f) = 1 \\ & W(T(a)g + T(b)f) \geq \mathbf{0} \\ & \mathbf{1}^T(T(a)g + T(b)f) \geq \epsilon \end{array} \quad (13)$$

Then choosing the coefficients f and g in controller (6) as $f = f^\star$ and $g = g^\star$ makes the closed-loop system (7) Schur stable with a monotonic step response.

Proof. The objective function in (13) is the bound (12) where $\hat{a}$ is expressed as in Theorem 3. The constraints in the optimization problem (13) are the same as in Theorem 3, which ensures a stable closed-loop system with a monotonic step response.

Corollary 4 is only applicable when it is possible to find a controller that satisfies the sufficient conditions for monotonicity derived in Theorem 1. The next corollary provides a formulation which works also when these conditions cannot be met. Specifically, it attempts to compute a stabilizing controller whose closed-loop transfer function is as close as possible to the desired family of transfer functions introduced in Theorem 1. As we will see later, this forrmulation can be quite useful in practice.

Corollary 5. Let $\epsilon > 0$, $\lambda \in [0, 1]$ and $[f^\star, g^\star, s_{\hat{b}}^\star, s_{\hat{a}}^\star]^T$ be the solution to the convex optimization problem

$$\begin{array}{ll} \text{minimize} & \lambda(\|s_{\hat{b}}\|_1 + \|s_{\hat{a}}\|_1) + (1-\lambda)\times \\ & \big(|{e_2}^T(T(a)g + T(b)f)| + \|W(T(a)g + T(b)f)\|_2\big) \\ \text{subject to} & \begin{cases} T(b)f + s_{\hat{b}} \geq \mathbf{0} \\ e_1^T(T(a)g + T(b)f) = 1 \\ W(T(a)g + T(b)f) + s_{\hat{a}} \geq \mathbf{0} \\ ||T(a)g + T(b)f||_1 \leq 2 - \epsilon \end{cases} \end{array} \quad (14)$$

If $s_{\hat{b}}^\star = s_{\hat{a}}^\star = \mathbf{0}$, then choosing the coefficients f and g in controller (6) as $f = f^*$ and $g = g^*$ makes the closed-loop system (7) stable with a monotonic step response.

Proof. The proof follows from Theorem 3 with a few changes. First, the conditions for monotonicity are relaxed using slack-variables $s_{\hat{b}}$ and $s_{\hat{a}}$. Second, the stability condition is re-written based on Cohn's diamond, since the closed-loop characteristic polynomial is no longer guaranteed to satisfy the sign restrictions which leads to a simplified stability condition (unless $s_{\hat{a}} = \mathbf{0}$). The cost function consists of a convex combination of the norms of the slack vectors, $\|s_{\hat{b}}\|_1 + \|s_{\hat{a}}\|_1$, and the decay rate bound from (10), $|\hat{a}_1| + \sqrt{\sum_{k=1}^{n+l} |\hat{a}_k|^2}$, where $\hat{a} = T(a)g + T(b)f$ is the closed-loop characteristic polynomial coefficients. The former encourages a near-monotonic response while the latter expression improves the decay rate.

If the optimal value of problem (14) is positive for $\lambda = 1$, then there is no stabilizing controller of degree l that makes the closed-loop system meet the conditions of Theorem 3 with the same value of ϵ. Unlike the conditions for monotonicity, the stability-related conditions are posed as hard constraints in Corollary 5. Thus when $\lambda = 1$, the formulation in Corollary 5 returns a stabilizing controller which renders the closed-loop system as close to the family of transfer functions introduced in Theorem 1 as possible. Using smaller values of λ compromises the monotonicity of the closed-loop in favor of a faster response. As shown in Section 5, Corollary 5 is not only beneficial for infeasible problems, but it is also useful for designing controllers of lower order than what is possible with Corollary 4, provided that one can accept a near-monotonic response.

Some additional remarks are in order. First, although our requirement that the closed-loop denominator should be monic implies that $g_0 = 1$, $f = \mathbf{0}$ could still be admissible. In Corollary 4, we can avoid this possibility by adding the condition $\mathbf{1}^T T(b) f \geq \epsilon_2$ for some (small) positive constant ϵ_2. The same condition avoids trivial solutions in Corollary 5, but may come at some degree of conservatism since positivity of $\hat{b}$ is not enforced. Although we have never encountered unbounded solutions in our experiments, we suggest to also include upper and lower bounds on the coefficient vectors f and g. Finally, our conditions guarantee that the closed-loop transfer function is Schur, but they do not guarantee asymptic stability of the controller itself. If one wishes to consider only stable compensators, then one can simply add the condition

$$\sum_{k=1}^{l} |g_k| \leq 1 - \epsilon_3 \quad (15)$$

for some positive constant ϵ_3. The corresponding design problems are still convex optimization problems.

4.2 Closed-loop systems with direct feedthrough

If both the plant and the controller transfer functions have direct feedthroughs, then so will the closed-loop transfer function. While Corollary 4 and 5 still hold in this case, one can argue that they are based on unnecessarily conservative conditions for positivity of C_s in Theorem 3. Indeed, if $\hat{b}_0 = b_0 f_0 \geq 0$ would be fixed, then we could replace the requirement $\hat{b} \geq 0$ with $\hat{b} - \hat{a}\hat{b}_0 \geq 0$ and still get linear conditions for positivity. However, it is difficult to fix f_0 without limiting the degrees of freedom in the design. Since it is the only one parameter that breaks convexity, it is feasible to make a one-parameter search over f_0 and evaluate the optimal performance for each f_0 by solving a convex problem. Since the condition that $\hat{A}(z)$ be monic implies that $g_0 = 1 - b_0 f_0$, the controllers will be proper for all values of $f_0 \neq 1/b_0$, non-trivial for $f_0 \neq 0$ and Cohn's criterion for stability of the controller transfer function

$$\sum_{k=1}^{l} |g_k| \leq |g_0| - \epsilon_3 = |1 - b_0 f_0| - \epsilon_3$$

is convex for all fixed values of f_0.

5. NUMERICAL EXAMPLES

In this section, we present several numerical examples which demonstrate the effectiveness of Corollaries 4 and 5. The first two examples are feasible with regard to Corollary 4, while Example 3 is not. There, we instead use Corollary 5 to design a controller resulting in a closed-loop behavior having most similar characteristics to the desired traits. Finally, Example 4 is provided to indicate how Corollary 5 can result in controllers with much lower order than what is feasible using Corollary 4. Finally the trade-off that exists in Corollary 5 between closed-loop near-monotonic behavior and high decay rate is studied.

Example 1. Consider the second-order unstable system

$$H(z) = \frac{-z^2}{z^2 + 2z + 2}$$

with an oscillatory impulse response. Corollary 4 with $l = 2$ results in the following controller

$$C(z) = \frac{9.68z^2 + 17.37z + 17.37}{8.68z^2} \tag{16}$$

which stabilizes the system and renders the closed-loop step response monotonic as shown in Figure 1.

Example 2. Consider the LTI system

$$H(z) = \frac{z^2 - 0.4z + 1.25}{(z - 1.7)(z^2 + 0.4z + 0.29)}$$

in Darbha and Bhattacharyya (2002), a more general three degree of freedom controller was used to render the closed-loop step response monotonic. However using Corollary 4 with $l = 2$, a simpler controller of lower order and with a faster step response is found. Its closed-loop step response is shown in Figure 2. For sake of comparison, the step response of the closed-loop system under the controller designed in Darbha and Bhattacharyya (2002) is also plotted in the same figure. A static pre-filter was used to equalize the steady state gains in this experiment.

Example 3. Consider the first-order LTI system

$$H(z) = \frac{z - 2}{2z - 2}.$$

According to the necessary conditions in Darbha and Bhattacharyya (2002), it is impossible to stabilize this system and ensure a monotonic step response. However using Corollary 5 with $\lambda = 1$, we find a stabilizing controller with a near-monotonic behavior. Increasing the emphasis on the decay rate leads to a faster response, but also a more pronounced undershoot. To avoid this effect, we consider controllers of higher order. The closed-loop step responses for controller orders $l \in \{2, 4, 6\}$ are shown in Figure 3. Here, to avoid tuning of λ for each value of l, we have instead increased the value of ε to 0.75. Note that a high value of ϵ serves as a proxy for $\bar{\rho}$ since the stability constraint is effectively imposing that $\sum_{k=1}^{n+1} |\hat{a}_k| \leq 1 - \epsilon$.

Example 4. Consider the second-order LTI system

$$H(z) = \frac{1}{z^2 - 0.25}.$$

This is a trivial problem with $l = 0$. However to also ensure error-free tracking of a step input, we enforce an integrator in the controller by virtually adding a plant pole at unity. We then design a controller for the extended system comprising the plant and the integrator dynamics in series. This tracking problem turns out to be difficult and was proven to be infeasible with PID controllers in Mohsenizadeh et al. (2012). However using Corollary 4 with $l = 14$ finds a controller with the desired tracking performance and a monotonic closed-loop step response; see Figure 4. Now we consider the same problem with Corollary 5 with $l = 2$ and $\lambda = 1$. Interestingly, the resulting closed-loop system does not satisfy the assumptions of Theorem 1, but it still has a monotonic response as seen in Figure 4. Both experiments used $\epsilon = 0.5$.

Example 5. Consider the second-order LTI system

$$H(z) = \frac{20z - 100}{(10z - 8)(2z - 3)}.$$

According to (Darbha and Bhattacharyya, 2002), It is impossible to stabilize this system and ensure a monotonic step response. Like Example 3 however, we will use Corollary 5 to find a second-order stabilizing controller with a near-monotonic behavior. This time we investigate the effect of λ in the controller design through the optimization in Corollary 5. The results of simulating the closed-loop step response using $\lambda \in \{0, 0.5, 1\}$ are shown in Figure 5, where the steady state gains are normalized for comparison. As can be deduced from Figure 5, choosing a smaller value of λ in this example gives a faster response but with greater undershoots and overshoots (as expected).

6. CONCLUSION

In this paper, we have developed a control design procedure which ensures asymptotic stability and monotonic step response of the closed-loop system. The method considers a convex subset of all stabilizing controllers of a desired order, and uses linear programming to ensure that the closed-loop impulse response is non-negative. However, as discussed in Darbha and Bhattacharyya (2002), not every closed-loop system can be rendered monotonic. In

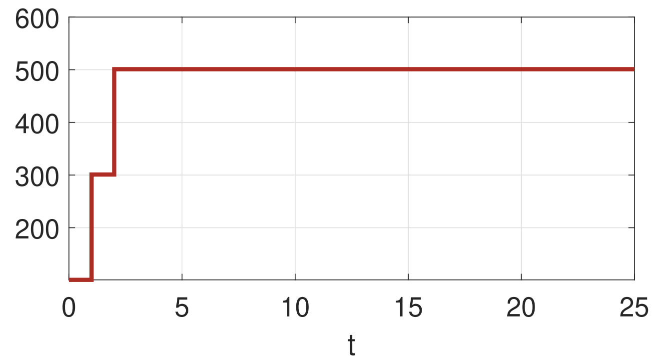

Fig. 1. Closed-loop step response in Example 1.

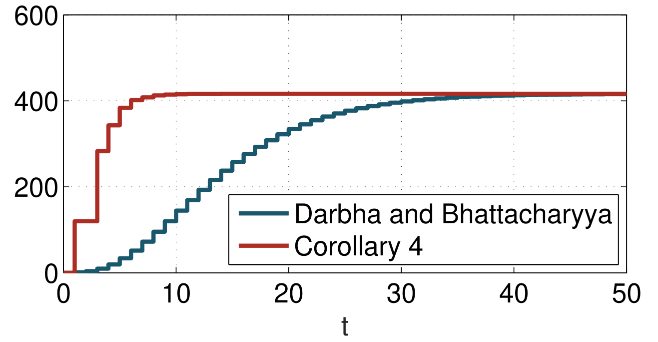

Fig. 2. Step response of the closed-loop system in Example 2 with the controller designed using the method from Darbha and Bhattacharyya (2002) in blue and our design from Corollary 4 in red.

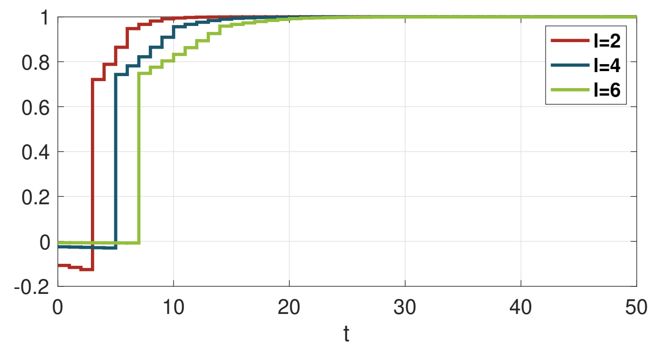

Fig. 3. Step response of the closed-loop system in Example 3 using a controller design detailed in Corollary 5 for different controller orders l.

addition, our synthesis conditions are only sufficient. To deal with instances when Corollary 4 is infeasible, we have also provided an alternative design methodology (Corollary 5) which attempts to find a controller which makes the closed-loop step response near-monotonic. The usefulness of this technique was shown in the numerical examples.

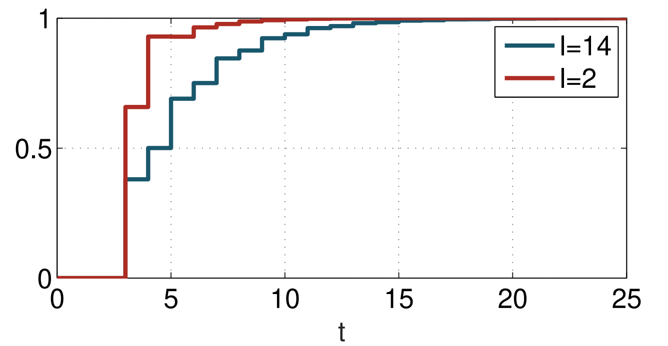

Fig. 4. Step responses of the system considered in Example 4 in closed-loop using controllers designed by Corollaries 4 and 5 with orders $l = 14$ and $l = 2$ respectively.

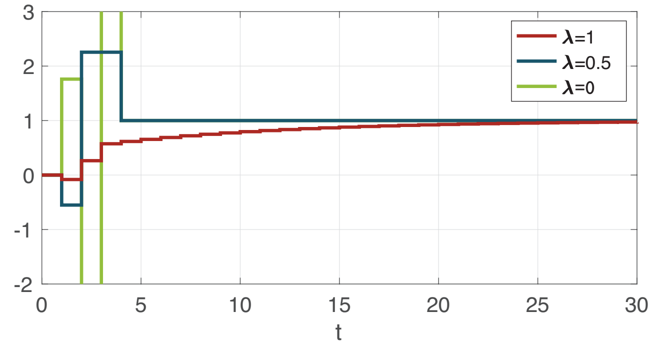

Fig. 5. Step responses of the system in Example 5 in closed-loop using controllers designed via Corollary 5 with parameters $\lambda = 0$, $\lambda = 0.5$ and $\lambda = 1$.

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