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Adaptive Fuzzy Control for Multivariable Nonlinear Systems with Indefinite Control Gain Matrix and Unknown Control Direction

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Abstract: In this paper, we focus on the direct adaptive fuzzy control design for a class of uncertain MIMO nonlinear systems with indefinite control gain matrix and unknown control direction. The control design is based on the approximation of an unknown ideal control law that can meet the control objective by using fuzzy systems. The adjustable parameters of the used fuzzy systems are adjusted online using the error between the unknown ideal controller and the fuzzy controller. In this paper, unlike most existing works, the Nussbaum gain technique is not used to overcome the obstacle of the unknown control direction. In fact, with the help of a matrix decomposition technique, the unknown control direction is redefined as an unknown constant vector, which is estimated online by a suitable update law. The stability of the closed-loop system is studied using the Lyapunov direct approach. Numerical simulation results are provided to illustrate the effectiveness of the proposed control design approach.

Keywords: Adaptive control, Fuzzy systems, Nonlinear systems, Control direction, Indefinite matrix.

1. INTRODUCTION

Over the past three decades, adaptive control of uncertain nonlinear systems using fuzzy systems has been extensively studied (Er and Mandal, 2016; Guerra *et al.*, 2015; Labiod and Guerra, 2007; Ordonez and Passino, 1999). The stability analysis of these adaptive control techniques is carried out by using the Lyapunov direct method. Theoretically speaking, there are two different approaches that have been used in conceiving adaptive fuzzy controllers for uncertain nonlinear systems: the indirect and the direct adaptive control approaches. In the first approach, fuzzy logic systems are employed to approximate the uncertain nonlinearities and these approximations are then used to design a control law (Ordonez and Passino, 1999). On the other hand, in the second approach, fuzzy logic systems are employed to approximate unknown ideal control laws (Labiod and Guerra, 2007; Ordonez and Passino, 1999).

However, most of the adaptive fuzzy control strategies have been designed for uncertain nonlinear systems with a priori-known control direction, i.e. the sign of the control gain coefficient of SISO systems, or the sign of the control gain matrix of MIMO systems is assumed a priori-known. In fact, without this assumption, adaptive fuzzy controller design becomes much more difficult because one cannot decide the direction along which the control operates. That is why the problem of designing effective adaptive fuzzy controllers for uncertain nonlinear systems with a priori-unknown control direction is receiving increasing attention. This problem has been mostly tackled by using the Nussbaum-type function technique (Chen, 2019; Nussbaum, 1983). This technique has been effectively used in adaptive control design for a class of uncertain multivariable nonlinear systems with an unknown sign of the control gain matrix, see for example, Boulkroune *et al.* (2010), Chen *et al.* (2017), Shi *et al.* (2017), Song *et al.* (2017), Zhang and Yang (2019). In Labiod and Guerra (2017), without utilizing the Nussbaum gain technique, a simple solution to the unknown control direction problem for a class of MIMO nonlinear systems was proposed. However, in the aforementioned papers, the control gain matrix is assumed either positive definite or negative definite. In fact, the control design problem is more challenging in the case of uncertain MIMO nonlinear systems with indefinite control gain matrix.

In this paper, inspired by the work of Labiod and Guerra (2017) and without using the Nussbaum gain technique, we propose a simple solution to the unknown control direction problem for a class of multi-input multi-output uncertain nonlinear systems with indefinite control gain matrix. The key idea is to use a matrix decomposition technique to define the sign of the control gain matrix as an unknown constant vector. Then, the entries of this vector are estimated in the same way as the unknown parameters of the fuzzy controller.

The remainder of this paper is structured as follows. Section 2 describes the problem formulation. Section 3 presents the used fuzzy logic systems. Section 4 presents the proposed direct adaptive fuzzy controller with the controller parameters and the control direction adaptive laws. Finally, section 5, gives some numerical simulation results to highlight the effectiveness of the proposed adaptive fuzzy control scheme.

2. PROBLEM FORMULATION

We consider multi-input multi-output nonlinear uncertain dynamic systems of the form

$$\begin{cases} y_1^{(r_1)} = f_1(\boldsymbol{x}) + \sum_{j=1}^{p} g_{1j}(\boldsymbol{x}) u_j \\ \vdots \\ y_p^{(r_p)} = f_p(\boldsymbol{x}) + \sum_{j=1}^{p} g_{pj}(\boldsymbol{x}) u_j \end{cases} \tag{1}$$

where $\boldsymbol{x} = \left[y_1,\ \dot{y}_1, \dots ,y_1^{(r_1-1)}, \dots, y_p,\ \dot{y}_p, \dots ,y_p^{(r_p-1)}\right]^T \in \mathbb{R}^n$ with $n = \sum_{i=1}^{p} r_i$, is the vector of the state variables which is assumed available for measurement, $\boldsymbol{y} = \left[y_1, \dots, y_p\right]^T \in \mathbb{R}^p$ is the vector of the output variables, $\boldsymbol{u} = \left[u_1, \dots, u_p\right]^T \in \mathbb{R}^p$ is the vector of the control input variables, and $f_i(\boldsymbol{x})$, $g_{ij}(\boldsymbol{x})$, $i, j = 1, \dots, p$ are smooth unknown nonlinear functions.

Let us denote

$$\boldsymbol{y}^{(r)} = \begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_p^{(r_p)} \end{bmatrix}; \boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} f_1(\boldsymbol{x}) \\ \vdots \\ f_p(\boldsymbol{x}) \end{bmatrix};$$

$$\boldsymbol{G}(\boldsymbol{x}) = \begin{bmatrix} g_{11}(\boldsymbol{x}) & \dots & g_{1p}(\boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ g_{p1}(\boldsymbol{x}) & \dots & g_{pp}(\boldsymbol{x}) \end{bmatrix}$$

Then, we can write system (1) compactly as

$$\boldsymbol{y}^{(r)} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} \tag{2}$$

The aim of this paper is to design an adaptive fuzzy control law $\boldsymbol{u}(t)$, without the knowledge of the control direction, such that the system output $\boldsymbol{y}(t)$ tracks a given desired trajectory $\boldsymbol{y}_d(t) = \left[y_{d1}(t), \dots, y_{dp}(t)\right]^T$ as closely as possible while all signals in the closed-loop system remain bounded.

In order to control uncertain nonlinear system (1), the following assumptions are required.

Assumption 1. Each desired trajectory $y_{di}(t)$, $i = 1, \dots, p$, and its time derivatives up to order r_i are continuous and bounded.

Assumption 2. The control gain matrix $\boldsymbol{G}(\boldsymbol{x})$ is nonsingular, and all its leading principal minors are nonzero and their signs are unknown.

Remark 1. Assumption 2 means that the control gain matrix may be non-symmetric and indefinite. Moreover, if some principal minors of the matrix $\boldsymbol{G}(\boldsymbol{x})$ are zero, an interchange of rows or columns may be needed before proceeding to controller design.

Since the matrix $\boldsymbol{G}(\boldsymbol{x})$ is nonsingular with nonzero leading principal minors, the following lemma (Boulkroune *et al.*, 2010; Tao, 2013) will be used in the controller design.

Lemma 1. Consider a real matrix $\boldsymbol{G}(\boldsymbol{x})$ with nonzero leading principal minors. Then it can be decomposed as:

$$\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{G}_s(\boldsymbol{x}) \boldsymbol{D}^* \boldsymbol{T}(\boldsymbol{x}) \tag{3}$$

where $\boldsymbol{G}_s(\boldsymbol{x})$ is a symmetric positive definite matrix, $\boldsymbol{D}^* = \text{diag}\left[d_1^*, d_2^*, \dots, d_n^*\right]$ where $d_1^* = \text{sgn}(\Delta_1)$, $d_2^* = \text{sgn}\left(\frac{\Delta_2}{\Delta_1}\right)$, $d_i^* = \text{sgn}\left(\frac{\Delta_i}{\Delta_{i-1}}\right)$ with Δ_i are the leading principal minors of the matrix $\boldsymbol{G}(\boldsymbol{x})$ and $\text{sgn}(\cdot)$ is the sign function; i.e. $\boldsymbol{D}^*$ is a diagonal matrix with diagonal entries $d_i^* = +1$ or -1, $\boldsymbol{T}(\boldsymbol{x})$ is a unity upper triangular matrix

Remark 2. The diagonal matrix $\boldsymbol{D}^*$ contains information on the control direction, i.e. the sign of the control gain matrix $\boldsymbol{G}(\boldsymbol{x})$. It is worth to note that the knowledge of the control direction $\boldsymbol{D}^*$ is crucial for constructing a stable controller parameter adaptation law. However, in this paper, the control direction $\boldsymbol{D}^*$ is assumed unknown and it will be estimated online (its diagonal entries) by an appropriate adaptation law.

Now, we define the tracking errors as follows

$$e_i(t) = y_{di}(t) - y_i(t)\ ; i = 1, \dots, p \tag{4}$$

and the following filtered tracking errors

$$s_i(t) = \left(\frac{d}{dt} + \lambda_i\right)^{r_i - 1} e_i(t)\ ; \lambda_i > 0\ ; i = 1, \dots, p \tag{5}$$

From (5), $s_i(t) = 0$ is a linear differential equation whose solution implies that $e_i(t)$ converges to zero with a time constant $(r_i - 1)/\lambda_i$. In addition, the time derivatives of $e_i(t)$ up to order $r_i - 1$ also converge to zero (Slotine and Li, 1991). Thus, the control objective becomes the design of a controller to enforce $s_i(t)$ to converge to zero, $i = 1, \dots, p$.

The time derivatives of the filtered errors (5) are

$$\dot{s}_i = v_i - f_i(\boldsymbol{x}) - \sum_{j=1}^{p} g_{ij}(\boldsymbol{x}) u_j\ ; i = 1, \dots, p \tag{6}$$

where $v_1, \dots, v_p$, are defined as

$$v_i = y_{di}^{(r_i)} + \beta_{i, r_i - 1} e_i^{(r_i - 1)} + \dots + \beta_{i,1} \dot{e}_i\ ; i = 1, \dots, p \tag{7}$$

with $\beta_{i,j} = \frac{(r_i - 1)!}{(r_i - j)!\ (j-1)!} \lambda_i^{r_i - j}$, $i = 1, \dots p$, $j = 1, \dots, r_i - 1$

Denote

$$\boldsymbol{s} = \left[s_1, \dots, s_p\right]^T;\ \boldsymbol{v} = \left[v_1, \dots, v_p\right]^T$$

Then we can represent (6) in the following compact form

$$\dot{\boldsymbol{s}} = \boldsymbol{v} - \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} \tag{8}$$

If the nonlinear functions $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{G}(\boldsymbol{x})$ are known, to accomplish the control objective, we can apply the following nonlinear control law (Labiod and Guerra, 2007)

$$\boldsymbol{u}^* = \boldsymbol{G}^{-1}(\boldsymbol{x})\left(-\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{v} + \boldsymbol{K}\,\boldsymbol{s} + \boldsymbol{K}_0 \tanh(\boldsymbol{s}/\varepsilon_0)\right) \tag{9}$$

where, $\boldsymbol{K} = diag\left[k_1,\ldots,k_p\right]$, $\boldsymbol{K}_0 = diag\left[k_{01},\ldots,k_{0p}\right]$, with $k_i > 0$ and $k_{0i} > 0$, for $i = 1,\ldots,p$, ε_0 is a small positive constant, and $\tanh(\cdot)$ is the hyperbolic tangent function.

Actually, when we choose the control input as $\boldsymbol{u} = \boldsymbol{u}^*$, equation (8) becomes

$$\dot{\boldsymbol{s}} = -\boldsymbol{K}\,\boldsymbol{s} - \boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right) \tag{10}$$

or, equivalently

$$\dot{s}_i = -k_i s_i - k_{0i}\tanh\left(s_i/\varepsilon_0\right),\quad i = 1,\ldots,p \tag{11}$$

From (11) one can conclude that $s_i(t) \to 0$ as $t \to \infty$ and, accordingly, $e_i(t)$ and all its time derivatives up to order $r_i - 1$ converge asymptotically to zero (Slotine and Li, 1991).

However, in this paper, the nonlinear function $\boldsymbol{f}(\boldsymbol{x})$ is assumed unknown and the control gain matrix $\boldsymbol{G}(\boldsymbol{x})$ is also assumed unknown with unknown sign. Consequently, the ideal nonlinear control law (9) cannot be used. In such a case, we propose a design method relying on the estimation of the sign of the control gain matrix (the diagonal entries of the matrix $\boldsymbol{D}^*$) and on the approximation of the entire unknown ideal control law (9) by using adaptive fuzzy systems.

3. DESCRIPTION OF THE USED FUZZY SYSTEMS

In this paper, we approximate the unknown continuous functions by using the zero-order Takagi-Sugeno fuzzy system. This fuzzy logic system performs a mapping from an input vector $\boldsymbol{z} = \left[z_1,\ldots,z_m\right]^T \in \Omega_z \subset \mathbb{R}^m$ to a scalar output $y_f \in \mathbb{R}$, where $\Omega_z = \Omega_{z_1}\times\cdots\times\Omega_{z_m}$ and $\Omega_{z_i} \subset \mathbb{R}$. Let us define M_i fuzzy sets F_i^j, $j = 1,\ldots,M_i$, for each input variable z_i, then the Takagi-Sugeno fuzzy logic system will be characterized by a set of if-then fuzzy rules of the following form (Labiod and Guerra, 2007; Wang, 1994)

$$\begin{aligned} &\mathrm{R}^k: \text{If } z_1 \text{ is } G_1^k \text{ and} \ldots \text{and } z_m \text{ is } G_m^k \\ &\qquad\qquad \text{Then } y_f \text{ is } y_f^k \quad (k = 1,\ldots,N) \end{aligned} \tag{12}$$

where $G_i^k \in \left\{F_i^1,\ldots,F_i^{M_i}\right\}$, $i = 1,\ldots,n$, y_f^k is the crisp output of the k-th rule, and N is the number of rules.

By considering the singleton fuzzifier strategy and the product inference engine, the output of the fuzzy system can be expressed as follows (Wang, 1994)

$$y_f(\boldsymbol{z}) = \frac{\sum_{k=1}^{N}\mu_k(\boldsymbol{z})\,y_f^k}{\sum_{k=1}^{N}\mu_k(\boldsymbol{z})} \tag{13}$$

where $\mu_k(\boldsymbol{z}) = \prod_{i=1}^{m}\mu_{G_i^k}(z_i)$ and $\mu_{G_i^k} \in \left\{\mu_{F_i^1},\ldots,\mu_{F_i^{M_i}}\right\}$, with $\mu_{F_i^j}(x_i)$ is the membership function of the fuzzy set F_i^j.

The output (13) can be written in the following compact form (Wang, 1994)

$$y_f(\boldsymbol{z}) = \boldsymbol{w}^T(\boldsymbol{z})\theta \tag{14}$$

where $\theta = \left[y_f^1,\ldots,y_f^N\right]^T$ is a vector grouping all consequent parameters, and $\boldsymbol{w}(\boldsymbol{z}) = \left[w_1(\boldsymbol{z}),\ldots,w_N(\boldsymbol{z})\right]^T$ is a set of fuzzy basis functions defined as

$$w_k(\boldsymbol{z}) = \frac{\mu_k(\boldsymbol{z})}{\sum_{j=1}^{N}\mu_j(\boldsymbol{z})},\ k = 1,\ \ldots, N \tag{15}$$

The fuzzy system (13) is assumed to be well-defined so that $\sum_{j=1}^{N}\mu_j(\boldsymbol{z}) \neq 0$ for all $\boldsymbol{z} \in \Omega_z$.

It has been proved in Wang (1994) that fuzzy systems in the form of (14) with Gaussian membership functions can approximate continuous functions over a compact set. The approximation accuracy is related to the number of fuzzy rules considered.

4. ADAPTIVE FUZZY CONTROL DESIGN

In order to design the adaptive control law, each entry of the ideal control law (9) will be approximated by a fuzzy system in the form of (14) as the following

$$u_i^*(\boldsymbol{z}) = \boldsymbol{w}_i^T(\boldsymbol{z})\theta_i^* + \varepsilon_i(\boldsymbol{z});\quad i = 1,\ldots,p \tag{16}$$

where $\boldsymbol{z} = \left[\boldsymbol{x}^T, \boldsymbol{s}^T\right]^T$, $\boldsymbol{w}_i(\boldsymbol{z})$ is a fuzzy basis function vector which is specified by the designer, θ_i^* is an unknown ideal parameter vector, and $\varepsilon_i(\boldsymbol{z})$ is the fuzzy approximation error which is assumed bounded.

Let us define the following vectors and matrices

$$\varepsilon(\boldsymbol{z}) = \left[\varepsilon_1(\boldsymbol{z}),\ldots,\varepsilon_p(\boldsymbol{z})\right]^T;\ \theta^* = \left[\theta_1^{*T},\ldots,\theta_p^{*T}\right]^T,$$
$$\boldsymbol{w}(\boldsymbol{z}) = diag\left[\boldsymbol{w}_1(\boldsymbol{z}),\ldots,\boldsymbol{w}_p(\boldsymbol{z})\right]$$

Using the above notation, one can write (16) in a compact format as

$$\boldsymbol{u}^* = \boldsymbol{w}^T(\boldsymbol{z})\theta^* + \varepsilon(\boldsymbol{z}) \tag{17}$$

Now, let θ be the estimate the unknown ideal parameter vector θ^* and consider the actual adaptive control law for system (1) as the following

$$\boldsymbol{u}(\boldsymbol{z}) = \boldsymbol{w}^T(\boldsymbol{z})\theta \tag{18}$$

After the specification of the control law, the next step should be the design of the controller parameter adaptation law to meet the control objective. However, because the control direction (i.e. the sign of the control gain matrix $\boldsymbol{G}(\boldsymbol{x})$ defined by the signs of the diagonal entries of the matrix $\boldsymbol{D}^*$) is unknown, to get a stable adaptation, the parameter adaptation law should be combined with an online control direction estimator.

Adding and subtracting $\boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}^*$ to the right-hand side of (8) leads to the following error dynamics

$$\dot{\boldsymbol{s}} = \boldsymbol{v} - \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}^* - \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}^* \tag{19}$$

Using (9), (19) becomes

$$\dot{\boldsymbol{s}} = -\boldsymbol{K}\,\boldsymbol{s} - \boldsymbol{K}_0\tanh\left(\boldsymbol{s}/\varepsilon_0\right) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} \tag{20}$$

where $\boldsymbol{e}_{\boldsymbol{u}} = \boldsymbol{u}^* - \boldsymbol{u}$.

By using Lemma 1, the matrix $\boldsymbol{G}(\boldsymbol{x})$ can be expressed as

$$\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{G}_{\boldsymbol{S}}(\boldsymbol{x})\boldsymbol{D}^*\boldsymbol{T}(\boldsymbol{x}) = \boldsymbol{G}_{\boldsymbol{S}}(\boldsymbol{x})\boldsymbol{D}^* + \Delta\boldsymbol{G}(\boldsymbol{x}) \quad (21)$$

where $\Delta\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{G}_{\boldsymbol{S}}(\boldsymbol{x})\boldsymbol{D}^*\left(\boldsymbol{T}(\boldsymbol{x}) - I_p\right)$ with I_p the identity matrix of size p.

Substituting (21) into (20) yields

$$\dot{\boldsymbol{s}} = -\boldsymbol{K}\,\boldsymbol{s} - \boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right) + \boldsymbol{G}_{\boldsymbol{S}}(\boldsymbol{x})\boldsymbol{D}^*\boldsymbol{e}_{\boldsymbol{u}} + \Delta\boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} \quad (22)$$

From (17) and (18), one can write $\boldsymbol{D}^*\boldsymbol{e}_{\boldsymbol{u}}$ as

$$\boldsymbol{D}^*\boldsymbol{e}_{\boldsymbol{u}} = \boldsymbol{D}^*\left(\boldsymbol{w}^T(z)\tilde{\theta} + \varepsilon(z)\right) \quad (23)$$

Using the fact that $\boldsymbol{D}^* = \boldsymbol{D}^* - \boldsymbol{D} + \boldsymbol{D} = \tilde{\boldsymbol{D}} + \boldsymbol{D}$, where $\tilde{\boldsymbol{D}} = \boldsymbol{D}^* - \boldsymbol{D}$ and $\boldsymbol{D}^* = \mathrm{diag}\left[d_1, d_2, \ldots, d_n\right]$ is the estimate of the unknown control direction $\boldsymbol{D}^* = \mathrm{diag}\left[d_1^*, d_2^*, \ldots, d_n^*\right]$, (23) can be further written as

$$\boldsymbol{D}^*\boldsymbol{e}_{\boldsymbol{u}} = -\tilde{\boldsymbol{D}}\,\boldsymbol{u} + \boldsymbol{D}\,\boldsymbol{w}^T(z)\tilde{\theta} + \tilde{\boldsymbol{D}}\,\boldsymbol{w}^T(z)\theta^* + \boldsymbol{D}^*\varepsilon(z) \quad (24)$$

By defining the following lumped disturbance term $\delta = \tilde{\boldsymbol{D}}\boldsymbol{w}^T(z)\theta^* + \boldsymbol{D}^*\varepsilon(z)$, it follows that

$$\boldsymbol{D}^*\boldsymbol{e}_{\boldsymbol{u}} = -\tilde{\boldsymbol{D}}\,\boldsymbol{u} + \boldsymbol{D}\,\boldsymbol{w}^T(z)\tilde{\theta} + \delta \quad (25)$$

Let use define $\boldsymbol{d}^* = \left[d_1^*, d_2^*, \ldots, d_n^*\right]^T$, $\boldsymbol{d} = \left[d_1, d_2, \ldots, d_n\right]^T$ and $\tilde{\boldsymbol{d}} = \boldsymbol{d}^* - \boldsymbol{d}$, so, we have $\boldsymbol{D}^* = \mathrm{diag}\left[\boldsymbol{d}^*\right]$, $\boldsymbol{D} = \mathrm{diag}[\boldsymbol{d}]$ and $\tilde{\boldsymbol{D}} = \mathrm{diag}\left[\tilde{\boldsymbol{d}}\right]$.

In order to obtain a stable adaptation without the knowledge of the control direction $\boldsymbol{D}^*$ and inspired by our previous works (Labiod and Guerra, 2007, 2016, 2017), the following cost function $J = \frac{1}{2}\left(\boldsymbol{D}^*\boldsymbol{e}_{\boldsymbol{u}}\right)^T \boldsymbol{G}_{\boldsymbol{S}}(\boldsymbol{x})\left(\boldsymbol{D}^*\boldsymbol{e}_{\boldsymbol{u}}\right)$ can be used to derive the following robust parameter adaptation laws

$$\dot{\theta} = \eta_1\, \boldsymbol{w}(z)\boldsymbol{D}\left(\dot{\boldsymbol{s}} + \boldsymbol{K}\,\boldsymbol{s} + \boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right)\right) - \eta_1\,\sigma_1\,\theta \quad (26)$$

$$\dot{\boldsymbol{d}} = -\eta_2\,\mathrm{diag}[\boldsymbol{u}]\left(\dot{\boldsymbol{s}} + \boldsymbol{K}\,\boldsymbol{s} + \boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right)\right) - \eta_2\sigma_2\,\boldsymbol{d} \quad (27)$$

where η_1 and η_2 are two strictly positive design constants, σ_1 and σ_2 are two small positive design constants.

Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{s} + \frac{1}{2\eta_1}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2\eta_2}\tilde{\boldsymbol{d}}^T\tilde{\boldsymbol{d}} \quad (28)$$

The time derivative of V is

$$\dot{V} = \boldsymbol{s}^T\dot{\boldsymbol{s}} - \frac{1}{\eta_1}\tilde{\theta}^T\dot{\theta} - \frac{1}{\eta_2}\tilde{\boldsymbol{d}}^T\dot{\boldsymbol{d}} \quad (29)$$

Substituting (20), (26) and (27) into (29), and using the equality $\dot{\boldsymbol{s}} + \boldsymbol{K}\,\boldsymbol{s} + \boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right) = \boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}}$, one gets

$$\begin{aligned}\dot{V} &= -\boldsymbol{s}^T\boldsymbol{K}\,\boldsymbol{s} - \boldsymbol{s}^T\boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right) + \boldsymbol{s}^T\boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} \\ &\quad -\left(-\tilde{\boldsymbol{D}}\,\boldsymbol{u} + \boldsymbol{D}\,\boldsymbol{w}^T(z)\tilde{\theta}\right)^T \boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} + \sigma_1\tilde{\theta}^T\theta + \sigma_2\tilde{\boldsymbol{d}}^T\boldsymbol{d}\end{aligned} \quad (30)$$

By using (25) one obtains

$$\begin{aligned}\dot{V} &= -\boldsymbol{s}^T\boldsymbol{K}\,\boldsymbol{s} - \boldsymbol{s}^T\boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right) - \boldsymbol{e}_{\boldsymbol{u}}^T\boldsymbol{G}_s(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} + \boldsymbol{s}^T\boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} \\ &\quad + \boldsymbol{e}_{\boldsymbol{u}}^T\boldsymbol{D}^*\Delta\boldsymbol{G}^T(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} + \delta^T\boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} + \sigma_1\tilde{\theta}^T\theta + \sigma_2\tilde{\boldsymbol{d}}^T\boldsymbol{d}\end{aligned} \quad (31)$$

By completion of squares, the following equalities hold

$$\sigma_1\tilde{\theta}^T\theta = \sigma_1\tilde{\theta}^T\left(-\tilde{\theta} + \theta^*\right) = -\frac{\sigma_1}{2}\left\|\tilde{\theta}\right\|^2 - \frac{\sigma_1}{2}\left\|\theta\right\|^2 + \frac{\sigma_1}{2}\left\|\theta^*\right\|^2 \quad (32)$$

$$\sigma_2\tilde{\boldsymbol{d}}^T\boldsymbol{d} = \sigma_2\tilde{\boldsymbol{d}}^T\left(-\tilde{\boldsymbol{d}} + \boldsymbol{d}^*\right) = -\frac{\sigma_2}{2}\left\|\tilde{\boldsymbol{d}}\right\|^2 - \frac{\sigma_2}{2}\left\|\boldsymbol{d}\right\|^2 + \frac{\sigma_2}{2}\left\|\boldsymbol{d}^*\right\|^2 \quad (33)$$

Now, by considering the fact that the $\sigma-$modification parameter adaptive laws (26) and (27) ensure the uniform boundedness of the adaptive parameters θ and $\boldsymbol{d}$, we can assume that the following inequality holds

$$\begin{aligned}&-\boldsymbol{s}^T\boldsymbol{K}_0 \tanh\left(\boldsymbol{s}/\varepsilon_0\right) + \boldsymbol{s}^T\boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} + \delta^T\boldsymbol{G}(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} + \frac{\sigma_2}{2}\left\|\boldsymbol{d}^*\right\|^2 \\ &+\boldsymbol{e}_{\boldsymbol{u}}^T\boldsymbol{D}^*\Delta\boldsymbol{G}^T(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} - \frac{\sigma_1}{2}\left\|\theta\right\|^2 + \frac{\sigma_1}{2}\left\|\theta^*\right\|^2 - \frac{\sigma_2}{2}\left\|\boldsymbol{d}\right\|^2 \le \\ &\kappa_1\boldsymbol{s}^T\boldsymbol{K}\,\boldsymbol{s} + \kappa_2\boldsymbol{e}_{\boldsymbol{u}}^T\boldsymbol{G}_s(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} + \frac{\kappa_3}{2}\left\|\tilde{\theta}\right\|^2 + \frac{\kappa_4}{2}\left\|\tilde{\boldsymbol{d}}\right\|^2 + \bar{\kappa}(t)\end{aligned} \quad (34)$$

where κ_1, κ_2, κ_3, κ_4 are positive constants and $\bar{\kappa}(t)$ a positive bounded function.

By using the above equations (31)-(34), the time derivative of V can be upper bounded as

$$\begin{aligned}\dot{V} &\le -\left(1-\kappa_1\right)\boldsymbol{s}^T\boldsymbol{K}\,\boldsymbol{s} - \left(1-\kappa_2\right)\boldsymbol{e}_{\boldsymbol{u}}^T\boldsymbol{G}_s(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} \\ &\quad -\frac{1}{2}\left(\sigma_1-\kappa_3\right)\left\|\tilde{\theta}\right\|^2 - \frac{1}{2}\left(\sigma_2-\kappa_4\right)\left\|\tilde{\boldsymbol{d}}\right\|^2 + \bar{\kappa}\end{aligned} \quad (35)$$

If the following inequalities are satisfied: $\kappa_1 < 1$, $\kappa_2 < 1$, $\kappa_3 < \sigma_1$ and $\kappa_4 < \sigma_2$, (35) can be rewritten as

$$\dot{V} \le -\alpha V + c \quad (36)$$

where $\alpha = \min\left(2\left(1-\kappa_1\right)\lambda_{\min}(\boldsymbol{K}), \eta_1\left(\sigma_1-\kappa_3\right), \eta_2\left(\sigma_2-\kappa_4\right)\right)$, $c = \sup_t\left(-\left(1-\kappa_2\right)\boldsymbol{e}_{\boldsymbol{u}}^T\boldsymbol{G}_s(\boldsymbol{x})\boldsymbol{e}_{\boldsymbol{u}} + \bar{\kappa}(t)\right)$.

We are now ready to prove the following theorem.

Theorem 1. For uncertain MIMO nonlinear system (1), under Assumptions 1 and 2, the control law (18) with parameter adaptation laws (26) and (27) guarantees the boundedness of all the signals in the closed-loop system and the asymptotic convergence of the tracking error to a small neighborhood of the origin.

Proof. By integrating (36) over $[0,t]$, one can obtain the following inequality

$$0 \le V(t) \le \frac{c}{\alpha} + \left(V(0) - \frac{c}{\alpha}\right)e^{-\alpha t} \quad (37)$$

From inequality (37), one can conclude that all signals of the closed-loop system are uniformly bounded and that the filtered tracking error $\boldsymbol{s}(t)$ is uniformly ultimately bounded with respect to the set: $\left\{ \boldsymbol{s} \in \mathbb{R}^p : \|\boldsymbol{s}(t)\| \le \sqrt{\frac{2c}{\alpha}} \right\}$. This in turn implies that each tracking error $e_i(t)$ and its first $(r_i - 1)$ derivatives converge to residual sets defined as: $\left|e^{(j)}(t)\right| \le 2^j \lambda^{j-n+1} \sqrt{\frac{2c}{\alpha}}$, $j = 0,...,r_i - 1$, $i = 1,...,p$.

Remark 3. In this paper, in order to improve the performance of the adaptive fuzzy controller, the parameter adaptation law is obtain by the minimization of a cost function that quantifies the error between $\boldsymbol{u}^*$ and $\boldsymbol{u}$ (Labiod and Guerra, 2007).

5. SIMULATION RESULTS

In this section, in order to validate the effectiveness of the presented adaptive fuzzy controller, we apply it to a two-link robotic manipulator moving in a vertical plane. The dynamic model of the two-link robotic manipulator is given by (Ordonez and Passino, 1999; Slotine and Li, 1991)

$$\ddot{\boldsymbol{q}} = \boldsymbol{M}^{-1}(\boldsymbol{q})\left(\boldsymbol{D}^*\boldsymbol{u} - \left(\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{F}(\dot{\boldsymbol{q}})\right)\right) \quad (38)$$

where $\boldsymbol{q} = [q_1, \ q_2]^T$ are the joint angles, $\boldsymbol{u} = [u_1, \ u_2]^T$ are the joint inputs, and

$$\boldsymbol{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \ \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix},$$

$$M_{11} = I_1 + I_2 + m_1\ell_{c1}^2 + m_2\ell_1^2 + \ell_{c2}^2 + 2\ell_1\ell_{c2}\cos(q_2)$$

$$M_{22} = I_2 + m_2\ell_{c2}^2$$

$$M_{21} = M_{12} = I_2 + m_2\ell_{c2}^2 + m_2\ell_1\ell_{c2}\cos(q_2)$$

$$h = m_2\ell_1\ell_{c2}\sin(q_2)$$

$$\boldsymbol{G}(\boldsymbol{q}) = \begin{bmatrix} (m_1\ell_{c1} + m_2\ell_1)g\cos(q_1) + m_2\ell_{c2}g\cos(q_1 + q_2) \\ m_2\ell_{c2}g\cos(q_1 + q_2) \end{bmatrix}$$

$$\boldsymbol{F}(\dot{\boldsymbol{q}}) = \begin{bmatrix} 6\dot{q}_1 + 0.5\,\mathrm{sgn}(\dot{q}_1) \\ 6\dot{q}_2 + 0.5\,\mathrm{sgn}(\dot{q}_2) \end{bmatrix}$$

The following parameter values are used in this paper: $I_1 = 0.2\,\mathrm{kgm}^2$, $I_2 = 0.2\,\mathrm{kgm}^2$, $m_1 = 1.0\mathrm{kg}$, $m_2 = 1.0\mathrm{kg}$, $\ell_1 = 1.0\,\mathrm{m}$, $\ell_2 = 1.0\,\mathrm{m}$, $\ell_{c1} = 0.5\,\mathrm{m}$, $\ell_{c2} = 0.5\,\mathrm{m}$, $g = 9.8\,\mathrm{m/s}^2$. Let $\boldsymbol{y} = \boldsymbol{q}$, $\boldsymbol{x} = [q_1, \ \dot{q}_1, \ q_2, \ \dot{q}_2]^T$, and

$$\boldsymbol{f}(\boldsymbol{x}) = -\boldsymbol{M}^{-1}\left(\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{F}(\dot{\boldsymbol{q}})\right), \ \boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{M}^{-1}\boldsymbol{D}^*,$$

Then, the robot manipulator dynamics given by (38) can be written as

$$\ddot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}, \quad (39)$$

Equation (39) is in the input-output form given by (2) and the matrix $\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{M}^{-1}\boldsymbol{D}^*$ is nonsingular. Note that the matrix $\boldsymbol{D}^*$ is a diagonal matrix with +1 or -1 on the diagonal and it is introduced to change the control direction.

The control objective is to force the robot manipulator outputs $y_1 = q_1$ and $y_2 = q_2$ to track the following desired trajectories $q_{1d} = \sin(t)$ and $q_{2d} = \cos(t)$, respectively.

Within this simulation, two fuzzy systems in the form of (14) are used to generate the control signals u_1 and u_2. The input vector of each fuzzy system is defined as $\boldsymbol{z} = \left[e_1(t), \dot{e}_1(t), e_2(t), \dot{e}_2(t)\right]^T$, and for each input variable z_i, $i = 1,\ldots,4$, we define three Gaussian membership functions centered at -1.25, 0, 1.25 with a variance equal to 0.6. The robot system initial conditions are $\boldsymbol{x}(0) = [0.25, 0, 0.5, 0]^T$, the initial values of the estimated parameters $\theta(0)$ are set equal to zero, and the initial values of the estimated control direction is $\boldsymbol{d}(0) = [-0.5, -0.5]^T$. The used design parameters are chosen as follows: $\lambda_1 = 2$, $\lambda_2 = 2$, $\boldsymbol{K} = diag[5,5]$, $\boldsymbol{K}_0 = diag[5,5]$, $\eta_1 = 2$, $\eta_2 = 2$, $\varepsilon_0 = 0.01$, $\sigma_1 = 0.001$, and $\sigma_2 = 0.001$.

Simulation results for the case $\boldsymbol{D}^* = \mathrm{diag}[-1,1]$ are shown in Figs. 1–3. Fig. 1 shows actual and desired joint positions of links 1 and 2. Fig. 2 shows joint control torque inputs. Fig. 3 shows the estimated control directions d_1 and d_2. We can see that actual trajectories converge to the desired ones, and that the control direction is correctly identified.

Simulation results for the case $\boldsymbol{D}^* = \mathrm{diag}[+1,-1]$ are shown in Figs. 4–6. Fig. 4 shows actual and desired joint positions of links 1 and 2. Fig. 5 shows joint control torque inputs. Fig. 6 shows the estimated control directions d_1 and d_2. We can see that we obtain similar control tracking performance.

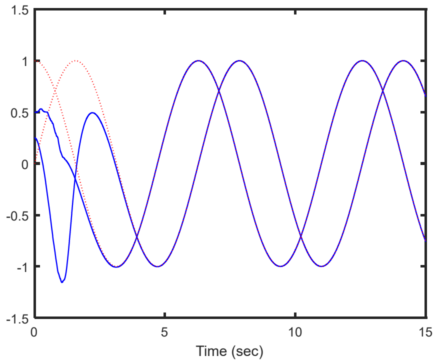

Fig. 1. Output tracking performance of links 1 and 2: actual (solid lines); desired (dotted lines).

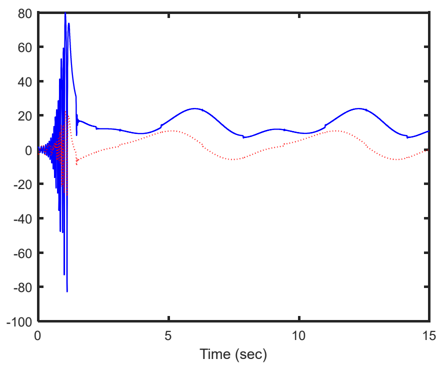

Fig. 2. Control input signals; u_1 solid line, u_2 dotted line.

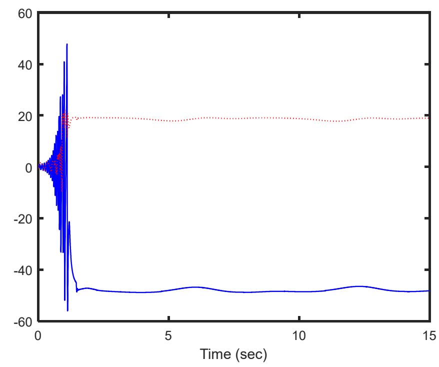

Fig. 3. Control direction; d_1 solid line, d_2 dotted lines.

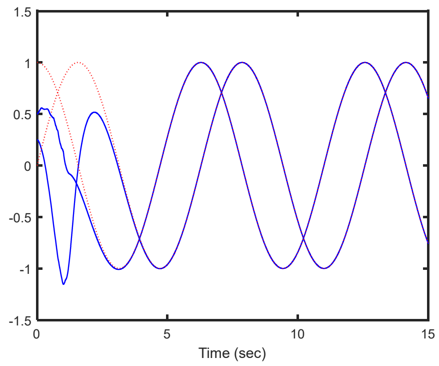

Fig. 4. Output tracking performance of links 1 and 2: actual (solid lines); desired (dotted lines).

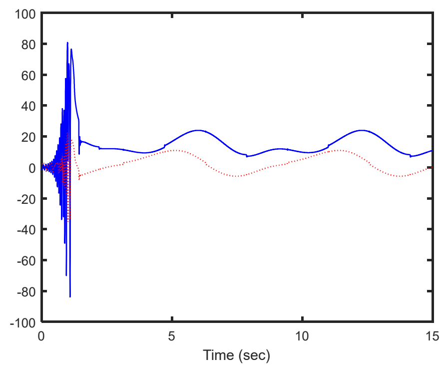

Fig. 5. Control input signals; u_1 solid line, u_2 dotted line.

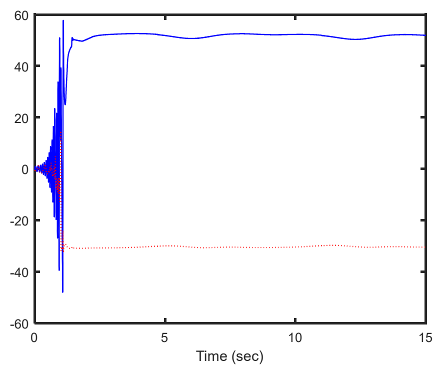

Fig. 6. Control direction; d_1 solid line, d_2 dotted lines

6. CONCLUSIONS

For a class of uncertain MIMO nonlinear systems with indefinite control gain matrix and unknown control direction, this paper has proposed a direct adaptive controller by using fuzzy systems. The fuzzy systems are used to approximate an unknown ideal nonlinear control law that ensures the control objective. Under the assumption that the unknown control gain matrix is indefinite, by using a matrix decomposition technique, the unknown control direction and the adjustable parameters of the fuzzy controller are adjusted online by a suitable parameter adaptive law that minimizes the error between the ideal control law and the fuzzy controller. Using the Lyapunov direct approach, all the closed-loop signals have been shown to be uniformly bounded. The effectiveness of the proposed adaptive control scheme has been demonstrated by simulation studies.

REFERENCES

Boulkroune, A., Tadjine, M., M'Saad, M., and Farza, M. (2010). Fuzzy adaptive controller for MIMO nonlinear systems with known and unknown control direction. *Fuzzy Sets and Systems*, 161(6), 797–820.

Chen, Z. (2019). Nussbaum functions in adaptive control with time-varying unknown control coefficients, *Automatica*, 102, 72–79.

Chen, C., Liu, Z., Xie, K., Liu, Y., Zhang, Y., and Chen, C. L. P. (2017). Adaptive fuzzy asymptotic control of MIMO systems with unknown input coefficients via a robust Nussbaum gain-based approach, *IEEE Transactions on Fuzzy Systems*, 25(5), 1252–1263.

Er, M.J., and Mandal, S. (2016). A survey of adaptive fuzzy controllers: Nonlinearities and classifications, *IEEE Transactions on Fuzzy Systems*, 24(5), 1075–1107.

Guerra, T.M., Sala, A., Tanaka, K. (2015). Fuzzy control turns 50: 10 years later. Fuzzy Sets and Systems, 281(15), 168–182.

Labiod, S., and Guerra, T.M. (2007). Direct adaptive fuzzy control for a class of MIMO nonlinear systems. *International Journal of Systems Science*, 38(8), 665–675.

Labiod, S., and Guerra, T.M. (2016). Direct adaptive fuzzy control for a class of nonlinear systems with unknown control gain sign. In *Proceedings of the IEEE International Conference on Fuzzy Systems*, Vancouver, Canada.

Labiod, S. and Guerra, T.M. (2017). Adaptive Fuzzy Control for a Class of Multivariable Nonlinear Systems with Unknown Control Direction. In *Proceedings of 20th IFAC World Congress*, Toulouse, France.

Nussbaum, R.D. (1983). Some remarks on the conjecture in parameter adaptive control. *Systems Control Letters*, 3(5), 243–246.

Ordonez, R., and Passino, K.M. (1999). Stable multi-input multi-output adaptive fuzzy/neural control. *IEEE Trans. Fuzzy Systems*, 7(3), 345–353.

Shi, W., Wang, D., Li, B. (2017). Indirect adaptive fuzzy prescribed performance control of feedback linearisable MIMO non-linear systems with unknown control direction, *IET Control Theory Appl.*, 11(7), 953–961.

Slotine, J. E. and Li, W. (1991). *Applied nonlinear control*. Prentice Hall, Englewood Cliffs, NJ.

Song, Y., Huang, X. and Wen, C. (2017). Robust adaptive fault-tolerant PID control of MIMO nonlinear systems with unknown control direction, *IEEE Transactions on Industrial Electronics*, 64(6), 4876–4884.

Tao, G. (2013). *Adaptive control design and analysis*. Hoboken, NJ, USA: John Wiley & Sons.

Wang, L. X. (1994). *Adaptive fuzzy systems and control: Design and stability analysis*. Prentice-Hall, Englewood Clifs, NJ.

Zhang, J. X. and Yang, G. H. (2019). Low-complexity adaptive tracking control of MIMO nonlinear systems with unknown control directions, *Int J Robust Nonlinear Control*, 29(7), 2203–2222.